

Seismic responses of transmission tower-line system under coupled horizontal and tilt ground motion

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Abstract. Tests and theoretical studies for seismic responses of a transmission tower-line system under coupled horizontal and tilt (CHT) ground motion were conducted. The method of obtaining the tilt component from seismic motion was based on comparisons from the Fourier spectrum of uncorrected seismic waves. The collected data were then applied in testing and theoretical analysis. Taking an actual transmission tower-line system as the prototype, shaking table tests of the scale model of a single transmission tower and towers-line systems under horizontal, tilt, and CHT ground motions were carried out. Dynamic equations under CHT ground motion were also derived. The additional $P-\Delta$ effect caused by tilt motion was considered as an equivalent horizontal lateral force, and it was added into the equations as the excitation. Test results were compared with the theoretical analysis and indicated some useful conclusions. First, the shaking table test results are consistent with the theoretical analysis from improved dynamic equations and proved its correctness. Second, the tilt component of ground motion has great influence on the seismic response of the transmission tower-line system, and the additional $P-\Delta$ effect caused by the foundation tilt, not only increases the seismic response of the transmission tower-line system, but also leads to a remarkable asymmetric displacement effect. Third, for the tower-line system, transmission lines under ground motion weaken the horizontal displacement and acceleration responses of transmission towers. This weakening effect of transmission lines to the main structure, however, will be decreased with consideration of tilt component.

Keywords: transmission tower-line system; tilt ground motion; foundation tilt; shaking table test

1. Introduction

Transitional ground motions are the primary objects in seismic observations and research; however, when seismic waves travel along the ground, the velocity, period, and phase of each point in waves differs, leading to rocking motions accompanied by horizontal movements, the laws for the rocking motion become difficult to obtain. Nowadays, the researchers have not paid enough attention to the tilt component of ground motion. Therefore, when calculating the seismic motion, they only considered the horizontal seismic effect, neglected the tilt component. One reason is the lack of measured records of the tilt component that can be used in practical engineering design, and restricting the structure design. Another important reason is that little research has taken the rocking component into account.

Trifunac (2009) pointed out that the recognition of rotational components in earthquakes began in the 1960s, and in the 1990s the research of rocking or tilt motion increase; Nigbor (1994) utilized a type of angle measuring

sensor to obtain the rotation component from a non-proliferation experiment in Nevada; Stedman et al. (1995) also observed the rotation effect near an earthquake with a magnitude of 6.3 in Kelburn, Wellington, New Zealand, using a ring laser device; Takeo (1998) first succeeded in recording rocking components around site Ito in the Izu peninsula; he (1997) also pointed out that the angular sensor was able to detect rotational motions in cases where the magnitude of an earthquake was 6 or greater and the hypocenter distance was shorter than 25 km. Besides the recording and research with special purpose, the effect of rocking motions has been apparent in several earthquakes in recent decades, such as the Northridge earthquake in the United States described by Gomberg (1997), the Chi-chi earthquake cited by Huang (2003), and the Wenchuan earthquake in China in article of Peng and Li (2012). The dip angle caused by the earthquakes in tilting ground motion can reach 0.8° to 3.1° , so the effect of rotational motions cannot be ignored.

Rocking component can be divided into two types based on the direction. The rocking motion along the vertical direction is known as torsion, and the tilt effects along x and y directions are indicated in Fig. 1.

As described above, some rocking motions of earthquakes have already been recorded. However, compared with translation motions, there is still a lack of enough records to be used in researching and engineering. Therefore, many scholars use methods of extracting

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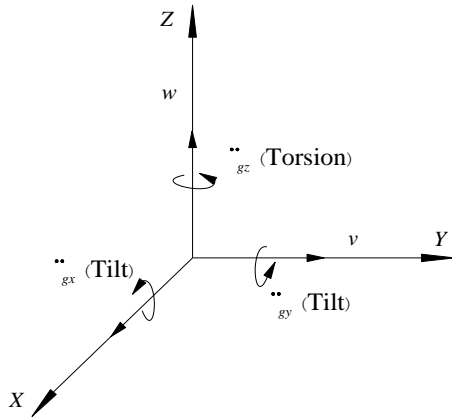


Fig. 1 Six directions of seismic movements

rotational components from translation records to study. The main methods are as follows:

One method to acquire the rocking component is based on the theory of elastic waves. The Fourier spectrum of amplitude in the rocking component was derived from the horizontal component by Trifunac (1982), but the surface wave (with no contribution to the rocking component) was not considered; Lee and Trifunac (1985) expanded an extracting horizontal acceleration method to torsional acceleration records, where wave passage effects and dispersion were also considered; Castellani and Boffi (1986) discussed the effects of surface wave on the rocking component and built mathematical models to study the tilt spectrum, and the results were compared with the response spectrum; Li (2004) applied the elastic wave theory to calculate tilt motion based on rotation degree of freedom for P, SV waves and Rayleigh waves; Wei and Luo (2010) and Zembaty (2009) calculated and analyzed the responses for the spectrum of ground motion rotational component based on the measured records of earthquakes, using the theory of elastic plane wave propagation.

In addition to the first method mentioned above, records from multi-station procedures (MPS) or dense array are also feasible approaches to obtain tilt components, and this is the second method. Niazi (1986) estimated the rotational motion in earthquakes to be a long, narrow, rigid foundation, based upon differential array record from El Centro seismic waves; Castellani and Zembaty (1986) indicated a correlation in vertical motion and rocking movement spectrum, noting that the ratio of rocking spectrum and summit translational spectrum differed by order of magnitude; Laouami and Labbe (2002) obtained the rocking components using data from stations of large scale seismic testing arrays in Lotuong, Taiwan. They calculated the rocking component for each station with dividing differences between translational acceleration along the travelling line by the distance between two stations.

The third method is proposed by Graizer (2006), and is based on different responses caused by tilt ground motion between the horizontal and vertical spectrum from the single-pendulum; this method ensures the cut off frequency by comparing the horizontal and vertical seismic waves of the Fourier spectrum, and obtains the rocking movement

time-history by fitting the horizontal component.

The methods outlined above are all able to extract or calculate the rocking component of earthquakes, but there are many shortcomings, such as inadequate accuracy, underestimation of rotation effects, and the inability of data to be directly applied to engineering, which needs further research and improvement.

