Earthquakes and Structures, *Vol. 16, No. 5 (2019) 601-609*
DOI: https://doi.org/10.12989/eas.2019.16.5.601

Effect of distribution shape of the porosity on the interfacial stresses of the FGM beam strengthened with FRP plate

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(Received August 20, 2018, Revised March 18, 2019, Accepted April 1, 2019)

Abstract. The effect of the porosity and its distribution shape on the normal and shear interfacial stresses of the FGM beam strengthened with FRP plate subjected to a uniformly distributed load are investigated analytically in the present paper. Basically, the governing equations of FGM beams with porosity strengthened with composite plates are identical to the ones without porosity. Nonetheless, when the effect of the porosity and its distribution shape are taken into account, the rule of mixture was reformulated to assess the material characteristics with the porosity phases and its distribution shape. This work discusses the influence of the gradient index, the porosity and its distribution shape on the interfacial stresses.

Keywords: interfacial stresses; FGM beam; FRP composites; porosity

1. Introduction

Functionally graded materials (FGMs) are composed of two or more elated materials. It is one of the most functional forms of composite structures developed by the composite industry. It has attained broad acceptance in aerospace and many other industries and it is widely employed in aircraft and space vehicles, ships, boats, cargo containers, and residential constructions. The technique of grading ceramics along with metals initiated by the Japanese material scientist in Sendai has marked the beginning of exploring the possibility of using FGMs for various structural applications (Reddy 2000).

Damage of various types often initiates and propagates in beams made of functionally graded materials (FGM) during their service life (Abualnour 2018, Attia 2018, Benchora 2018, Benferhat 2016, Tahar 2016b, Adim 2016, Attia 2018, Benaouda 2017, Benoun 2016, Bellifa 2017, Bousahla 2016, Bouafia 2017, Bounouara 2016, Bouadi 2018, Boukhari 2016, ElHaina 2017, Hachemi 2017, Karami 2018, Belabed 2018, Hassaine Daouadji 2012, Benhenni 2018, Belkacem 2016a, Tahar 2016d, Rabia 2016b, Bouhadra 2018). The presence of any damage results in the degradation of the mechanical properties of the beams. To insure structural integrity, damaged beams should be replaced or repaired. However, in many cases the replacement of these beams is impossible or costly. Consequently, composite repair has developed into an important technology in order to restore the strength, stiffness and durability of damaged beams (Abdelhak 2016, Adim 2018, Ashraf 2018, Kaci 2018, Khatir 2017, Menasria 2017, Moukhtari 2017, Mouffoki 2017, Tounsi 2013, Yazid 2018, Yousfi 2018, Ziani 2018, Awad 2018, Khelifa 2018, Hadji 2015, Hachemi 2017, Belkacem 2016b, Zohra 2016, Rabia 2016a, Tahar 2016a, Tahar 2016e, Benferhat 2015, Fourn 2018, Fu 2018).

The externally bonded plate reinforcement is an effective method of repairing cracked and damaged beams. The success of the technique relies heavily on the physical properties of the material used to attach the new reinforcement and the long-term durability of the reinforcement material. However, both tests and theoretical work have shown that the strength of the adhesive joint is governed by interfacial stress concentrations which occur due to the mismatch of strains in the beam and the plate across the adhesive layer (Meradjah 2010, Benferhat 2018, Tahar 2016c, Hassaine Daouadji 2017).

During the last two decade there has been a considerable research reports on the interfacial stresses of the beam strengthened with FRP plate. Tounsi (2007, 2008) developed an analytical method to predict the distributions of interfacial stress in concrete beams strengthened by composite plates including the variation in FRP plate fibre orientation (Benyoucef 2006, Bouakaz 2014, Tahar 2012, Chadad 2018, El Mahi2014, Guenaneche 2014, Hassaine Daouadji 2013, Rabahi 2018, Jian 2007, Tayeb 2018, Bensatallah 2018, Jian 2010, Tounsi 2006, Smith 2001, Shen 2001, Wensu 2018, Xu 2018, Yuan 2018, Yuan 2019, Kongian 2018). Yang (2007, 2009) gives a new analytical solution for the interfacial stresses in plated beams subjected to arbitrary mechanical and thermal loads which are symmetrically positioned about the mid-span, The solution is represented by Fourier series and is based on the minimisation of complementary energy. The new solution takes into consideration the non-uniform stress distribution in the adhesive layer and the stress-free boundary condition at the ends of the plate. Rabahi (2016) presents an improved solution for interfacial stresses in a concrete beam bonded with the FRP plate by including the effect of the adherend

