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Effect of nonlinear elastic foundations on dynamic behavior of FG plates using four-unknown plate theory

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Abstract. This present paper concerned with the analytic modelling for vibration of the functionally graded (FG) plates resting on non-variable and variable two parameter elastic foundation, based on two-dimensional elasticity using higher shear deformation theory. Our present theory has four unknown, which mean that have less than other higher order and lower theory, and we denote do not require the factor of correction like the first shear deformation theory. The indeterminate integral are introduced in the fields of displacement, it is allowed to reduce the number from five unknown to only four variables. The elastic foundations are assumed a classical model of Winkler-Pasternak with uniform distribution stiffness of the Winkler coefficient (kw), or it is with variables distribution coefficient (kw). The variable's stiffness of elastic foundation is supposed linear, parabolic and trigonometry along the length of functionally plate. The properties of the FG plates vary according to the thickness, following a simple distribution of the power law in terms of volume fractions of the constituents of the material. The equations of motions for natural frequency of the functionally graded plates resting on variables elastic foundation are derived using Hamilton principal. The government equations are resolved, with respect boundary condition for simply supported FG plate, employing Navier series solution. The extensive validation with other works found in the literature and our results are present in this work to demonstrate the efficient and accuracy of this analytic model to predict free vibration of FG plates, with and without the effect of variables elastic foundations.

Keywords: free vibration; variables elastic foundations; Functionally graded plate; higher shear deformation theory

1. Introduction

Functionally graded materials (FGM) are components of two or more materials, with continuous changes in properties materials from one surface to another (Koizumi 1997, Benachour *et al.* 2011, Daouadji *et al.* 2012, Ahmed 2014, Al-Basyouni *et al.* 2015, Mahi *et al.* 2015, Aldousari 2017, Abdelaziz *et al.* 2017, Bellifa *et al*. 2017a, Faleh *et al.* 2018, Dash *et al*. 2018a, Younsi *et al.* 2018, Chandra Mouli *et al*. 2018, Karami *et al*. 2018ab, Zarga *et al*. 2019, Avcar 2019).

Many studies have been carried out, to predict and asses the mechanical behavior of functionally graded structures, as beams, plates and shells, employing various theories. Reddy (2000) presented theoretical formulation and finite element models for FG plates through third-order deformation theory of hearing. Formulations take account of the thermo-coupling, the time dependence and the geometric nonlinearity von Karman type of FG plates. Matsunaga (2008) examined an analysis of the free vibration and stability of FGM plates by the two-dimensional (2D) theory of higher order deformation. Jha *et al.* (2013) studied a free vibration response of functionally graded rectangular plates based on higher order shear and normal deformations theories. Mahmoudi *et al.* (2019) studied the flexure response of FG plates under thermomechanical loading. By dividing the transverse displacement into bending, shear, and thickness stretching parts, Bennai *et al.* (2015) studied vibration and stability for functionally graded sandwich beams using a novel higher-order shear and normal deformation theory. Boukhari *et al.* (2016) presented refined shear deformation theory for wave propagation of functionally graded material plates based neural surface position.

The bending and free vibration analysis using a simple two-variable shear deformation theory with the concept the neutral surface position was treated by Bellifa *et al.* (2016). Brischetto *et al.* (2016) examine the vibration behavior of plates and cylinders with functional gradation using three-dimensional and two-dimensional generalized quadrature differential models. The buckling and vibration analysis of thermally pre-stressed functionally graded carbon-nanotube-reinforced composite annular sector plates using the variational differential quadrature method were investigated by Ansari *et al.* (2017). Dynamic behavior of functionally graded plates and beams was studied using various higher shear deformation theories by (Atmane *et al.* 2015, Kolahchi *et al*. 2015, Yahia *et al.* 2015, Bounouara *et al.* 2016, Menasria *et al*. 2017, Benadouda *et al.* 2017, Bouhadra *et al.* 2018, Bourada *et al.* 2018, Ayache 2018, Yousfi *et al.* 2018, Boukhlif *et al.* 2019, Berghouti *et al.*

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2019, Addou *et al.* 2019, Hellal *et al*. 2019, Khiloun *et al.* 2019, Meksi *et al.* 2019, Boulefrakh *et al.* 2019, Nebab *et al.* 2019). Fourn *et al.* (2018) are suggested a high order hyperbolic shear deformation theory for wave propagation in functionally graded material plates with power and sigmoid law distribution. The static and free vibration of multilayered plates based on isogeometric analysis and higher-order shear and normal deformation theory were studied by Tran and Kim (2018). Abualnour *et al.* (2018) presented a new theory of shear deformation with stretching effect for the free vibration of functionally graded plates.

Structures, as plates and beams, resting elastic foundation can be found in different structural engineering fields. Winkler (1867) proposed layer springs to describe elastic foundations. Pasternak (1954) presents a shear layer for the interconnection between layers of Winkler elastic foundation. Rashed *et al.* (1998) studied bending of thick homogenous plates resting Winkler elastic foundation using boundary element method. Ying *et al.* (2008) presented exact solutions for bending and free vibration of functionally graded beams resting on a Winkler-Pasternak elastic foundation based on the two-dimensional theory of elasticity. The problem is solved using the state space method. Lü *et al.* (2009) investigated on dynamic behavior functionally graded thick plates on resting elastic foundation based on three-dimensional elasticity, by employing state space method to derive an exact solution for a simply supported plate. Malekzadeh (2009) studied three-dimensional free vibration analyses of functionally graded resting or without elastic foundations using the differential quadrature method and series solution, with two opposite edges simply supported and arbitrary boundary conditions at other edges. Zenkour (2009) presented a refined sinusoidal shear deformation plate theory for bending analysis of functionally graded plates resting on Pasternak-Winkler elastic foundations, with simply supported edges, subjected to a transverse uniform load and a temperature field. Ait Atmane *et al.* (2010) investigated on the influence of the two parameter elastic foundations and distribution of material properties, on free vibration of functionally graded plates using higher shear deformation theory. Farid *et al.* (2010) treated the free vibration analysis of initial stressed, thick simply supported functionally graded curved panel resting on elastic foundation subjected in thermal environment using the three dimensional elasticity formulation. Thai and Choi (2011) developed a refined plate theory for functionally graded plates resting on elastic foundation using exact solutions for simply supported case, and the Levy solution for various type boundary conditions. Neves *et al.* (2013) presented a quasi-three dimensional, higher-order shear deformation theory and a meshless technique for mechanical and dynamic behavior of isotropic and sandwich functionally graded plates. Sobhy (2013) studied the effect of elastic foundations on buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Al Khateeb and Zenkour (2014) presented a refined sinusoidal shear and normal deformation plate theory for advance composite plates resting on elastic foundations including the effects of temperature and moisture concentration. An accurate refined shear deformation theory for the vibration and buckling of functionally graded sandwich plate resting on elastic foundations under various boundary conditions was treated by Meziane *et al.* (2014). Taibi *et al.* (2015) studied the thermo-mechanical deformation behavior of functionally graded sandwich plates resting on a Pasternak- Winkler two-parameter elastic foundation based on refined shear deformation. Ait Atmane *et al.* (2015) presented a refined transverse shear and normal Deformation theory for bending, free vibration and buckling analysis of functionally graded beams porous on elastic foundation based on neutral surface position. Lee *et al.* (2015) presented a refined higher order shear and normal deformation theory for exponential, power-law and sigmoid functionally graded plates on elastic foundation. Zhang *et al.* (2016) investigated dynamic behavior of thin-to-moderately thick functionally graded carbon nanotube reinforced composite quadrilateral plates resting on elastic foundations employing improved moving least-squares-Ritz method. Chakraverty and Pradhan (2017) investigated free vibration of functionally graded thin skew plates resting on elastic foundation and subjected to different classical edge supports with various skew angles. Duc *et al.* (2017) studied static response and free vibration of functionally graded carbon nanotube reinforced composite plate resting on elastic foundations using an analytical approach.

