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Free vibration response of functionally graded Porous plates using a higher-order Shear and normal deformation theory

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Abstract. In this work, a new analytical approach using a theory of a high order hyperbolic shear deformation theory (HSDT) has been developed to study the free vibration of plates of functionally graduated material (FGM). This theory takes into account the effect of stretching the thickness. In contrast to other conventional shear deformation theories, the present work includes a new displacement field that introduces indeterminate integral variables. During the manufacturing process of these plates defects can appear as porosity. The latter can question and modify the global behavior of such plates. The materials constituting the plate are assumed to be gradually variable in the direction of height according to a simple power law distribution in terms of the volume fractions of the constituents. The motion equations are derived by the Hamilton principle. Analytical solutions for free vibration analysis are obtained for simply supported plates. The effects of stretching, the porosity parameter, the power law index and the length / thickness ratio on the fundamental frequencies of the FGM plates are studied in detail.

Keywords: functionally graded plate; shear deformation theory; free vibration; porosity; stretching effect

1. Introduction

Functionally Graded Materials (FGM) are a new class of composite materials whose microstructure and composition gradually and continuously vary with position to optimize the mechanical and thermal performance of the structure. They are considered intelligent materials whose desired functions are integrated, from the design stage, into the very heart of the material. At each interface, the material is chosen according to specific applications and environmental loads. These materials have multiple advantages that can make them attractive from the point of view of their application potential. It can be improved rigidity, fatigue strength, corrosion resistance or thermal conductivity in addition to having a gradation of properties to increase or modulate performance such as reducing local stresses or improving heat transfer. This new concept marks the beginning of a revolution in the fields of materials science and mechanics. The reason for the increasing use of FGMs in a variety of aerospace, automotive, civil engineering, and mechanical engineering structures is that their material properties can be tailored to different applications and working environments (Qian and Batra 2005, Yaghoobi *et al.* 2011, Bachir Bouiadjra *et al.* 2013, Bouderba *et al.* 2013, Yaghoobi and Torabi 2013a, Tounsi *et al.* 2013, Yaghoobi and Fereidoon 2014, Yaghoobi *et al.* 2014, Yaghoobi *et al.* 2015, Attia *et al.* 2015, Hamidi *et al.* 2015, Larbi Chaht *et al.* 2015, Darılmaz 2015, Ebrahimi and Dashti 2015, Bouguenina *et al.* 2015, Akbaş 2015, Arefi 2015, Pradhan and Chakraverty 2015, Kar and Panda 2015ab, Beldjelili *et al.* 2016, Ebrahimi and Habibi 2016, Hadji *et al.* 2016, Moradi-Dastjerdi 2016, Laoufi *et al.* 2016, Bousahla *et al.* 2016, Ebrahimi and Salari 2016, Trinh *et al.* 2016, Kar and Panda 2016, El-Haina *et al.* 2017, Kar and Panda 2017, Attia *et al.* 2018, Karami *et al.* 2019a,b).

Numerous research studies have dealt with the mechanical behavior of the plates functionally graduated, the demonstration of the effect of the transverse shear and normal deformation in the study of the vibration behavior of the FGM plates makes it possible to describe with a good precision the field's stresses and deformations induced through their thickness. Reissner (1945), Cranch and Adler (1956), Ambartsumyan (1969), Bresse (1859) were the pioneer investigators in studying the different behavior of structures made with isotropic materials under different stresses. With the development of the FGM concept, many works have been studied in literature. Reddy (2000) is one of the first to analyzed the static behavior of FGM rectangular plates based on his plate theory. Cheng and Batra (2000) have found correspondence between eigen values of membranes and functionally graded simply supported polygonal plate. The same membrane analogy was later applied to FGM plate and shell analysis based on a third order theory of plates by Reddy (2002). Vel and Batra (2004) has come closer to real behavior of structure