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ISSN: 2092-7614 (Print), 2092-7622 (Online)

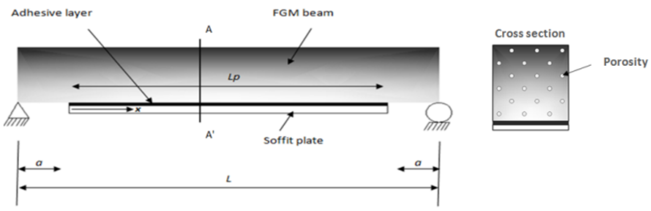


Fig. 1 Simply supported beam strengthened with bonded plate

shear deformations. The analysis is based on the deformation compatibility approach where both the shear and normal stresses are assumed to be invariant across the adhesive layer thickness. Meradjah (2010) presented a method for determining the elastic shear and peel stresses in an adhesive joint between a strengthening plate and a functionally graded beam (FGB). The beam is assumed to be isotropic with a constant Poisson's ratio and exponentially-varying elastic modulus through the beam thickness. Benachour (2008) presented a method to calculate the closed form solutions of the interfacial stresses in steel beams strengthened by prestressed bonded FRP plates.

However, in FGM fabrication, micro voids or porosities can occur within the materials during the process of sintering. This is because of the large difference in solidification temperatures between material constituents (Zhu *et al*. 2001). Wattanasakulpong (2012, 2014) also gave the discussion on porosities happening inside FGM samples fabricated by a multi-step sequential infiltration technique. Therefore, it is important to take into account the porosity effect and its distribution shape in the study of the interfacial stresses of the FGM beam strengthened with FRP plate.

The objective of this study is to present the effect of the porosity and its distribution shape on the normal and shear interfacial stresses of the simply supported FGM beam with porosities strengthened with different type of the composite plate and subjected to a uniformly distributed load. The material properties of the functionally graded beams are assumed to vary continuously through the thickness of the beam, according to the power law distribution. the rule of mixture which was reformulated to assess the material characteristics with the porosity phases and its distribution shape.

2. Theoretical approach

The derivation of the new solution below is described in terms of adherends 1 and 2 (Fig. 1), where adherend 1 is the FGM beam with porosity and adherend 2 is the soffit plate. Adherend 2 can be either steel or FRP but not limited to these two.

- The following assumptions are made:
- The FGM, adhesive, and FRP materials behave elastically and linearly.
- No slip is allowed at the interface of the bond (i.e., there is a perfect bond at the adhesive-FGM interface and at the adhesive-plate interface).