Recently, Haciyev *et al.* (2018) studied the vibration of bi-directionally exponentially graded orthotropic plates resting on the two-parameter elastic foundation. A four-unknown shear deformation theory for hygrothermal bending response of functionally graded sandwich plates lying on visco-Pasternak foundation were investigated by Sobhy and Alotebi (2018). Songsuwan *et al.* (2018) investigated the free vibration and dynamic response of functionally graded sandwich beams resting on an elastic foundation under the action of a moving harmonic load using Ritz and Newmark methods. Torabi *et al.* (2018) studied the buckling of sandwich annular plates with carbon nanotube-reinforced face sheets resting on an elastic foundation by higher-order shear deformation theory, with employing the variational differential quadrature method.

The main goal of the present paper is examined the effect of variables elastic foundation on free vibration on functionally graded plates using four variables shear deformation theory. The variables elastic foundation is purposed to Winkler- Pasternak with variation in stiffness Winkler coefficient that is linear, parabolic and trigonometry. The exact solutions are present for resolve the equations of motion of simply supported functionally graded plates. The results and validation are present in this paper.

2. Theoretical formulation

2.1 Geometry and material properties

In this paper, the configurations of the functionally graded plate are a length, b width and uniform thickness h, in coordinate system (x,y,z).

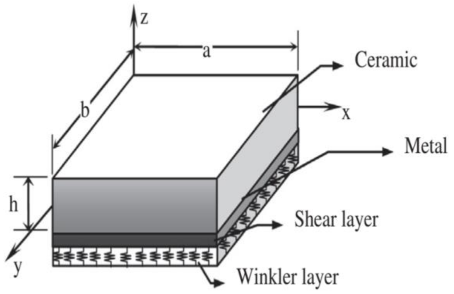


Fig. 1 Geometry and coordinate of FG plate resting on elastic foundation

The effective material properties of functionally graded plates such as Young's modulus E and mass density ρ are considered to vary gradually through the thickness according to a power law distribution, which is given by (Benchohra *et al*. 2018)

$$p(z) = p_m + (p_c - p_m)\left(\frac{1}{2}+\frac{z}{h}\right)^p \tag{1a}$$

$$E(z) = E_m + (E_c - E_m)\left(\frac{1}{2}+\frac{z}{h}\right)^P \tag{1b}$$

Where (E_c,p_c) and (E_m,p_m) are corresponding properties of the ceramic and metal, respectively, and p is the power law exponent that defines the gradation of material properties across the thickness direction. The Poisson's ratio v is considered constant.

2.2 Elastic foundation

The FG plates are supposed resting on two parameter elastic foundation. The parameter of Winkler is modeling with springs. The second parameter is presented from the Pasternak shear layer, which is interconnected the Winkler's spring (Pradhan and Murmu 2009, Sobhy 2015, Beldjelili *et al.* 2016, Attia *et al.* 2018)

$$R(x) = K_w(x)w - K_s\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \tag{2}$$

Where reaction of the elastic foundation is $R(x)$, and $K_w(x)$ is the variable Winkler parameter depends only in x direction.

$$\mathrm{K_w(x)} = \frac{\mathrm{J_1 D_i}}{\mathrm{a}^4}\begin{cases} 1+\zeta\dfrac{\mathrm{x}}{\mathrm{a}} & \text{Linear} \\ 1+\zeta\left(\dfrac{\mathrm{x}}{\mathrm{a}}\right)^2 & \text{Parabolic} \\ 1+\zeta\sin\left(\dfrac{\pi\cdot\mathrm{x}}{\mathrm{a}}\right) & \text{Sinusoidal} \end{cases} \tag{3}$$

Where J_1 is a constant and is a varied parameter. K_s is the base stiffness of the shear layer. Where that if ζ is zero, the elastic foundation becomes uniform foundation Pasternak of and if the rigidity of the shear layer is neglected, the foundation of Pasternak becomes the Winkler Foundation.

2.3 Kinematics and constitutive equations

Based on the theory of thick plates, the basic assumption of the displacement field of the higher order shear theory can be described by four variables

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + \varphi_x(x,y,t), \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + \varphi_y(x,y,t), \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{4}$$

Where u, v, w, φ_x and φ_y are five unknown displacements of the mid- plane of the plate, $f(z)$ describes a function of form representing the variation of the deformations and transverse shear stresses in the thickness. By considering that $\varphi_x = \int\theta(x,y,t)dx$ and $\varphi_y = \int\theta(x,y,t)dy$, displacement of the current model can be expressed in a simpler form

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int\theta(x,y,t)dx, \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int\theta(x,y,t)dx, \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{5}$$

The current theory of the higher order shear deformation is obtained by defining

$$f(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{6}$$

According to the displacement fields of higher shear deformation theory, non-zero strains can be given as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \varepsilon_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{7}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \varepsilon_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial y}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w}{\partial^2 x}\\ -\dfrac{\partial^2 w}{\partial^2 y}\\ -2\dfrac{\partial^2 w}{\partial^2 x\,\partial^2 y}\end{Bmatrix}, \tag{8a}$$

$$\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta dx + k_2\dfrac{\partial}{\partial x}\int\theta dy\end{Bmatrix}, \tag{8b}$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}k_2\int\theta dy\\ k_1\int\theta dx\end{Bmatrix}, \textit{and} \quad g(z) = \frac{\partial f(z)}{\partial z} \tag{8c}$$

The undefined integrals in the equations below must be solved by a Navier type method and can be written as follows

$$\frac{\partial}{\partial y}\int \theta dx = A'\frac{\partial^2\theta}{\partial y\,\partial x},\quad \frac{\partial}{\partial x}\int \theta dy = B'\frac{\partial^2\theta}{\partial y\,\partial x},\quad \int \theta dx = A'\frac{\partial\theta}{\partial x},\quad \int \theta dy = B'\frac{\partial\theta}{\partial y} \tag{9}$$

Where the coefficient A' and B' are considered according to the type of solution used, in this case via the Navier method. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2},\quad B' = -\frac{1}{\beta^2},\quad k_1 = \alpha^2,\quad k_2 = \beta^2 \tag{10}$$

Where α and β are define in Eq. (21).