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by studying free vibration of FGM rectangular plates with three-dimensional solution. Zenkour (2006) presented a generalized shear deformation theory in which function across the thickness. Woo *et al.* (2006) studied the non-linear free vibration behavior of plates made of FGMs using the Von Karman theory for large transverse deflection. Ait atmane *et al*. (2010) proposed a new model of shear deformation to analyze the free vibration of FGM plates rested on elastic foundation. Also, Arefi and Rahimi (2011) investigated the nonlinear response of a FG square plate with two smart layers as a sensor and actuator under pressure. Arefi (2013) analyzed the nonlinear thermo-elastic behavior of thick-walled functionally graded piezoelectric cylinder. Sobhy (2013) studied the vibration and buckling behavior of exponentially graded material sandwich plate resting on elastic foundations under various boundary conditions. The first-order shear deformation theory (FSDT), including the effects of transverse shear deformation, was employed by some researches to analyze buckling behavior of moderately thick FGM plates (Yaghoobi and Yaghoobi 2013, Bouazza *et al.* 2010). By using an efficient and simple refined theory, Ait Amar Meziane *et al*. (2014) studied the buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Hebali *et al.* (2014) proposed a new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of FG plates. Bousahla *et al*. (2014) presented a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Zidi *et al.* (2014) employed a four variable refined plate theory for bending analysis of FG plates under hygro-thermo-mechanical loading. A new simple shear and normal deformations theory was developed by Bourada *et al*. (2015) for the analysis of the behavior of functionally graded beams. Yahia *et al.* (2015) studied the wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories. Belabed *et al.* (2014) used a hyperbolic function based higher-order shear deformation theory to analysis the vibration characteristics of FGM plate. Bennai *et al.* (2015) proposed a novel higher-order shear and normal deformation theory for the study of vibration and stability for FG sandwich beams. Mahi *et al.* (2015) developed a novel hyperbolic shear deformation model for static and dynamic analysis of isotropic, functionally graded, sandwich and laminated composite plates. Belkorissat *et al.* (2015) studied the dynamic properties of FG nanoscale plates using a novel nonlocal refined four variable theory. Recently, Tounsi *et al.* (2016) proposed a new 3-unknowns non-polynomial plate theory for buckling and vibration of FG sandwich plate. Bouderba *et al*. (2016) studied the thermal stability of FG sandwich plates using a simple shear deformation theory. Bellifa *et al.* (2016) presented static bending and dynamic analysis of FG plates using a simple shear deformation theory and the concept the neutral surface position. Houari *et al.* (2016) presented a new simple three-unknown sinusoidal shear deformation theory for FG plates. Draiche *et al.* (2016) used a refined theory with stretching effect for the flexure analysis of laminated composite plates. Bennoun *et al*. (2016) studied the vibration response of FG sandwich plates using a novel five variable refined plate theory. Using higher-order equivalent single-layer theory, Katariya *et al.* (2017) studied the nonlinear Eigen frequency of laminated curved sandwich structure. Bellifa *et al.* (2017a) proposed a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Katariya *et al.* (2018) used HSDT, FEM to study the bending, and vibration of skew sandwich plate. A study of the dynamic response of functionally graduated plates based on elastic foundations by a high order theory was realized by Nebab *et al.* (2019). Other works on shear deformation theories such as HSDT and FSDT can be documented in references (Kar and Panda 2013, Panda and Katariya 2015, Abdelaziz *et al.* 2017, Zidi *et al.* 2017, Bouafia *et al.* 2017, Sekkal *et al.* 2017a, b, Karami *et al.* 2017, Katariya and Panda 2018, Zine *et al.* 2018, Mehar *et al.* 2018, Abualnour *et al.* 2018, Dash *et al.* 2018, Mokhtar *et al.* 2018, Karami *et al.* 2018abce, Bouadi *et al.* 2018, Yazid *et al.* 2018, Kadari *et al.* 2018, Sahoo *et al.* 2018, Karami *et al.* 2019c, Bourada *et al.* 2019, Boukhlif *et al.* 2019).