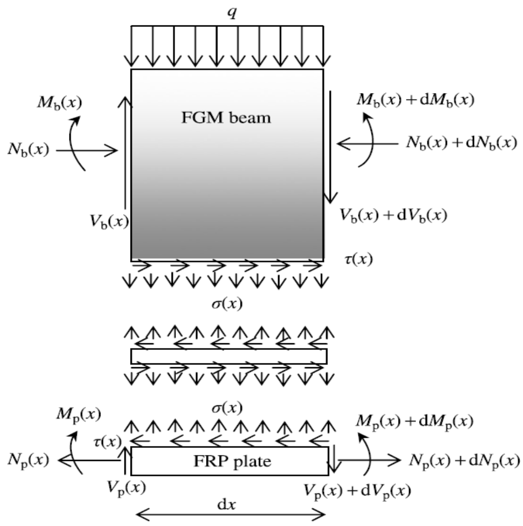


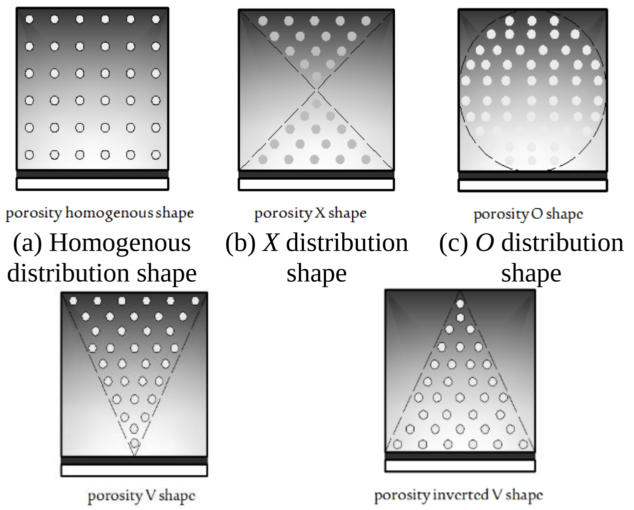
Fig. 2 forces in infinitesimal element of a soffit plated FGM beam

- Stresses in the adhesive layer do not change with the thickness.
- Les déformations des adhérences 1 et 2 sont dues aux moments de flexion et aux forces axiales.
- The shear stress analysis assumes that the curvatures in the beam and plate are equal (since this allows the shear stress and peel stress equations to be uncoupled). However, this assumption is not made in the peel stress solution. This assumption is used in several works e.g., Smith and Teng (2001), Tounsi (2007).

2.1 Properties of the FGM constituent materials

The functionally graded material (FGM) can be produced by continuously varying the constituents of multi-phase materials in a predetermined profile. The most distinct features of an FGM are the non-uniform microstructures with continuously graded macro properties. An FGM can be defined by the variation in the volume fractions. Most researchers use the power-law function or exponential function to describe the volume fractions. However, only a few studies used sigmoid function to describe the volume fractions. Therefore, FGM beams with sigmoid function will be considered in this paper in detail. Consider an elastic FGM plate of uniform thickness h, which is made of a ceramic and metal, is considered in this study. The material properties, Young's modulus and the Poisson's ratio, on the upper and lower surfaces are different but are preassigned according to the performance demands. However, the Young's modulus and the Poisson's ratio of the beams vary continuously only in the thickness direction (z-axis) i.e., $E=E(z)$, $\nu=\nu(z)$. Hassaine daouadji (2015) indicated that the effect of Poisson's ratio on the deformation is much less than that of Young's modulus. Thus, Poisson's ratio of the plate is assumed to be constant. However, the Young's modules in the thickness direction of the FGM beams vary with power-law functions (P-FGM) or with exponential functions (E-FGM).

The properties of the functionally graded material



(a) Homogenous distribution shape (b) *X* distribution shape (c) *O* distribution shape

(d) *V* distribution shape (e) Inverted *V* distribution shape

Fig. 3 Distribution shape of the porosity

(FGM) are assumed to vary according to the thickness direction of the beam and the rule of the mixture that has been reformulated to evaluate the characteristics of the materials with the phases and the shape distribution of porosity. Consider an imperfect FGM with a porosity volume fraction, α (α<<1), distributed evenly among the metal and ceramic, the modified rule of mixture proposed by Wattanasakul pong and Ungbhakorn (2014), the Young's modulus $E_1(z)$ equations of the imperfect FGM beam can be expressed as:

- Homogenous distribution shape of the porosity (Fig. 3(a))

$$E_1(z)=(e_c-e_m)((\frac{z}{t_1}+0.5))^p+e_m-(e_c+e_m)\frac{\alpha}{2} \tag{1a}$$

- *X* distribution shape of the porosity (Fig. 3(b))

$$E_1(z)=(e_c-e_m)((\frac{z}{t_1}+0.5))^p+e_m-(e_c+e_m)\frac{\alpha}{2}(2\frac{z}{t_1}) \tag{1b}$$

- Distribution shape of the porosity (Fig. 3(c))

$$E_1(z)=(e_c-e_m)((\frac{z}{t_1}+0.5))^p+e_m-(e_c+e_m)\frac{\alpha}{2}(1-2\frac{z}{t_1}) \tag{1c}$$