The stress-strain relations for the FGM plate are given by

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{21} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0\\ 0 & 0 & 0 & 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\\ \gamma_{xy}\\ \gamma_{xy}\end{Bmatrix} \tag{11}$$

Where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_{xx}, ε_{yy}, ε_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. The elastic constants Q_{ij} are defined as

$$Q_{11} = \frac{E(z)}{1-\nu^2},\quad Q_{11} = Q_{22} \tag{12a}$$

$$Q_{12} = \frac{\nu E(z)}{1-\nu^2},\quad Q_{12} = Q_{12} \tag{12b}$$

$$Q_{44} = \frac{E(z)}{2(1+\nu)},\quad Q_{44} = Q_{55} = Q_{66} \tag{12c}$$

2.4 Governing equations

The Hamilton Energy Principle applies to derive the Motion Equations of FG Plates; the principle can be stated in analytical form as (Attia *et al.* 2015, Fahsi *et al.* 2017, Karami *et al.* 2018c, Bendaho *et al.* 2019)

$$\int_0^t (\delta U + \delta U_{ef} - \delta K)dt = 0 \tag{13}$$

Where δ indicates a variation, and U, U_{ef} and Krepresent, respectively, the strain energy, the strain energy of elastic foundations and the kinetic energy of the FG plate.

2.4.1 Strain energy

The variation of the deformation energy of the plate indicated as (Bousahla *et al.* 2016, El-Haina *et al.* 2017, Ait Atmane *et al.* 2017, Boussoula *et al.* 2019, Boutaleb *et al.* 2019)

$$\delta U = \int_V (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \tau_{xy}\delta\tau_{xy} + \tau_{yz}\delta\tau_{yz} + \tau_{xz}\delta\tau_{xz})dV \tag{14a}$$

$$= \int_A \begin{pmatrix} N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\varepsilon_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b \\ + M_{xy}^b\delta k_{xy}^b + M_x^s\delta k_x^s + M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s \\ + R_{yz}^s\delta R_{yz}^s + R_{xz}^s\delta R_{xz}^s \end{pmatrix} dA \tag{14b}$$

Where

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz,\quad (i = x, y, xy) \tag{15a}$$

$$\left(R_{xz}^s, R_{yz}^s\right) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz})g\ dz \tag{15b}$$

2.4.2 Strain energy of elastic foundations

The variation of the deformation energy of elastic foundations indicated as

$$\delta U_{ef} = \int_A (K_w w_0\delta w_0 - K_s\left(\frac{d^2w_0}{dx^2} + \frac{d^2w_0}{dy^2}\right)\delta w)\ dA \tag{16}$$

2.4.3 Kinetic energy

The variation in kinetic energy based on the high theory of shear deformation can be expressed as follows (Zine *et al.*2018, Bourada *et al.*2019, Chaabane *et al.* 2019, Medani *et al.* 2019)

$$\delta K = \int_V \rho(z)\left(\dot u\delta\dot u + \dot v\delta\dot v + \dot w\delta\dot w\right)dV \tag{17a}$$

$$= \int_A \left\{\begin{aligned} & I_0(\dot u_0\delta\dot u_0 + \dot v_0\delta\dot v_0 + \dot w_0\delta\dot w_0) \\ & - I_1\begin{pmatrix}\dot u_0\dfrac{\partial\delta\dot w_0}{\partial x} + \dfrac{\partial\dot w_0}{\partial x}\delta\dot u_0 \\ + \dot v_0\dfrac{\partial\delta\dot w_0}{\partial y} + \dfrac{\partial w_0}{\partial y}\delta\dot v_0\end{pmatrix} \\ & + J_1\begin{pmatrix}\left(k_1A'\right)\left(\dot u_0\dfrac{\partial\delta\dot\theta}{\partial x} + \dfrac{\partial\dot\theta}{\partial x}\delta\dot u_0\right) \\ + \left(k_2B'\right)\left(\dot v_0\dfrac{\partial\delta\dot\theta}{\partial y} + \dfrac{\partial\dot\theta}{\partial y}\delta\dot v_0\right)\end{pmatrix} \\ & + I_2\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot w_0}{\partial x} + \frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}\right) \\ & + K_2\begin{pmatrix}\left(k_1A'\right)^2\left(\dfrac{\partial\dot\theta}{\partial x}\dfrac{\partial\delta\dot\theta}{\partial x}\right) \\ + \left(k_2B'\right)^2\left(\dfrac{\partial\dot\theta}{\partial y}\dfrac{\partial\delta\dot\theta}{\partial y}\right)\end{pmatrix} \\ & - J_2\begin{pmatrix}\left(k_1A'\right)\left(\dfrac{\partial\dot w}{\partial x}\dfrac{\partial\delta\dot\theta}{\partial x} + \dfrac{\partial\delta\dot w}{\partial x}\dfrac{\partial\dot\theta}{\partial x}\right) \\ + \left(k_2B'\right)\left(\dfrac{\partial\dot w}{\partial y}\dfrac{\partial\delta\dot\theta}{\partial y} + \dfrac{\partial\delta\dot w}{\partial y}\dfrac{\partial\dot\theta}{\partial y}\right)\end{pmatrix}\end{aligned}\right\} dA \tag{17b}$$

Where

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2} (1, z, z^2)p(z)dz \tag{18a}$$

$$\left(J_1, J_2, K_2\right) = \int_{-h/2}^{h/2} (f, zf, f^2) p(z) dz \tag{18b}$$

By substituting the expressions for, δU, δU_{ef} and δK from the Eqs. (14), (16) and (17) in Eq. (13) and integrating by parts and collecting the coefficients of (δu_0, δv_0, δw_0 and $\delta\theta$) the following equations of plate motion are obtained as

$$\begin{aligned}
\delta u_0 :\ & \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + J_1 k_1 A' \frac{\partial \ddot{\theta}}{\partial x} \\
\delta v_0 :\ & \frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + J_1 k_2 B' \frac{\partial \ddot{\theta}}{\partial y} \\
\delta w_0 :\ & \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + K_w w_0 \\
& -K_s\left(\frac{d^2 w_0}{dx^2} + \frac{d^2 w_0}{dy^2}\right) = I_0 \ddot{w}_0 + I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) \\
& - I_2 \nabla^2 \ddot{w}_0 + J_2\left(k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) \\
\delta\theta_0 :\ & -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x \partial y} \\
& + k_1 A' \frac{\partial R_{xz}^S}{\partial x} + k_2 B' \frac{\partial R_{yz}^S}{\partial y} = -J_1\left(k_1 A' \frac{\partial \ddot{u}_0}{\partial x} + k_2 B' \frac{\partial \ddot{v}_0}{\partial y}\right) \\
& - K_2\left(\left(k_1 A'\right)^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + \left(k_2 B'\right)^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) \\
& + J_2\left(k_1 A' \frac{\partial^2 \ddot{w}}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{w}}{\partial y^2}\right)
\end{aligned} \tag{19}$$

Substituting Eq. (7) in Eq. (11) and integrating into the thickness of the plate, the resulting stresses are presented as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ A_{21} & A_{66} & 0 & B_{21} & B_{22} & 0 & D_{21} & D_{66} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & D_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ B_{21} & B_{22} & 0 & D_{21} & D_{22} & 0 & D_{21}^s & D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{21}^s & B_{22}^s & 0 & D_{21}^s & D_{22}^s & 0 & H_{21}^s & H_{22}^s & 0 \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_x^0 \\ \varepsilon_{xy}^0 \\ k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{20a}$$

$$\begin{Bmatrix} R_{xz}^s \\ R_{yz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{20b}$$