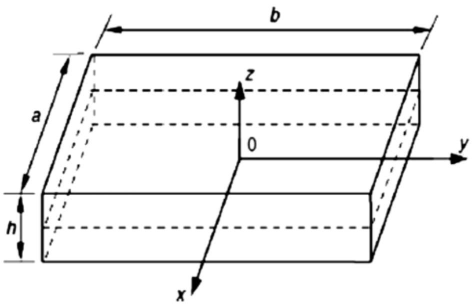


Fig. 1 coordinates and geometry of functionally graded plate

In the literature, studies of the porosity effect in the FG structures are as follows; Wattanasakulpong and Ungbhakorn (2014) examined dynamics of porous functionally graded beams. Ait Atmane *et al.* (2015) examined dynamics of FG porous beams with different beams theories. Jahwari and Naguib (2016) investigated FG porous plates with different plate theories and cellular distribution model. Recently, Mouaici *et al.* (2016) proposed an analytical solution for the vibration of FGM plates with porosities. Ait Atmane *et al.* (2017) is study the effect of stretching the thickness and porosity on the mechanical response of a FG beam resting on elastic foundations. Akbas SD (2017) studied the thermal effects on the vibratory behavior of FG beams with porosity. Yousfi *et al.* (2018) used a shear deformation theory with four variables for the analysis of the vibratory behavior of porous FGM plates.

The objective of this study is to develop a theory of high order hyperbolic shear strain (HSDT) to study the effect of normal deformation and porosity on the vibratory behavior of FG plates. Current theory has a displacement field that introduces indeterminate integral variables. The free vibration motion equations in the FG plate are obtained using the Hamilton principle, whose effects of shear deformation and inertia rotation are taken into account. To solve the problem, the Navier solution is also used. In the end, the numerical results of the current theory are compared to those predicted by the theory used by Mouaici *et al.* (2016), and the theory proposed by Belabed *et al.* (2014). The influence of the stretching effect, volume fraction index and porosity on the free vibration of functionally graduated plates is clearly discussed.

2. Theoretical formulation

2.1 Material properties

The FG plate is composed by a mixture of ceramic and metal components whose material characteristics change across the plate thickness with a power law distribution of the volume fractions of the constituents of the two materials as (Yaghoobi and Torabi 2013b, Kar and Panda 2014, Zemri *et al.* 2015, Ahouel *et al.* 2016, Bellifa *et al.* 2017b, Ayache *et al.* 2018)

$$P(z) = P_m + (P_c - P_m)\left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{1}$$

Where $\boldsymbol{P}$ denotes the effective material characteristic such as Young's modulus $\boldsymbol{E}$ and mass density $\boldsymbol{\rho}$ subscripts $\boldsymbol{m}$ and $\boldsymbol{c}$ denote the metallic and ceramic components, respectively; and $\boldsymbol{p}$ is the power law exponent. The value of $\boldsymbol{p}$ equal to zero indicates a fully ceramic plate, whereas infinite $\boldsymbol{p}$ represents a fully metallic plate. Since the influences of the variation of Poisson's ratio $\boldsymbol{\upsilon}$ on the behavior of FG, plates are very small (Yang *et al.* 2005, Kitipornchai *et al.* 2006), it is supposed to be constant for convenience Now, the total volume fraction of the metal and ceramic is $V_m+V_c=1$, and the power law of volume fraction of the ceramic is described as

$$V_C = \left(\frac{z}{h} + \frac{1}{2}\right)^p \tag{2}$$

The properties of the material with porosity should follow the power law in the present study and are expressed by the expression given by Ankit Gupta (2017)

$$P = P_c\left(V_c - \log\left(1 + \frac{\lambda}{2}\right) + P_m\left(V_m - \log\left(1 + \frac{\lambda}{2}\right)\right.\right. \tag{3}$$