- *V* distribution shape of the porosity (Fig. 3(d))

$$E_1(z)=(e_c-e_m)((\frac{z}{t_1}+0.5))^p+e_m-(e_c+e_m)\frac{\alpha}{2}(\frac{1}{2}+\frac{z}{t_1}) \tag{1d}$$

- Inverted *V* distribution shape of the porosity (Fig. 3(e))

$$E_1(z)=(e_c-e_m)((\frac{z}{t_1}+0.5))^p+e_m-(e_c+e_m)\frac{\alpha}{2}(\frac{1}{2}-\frac{z}{t_1}) \tag{1e}$$

where $E_1(z)$ is the Young's modulus of the homogeneous beam; E_m denote Young's modulus of the bottom (as metal) and top E_c (as ceramic) surfaces of the FGM beam, respectively; $E_1(z)$ is Young's modulus of the homogeneous FGM beam; and p is a parameter that indicates the material variation through the plate thickness. For the power law distribution P-FGM, the Young's modulus is given as Hassaine daouadji (2015):

The linear constitutive relations of a FG plate can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}\frac{E(z)}{1-\nu^2} & \frac{\nu E(z)}{1-\nu^2} & 0 & 0 & 0\\ \frac{\nu E(z)}{1-\nu^2} & \frac{E(z)}{1-\nu^2} & 0 & 0 & 0\\ 0 & 0 & \frac{E(z)}{2(1+\nu)} & 0 & 0\\ 0 & 0 & 0 & \frac{E(z)}{2(1+\nu)} & 0\\ 0 & 0 & 0 & 0 & \frac{E(z)}{2(1+\nu)}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix} \tag{2}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively, and A_{ij}, D_{ij} are the plate stiffness, defined by

$$A_{ij}=\int_{-h/2}^{h/2}Q_{ij}dz \quad D_{ij}=\int_{-h/2}^{h/2}Q_{ij}z^2dz \tag{3}$$

where A'_{11}, D'_{11} are defined as

$$A'_{11}=\frac{A_{22}}{A_{11}A_{22}-A_{12}^2} \quad D'_{11}=\frac{D_{22}}{D_{11}D_{22}-D_{12}^2} \tag{4}$$

2.2 Shear stress distribution along the FGM-FRP interface

The governing differential equation for the interfacial shear stress (Hassaine Daouadji 2016) is expressed as

$$\frac{d^2\tau(x)}{dx^2}-K_1\left(A'_{11}+\frac{b_2}{E_1A_1}+\frac{(y_1+t_2/2)(y_1+t_a+t_2/2)}{E_1I_1D'_{11}+b_2}b_2D'_{11}\right)\tau(x)+\frac{\left(\frac{(y_1+t_2/2)}{E_1I_1D'_{11}+b_2}D'_{11}\right)}{\frac{t_a}{G_a}+\frac{t_1}{4G_1}}V_T(x)=0 \tag{5}$$

For simplicity, the general solutions presented below are limited to loading which is either concentrated or uniformly distributed over part or the whole span of the beam, or both. For such loading, $d^2V_T(x)/dx^2=0$, and the general solution to Eq. (5) is given by

$$\tau(x)=B_1\cosh(\lambda x)+B_2\sinh(\lambda x)+m_1V_T(x) \tag{6}$$

Where

$$\lambda^2=\frac{\left(A'_{11}+\frac{b_2}{E_1A_1}+\frac{(y_1+t_2/2)(y_1+t_a+t_2/2)}{E_1I_1D'_{11}+b_2}b_2D'_{11}\right)}{\frac{t_a}{G_a}+\frac{t_1}{4G_1}} \tag{7}$$

$$m_1=\frac{1}{\lambda^2(\frac{t_a}{G_a}+\frac{t_1}{4G_1})}\left(\frac{(y_1+t_2/2)}{E_1I_1D'_{11}+b_2}D'_{11}\right) \tag{8}$$

And B_1 and B_2 are constant coefficients determined from the boundary conditions. In the present study, a simply supported beam has been investigated which is subjected to a uniformly distributed load (Fig. 1). The interfacial shear stress for this uniformly distributed load at any point is written as (Hassaine Daouadji 2016)

$$\tau(x)=\left[\frac{m_2a}{2}(L-a)-m_1\right]\frac{qe^{-\lambda x}}{\lambda}+m_1q\left(\frac{L}{2}-a-x\right) \quad 0\le x\le L_P \tag{9}$$

Where q is the uniformly distributed load and x; a; L and L_p are defined in Fig. 1.