In addition, and stiffness components are given as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} Q_{11}\left(1,\ z,\ z^2,\ f(z),\ z\ f(z),\ f(z)^2\right) dz \tag{21a}$$

$$\left(A_{22} \quad B_{22} \quad D_{22} \quad B_{22}^s \quad D_{22}^s \quad H_{22}^s\right) = \left(A_{11} \quad B_{11} \quad D_{11} \quad B_{11}^s \quad D_{11}^s \quad H_{11}^s\right) \tag{21b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} Q_{44}[g(z)]^2 dz \tag{21c}$$

2.5 Equations of motion in terms of displacements

Substituting Eq. (20) into Eq. (19), the equations of motion of the presented theory can be rewritten in terms of displacements (δu_0, δv_0, δw_0 and $\delta\theta_0$) as follow

$$\begin{aligned}
& A_{11} d_{11} u_0 + A_{66} d_{22} v_0 + \left(A_{11} + A_{66}\right) d_{12} v_0 - B_{11} d_{11} w_0 \\
& -\left(B_{12} + 2B_{66}\right) d_{122} w_0 + \left(B_{66}^s\left(k_1 A' + k_2 B'\right)\right) d_{122}\theta \\
& +\left(B_{11}^S k_1 + B_{12}^s k_2\right) d_1\theta = I_0 \ddot{u}_0 - I_1 d_1 \ddot{w}_0 + J_1 k_1 A' d_1 \ddot{\theta}
\end{aligned} \tag{22a}$$

$$\begin{aligned}
& A_{22} d_{22} v_0 + A_{66} d_{11} v_0 + \left(A_{12} + A_{66}\right) d_{12} u_0 - B_{22} d_{222} w_0 \\
& -\left(B_{12} + 2B_{66}\right) d_{112} w_0 + \left(B_{66}^s\left(k_1 A' + k_2 B'\right)\right) d_{112}\theta \\
& +\left(B_{22}^s k_2 + B_{12}^s k_1\right) d_2\theta = I_0 \ddot{v}_0 - I_1 d_2 \ddot{w}_0 + J_1 k_2 B' d_2 \ddot{\theta}
\end{aligned} \tag{22b}$$

$$\begin{aligned}
& B_{11} d_{111} u_0 + \left(B_{12} + 2B_{66}\right) d_{122} u_0 + \left(B_{12} + 2B_{66}\right) d_{112} v_0 \\
& + B_{22} d_{222} v_0 - D_{11} d_{222} w_0 - 2\left(D_{12} + 2D_{66}\right) d_{1122} w_0 \\
& - D_{22} d_{2222} w_0 - K_w w_0 + K_s\left(d_{11} w_0 + d_{22} w_0\right) \\
& +\left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11}\theta + 2\left(D_{66}^s\left(k_1 A' + k_2 B'\right)\right) d_{1122}\theta \\
& +\left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22}\theta = I_0 \ddot{w}_0 + I_1\left(d_1 \ddot{u}_0 + d_2 \ddot{v}_0\right) \\
& - I_2\left(d_{11} \ddot{w}_0 + d_{22} \ddot{w}_0\right) + J_2\left(A' k_1 d_{11} \ddot{\theta} + B' k_2 d_{22} \ddot{\theta}\right)
\end{aligned} \tag{22c}$$

$$\begin{aligned}
& \left(-B_{11}^s k_1 + B_{12}^s k_2\right) d_1 u_0 - \left(B_{66}^s\left(A' k_1 + B' k_2\right)\right) d_{122} u_0 \\
& -\left(B_{66}^s\left(A' k_1 + B' k_2\right)\right) d_{112} v_0 - \left(B_{12}^s k_1 + B_{22}^s k_2\right) d_2 v_0 \\
& -\left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11} w_0 + 2\left(D_{66}^s\left(A' k_1 + B' k_2\right)\right) d_{1122} w_0 \\
& +\left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22} w_0 - H_{11}^s k_1^2 \theta - H_{12}^s k_2^2 \theta \\
& -2H_{12}^s k_1 k_2 \theta - \left(H_{66}^s\left(A' k_1 + B' k_2\right)^2\right) d_{1122}\theta \\
& + A_{44}^s\left(B' k_2\right)^2 d_{22}\theta + A_{55}^s\left(A' k_1\right)^2 d_{11}\theta \\
& = -J_1\left(A' k_1 d_1 \ddot{u}_0 + B' k_2 d_2 \ddot{v}_0\right) + J_2\left(A' k_1 d_{11} \ddot{w}_0 + B' k_2 d_{22} \ddot{w}_0\right) \\
& - K_2\left(\left(A' k_1\right)^2 d_{11}\ddot{\theta} + \left(B' k_2\right)^2 d_{22}\ddot{\theta}\right)
\end{aligned} \tag{22d}$$

Where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ij} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \quad d_{ij} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_{ij} = \frac{\partial}{\partial x_i} \tag{23}$$

3. Analytical solutions

Analytical solution is considered for the free vibration of FG plates resting on a variable elastic base using the

Fourier series. The displacement field can be assumed

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \begin{Bmatrix} U_{mn} \cos(\alpha x)\sin(\beta y)\ e^{i\omega t} \\ V_{mn} \sin(\alpha x)\cos(\beta y)\ e^{i\omega t} \\ W_{mn} \sin(\alpha x)\sin(\beta y)\ e^{i\omega t} \\ X_{mn} \sin(\alpha x)\sin(\beta y)\ e^{i\omega t} \end{Bmatrix} \tag{24}$$

Where

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b} \tag{25}$$

Where $i = \sqrt{-1}$ and ω is the natural frequency, and U_{mn}, V_{mn}, W_{mn}, X_{mn} are the unknown maximum displacement coefficients.

Substituting Eq. (24) into Eq. (22), the following problem is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{41} & m_{43} & m_{44} \end{bmatrix} \right) \begin{Bmatrix} U \\ V \\ W \\ X \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{26}$$

Where

$$\begin{aligned}
S_{11} &= -\left(A_{11}\alpha^2 + A_{66}\beta^2\right), \\
S_{12} &= -\alpha\beta\left(A_{12} + A_{66}\right), \\
S_{13} &= \alpha(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2), \\
S_{14} &= \alpha\left(k_1 B_{11}^s + k_2 B_{12}^s - \left(k_1 A' + k_2 B'\right) B_{66}^s \beta^2\right), \\
S_{22} &= -\left(A_{66}\alpha^2 + A_{22}\beta^2\right), \\
S_{23} &= \beta(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2), \\
S_{24} &= \beta\left(k_2 B_{22}^s + k_1 B_{12}^s - \left(k_1 A' + k_2 B'\right) B_{66}^s \alpha^2\right), \\
S_{33} &= -\left(D_{11}\alpha^4 + 2\left(D_{12} + 2D_{66}\right)\alpha^2\beta^2 + D_{22}\beta^4\right) \\
&\quad - \bar{K}_W - K_P(\alpha^2 + \beta^2) \\
S_{34} &= -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2\left(k_1 A' + k_2 B'\right) D_{66}^s \alpha^2\beta^2 \\
&\quad - k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right) \\
S_{44} &= -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - \left(k_1 A' + k_2 B'\right)^2 H_{66}^s \alpha^2\beta^2 \\
&\quad - k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - \left(k_1 A'\right)^2 A_{55}^s \alpha^2 - \left(k_2 B'\right)^2 A_{44}^s \beta^2
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
m_{11} &= -I_0, \quad m_{13} = \alpha I_1, \quad m_{14} = -J_1 k_1 A' \alpha, \\
m_{22} &= -I_0, \quad m_{23} = \beta I_1, \quad m_{24} = -J_1 k_2 B' \beta, \\
m_{33} &= -I_0 - I_2\left(\alpha^2 + \beta^2\right) \quad , \\
m_{34} &= J_2\left(A' k_1 \alpha^2 + B' k_2 \beta^2\right), \\
m_{44} &= -K_2\left(\left(A' k_1\right)^2 \alpha^2 + \left(B' k_2\right)^2 \beta^2\right)
\end{aligned}$$