'$\boldsymbol{\lambda}$' is termed as porosity volume fraction ($\lambda<1$). $\lambda=0$ indicates the non-porous functionally graded plate. Thus, the Young's modulus ($\boldsymbol{E}$) and material density ($\boldsymbol{\rho}$) equations of the imperfect FGM plate can be expressed as:

$$E(z) = (E_c - E_m)\left(1 + \frac{h}{2z}\right)^p - \log\left(1 + \frac{\lambda}{2}\right)(E_c + E_m)\left(1 - \frac{2|z|}{h}\right) + E_m \tag{4}$$

$$\rho(z) = (\rho_c - \rho_m)\left(1 + \frac{h}{2z}\right)^p - \log\left(1 + \frac{\lambda}{2}\right)(\rho_c + \rho_m)\left(1 - \frac{2|z|}{h}\right) + \rho_m \tag{5}$$

2.2 Constitutive equations

For elastic and isotropic FGMs, the constitutive relations are given as follows

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{6}$$

Where $(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{xz})$ and $(\varepsilon_x, \varepsilon_y, \varepsilon_z, \gamma_{xy}, \gamma_{yz}, \gamma_{xz})$ are the stress and strain components, respectively. Using the material properties defined in Eq. (1), stiffness coefficients Q_{ij} can be given as

$$Q_{11} = Q_{22} = Q_{33} = \frac{E(z)}{1-\nu^2}\,;\quad Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}$$
$$Q_{12} = Q_{13} = Q_{23} = \frac{\nu E(z)}{1-\nu^2} \tag{7}$$

If $\varepsilon_z \neq 0$ (thickness stretching), then Q_{ij} are 3D elastic constants, given by

$$Q_{11} = Q_{22} = Q_{33} = \frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}$$
$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)} \tag{8}$$
$$Q_{12} = Q_{13} = Q_{23} = \frac{\nu E(z)}{(1-2\nu)(1+\nu)}$$

Based on the thick plate theory and including the effect of transverse normal stress (thickness stretching effect), the basic assumptions for the displacement field of the plate can be described as (Bourada *et al.* 2016, Fahsi *et al.* 2017, Khetir *et al.* 2017, Menasria *et al.* 2017, Benchohra *et al.* 2018, Fourn *et al.* 2018, Bourada *et al.* 2018, Younsi *et al.* 2018, Bouhadra *et al.* 2018, Zaoui *et al.* 2019, Bennai *et al.* 2019 and Meksi *et al.* 2019)

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\,dx \tag{9a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy \tag{9b}$$

$$w(x,y,z,t) = w_0(x,y,t) + w_0(x,y,t) + g(z)\varphi_z(x,y,t) \tag{9c}$$

The coefficients $\boldsymbol{k_1}$and $\boldsymbol{k_2}$ depends on the geometry. In this article, the shape function is considered based on the hyperbolic function given by Reissner (1975) as

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{10}$$

It can be observed that the kinematic in Eq. (9) uses only five unknowns (u_0; v_0; w_0; θ andφ_z). Nonzero strains of the five variable plate model are expressed as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix};$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \text{ and } \varepsilon_z = g'(z)\varepsilon_z^0 \tag{11}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix};\ \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix};$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx + k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix} \text{ and} \tag{12}$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}k_2\int\theta\,dy + \dfrac{\partial\varphi_z}{\partial y}\\ k_1\int\theta\,dx + \dfrac{\partial\varphi_z}{\partial x}\end{Bmatrix};\varepsilon_z^0 = \varphi_z$$

and

$$g(z) = \frac{df(z)}{dz} \tag{13}$$

It can be observed from Eq. (6) that the transverse shear strains (γ_{xz} , γ_{yz}) are equal to zero at the upper ($z=h/2$) and lower ($z=-h/2$) surfaces of the plate (Fig. 1). A shear correction coefficient is, hence, not required.