2.3 Normal stress distribution along the FGM - FRP interface

The following governing differential equation for the interfacial normal stress (Hassaine Daouadji 2016)

$$\frac{d^4\sigma_n(x)}{dx^4} + K_n\left(D'_{11} + \frac{b_2}{E_1 I_1}\right)\sigma_n(x) - K_n\left(D'_{11}\frac{t_2}{2} - \frac{y_1 b_2}{E_1 I_1}\right)\frac{d\tau(x)}{dx} + \frac{qK_n}{E_1 I_1} = 0 \tag{10}$$

The general solution to this fourth-order differential equation is

$$\begin{aligned}\sigma_n(x) &= e^{-\beta x}\left[C_1\cos(\beta x) + C_2\sin(\beta x)\right] + \\ &\quad e^{\beta x}\left[C_3\cos(\beta x) + C_4\sin(\beta x)\right] - n_1\frac{d\tau(x)}{dx} - n_2 q\end{aligned} \tag{11}$$

For large values of x it is assumed that the normal stress approaches zero and, as a result, $C_3=C_4=0$. The general solution therefore becomes

$$\sigma_n(x) = e^{-\beta x}\left[C_1\cos(\beta x) + C_2\sin(\beta x)\right] - n_1\frac{d\tau(x)}{dx} - n_2 q \tag{12}$$

Where

$$\beta = \sqrt[4]{\frac{K_n}{4}\left(D'_{11} + \frac{b_2}{E_1 I_1}\right)} \tag{13}$$

$$n_1 = \left(\frac{y_1 b_2 - D'_{11}E_1 I_1 t_2/2}{D'_{11}E_1 I_1 + b_2}\right) \tag{14}$$

$$n_2 = \frac{1}{D'_{11}E_1 I_1 + b_2} \tag{15}$$

As is described by Hassaine Daouadji (2016), the constants C_1 and C_2 in Eq. (12) are determined using the appropriate boundary conditions and they are written as follows

$$\begin{aligned}C_1 &= \frac{K_n}{2\beta^3 E_1 I_1}\left[V_T(0) + \beta M_T(0)\right] - \frac{n_3}{2\beta^3}\tau(0) \\ &\quad + \frac{n_1}{2\beta^3}\left(\frac{d^4\tau(0)}{dx^4} + \beta\frac{d^3\tau(0)}{dx^3}\right)\end{aligned} \tag{16}$$

$$C_2 = -\frac{K_n}{2\beta^2 E_1 I_1}M_T(0) - \frac{n_1}{2\beta^2}\frac{d^3\tau(0)}{dx^3} \tag{17}$$

$$n_3 = b_2 K_n\left(\frac{y_1}{E_1 I_1} - \frac{D'_{11}t_2}{2b_2}\right) \tag{18}$$

The above expressions for the constants C_1 and C_2 has been left in terms of the bending moment $M_T(0)$ and shear force $V_T(0)$ at the end of the soffit plate. With the constants C_1 and C_2 determined, the interfacial normal stress can then be found using Eq. (12).

3. Results and discussions

Material used: The material used for the present studies is an RC beam and FGM beam bonded with a GFRP, CFRP and Steel plate. The beam is simply supported and subjected to a uniformly distributed load. A summary of the geometric and material properties is given in Table 1.

Table 1 Geometric and material properties

Materials	Young's modulus E (GPa)	Shear modulus G (GPa)	Width (mm)	Depth (mm)
Concrete	30	/	b_1=200	t_1=300
Ceramique	380	/	b_1=200	t_1=300
Metal	70	/	b_1=200	t_1=300
CFRP plate (bonded FGM beam)	140	5	b_2=200	t_2=4
CFRP plate (bonded FGM beam)	50	5	b_2=200	t_2=4
Steel plate (bonded FGM beam)	200	/	b_2=200	t_2=4
Adhesive layer FGM beam	3	/	b_2=200	t_a=2