1.2 The effect of tilt motions on the structures

For the high-rise structure, especially for high-rise and flexible structures such as transmission towers, the effects of tilt motion should not be ignored. Many scholars have already researched the seismic responses of transmission tower-line systems and problems about the influences of tilt motion. Li and Wang (1991) studied the response of a system consisting of long-span transmission lines and their supporting towers to horizontal and rocking seismic motions. The results revealed significant effects on high-rise structures, such as transmission towers, whose tilt motion effect should not be ignored. The progression of collapse for a transmission line system was discussed by Tian and Li (2013), Tian (2017a, 2017b), and comparisons between one-dimensional and multi-dimensional seismic responses were made. The results indicated significant differences between them. Their research also pointed out that the influence of multi-dimensional seismic responses to the progressive collapse of a transmission line should be considered. Research conducted by Sun (2015) on seismic stability analysis of a long-span transmission tower-line system under multiple support excitation revealed that, besides the observation of multiple support excitations, the comparison between multiple input and uniform excitation should also be observed. Li (2015) revealed the mass effect and nonlinear vibration effect of a transmission line to a tower by using the shaking table test, which indicated that both effects would greatly reduce the dynamic response of the tower.

The research discussed above indicated that, due to the features of significant height and low lateral rigidity, the dynamic responses of transmission tower-line systems, with considering tilt components, differed greatly from studies that only considered horizontal motion. It indicated that the effects of rocking components in the structures were notable. Furthermore, the effects of angular displacement on basements should also be considered, since this form of displacement would lead to additional $P-\Delta$ effects, which resulted in displacements of structure coupled with greater horizontal and tilt (CHT) motion than those that only considered horizontal motion, even in the case of a small rotation angle coupled with tilt motion.

Thus, further research on the seismic responses under tilt and CHT motions were needed. The research presented in this paper used shaking table tests and dynamic equations to calculate the tilt effects of an earthquake. Comparisons between theoretical and test results were set to analyze the effects. Scale models of a single tower and a system of two-line three towers were observed in the shaking table test, and results under horizontal, tilt, and coupled horizontal and tilt (CHT) ground motions were obtained. Dynamic equations considering tilt motions and additional $P-\Delta$

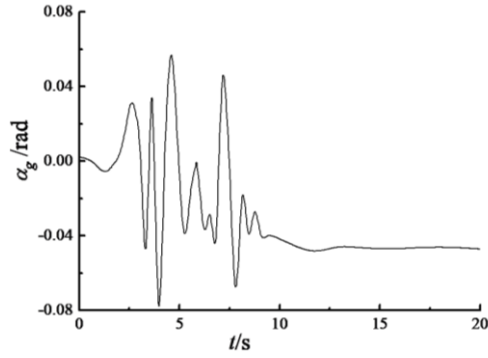


Fig. 2 Time history of tilt displacement of ground motion

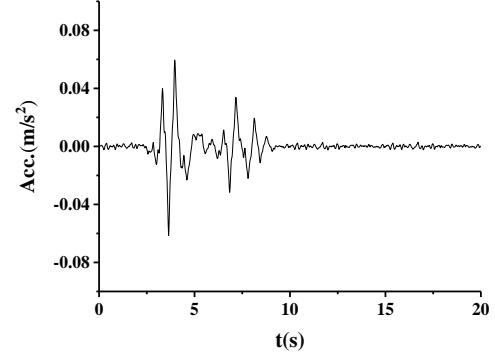


Fig. 3 Time history of tilt acceleration of ground motion

effects were also derived. The results of the shaking table test and the equations were compared, and references for approaching the seismic design of transmission line systems were proposed.

2. Improved method for measuring tilt motion of an earthquake

Multi-station and dense array records are generally only suitable for extracting the sway components of ground motion in special areas where conditions suitable. At the same time, due to various reasons, it is still impossible to obtain data that can be used in engineering applications. Therefore, there are two main methods to gain rocking component by extracting the general ground motion, the method based on the theory of elastic wave, and the method in comparison to the Fourier spectrum.

For the first method, the ideas and concepts are clear, but the defects are as follows: the precondition of this method is to assume that the propagation medium of seismic wave is isotropic, homogeneous and horizontal layered medium. Therefore, when dealing with seismic wave far away from the source, satisfactory results can be obtained, for the assumed conditions are satisfied. However, when dealing with seismic waves near the source, the results obtained are not ideal because of the large differences between the near-field site and the assumed conditions. Peng and Li (2012) pointed out that the tilt motion using this method may be underestimated near the source, and the rocking component near the site is usually more significant. The actual rotating component obtained is worth considering.

Another method that applied the comparison of the Fourier spectrum was proposed by Graizer (2006), which was based on different responses caused by tilt ground motion between the horizontal and vertical pendulum. However, the Fourier spectrum could only characterize the frequency domain characteristics of seismic waves, and failed to achieve the real comparison in the time-frequency window. There is a lack of frequency determination criteria. Meanwhile, a few horizontal accelerations are included in rocking components obtained by filtering, which lead to inaccuracy of the results, and often offset when fitting the time histories of rocking angular acceleration. Therefore, the accuracy of the time histories is worth discussing, too.

Wavelet analysis method, which is able to resolve the different frequency and also has capability in comparison in overall range, could be used in revising the insufficiency of the Fourier spectrum method. The tilt component could be derived from the horizontal component based on the uncorrected seismic waves by Wei(2015). To validate the method, the seismic wave data from the left point of Pacoima in the 1994 Northridge earthquake was analyzed and tested. The time history of tilt displacement of ground motion can be observed in Fig.2, and the residential rotation displacement is about $3.1^\circ(0.054\text{rad})$, which coincides with actual observation values, and this is the effect of tilt motions. After two time derivations for rotation displacement time history, the resulting rotation acceleration time history can be observed in Fig. 3. The tilt motion component using the method above based on data from the Northridge earthquake can be used in theoretical analysis and testing.

3. Shaking table tests of transmission system under CHT ground motion

To understand the effect of tilt motion on a transmission tower line system in an earthquake, a true transmission tower-line system was utilized as the prototype, and shaking table tests of the scale model of a single transmission tower and a three-tower two-line system under horizontal, tilt, and CHT ground motions were carried out. The actual parameters of the tower are as follows: the practical transmission system consists of four transmission lines arranged side by side; the tower height is 81.80 m, while space between tower feet at the bottom is 17.18 m, and the distances of system between towers is 500 m. The tests took place in the structural dynamics laboratory at the Chongqing Traffic Research and Design Institute. The lab is equipped with an earthquake simulation shaking table with 6 freedoms on 3 different axes. The largest acceleration of the shaking table is $\pm 1.0\text{ g}$, and the size of the table is 3 m×6 m; the lab is also equipped with an advanced system for controlling, data acquisition and analysis, which ensures a smooth test process and accuracy of the results.