Table 1 Material properties of the functionally graded plates

Material	Properties		
	E (GPa)	ν	ρ (Kg/m^3)
Aluminum (Al)	70	0.3	2702
Alumina (Al_2O_3)	380	0.3	3800

Table 2 Non-dimensional fundamental frequencies β for simply supported isotropic square plates

a/h	Source	Mode						
		1,1	1,2	2,1	2,2	3,1	1,3	3,2
1000	Leissa (1973)	19.7392	49.3480	49.3480	78.9568	98.6960	98.6960	128.3021
	Zhou *et al.* (2002)	19.7115	49.3470	49.3470	78.9528	98.6911	98.6911	128.3048
	Akavci (2014)	19.7391	49.3476	49.3476	78.9557	98.6943	98.6943	128.3020
	Mantari *et al.* (2014)	19.7391	49.3475	49.3475	78.9557	98.6942	98.6942	128.3018
	Meftah *et al.* (2017)	19.7391	49.3476	49.3476	78.9557	98.6943	98.6943	128.3019
	Zaoui *et al.* (2019) (2D)	19.7391	49.3476	49.3476	78.9557	98.6942	98.6942	128.3018
	Zaoui *et al.* (2019) (quasi-3D)	19.7712	49.4278	49.4278	79.0841	98.8548	98.8548	128.5106
	Present	19.7391	49.3476	49.3476	78.9557	98.6942	98.6942	128.3018
100	Liu and Liew (1999)	19.7319	49.3027	49.3027	78.8410	98.5150	98.5150	127.9993
	Nagino *et al.* (2008)	19.7320	49.3050	49.3050	78.8460	98.5250	98.5250	128.0100
	Akavci (2014)	19.7322	49.3045	49.3045	78.8456	98.5223	98.5223	128.0120
	Mantari *et al.* (2014)	19.7320	49.3032	49.3032	78.8422	98.5170	98.5170	128.0025
	Meftah *et al.* (2017)	19.7321	49.304	49.304	78.8442	98.5202	98.5202	128.008
	Zaoui *et al.* (2019) (2D)	19.732	49.3032	49.3032	78.8422	98.5171	98.5171	128.0028
	Zaoui *et al.* (2019) (quasi-3D)	19.7644	49.3853	49.3853	78.9754	98.685	98.685	128.2239
	Present	19.7320	49.3032	49.3032	78.8421	98.5169	98.5169	128.0024
10	Liu and Liew (1999)	19.0584	45.4478	45.4478	69.7167	84.9264	84.9264	106.5154
	Hosseini-Hashemi *et al.* (2011)	19.0653	45.4869	45.4869	69.8093	85.0646	85.0646	106.7350
	Akavci (2014)	19.0850	45.5957	45.5957	70.0595	85.4315	85.4315	107.3040
	Mantari *et al.* (2014)	19.0656	45.4890	45.4890	69.8146	85.0729	85.0729	106.7491
	Meftah *et al.* (2017)	19.0775	45.5548	45.5548	69.9664	85.2958	85.2958	107.0953
	Zaoui *et al.* (2019) (2D)	19.0661	45.4917	45.4917	69.8213	85. 0829	85.0829	106.7652
	Zaoui *et al.* (2019) (quasi-3D)	19.1248	45.7152	45.7152	70.2709	85.7067	85.7067	107.6744
	Present	19.0653	45.4869	45.4869	69.8093	85.0646	85.0646	106.7350
5	Shufrin and Eisenberger (2005)	17.4524	38.1884	38.1884	55.2539	65.3130	65.3130	78.9864
	Hosseini-Hashemi *et al.* (2011)	17.4523	38.1883	38.1883	55.2543	65.3135	65.3135	78.9865
	Akavci (2014)	17.5149	38.4722	38.4722	55.8358	66.1207	66.1207	80.1637
	Mantari *et al.* (2014)	17.4537	38.1965	38.1965	55.2748	65.3446	65.3446	79.0371
	Present	17.4523	38.1883	38.1883	55.2543	65.3135	65.3135	78.9865

Table 3 Comparison of influence elastic foundation on non-dimensional natural frequencies $\tilde{\omega}$ for isotropic and functionally graded square plates, with various index powe

(k_w/k_s)	*h/a*	Source	*p*				
			0	0.5	1	2	5
(0,0)	0.1	Hasani Baferani *et al.* (2011)	0.0291	0.0249	0.0227	0.0209	0.0197
		Zaoui *et al.* (2019) (2D)	0.0291	0.0246	0.0222	0.0202	0.0191
		Zaoui *et al.* (2019) (quasi-3D)	0.0291	0.0248	0.0226	0.0207	0.0195
		Benahmed *et al.* (2017)	0.0291	–	0.0226	0.0207	–
		Shahsavari *et al.* (2018)	0.0291	0.0248	0.0226	0.0206	0.0195
		Present	0.0291	0.0246	0.0222	0.0202	0.0191
	0.1	Hasani Baferani *et al.* (2011)	0.1134	0.0975	0.0891	0.0819	0.0767
		Zaoui *et al.* (2019) (2D)	0.1134	0.0963	0.0868	0.0788	0.074
		Zaoui *et al.* (2019) (quasi-3D)	0.1137	0.0972	0.0883	0.0807	0.0756
		Benahmed *et al.* (2017)	0.1136	–	0.0883	0.0807	–
		Shahsavari *et al.* (2018)	0.1135	0.097	0.0882	0.0806	0.0755
		Present	0.1134	0.0963	0.0868	0.0788	0.0740
	0.2	Hasani Baferani *et al.* (2011)	0.2454	0.2121	0.1939	0.1778	0.1648
		Zaoui *et al.* (2019) (2D)	0.2452	0.209	0.1885	0.1706	0.1588
		Zaoui *et al.* (2019) (quasi-3D)	0.2464	0.2112	0.1919	0.1749	0.1623
		Benahmed *et al.* (2017)	0.2461	–	0.1918	0.1748	–
		Shahsavari *et al.* (2018)	0.2459	0.2108	0.1916	0.1746	0.1622
		Present	0.2452	0.2090	0.1885	0.1706	0.1588
	0.2	Hasani Baferani *et al.* (2011)	0.4154	0.3606	0.3299	0.3016	0.2765
		Zaoui *et al.* (2019) (2D)	0.4151	0.3551	0.3205	0.2892	0.2665
		Zaoui *et al.* (2019) (quasi-3D)	0.4178	0.3593	0.3267	0.2968	0.2725
		Benahmed *et al.* (2017)	0.4174	–	0.3264	0.2965	–
		Shahsavari *et al.* (2018)	0.4168	0.3586	0.326	0.2961	0.2722
		Present	0.4151	0.3551	0.3205	0.2892	0.2667
(0,100)	0.1	Hasani Baferani *et al.* (2011)	0.0406	0.0389	0.0382	0.038	0.0381
		Zaoui *et al.* (2019) (2D)	0.0406	0.0386	0.0378	0.0374	0.0377
		Zaoui *et al.* (2019) (quasi-3D)	0.0406	0.0387	0.038	0.0376	0.0378
		Benahmed *et al.* (2017)	0.0406	-	0.038	0.0376	-
		Present	0.0406	0.0386	0.0378	0.0374	0.0377
	0.1	Hasani Baferani *et al.* (2011)	0.1599	0.154	0.1517	0.1508	0.1515
		Zaoui *et al.* (2019) (2D)	0.1597	0.1526	0.1494	0.1478	0.1487
		Zaoui *et al.* (2019) (quasi-3D)	0.1594	0.1525	0.1497	0.1483	0.1489
		Benahmed *et al.* (2017)	0.1594	-	0.1497	0.1483	-
		Present	0.1597	0.1526	0.1494	0.1478	0.1487
	0.2	Hasani Baferani *et al.* (2011)	0.3515	0.3407	0.3365	0.3351	0.3362
		Zaoui *et al.* (2019) (2D)	0.3512	0.3369	0.3304	0.3269	0.3286
		Zaoui *et al.* (2019) (quasi-3D)	0.3494	0.3354	0.3296	0.3266	0.3275
		Benahmed *et al.* (2017)	0.3492	-	0.3295	0.3265	-
		Present	0.3512	0.3369	0.3304	0.3269	0.3286
	0.2	Hasani Baferani *et al.* (2011)	0.608	0.5932	0.5876	0.5861	0.5879
		Zaoui *et al.* (2019) (2D)	0.6075	0.5857	0.5753	0.5694	0.5722
		Zaoui *et al.* (2019) (quasi-3D)	0.6015	0.5795	0.5701	0.5652	0.5662
		Benahmed *et al.* (2017)	0.6011	-	0.5699	0.565	-
		Present	0.6075	0.5858	0.5753	0.5695	0.5722