Table 2 Comparison of the interfacial stresses for different type of the reinforced beam reinforced with GFRP, CFRP and steel plates under uniformed distributed load

Theory		Shear stress (MPa)	Normal stress (MPa)
FGM beam bonded with CFRP plate			
Tounsi (2007)	RC beam	1.0898	0.831
Hassaine Daouadji (2016)	RC beam	1.0880	0.824
Present Model	RC beam	1.0885	0.826
	Steel beam	1.477	1.055
	Ceramic beam (P=0)	0.16306	0.11574
	FGM beam (P=5)	0.46408	0.33880
	Aluminium beam (P=∞)	0.62786	0.46422
FGM beam bonded with CFRP plate			
Tounsi (2007)	RC beam	1.799	1.082
Hassaine Daouadji (2016)	RC beam	1.788	1.073
Present Model	RC beam	1.794	1.078
	Steel beam	2.385	1.355
	Ceramic beam (P=0)	0.28217	0.15993
	FGM beam (P=5)	0.79577	0.46150
	Aluminium beam (P=∞)	1.0686	0.62638
FGM beam bonded with Steel plate			
Tounsi (2007)	RC beam	2.127	1.179
Hassaine Daouadji (2016)	RC beam	2.118	1.171
Present Model	RC beam	2.120	1.175
	Steel beam	2.875	1.499
	Ceramic beam (P=0)	0.35719	0.18606
	FGM beam (P=5)	0.98215	0.52385
	Aluminium beam (P=∞)	1.3034	0.70290

Comparison studies: The present simple solution is compared, in this section, with some approximate solutions available in the literature. These include Tounsi (2007), Hassaine Daouadji (2016) solutions uniformly distributed loads.

A comparison of the interfacial shear and normal stresses from the different existing closed - form solutions and the present solution is undertaken in this section. An undamaged beams bonded with CFRP, GFRP and steel plate soffit plate is considered. The beam is simply supported and subjected to a uniformly distributed load. A summary of the geometric and material properties is given in Table 1.

Table 3 Effect of the porosity on the interfacial stresses of the FGM beam reinforced with GFRP, CFRP and steel plates under uniformed distributed load

FGM beam	Porosity	FGRP plate (bonded FGM beam)		CFRP plate (bonded FGM beam)		Steelplate (bonded FGM beam)	
		Shear stress (MPa)	Normal stress (MPa)	Shear stress (MPa)	Normal stress (MPa)	Shear stress (MPa)	Normal stress (MPa)
FGM beam P=5	a=0	0.46408	0.33880	0.79577	0.46150	0.98215	0.52385
	a=0.1	0.50627	0.37085	0.86660	0.50401	1.0663	0.57040
	a=0.2	0.50828	0.37238	0.86997	0.50602	1.0703	0.57263
FGM beam P=10	a=0	0.61219	0.45211	0.61055	0.61055	1.2733	0.68594
	a=0.1	0.76782	0.57329	0.76654	0.76654	1.5665	0.85216
	a=0.2	0.95948	0.72492	0.95573	0.95573	1.9081	10.498

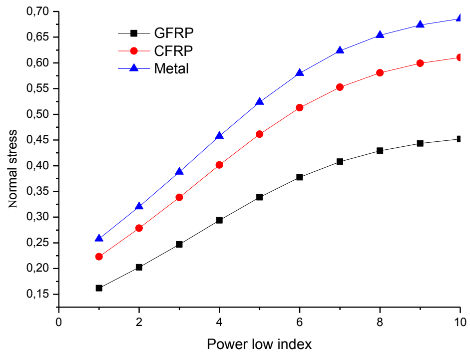


Fig. 4 Effect of the power law index on the normal stress of the FGM beam reinforced with GFRP, CFRP and steel plates under uniformed distributed load

The results of the peak interfacial shear and normal stresses are given in Table 2 for the beams strengthened by bonding CFRP, GFRP and steel plate. As it can be seen from the results, the peak interfacial stresses assessed by the present theory are smaller compared to those given by Tounsi (2007), Hassaine Daouadji (2016) solutions. This implies that adherend shear deformation is an important factor influencing the adhesive interfacial stresses distribution and it to be noted that the stresses are more important when the beam is reinforced by CFRP or steel plate and becomes very weak when the beam is from FGM or Aluminum.