3.1 Model design and arrangement of vibration pickup

Table 1 Similarity parameters of the test model

Quantity	Relationship equation	Similarity coefficient
Length l	S_l	1/30
Elastic modulus E	S_E	1.0
Stiffness K	$S_K = S_E S_l$	1/30
Mass m	$S_m = S_l^2$	1/900
Time t	$S_t = S^{1/2}$	$1/\sqrt{30}$
Acceleration \ddot{x}	$S_{\ddot{x}} = S_l / S_t^2$	1.0
Rotation displacement α	$S_\alpha = 1.0$	1.0
Rotation acceleration $\ddot{\alpha}$	$S_{\ddot{\alpha}} = S_\alpha / S_t^2$	30

Table 2 Comparisons of transmission parameters

Transmission line	Displacement (m)	1 st order frequency	2 nd order frequency	3 rd order frequency
Prototype	500	0.13	0.25	0.26
Theoretical model	16.67	0.71	1.37	1.42
Practical model	2	0.75	1.42	1.51

The transmission line system model was designed as a scale of 1:30, and the similarity parameters of the test model are presented in Table 1.

Considering that the size of the table is 6m, the horizontal spacing of the shaking table cannot be designed according to the theoretical similarity ratio. Based on the actual installation, the horizontal spacing of the scale model is set to 2 m. At the same time, the frequency of the calculation model and the actual test model is consistent with each other. The transmission line is simulated by a steel strand with a nominal diameter of 3 mm, and the galvanized iron chain is used as the counterweight. According to the consistent relationship between the basic period and the theoretical scale model, the mass of the additional chain on a single conductor is 2.2 kg.

Based on the above design, the prototype of the actual transmission line, the calculation model of transmission line scale based on the similarity theory, and the frequency comparison of the actual test model are shown in Table 2.

Table 2 shows that the first three frequencies of the actual transmission line are in good agreement with the theoretical values, and the test model of transmission tower-line system meets the dynamic characteristics requirements of shaking table tests.

In the test model, the height of the main tower was 2.73 m, the spaces between towers' feet were 0.3 m, and the horizontal bars were 1.2 m above the tower. The equilateral roof steel used in the test model as the main and web members were L30×2.5 and the diagonal braces were round bars with a diameter of 3 mm, using Q345 steel welded each member bar together. Counterweight boxes weighing 10.5 kg were set at the top and middle layers of the tower, which could be used to adjust the fundamental frequency of the tower by changing the weight of each box. Figs. 4 and 5 present the design drawings of the front elevation and side facade for the transmission system model.

The tower feet were fixed firmly to the shaking table by four bolts, and the main structures were connected by welding. The final design of the transmission tower model

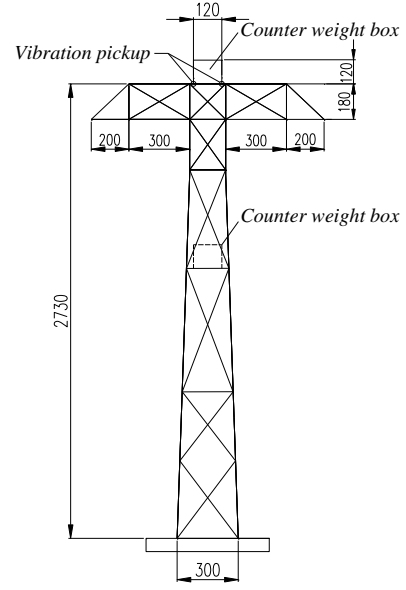


Fig. 4 Front evaluation view of transmission tower-line model

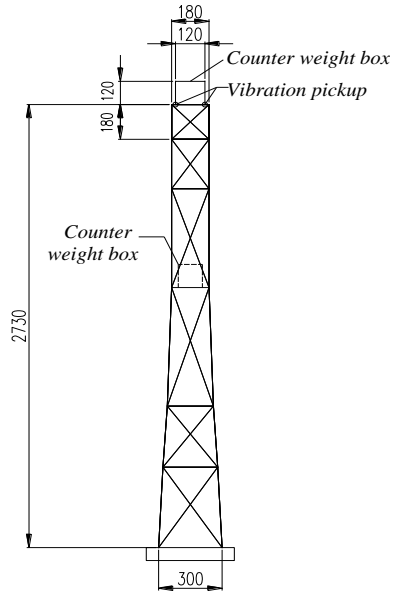


Fig. 5 Side evaluation view of transmission tower-line model

is presented in Fig. 6.

The designs of transmission lines and insulators were also needed. The main factors to consider included the counterweight and the connection. The determination of counterweight was based on similarity relationship. Steel strand wires were used to simulate the transmission lines in the test, and additional weights using steel chains were added to the line to meet the requirement. Insulators were simulated as rigid chain rods without consideration of their stiffness, and were also designed considering the demands of similarity ratio.

The three-tower two-line models and the arrangement of the vibration pickup are presented in Figs. 7 and 8. The type of accelerometer set on top of the model for transmission system is KISTLER 8310A10 with measurement range is 5



Fig. 6 Final design of transmission tower model



Fig. 7 Three-tower two-line model



(a) Arrangement of accelerometer



(b) Arrangement of displacement meter

Fig. 8 Test model details

g, while the largest frequency response is 2 100 Hz; and the type of accelerometer set on top of the test table is KISTLER 8310A2 with measurement range is 2 g, while the largest frequency response is 1400 Hz. The laser displacement meter opto NCDT was set on top of the transmission tower, and its type is ILD1401-200 (000) with a range of 200 mm.

3.2 Test conditions

The horizontal acceleration time history of seismic waves used in the test was from the left point of Pacoima in the 1994 Northridge earthquake, and the rotation angle of time history was from the same earthquake. The duration of the earthquake was compressed based on similarity parameters of the test model, which can be found in Table 1, and the time of acceleration time history was 3.65s.

Table 3 Test conditions

Condition number	Test model	Test model	Earthquake input	
			Horizontal movement (m/s ²)	Tilt movement (rad/s ²)
1	Single tower	W1	0.5	-
2		UX1	6.2	-
3		RY1	-	0.68
4		UX1 +RY1	6.2	0.68
5		UY1	6.2	-
6		RX1	-	0.68
7		UY1+RX1	6.2	0.68
8	Tower line system	W2	0.5	-
9		UX2	6.2	-
10		RY2	-	0.68
11		UX2 +RY2	6.2	0.68
12		UY2	6.2	-
13		RX2	-	0.68
14		UY2 +RX2	6.2	0.68

Table 4 Vibration frequency of the single tower

Mode of vibration	Mode of vibration of the original model	Theoretical value of the scale model	Measured value of the practical model
1 st order in longitudinal direction	1.75	9.59	9.82
1 st order in transverse direction	1.72	9.42	9.75

Based on the Code for Seismic Design of Buildings (2016 Edition), the summit of acceleration time history used for rare earthquakes of 9 degree is 6.2 m/s². Therefore, to adjust to such value, a proportional coefficient should be multiplied, and the same coefficient was used in the tilt of seismic waves, with a summit value is 0.68 rad/s². After adjustment, experiments under three different conditions were carried out, including horizontal, tilt, and CHT, and the details are presented in Table 3.