4. Results and discussion

Here in this study of free vibration of the functionally graded plates without or resting on variable elastic foundation, a various numerical examples are carried out to validate the results of the present numerical model. A type of FGM plates of Al/Al2O3 is used in this study. The material properties of P-FGM plates are presented in Table 1.

The used non-dimensional parameters are

$$D_c = \frac{E_c h^3}{12\left(1-2\nu^2\right)},\; D_m = \frac{E_m h^3}{12\left(1-2\nu^2\right)}$$

$$\beta = \omega a^2 \sqrt{\frac{\rho h}{D_c}} \;,\; \tilde{\omega} = \omega h \sqrt{\frac{\rho_m}{E_m}} \;,$$

$$\vartheta = \omega (a^2/h) \sqrt{\frac{\rho_c}{E_c}} \;,\; K_s = \frac{k_s D_i}{a^2},\; i = m,c$$

4.1 Free vibration of homogenous isotropic plates

Table 3 Continued

(k_w/k_s)	*h/a*	Source	*p* = 0	0.5	1	2	5
(100,0)	0.1	Hasani Baferani *et al.* (2011)	0.0298	0.0258	0.0238	0.0221	0.021
		Zaoui *et al.* (2019) (2D)	0.0298	0.0255	0.0233	0.0214	0.0204
		Zaoui *et al.* (2019) (quasi-3D)	0.0298	0.0257	0.0236	0.0219	0.0208
		Benahmed *et al.* (2017)	0.0298	–	0.0236	0.0218	–
		Shahsavari *et al.* (2018)	0.0298	0.0257	0.0236	0.0218	0.0208
		Present	0.0298	0.0255	0.0233	0.0214	0.0204
	0.1	Hasani Baferani *et al.* (2011)	0.1162	0.1012	0.0933	0.0867	0.0821
		Zaoui *et al.* (2019) (2D)	0.1162	0.0999	0.091	0.0836	0.0795
		Zaoui *et al.* (2019) (quasi-3D)	0.1164	0.1007	0.0924	0.0854	0.0809
		Benahmed *et al.* (2017)	0.1164	–	0.0924	0.0854	–
		Shahsavari *et al.* (2018)	0.1163	0.1006	0.0923	0.0853	0.0809
		Present	0.1162	0.0999	0.0910	0.0836	0.0795
	0.2	Hasani Baferani *et al.* (2011)	0.2519	0.2204	0.2036	0.1889	0.1775
		Zaoui *et al.* (2019) (2D)	0.2516	0.2173	0.1982	0.1818	0.1715
		Zaoui *et al.* (2019) (quasi-3D)	0.2526	0.2192	0.2012	*0.1856*	*0.1745*
		Benahmed *et al.* (2017)	0.2524	–	0.2011	0.1855	–
		Shahsavari *et al.* (2018)	0.2522	0.219	0.201	0.1855	0.1745
		Present	0.2517	0.2173	0.1982	0.1818	0.1715
	0.2	Hasani Baferani *et al.* (2011)	0.4273	0.3758	0.3476	0.3219	0.2999
		Zaoui *et al.* (2019) (2D)	0.4269	0.3702	0.3381	0.3097	0.29
		Zaoui *et al.* (2019) (quasi-3D)	0.429	0.3737	0.3433	0.3161	0.2948
		Benahmed *et al.* (2017)	0.4286	–	0.3431	0.3158	–
		Shahsavari *et al.* (2018)	0.4284	0.3734	0.3431	0.3159	0.295
		Present	0.4269	0.3702	0.3381	0.3097	0.2901
(100,100)	0.1	Hasani Baferani *et al.* (2011)	0.0411	0.0395	0.0388	0.0386	0.0388
		Zaoui *et al.* (2019) (2D)	0.0411	0.0392	0.0384	0.0381	0.0384
		Zaoui *et al.* (2019) (quasi-3D)	0.0411	0.0393	0.0386	0.0383	0.0385
		Benahmed *et al.* (2017)	0.0411	–	0.0386	0.0383	–
		Shahsavari *et al.* (2018)	0.0411	0.0393	0.0386	0.0383	0.0385
		Present	0.0411	0.0392	0.0384	0.0381	0.0384
	0.1	Hasani Baferani *et al.* (2011)	0.1619	0.1563	0.1542	0.1535	0.1543
		Zaoui *et al.* (2019) (2D)	0.1617	0.1549	0.1519	0.1505	0.1515
		Zaoui *et al.* (2019) (quasi-3D)	0.1614	0.1548	0.1522	0.1509	0.1517
		Benahmed *et al.* (2017)	0.1614	–	0.1521	0.1509	–
		Shahsavari *et al.* (2018)	0.1616	0.1551	0.1525	0.1512	0.1521
		Present	0.1617	0.1549	0.1519	0.1505	0.1515
	0.2	Hasani Baferani *et al.* (2011)	0.356	0.346	0.3422	0.3412	0.3427
		Zaoui *et al.* (2019) (2D)	0.3557	0.3421	0.3359	0.3329	0.3349
		Zaoui *et al.* (2019) (quasi-3D)	0.3538	0.3405	0.335	0.3324	0.3337
		Benahmed *et al.* (2017)	0.3537	–	0.3349	0.3323	–
		Shahsavari *et al.* (2018)	0.356	0.346	0.3422	0.3412	0.3427
		Present	0.3557	0.3421	0.3360	0.3329	0.3349
	0.2	Hasani Baferani *et al.* (2011)	0.6162	0.6026	0.5978	0.597	0.5993
		Zaoui *et al.* (2019) (2D)	0.6156	0.595	0.5852	0.58	0.5833
		Zaoui *et al.* (2019) (quasi-3D)	0.6093	0.5884	0.5797	0.5754	0.577
		Benahmed *et al.* (2017)	0.6089	–	0.5794	0.5752	–
		Shahsavari *et al.* (2018)	0.6137	0.594	0.5856	0.5815	0.5843
		Present	0.6156	0.5950	0.5852	0.5800	0.5834