3.1 Parametric studies

3.1.1 Effect of plate stiffness on interfacial stress:

The effect of the porosity and the power law index on the normal and shear stresses of the FGM beam reinforced with CFRP plate are presented in Table 3. The results show that the effect of power law index (P=5 and P=10) on the stresses is very interesting. Increasing value of porosity coefficient causes an increase in the normal and shear stresses.

Figs. 4 and 5 shows the effect of the power law index on the normal and shear interfacial stresses of the FGM beam reinforced with GFRP, CFRP and steel plates under uniformed distributed load, respectively. It's clear that the interfacial shear stresses increase with the increases of the volume fraction exponent of the FG beam.

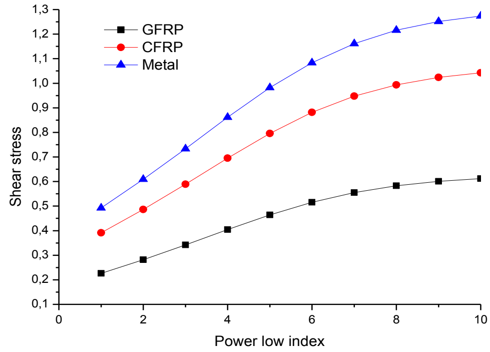


Fig. 5 Effect of the power law index on the shear stress of the FGM beam reinforced with GFRP, CFRP and steel plates under uniformed distributed load

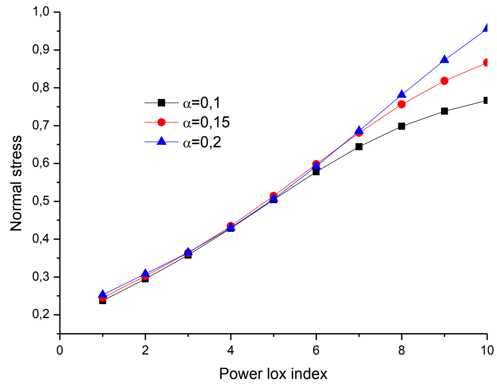


Fig. 6 Effect of the volume fraction of the porosity on the normal stress of the FGM beam reinforced with CFRP plate under uniformed distributed load

Table 4 Effect of the distribution shape of the porosity on the interfacial stresses of the FGM beam reinforced with CFRP plate under uniformed distributed load

FGM beam	Distribution shape of the porosity	Shear stress (MPa)	Normal stress (MPa)	Shear stress (MPa)	Normal stress (MPa)	Shear stress (MPa)	Normal stress (MPa)
		a=0.1		a=0.15		a=0.2	
FGM beam P=5	Homogeneous	0.86660	0.50401	0.88328	0.51407	0.86997	0.50602
	'O' shape	0.81508	0.47307	0.79646	0.46192	0.74779	0.43282
	'X' shape	0.78321	0.45399	0.76791	0.44483	0.73990	0.42811
	'V' shape	0.82273	0.47766	0.82129	0.47680	0.79935	0.46365
	Inverted 'V' shpae	0.77638	0.44989	0.74632	0.43194	0.69572	0.40179
FGM beam P=10	Homogeneous	1.2965	0.76654	1.4570	0.86635	1.5991	0.95573
	'O' shape	1.1962	0.70457	1.2467	0.73573	1.2286	0.72454
	'X' shape	1.0945	0.64221	1.1288	0.66322	1.1599	0.68229
	'V' shape	1.1862	0.69840	1.2699	0.75003	1.3406	0.79381
	Inverted 'V' shpae	1.1030	0.64743	1.1108	0.65222	1.0771	0.63159

3.1.2 Effect of the distribution shape of the porosity

The influence of the distribution form of the porosity on the interfacial stresses of the FGM beam reinforced with CFRP plate is presented in the Table 4 for different values of the power law index (P=5 and P=10) and volume fraction