In Table 3, “W” represents white noise, while UX represents horizontal movement along the *x*-direction; RX or RY represent rocking movements along the *x*-direction or *y*-direction. The referenced direction of loading axis can be observed in Fig. 9. Furthermore, to obtain the actual acceleration time history of the shaking table, a vibration pickup was also arranged on the shaking table.

3.3 Test results

3.3.1 Dynamic factors of structure

Through numerical calculation and white noise sweep, the frequency of the original actual transmission tower and the theoretical calculation value of the scale model can be obtained. At the same time, the practical frequency of the single tower model can also be gained, and the above data are listed in Table 4.

Table 4 indicates that the theoretical and measured models of scale single tower are close. Based on the

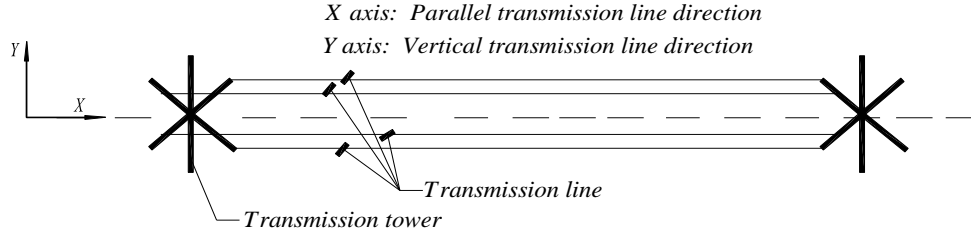
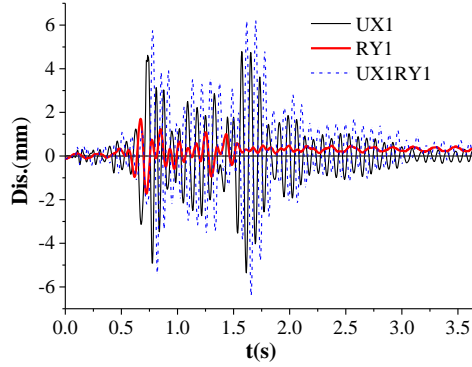
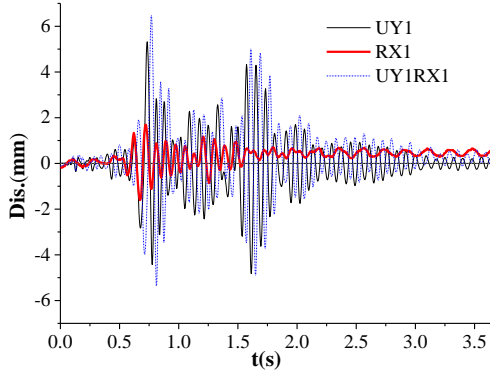


Fig. 9 Diagrammatic sketch of transverse, longitudinal direction of transmission tower system



(a) Displacement time history curves in longitudinal direction



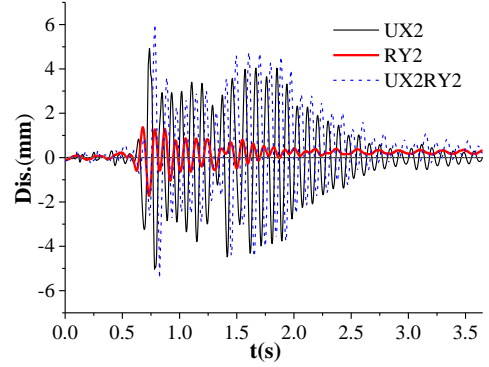
(b) Displacement time history curves in transverse direction
Fig. 10 Displacement time history curves on top for single tower in different directions

similarity theory, the basic natural vibration period of the original tower line system is 0.55s, while the calculated results of the tower-line structure are 0.57s, with little difference. Thus, the integral tower-line structure model meets the requirement of dynamic characteristics for shaking table tests.

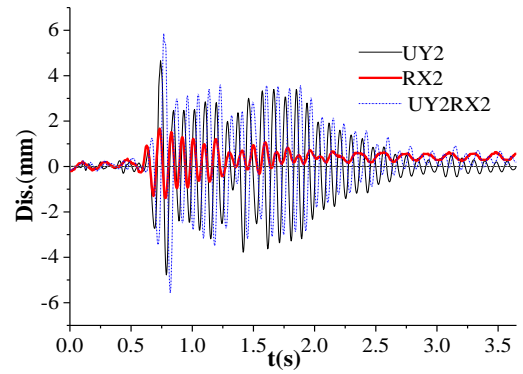
3.3.2 Horizontal displacement of model top layer

The transverse and longitudinal directions for the transmission tower system and their relationship with the single tower and transmission lines are indicated in Fig. 9, where the longitudinal direction or X-axis represents the direction of parallel transmission lines, and the transverse direction or Y-axis represents the direction perpendicular to the transmission lines.

Displacement time history curves on top of the model under horizontal, tilt, and CHT motions in longitudinal and transverse directions for the single tower are presented in Fig. 10, and the displacement time history curve in



(a) Displacement time history curves in longitudinal direction



(b) Displacement time history curves in transverse direction
Fig. 11 Displacement time history curves on top for tower-line system in different directions

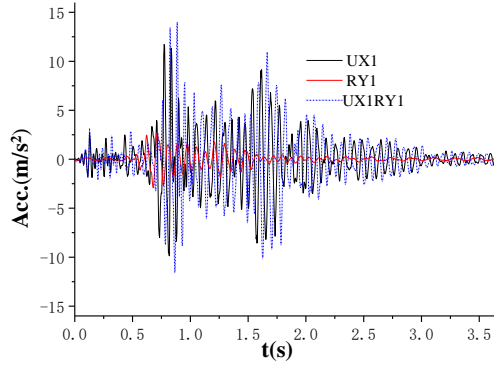
longitudinal and transverse directions on the top layer of the tower-line system are presented in Fig. 11.

3.3.3 Horizontal acceleration of model top layer

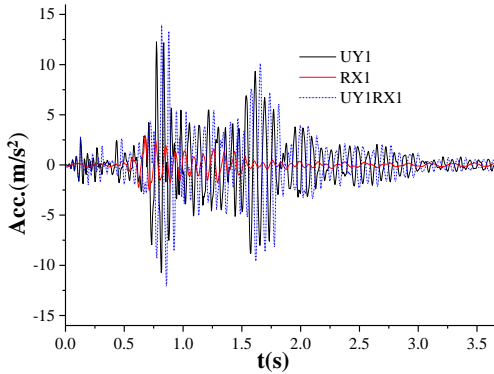
The acceleration time history curve on top of the single tower under horizontal, tilt, and CHT motions in longitudinal and transverse directions are presented in Fig. 12, and acceleration time history curves in longitudinal and transverse directions on top of the tower line system are presented in Fig. 13.