In the Table 2 and the first examples, we start to check the efficiency and convergence of the current method for homogeneous plates simply supported with different ratios (*h/a*). The first eight non-dimensional frequencies are presented for different, thickness to length ratios (*a/h*). The present results are compared to the other results found in the literature. The results reported by Leissa (1973) were based on three dimensional exact solution, Zhou *et al.* (2002) were based on a three dimensional Ritz method with Chebyshev polynomials, Liu and Liew (1999) were employing a differential quadrature method, Nagino *et al.* (2008) were using a three dimensional B-spline Ritz method. Hosseini-Hashemi *et al.* (2011) using an exact closed form Levy-type solution. Shufrin and Eisenberger

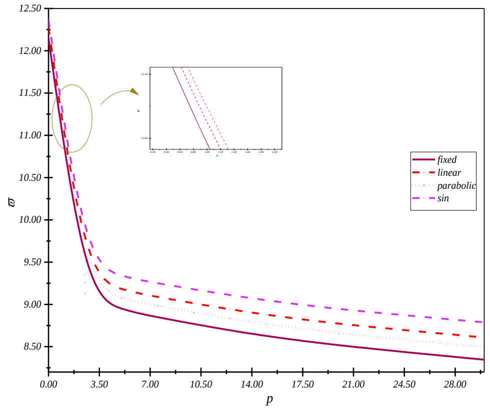


Fig. 2 Effect of the various types elastic foundation on non-dimensional frequencies V of FG square plates, (a/h=10, k_w=100, k_s=10 and z=1)

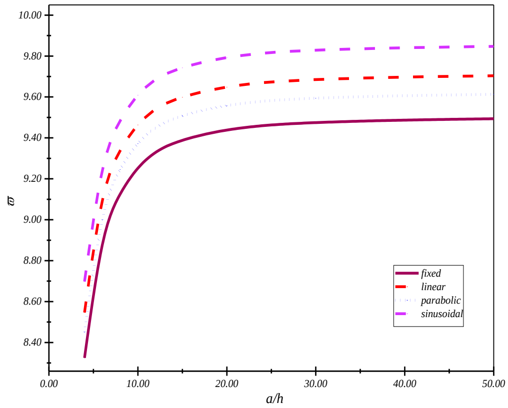


Fig. 3 Effect of the various types elastic foundation on non-dimensional frequencies V of FG square plates (p=2, k_w=100, k_s=10 and z=1)

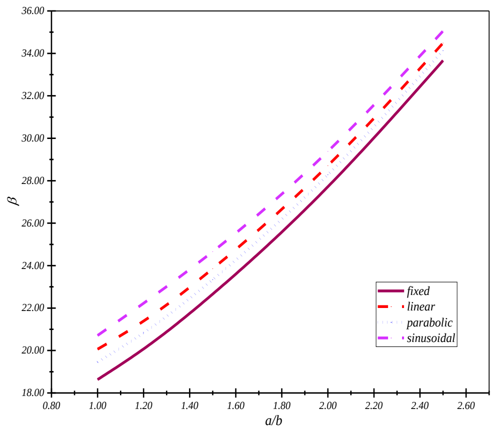
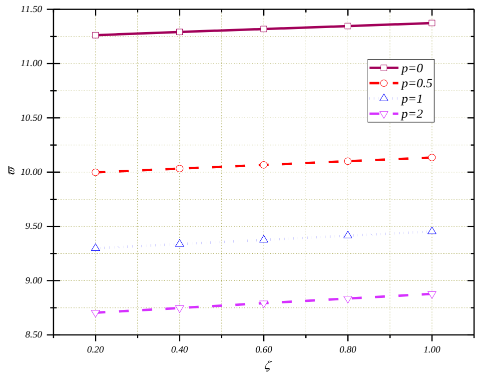
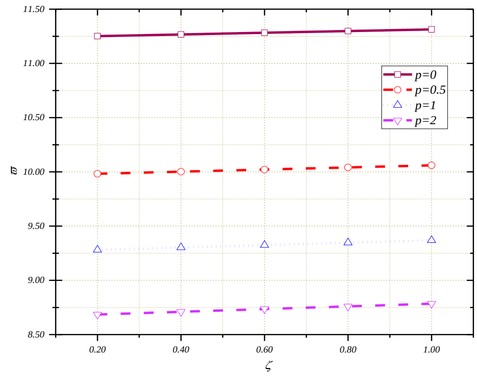


Fig. 4 Effect of the various types elastic foundation non-dimensional frequencies ϑ FG plates (p=2, a/h=10, k_w=10^3, k_s=10^2 and z=1)

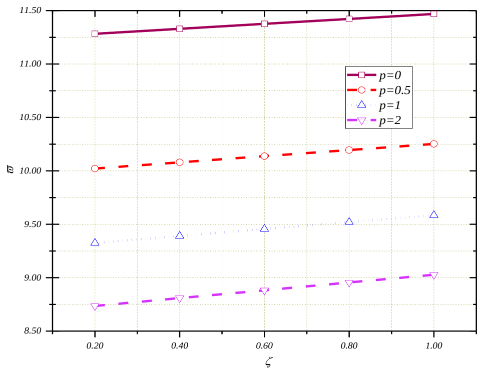
(2005) were based on 2D higher order theories, and HSDTs of (Mantari *et al.* 2014), (Akavci 2014), and (Meftah *et al.* 2017). Results produced by Zaoui *et al.* (2019) using the analytical solutions, based on 2D and quasi-3D shear deformation theory. As can be seen in the current results in



(a) Linear variation



(b) Parabolic variation



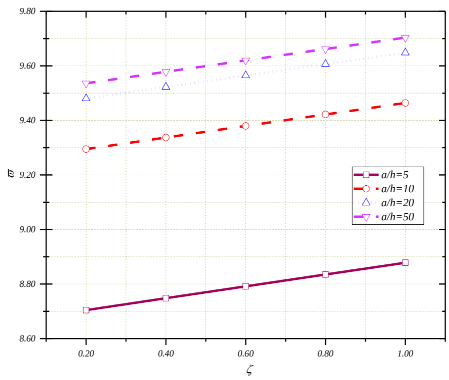
(c) Sinusoidal variation

Fig. 5 Influence of z parameter of the variable elastic foundation on non-dimensional frequencies $\tilde{\omega}$ FG square plates (a/h=10, k_w=100, k_s=10)