The integrals used in the above equations shall be resolved by a Navier type solution and can be written as follows

$$\begin{gathered}\frac{\partial}{\partial y}\int\theta\,dx = A'\frac{\partial^2\theta}{\partial x\partial y},\ \frac{\partial}{\partial x}\int\theta\,dy = B'\frac{\partial^2\theta}{\partial x\partial y}\\ \int\theta\,dx = A'\frac{\partial\theta}{\partial x},\ \int\theta\,dy = B'\frac{\partial\theta}{\partial y}\end{gathered} \tag{14}$$

Where the coefficients A' and B' are expressed according to the type of solution employed, in this case by using Navier method. Therefore, $\boldsymbol{A'}$ and $\boldsymbol{B'}$ are expressed as follows

$$A' = -\frac{1}{\kappa_1^{\,2}},\ B' = -\frac{1}{\kappa_2^{\,2}},\ k_1 = \kappa_1^{\,2},\ k_2 = \kappa_2^{\,2} \tag{15}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ defined in expression (28).

3.2 Equations of motion

To determine the equations of motion, we apply the principle of Hamilton (Meziane *et al.* 2014, Al-Basyouni *et al.* 2015, Bounouara *et al.* 2016, Benadouda *et al.* 2017, Hachemi *et al.* 2017, Benahmed *et al*. 2017, Besseghier *et al*. 2017, Mouffoki *et al.* 2017,Klouche *et al.* 2017, Bakhadda *et al.* 2018, Cherif *et al.* 2018, Yousfi *et al.* 2018, Youcef *et al*. 2018, Kaci *et al.* 2018, Tlidji *et al.* 2019, Semmah *et al.* 2019, Khiloun *et al.* 2019)

$$0 = \int_0^t (\delta U + \delta V - \delta K)\,dt \tag{16}$$

Where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the plate; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned}\delta U &= \int_V\left[\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_z\delta\varepsilon_z + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right]dV\\ &= \int_A\left[N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_z\delta\varepsilon_z^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b\right.\\ &\quad + M_{xy}^b\delta k_{xy}^b + M_x^s\delta k_x^s + M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + S_{yz}^s\delta\gamma_{yz}^0\\ &\quad \left.+ S_{xz}^s\delta\gamma_{xz}^0\right]dA = 0\end{aligned} \tag{17}$$

Where A is the top surface and the stress resultants N, M, and S are defined by

$$\begin{gathered}\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2}(1, z, f)\sigma_i\,dz\ \left(i = x, y, xy\right)\\ \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right)dz \text{ and } N_z = \int_{-h/2}^{h/2} g'(z)\sigma_z\,dz\end{gathered} \tag{18}$$

The variation of the external work can be expressed as

$$\delta V = -\int_A q\delta w_0\,dA + \int_A\left(\overline{N}\frac{\partial(w_0 + g_0\varphi_z)}{\partial x}\frac{\partial\delta(w_0 + g_0\varphi_z)}{\partial x}\right)dA \tag{19}$$

Where q and $\overline{N}$ are transverse and in-plane applied loads, respectively.

The variation of kinetic energy of the plate can be expressed as

$$\begin{aligned}\delta K &= \int_V\left[\dot{u}\,\delta\dot{u} + \dot{v}\,\delta\dot{v} + \dot{w}\,\delta\dot{w}\right]\rho(z)\,dV\\ &= \int_A\Bigg\{I_0\left(\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right) + J_0\left(\dot{\varphi}_z\delta\dot{w}_0 + \dot{w}_0\delta\dot{\varphi}_z\right)\\ &\quad - I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right)\\ &\quad + J_1\left(\left(k_1A'\right)\left(\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\delta\dot{u}_0\right) + \left(k_2B'\right)\left(\dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\delta\dot{v}_0\right)\right)\\ &\quad + I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) + K_2\left(\left(k_1A'\right)^2\left(\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}\right) + \left(k_2B'\right)^2\left(\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\right)\\ &\quad - J_2\left(\left(k_1A'\right)\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right) + \left(k_2B'\right)\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\right)\\ &\quad + K_0\left(\dot{\varphi}_z\delta\dot{\varphi}_z\right)\Bigg\}dA\end{aligned} \tag{20}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eq. (5); and (I_t, J_i, K_i) are mass inertias expressed by

$$\left(I_0, I_1, I_2\right) = \int_{-h/2}^{h/2}\left(1, z, z^2\right)\rho(z)\,dz \tag{21a}$$

$$\left(J_0, J_1, J_2\right) = \int_{-h/2}^{h/2}\left(g, f, z\,f\right)\rho(z)\,dz \tag{21b}$$

$$\left(K_0, K_2\right) = \int_{-h/2}^{h/2} \left(g^2, f^2\right) \rho(z)dz \tag{21c}$$