3.3.4 Responses of amplitude on top tower

The responses of amplitude for displacement and acceleration on top of the tower under horizontal, tilt, and CHT motions in transverse and longitudinal alignment are presented in Table 5. In this table, the single tower in the longitudinal direction represents the loading condition under horizontal direction X1, rotational direction RY1, and loading conditions in both UX1 and RY1, while the single



(a) Acceleration time history curves in longitudinal direction



(b) Acceleration time history curves in transverse direction
Fig. 12 Acceleration time history curves on top for single tower in different directions

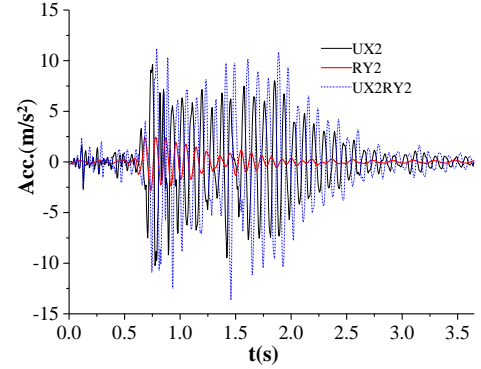
Table 5 Horizontal reaction amplitude of top of transmission tower

Model	Response	Horizontal	CHT	Amplification
Single tower in longitudinal direction	Displacement (mm)	5.27	6.41	21.6%
	Acceleration (m/s ²)	11.71	13.99	19.5%
Single tower in transverse direction	Displacement (mm)	5.36	6.68	24.6%
	Acceleration (m/s ²)	11.37	14.55	27.9%
Tower lines system in longitudinal direction	Displacement (mm)	4.90	6.01	22.6%
	Acceleration (m/s ²)	10.22	12.47	22.0%
Tower lines system in transverse direction	Displacement (mm)	4.81	6.27	30.3%
	Acceleration (m/s ²)	10.06	13.56	36.1%

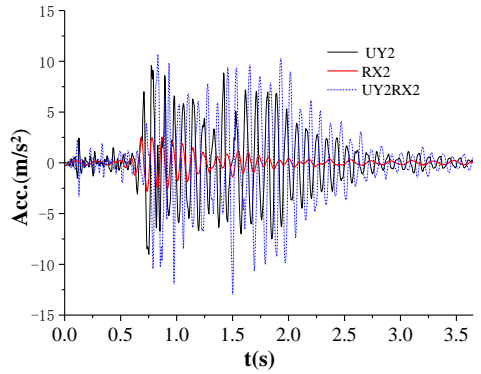
tower in the transverse direction represents the loading condition under horizontal direction Y1, rotational direction RX1, and loading conditions in both UY1 and RX1. The tower line system is represented in the same way, as shown in Table 5.

Reduction in response amplitude between single tower and tower line system under horizontal and CHT motions can be found in Table 6, Table 7.

3.4 Analysis and discussion of the test results



(a) Acceleration time history curves in longitudinal direction



(b) Acceleration time history curves in transverse direction
Fig. 13 Acceleration time history curves on top for tower-line system in different directions

Table 6 Reduction in response amplitude between single tower and tower line system

Model	Response	Single tower	Tower line system	Decreasing amplitude
longitudinal direction	Displacement (mm)	5.27	4.85	7.9%
	Acceleration (m/s ²)	11.71	10.65	9.1%
transverse direction	Displacement (mm)	5.36	4.92	8.2%
	Acceleration (m/s ²)	11.37	10.21	8.8%

Table 7 Reduction in response amplitude between single tower and tower line system

Model	Response	Single tower	Tower line system	Decreasing amplitude
longitudinal direction	Displacement (mm)	6.41	6.01	6.2%
	Acceleration (m/s ²)	13.99	12.84	8.9%
transverse direction	Displacement (mm)	6.68	6.27	6.1%
	Acceleration (m/s ²)	14.55	13.56	6.8%

Based on observations of the data presented in Figs. 10 to 13 and Tables 5 to 7, conclusions can be made as follows:

1. Compared with the horizontal situation, the displacement and acceleration of CHT movement on top of each model increased by a certain degree. The increase in displacement amplitude of the single tower and tower transmission system was more than 20% (Only the increase in the second line of table 5 is exceptional). This increase indicated as follows: horizontal movement and acceleration



Fig. 14 A series of multi-particle models for the single tower

response increase greatly in a high-rise structure, such as the top of a transmission tower. For the reasons outlined above, the effects of tilt motion should not be ignored.

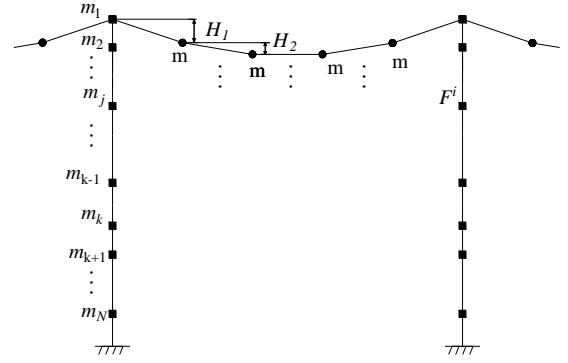
2. Under the condition of tilt and CHT, the time history of each model on the top layer indicated movement from baseline 0 from the Figs. 10 and 11. The reasons for this were the residual displacement, and additional $P-\Delta$ effects generated in structures that led to the serious asymmetric effect. Certain differences between horizontal displacement in positive and negative directions were observed. Horizontal acceleration time history curves fluctuated above and down along baseline 0, and it showed vague asymmetry of the curves.

3. The horizontal displacement and acceleration responses of the tower-line system are lower than those of the single-tower model under the same ground motions. Seismic effect decreased to a certain degree considering mass and nonlinear vibration effect, and the decreasing effect in the transverse direction was more easily observed than that in the longitudinal direction.

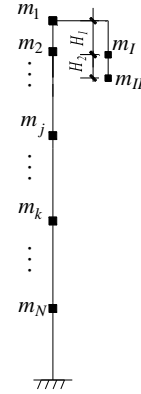
4. Compared with the horizontal condition, slighter decrease of displacement and acceleration appeared between the tower transmission line and the single-tower model under CHT movement in both longitudinal and transverse directions. This indicated that the weakening effect of transmission lines to the main structure of the system would decrease consideration tilt component, which means tilt movement should be examined more closely.