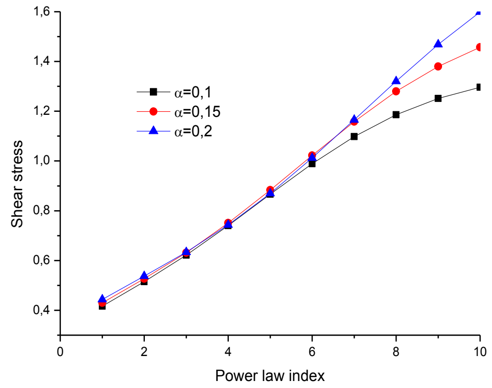


Fig. 7 Effect of the volume fraction of the porosity on the shear stress of the FGM beam reinforced with CFRP plate under uniformed distributed load

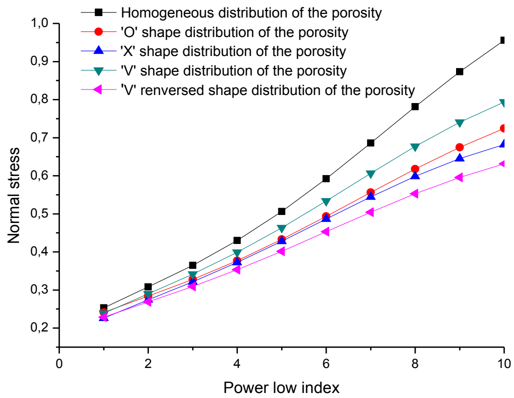


Fig. 8 Effect of the distribution shape of the porosity on the normal stress of the FGM beam reinforced with CFRP plate under uniformed distributed load

of the porosity (a=0, a=0.1 and a=0.2). It can be seen that the interfacial stresses decreases when the distribution of the porosity is not homogeneous in the FGM beam.

The effect of the volume fraction of the porosity on the normal and shear interfacial shear stresses of the FGM beam reinforced with CFRP plate under uniformed distributed load are presented in Figs. 6 and 7, respectively. The figures show that increasing value of the gradient index increases the effect of the porosity on the interfacial stresses of the FGM beam.

Figs. 8 and 9 contains the plots of the normal and shear interfacial stresses of the FGM beam reinforced with CFRP plate under uniformed distributed load for different porosity distribution shapes, respectively. As it can be seen, increasing value of the gradient index increases the effect of the porosity distribution shapes on the interfacial stresses of the FGM beam.

4. Conclusions

In this article, we presented the effect of distribution shape of the porosity on the interfacial stresses of the functionally grade materials FGM beam strengthened with fiber reinforced polymer FRP plate. Such interfacial stresses

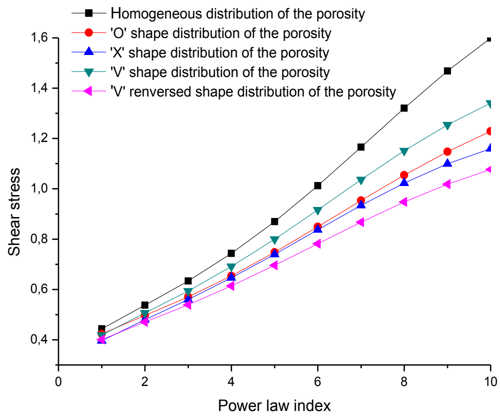


Fig. 9 Effect of the distribution shape of the porosity on the shear stress of the FGM beam reinforced with CFRP plate under uniformed distributed load

provide the basis for understanding debonding failures in such FGM beams and for the development of suitable design rules. It is shown that the in homogeneities play an important role in interfacial stresses. In this analysis, The obtained solution could serve as a basis for establishing simplified FGM theories or as a benchmark result to assess other approximate methodologies. Compared with the existing solutions, the results show that there exists a high concentration of shear and peeling stress at the ends of the FRP plate. We can conclude that, This research is helpful for the understanding on mechanical behavior of the interface and design of the FGM-FRP hybrid structures. The new solution is general in nature and may be applicable to all kinds of materials.

Acknowledgments

This research was supported by the French Ministry of Foreign Affairs and International Development (MAEDI) and Ministry of National Education, Higher Education and Research (MENESR) and by the Algerian Ministry of Higher Education and Scientific Research (MESRS) under Grant No. PHC Tassili 17MDU992. Their support is greatly appreciated.

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