Substituting the expressions for $\boldsymbol{\delta U}$, $\boldsymbol{\delta V}$ and $\boldsymbol{\delta K}$ from Eqs. (17), (19) and (20) into Eq. (16) and integrating by parts and collecting the coefficients of $\boldsymbol{u_0}$; $\boldsymbol{v_0}$; $\boldsymbol{w_0}$; $\boldsymbol{\theta}$ and $\boldsymbol{\varphi_z}$, the following equations of motion of the plate are obtained as

$$\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + k_1 A' J_1 \frac{\partial \ddot{\theta}}{\partial x} \\
&\delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + k_2 B' J_1 \frac{\partial \ddot{\theta}}{\partial y} \\
&\delta w_0: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q + \overline{N} \frac{\partial^2 (w_0 + g_0 \varphi_z)}{\partial x^2} \\
&= I_0 \ddot{w}_0 + J_0 \ddot{\varphi}_z + I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_2 \nabla^2 \ddot{w}_0 + J_2 \left(k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) \\
&\delta \theta: -\mathrm{k}_1 M_x^s - \mathrm{k}_2 M_y^s - \left(k_1 A' + k_2 B'\right) \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = \\
&- J_1 \left(k_1 A' \frac{\partial \ddot{u}_0}{\partial x} + k_2 B' \frac{\partial \ddot{v}_0}{\partial y} \right) - K_2 \left(\left(k_1 A'\right)^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + \left(k_2 B'\right)^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2} \right) + \\
&J_2 \left(k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2} \right) \\
&\delta \varphi_z: -N_z + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} + \overline{N} g_0 \frac{\partial^2 (w_0 + g_0 \varphi_z)}{\partial x^2} = \ J_0 \ddot{w}_0 + K_0 \ddot{\varphi}_z
\end{aligned} \tag{22}$$

Using Eq. (11) in Eq. (6), the stress resultants of a FG plate can be related to the total strains by

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \\ N_z \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 & X_{13} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 & X_{23} \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s & 0 \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 & Y_{13} \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 & Y_{11} \\ 0 & 0 & B_{66} & 0 & 0 & D_{11} & 0 & 0 & D_{12}^s & 0 \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 & Y_{13}^s \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 & Y_{23}^s \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s & 0 \\ X_{13} & X_{23} & 0 & Y_{13} & Y_{23} & 0 & Y_{13}^s & Y_{23}^s & 0 & Z_{33} \end{bmatrix} \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \\ \frac{-\partial^2 w_0}{\partial x^2} \\ \frac{-\partial^2 w_0}{\partial y^2} \\ \frac{-\partial^2 w_0}{\partial x \partial y} \\ k_1 \theta \\ k_2 \theta \\ (k_1 A' + k_1 B') \frac{\partial^2 \theta}{\partial x \partial y} \\ \varphi_z \end{Bmatrix} \tag{23}$$

and

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} k_2 B' \frac{\partial \theta}{\partial y} + \frac{\partial \varphi_z}{\partial y} \\ k_1 A' \frac{\partial \theta}{\partial x} + \frac{\partial \varphi_z}{\partial x} \end{Bmatrix} \tag{23b}$$

Where

$$\begin{aligned}
&(A_{ij}, A_{ij}^s, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s) = \\
&\qquad \int_{-h/2}^{h/2} Q_{ij} (1, g^2(z), z, z^2, f(z), zf(z), f^2(z)) dz
\end{aligned} \tag{24a}$$

$$(X_{ij}, Y_{ij}, Y_{ij}^s, Z_{ij}) = \int_{-h/2}^{h/2} Q_{ij} (1, z, f(z), g'(z)) g'(z) dz \tag{24b}$$