4. Theoretical analyses for transmission tower system under CHT effect

In the transmission tower line system, the influence of the transmission line to the transmission tower should be considered, and the lines should be simplified before calculation and analysis. When the tower line system vibrated in the longitudinal direction, the vertical projection of each transmission line could be divided into five parts, corresponding to four turning points with lumped mass, and this type of system is three degrees of freedom. When tower system vibration appeared in the transverse direction, each transmission line could be simplified into a vertical link.



(a) Simplification of models with transmission lines in the longitudinal direction



(b) Simplification of models with transmission lines in the transverse direction

Fig. 15 Simplification of models with transmission lines in different directions

Longitudinal and transverse definitions are the same as those in Fig. 9 of Section 3.3. The longitudinal direction means along the transmission line while transverse direction means perpendicular to the lines. To explain the influence of rotation effect conveniently, the single transmission tower can be simplified to a series multi-particle model indicated in Fig. 14. The simplifications of the transmission tower system in longitudinal and transverse directions are indicated in Fig. 15, where N is the total number of particles. For this experimental model, N is 6.

The motions of the series multi-particle model of transmission tower under horizontal actions are shown in Fig. 16.

In the figure, X_i represents the relative horizontal displacement of particle i to the ground; and H_i represents the relative height of particle to the ground; while β_i represents the relative rotation angle when the horizontal particle displacement occurs, and within a small deformation range, value β_i is equal to the ratio of the horizontal displacement for particle i to the ground surface height, which means $\beta_i = X_i/H_i$; and \ddot{X}_g represents the horizontal acceleration caused by earthquake.

The relative displacement of the structure to the ground is generated under horizontal motion. Under the action of gravity and other vertical loads, the lateral displacement of the structure will be further increased, and additional internal forces will be generated in the internal components

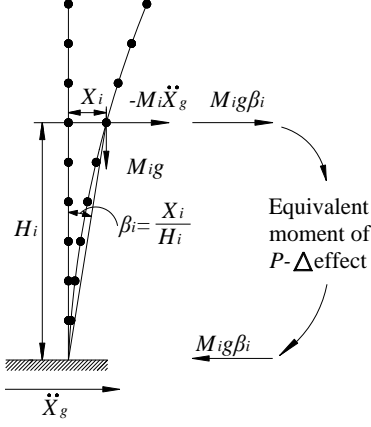


Fig. 16 Multi-particle model of transmission tower considering the $P-\Delta$ effect

of the structure, resulting in non-linear geometric effect, which is also called the second-order effect of gravity or $P-\Delta$ effect. The lateral displacement of high-flexible structure, such as transmission tower, is large under earthquake action, and the $P-\Delta$ effect on the internal force and deformation of the structure is more obvious. Therefore, the $P-\Delta$ effect should be considered in the seismic design calculation of transmission tower-line system.

In this paper, we used the finite element method based on stiffness modification to calculate the $P-\Delta$ effect of the structure, and the $P-\Delta$ effect can be equivalent to a geometric stiffness $V_i = M_i g X_i / H_i$. The equivalent moment of the effect of the particle in Fig. 16 is that if the additional moment is equivalent to a couple of horizontal forces, then the horizontal force is equal to the additional equivalent horizontal force on the particle can be expressed as $M_i g \beta_i$.

With consideration of the $P-\Delta$ effect, the dynamic equation of the transmission line system under horizontal motion was obtained as Eq. (1)

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K_0]\{X\} = [M]\{I\}\ddot{X}_g + [M]\{\beta\}g \quad (1)$$

In the left part of Eq. (1), $[M]$, $[C]$, $[K_0]$ represent the matrix of mass, damping, stiffness, and the right part of the equation represent the horizontal motions and equivalent second-order effect. $[C]$ represents the matrix of Rayleigh damping, which also can be described as $[C] = \alpha[M] + \beta[K]$. However, α and β are determined by the first frequency and corresponding damping ratio, and the damping ratio for calculation is 0.01.

The equivalent horizontal force for $P-\Delta$ effect, $[M]\{\beta\}g$ can also be expressed by multiplying the geometric stiffness $M_i g / H_i$ with the horizontal displacement X_i of the particle i , which can be expressed in Eq. (2)

$$[M]\{\beta\}g = \text{diag}[M_1 g / H_1, M_2 g / H_2 \dots M_N g / H_N]\{X\} \quad (2)$$

Moreover, the right part of the Eq. (2) can be expressed as

$$\text{diag}[M_1 g / H_1, M_2 g / H_2 \dots M_N g / H_N] = [K_G] \quad (3)$$

Where $[K_G]$ is the geometric stiffness.

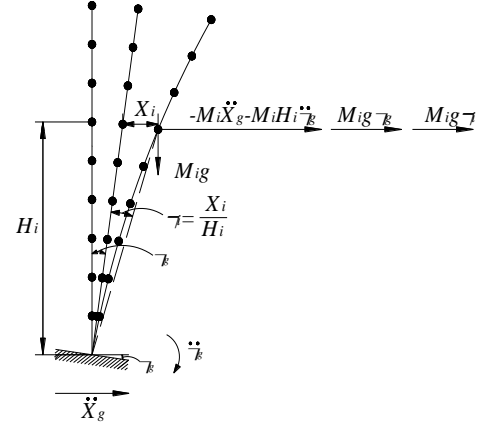


Fig. 17 Multi-particle model of transmission tower under CHT motion

Bring Eqs. (2) and (3) into Eq. (1), then move the item from right to left, and Eq. (4) is obtained

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + ([K_0] - [K_G])\{X\} = -[M]\{I\}\ddot{X}_g \quad (4)$$

To calculate tilt motion of an earthquake, the angular acceleration of the rocking earthquake $\ddot{\alpha}_g$ can be added to the mass i in the form of inertial force F_{ai} , which can be expressed as Eq. (5)

$$F_{ai} = M_i H_i \ddot{\alpha}_g \quad (5)$$

With consideration of the direct impact of rocking motion on the structure, the effect caused by the inclined deformation of the foundation also needs to be considered. At this time, assuming that the ground foundation is ideally rigid, and the axis of the structure will deviate due to the swing and rotation of the ground foundation, and its inclination angle is α_g . The effect of the ground tilt displacement to the structure can be similarly equivalent to a lateral horizontal force $M_i g \alpha_g$, as indicated in Fig. 17.