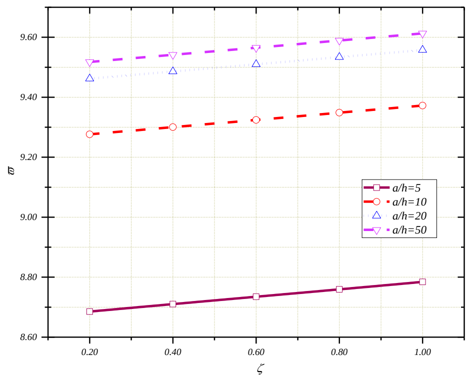
Table 2, Eigen frequencies of isotropic square plates are predicted with optimal efficiency, not for isotropic thin plates, but also for thick plates. The currently work are in good agreement with other methods

4.2 Free vibration of homogenous and FG plates resting of elastic foundations

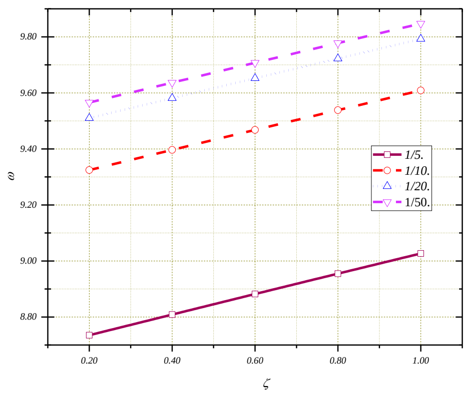
In the second part of this study and in Table 3, it shows the effect of Pasternak-Winkler of non-dimensional frequencies of FG plates with numerous two-parameter elastic foundation values (k_w, k_s), thickness to length ratio (h/a) and a volume fraction with five indices p=(1, 0.5, 1, 2 and 5). The present results are compared with those of



(a) Linear variation

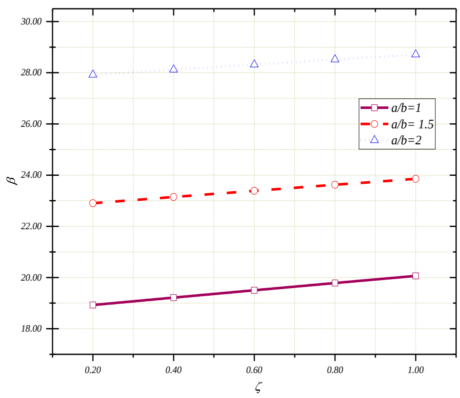


(b) Parabolic variation

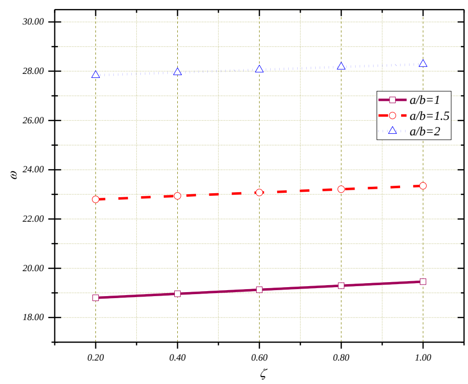


(c) Sinusoidal variation

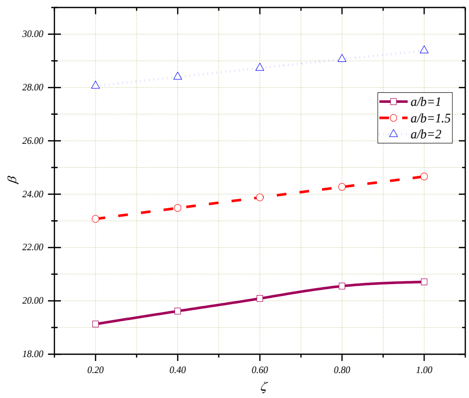
Fig. 6 Influence of z parameter of the variable elastic foundation on non-dimensional frequencies V FG square plates (p=2, k_w=100, k_s=10)



(a) Linear variation



(a) Parabolic variation



(c) Sinusoidal variation

Fig. 7 Influence of z parameter of the variable elastic foundation on non-dimensional frequencies ϑ FG plates, (p=2, a/h=10, k_w=10^3, k_s=10^2)

Hasani Baferani *et al.* (2011), Zaoui *et al.* (2019), Benahmed *et al.* (2017) and (Shahsavari *et al.* 2018) using a theory of third order shear deformation. From the results, we can observe that the Eigen frequencies decrease with the increase of the power law indices in with or without of elastic base and that the current theory is in good agreement with Zaoui *et al.* (2019) in part two dimensional 2D. There is little difference between this and other theories.

4.3 Free vibration of homogenous and FG plates resting of variables elastic foundations

In order to verify the effectiveness of the work in progress, here in this part of the results, the free vibrations of the *P*-FG plates based on variable elastic foundations are examined with different study parameters such as the power law index (p), the thickness-to-length ratio (h/a) and the width ratio on length (a/b), respectively. This study is based on a comparison between different types of variable elastic foundations. Figs. 2, 3 and 4 show the comparison of the effect of the elastic foundation (linear, parabolic, sinusoidal and non-variable module) on the vibratory behavior of FG plates. In Fig. 2, non-dimensional natural frequencies in terms of (p) decrease with the increase of effect elastic foundations, as well as Figs. 3-4, and the no dimensional natural frequencies ,in terms of (a/h) and (a/b), increase with increasing effect of the elastic foundation. From Figs. 5, 6 and 7, can be observed the influence of parameter z of variables foundations on the frequencies of the FG plates in terms the index (p), (h/a) and (a/b). It can be seen that the frequencies increase with increasing the parameter z. Finally, the current work of this part has proved its accuracy and efficiency in predicting the dynamic behavior of FGM plates based on various types of variable elastic foundations.

5. Conclusions

Free vibration of functionally graded isotropic plates resting on variables elastic foundations are studied using shear deformation theory. The present theory have only four variables which means are able to reduced time of calculate. The elastic foundations are supposed with two parameters; The Winkler modulus is assumed to be variable in this current study (linear, parabolic and sinusoidal) through length of FGM plates and Pasternak shear layer is to be constant. The equations of motions for FG plates are derived employing principle of Hamilton. Natural frequencies of FG plates resting two parameter foundations are resolved using exact solutions approach .The effect of variables elastic foundation on dynamic behavior of homogeneous and FG plates was investigated in terms of proprieties of materials ,thickness to length ratios and side to length ratio (a/b) with succeed. An improvement of the present formulation will be considered in the future work to consider other type of materials (Panda 2016, Draiche *et al*. 2016, 2019, Hirwani *et al*. 2016, Mehar *et al*. 2016, Chikh *et al*. 2017, Fadoun *et al*. 2017, Karami *et al*. 2017, Hirwani *et al*. 2017a,b,c, Mehar *et al*. 2017a,b, Bellifa *et al*. 2017b, Panjehpour *et al*. 2018, Youcef *et al*. 2018, Hirwani and Panda 2018, Behera and Kumari 2018, Nebab *et al*. 2018, Dash *et al*. 2018b, Cherif *et al*. 2018, Shahadat *et al*. 2018, Karami *et al*. 2018d, Bouadi *et al*. 2018, Ayat *et al*. 2018, Bakhadda *et al*. 2018, Hirwani *et al*. 2018a,b, Sharma *et al*. 2018a,b, Bisen *et al*. 2018, Semmah *et al*. 2019, Benmansour *et al*. 2019, Hussain and Naeem 2019, Tlidji *et al*. 2019, Karami *et al*. 2019a,b,c, Hirwani and Panda 2019a,b, Draoui *et al*. 2019).

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