The dynamic equation of the transmission tower under CHT motion considering the inclined foundation deformation can be obtained as Eq. (6), where $[H]$ and $[I]$ represents the height and identity matrix

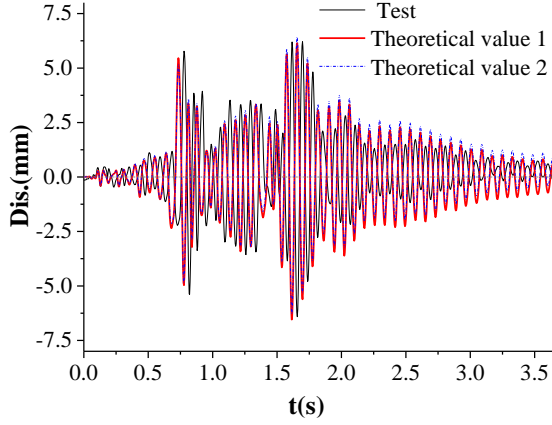
$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + ([K_0] - [K_G])\{X\} = [M]\{I\}\ddot{X}_g - [M][H]\ddot{\alpha}_g + [M][I]g\alpha_g \quad (6)$$

Based on Eq. (6), the dynamic equation of the transmission tower-line system in longitudinal and transverse directions considering additional $P-\Delta$ effects under coupled horizontal and tilt motions can be expressed in Eqs. (7) and (8)

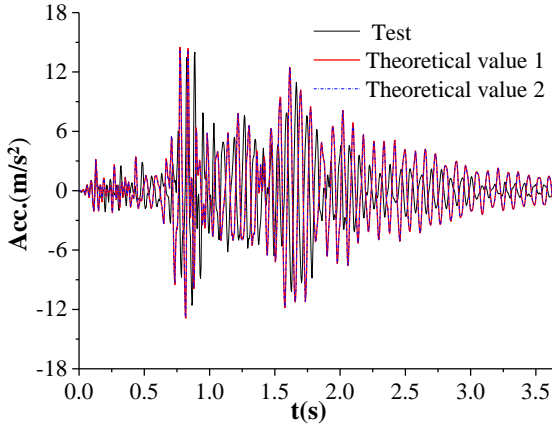
$$[M]_s\{\ddot{X}\} + [C]_s\{\dot{X}\} + ([K_0]_s - [K_G]_s)\{X\} = -[M]_s\{\ddot{X}_g\} - [M]_s[H]_s\{\ddot{\alpha}_g\}_h + [M]_s g\{\alpha_g\}_s \quad (7)$$

$$[M]_h\{\ddot{X}\} + [C]_h\{\dot{X}\} + ([K_0]_h - [K_G]_h)\{X\} = -[M]_h\{\ddot{X}_g\} - [M]_h[H]_h\{\ddot{\alpha}_g\}_h + [M]_h g\{\alpha_g\}_h \quad (8)$$

The representation of the main matrix and vector in these equations are as follows in the longitudinal direction



(a) Comparison of horizontal displacement



(b) Comparison of horizontal acceleration

Fig. 18 Comparison of horizontal displacement/acceleration for single tower under CHT motions

$$\begin{aligned}
 [M]_s &= \begin{bmatrix} [M]_{line} & [M]_{coupled} \\ [M]_{coupled} & [M]_{tower} \end{bmatrix}_{(3+N) \times (3+N)} ; \\
 [K_0]_s &= \begin{bmatrix} [K]_{line} & [O] \\ [O]^T & [M]_{tower} \end{bmatrix}_{(3+N) \times (3+N)} ; \\
 [H]_{(s)} &= diag[0, 0, 0, h_1, h_2, \dots, h_N]_{(3+N) \times (3+N)} ; \\
 \{\ddot{X}_g\} &= \begin{bmatrix} 0, 0, 0, \underbrace{\ddot{X}_g, \dots, \ddot{X}_g}_{N \uparrow} \end{bmatrix}_{(3+N) \times 1}^T, \quad \{\alpha_g\} = \begin{bmatrix} 0, 0, 0, \underbrace{\alpha_g, \dots, \alpha_g}_{N \uparrow} \end{bmatrix}_{(3+N) \times 1}^T
 \end{aligned}$$

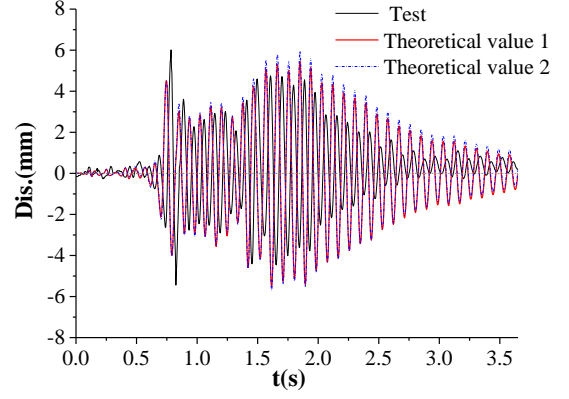
Moreover, in the transverse direction

$$\begin{aligned}
 [K_0]_h &= \begin{bmatrix} [K]_{line} & [K]_{coupled} \\ [K]_{coupled}^T & [K]_{line} \end{bmatrix}_{(2+N) \times (2+N)} \\
 [M]_h &= diag[m_1^1, m_1^1, m_1, m_2, \dots, m_N]_{(2+N) \times (2+N)} \\
 [H]_{(h)} &= diag[h_1^1, h_1^1, h_1, h_2, \dots, h_N]_{(2+N) \times (2+N)} \\
 \{\ddot{X}_g\}_h &= [\ddot{X}_g \dots \ddot{X}_g]_{(2+N) \times 1}^T, \quad \{\alpha_g\}_h = [\alpha_g \dots \alpha_g]_{(2+N) \times 1}^T \\
 \{\alpha_g\}_h &= [\alpha_g \dots \alpha_g]_{(2+N) \times 1}^T
 \end{aligned}$$

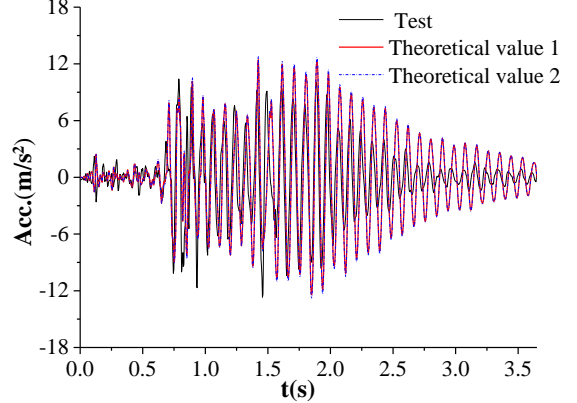
Other detailed parameters of these equations can be found research by Li (1997).

Table 8 Horizontal amplitude of responses on the top of the single tower

Model	Response	Test result	Theoretical result 1	Theoretical result 2
Single tower	Displacement(mm)	6.41	6.14	6.59
	Acceleration(m/s²)	13.99	13.01	13.82



(a) Comparison of horizontal displacement



(b) Comparison of horizontal acceleration

Fig. 19 Comparison of horizontal displacement/acceleration for tower-line system under CHT motions

5. Comparison of theoretical analysis and test results

Based on the Eq. (6) to Eq. (8) of the theoretical analysis in Section 4, the displacement and acceleration time history curves of the transmission tower system can be obtained. To seek additional seismic excitation effect on the structure with consideration of basement rotation and rocking component, the shaking table test was used as a reference, and comparisons were made between theoretical analysis and test results. Theoretical results without consideration of angle displacement were marked as **theoretical result 1**, and with consideration of angle, displacement were marked as **theoretical result 2**. Comparisons were made in the single tower, transmission line system, and seismic response in longitudinal and transverse directions.

The comparison of displacement and accelerations on the top of the single-tower model under CHT movement

Table 9 Horizontal amplitude of responses on the top of the tower line system (mm)

Model	Response	Test result	Theoretical result 1	Relative decrease of theoretical result 1	Theoretical result 2	Relative decrease of theoretical result 2
Tower line system	Displacement (mm)	6.01	5.54	10.1%	5.95	9.7%
	Acceleration (m/s^2)	12.84	11.58	11.6%	12.34	10.6%

Table 10 Horizontal amplitude of responses on the top layer in transverse directions

Model	Response	Test result	Theoretical result 1	Theoretical result 2
Transverse directions	Displacement (mm)	6.27	5.71	6.06
	Acceleration (m/s^2)	13.56	11.97	12.89

between theoretical analysis and test results is presented in Fig. 18, and the results of the comparison in amplitude are presented in Table 8.

The comparison of displacement and acceleration on top of the transmission tower-line system model under CHT movement between theoretical analysis and test results in longitudinal direction is presented in Fig. 19 and the results of the comparison in amplitude are presented in Table 9.

Displacement and acceleration of time history curves on the top layer under CHT movement in the transverse direction are presented in Fig. 20, and the comparison in amplitude is presented in Table 10.

From Figs. 18 to 20 and Table 10, the following conclusions can be made:

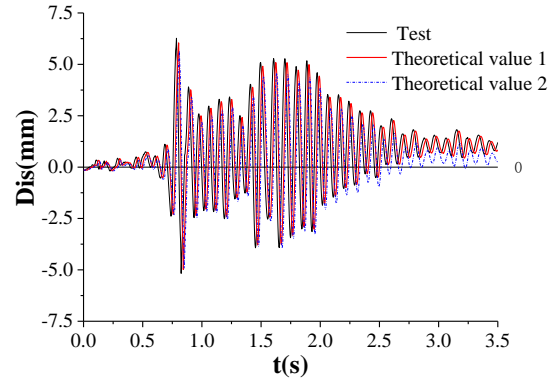
1. **Theoretical result 2** is closer to the test result compared with **theoretical result 1**, which implies that the dynamic Eq. (6) to Eq. (8) with consideration of tilt movement for transmission tower-line system is reasonable.

2. Under tilt movement, the amplitude increase in displacement between **theoretical result2** and **result1** is 7.3%, and the acceleration increase is 6.2%. There are some observable differences in the amplitude of displacement between **theoretical result1** and the test results, indicating that the additional $P-\Delta$ effect caused by basement angle rotation should not be neglected.

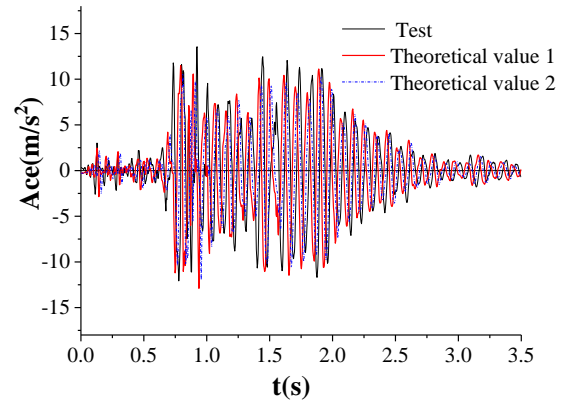
3. Similar to the test, the displacement and acceleration of the transmission line system are lower than those of the single tower, and the relative decreases of **theoretical result1** and **result2** are close to those of the single tower, mainly because the additional terms are different and the other terms are the same. The results of **theoretical result2** are closer to the test results, indicating that the Eq. (7) and Eq. (8) which the tower transmission line system is based on, are more accurate and more suitable for theoretical calculation and analysis.

4. The integral regulation in the transverse direction is equal to that in the longitudinal direction, but the range of tilt increase is smaller than that in the longitudinal direction. Thus, the additional second-order effect proposed by tilt is smaller in the transverse direction, which is the same in the theoretical analysis.

There is a significant effect from tilt movement and



(a) Comparison of displacement



(b) Comparison of acceleration

Fig. 20 Comparison of displacement/acceleration for tower-line system under CHT motions

residual tilt of basement from the seismic wave caused by the Northridge earthquake at Pacoima point; thereby the ratio for tilt movement of the test result base upon the earthquake is also great.

6. Conclusions

In this paper, shaking table tests and theoretical analysis are made for the response of transmission tower-line system under CHT motion. The tilt motion was obtained from the seismic wave by wavelet analysis and then used for shaking table tests and theoretical calculation. The shaking table tests for single tower and tower line system were carried out in the transverse and longitudinal directions, under the action of horizontal, tilt, and CHT motions. The dynamic equations of transmission tower-line system under horizontal, tilt and CHT motions were also obtained. Based on the above research results, the following conclusions are drawn:

- Great influence could be observed on the transmission line system because of the tilt movement, which cannot be neglected. Compared with horizontal movement, there were observable increases in displacement and acceleration responses on top of the tower model under CHT movement. The increase in the amplitude of horizontal displacement on top of the tower was about or more than 20%, meaning the effects of tilt motions

could not be ignored.

- The amplitude of displacement and acceleration for the three-tower line system was smaller than that of the single tower under horizontal and CHT movement. This was because there was an observable weakening effect caused by transmission line to tower structure in horizontal displacement and acceleration. The weakening effect from transmission lines to towers would decrease considering tilt movement compared with horizontal movement only.

- Dynamic equations of the transmission system under horizontal, tilt, and CHT movement were proposed. After comparisons between theoretical analysis and test results, the result from analysis considering the additional $P-\Delta$ effect was more closely aligned with the test results, which indicated the reasonable and preciseness of the derived equation.

- The additional $P-\Delta$ effect caused by basement angle displacement would increase the dynamic response of the transmission line system, and asymmetrical movement effect of the structure appeared which would increase the horizontal displacement further. This phenomenon should be explored more closely in actual seismic design and calculation. No obvious asymmetry appeared in acceleration time history curves of the structure.

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