By substituting Eq. (23) into Eq. (22), the equilibrium equations can be expressed in terms of displacements ($\boldsymbol{u_0}$; $\boldsymbol{v_0}$; $\boldsymbol{w_0}$; $\boldsymbol{\theta}$ and $\boldsymbol{\varphi_z}$) as

$$\begin{aligned}
&A_{11} d_{11} u_0 + A_{66}\, d_{22} u_0 + \left(A_{12} + A_{66}\right) d_{12} v_0 - B_{11} d_{111} w_0 - \left(B_{12} + 2B_{66}\right) d_{122} w_0 \\
&\quad + \left(B_{66}^s \left(k_1 A' + k_2\, B'\right)\right) d_{122} \theta + \left(B_{11}^s k_1 + B_{12}^s\, k_2\right) d_1 \theta + X_{13} d_1 \varphi_z = \\
&\quad I_0 \ddot{u}_0 - I_1\, d_1 \ddot{w}_0 + J_1\, A' k_1 d_1 \ddot{\theta},
\end{aligned} \tag{25a}$$

$$\begin{aligned}
&A_{22}\, d_{22} v_0 + A_{66}\, d_{11} v_0 + \left(A_{12} + A_{66}\right) d_{12} u_0 - B_{22}\, d_{222} w_0 \\
&- \left(B_{12} + 2B_{66}\right) d_{112} w_0 + \left(B_{66}^s \left(k_1 A' + k_2\, B'\right)\right) d_{112} \theta + \left(B_{22}^s k_2 + B_{12}^s k_1\right) d_2 \theta \\
&+ X_{23} d_2 \varphi_Z = I_0 \ddot{v}_0 - I_1\, d_2 \ddot{w}_0 + J_1\, B' k_2\, d_2 \ddot{\theta},
\end{aligned} \tag{25b}$$

$$\begin{aligned}
&B_{11}\, d_{111} u_0 + \left(B_{12} + 2B_{66}\right) d_{122} u_0 + \left(B_{12} + 2B_{66}\right) d_{112} v_0 + B_{22}\, d_{222} v_0 \\
&- D_{11} d_{1111} w_0 - 2\left(D_{12} + 2D_{66}\right) d_{1122} w_0 - D_{22}\, d_{2222} w_0 + \left(D_{11}^s\, k_1 + D_{12}^s\, k_2\right) d_{11} \theta \\
&+ 2\left(D_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{1122} \theta + \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22} \theta + \\
&N_x^0\, d_{11} w_0 + 2\ N_{xy}^0\, d_{12} w_0 + N_y^0\, d_{22} w_0 + (Y_{13} d_{11} + Y_{13} d_{11}) \varphi_Z \\
&+ q = I_0 \ddot{w}_0 + I_1 \left(d_1 \ddot{u}_0 + d_2\, \ddot{v}_0\right) - I_2 \left(d_{11} \ddot{w}_0 + d_{22} \ddot{w}_0\right) + \\
&J_2 \left(k_1 A'\, d_{11} \ddot{\theta} + k_2\, B'\, d_{22} \ddot{\theta}\right)
\end{aligned} \tag{25c}$$

$$\begin{aligned}
&- \left(B_{11}^s k_1 + B_{12}^s k_2\right) d_1 u_0 - \left(B_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{122} u_0 - \left(B_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{112} v_0 \\
&- \left(B_{12}^s k_1 + B_{22}^s k_2\right) d_2 v_0 + \left(D_{11}^s k_1 + D_{12}^s k_2\right) d_{11} w_0 + 2\ \left(D_{66}^s \left(k_1 A' + k_2 B'\right)\right) d_{1122} w_0 \\
&+ \left(D_{12}^s k_1 + D_{22}^s k_2\right) d_{22} w_0 - H_{11}^s\, k_1^2\ \theta - H_{22}^s\ k_2^2\, \theta - 2\, H_{12}^s\, k_1\, k_2 \theta - \\
&\left(\left(k_1 A' + k_2 B'\right)^2 H_{66}^s\right) d_{1122} \theta + A_{44}^s \left(k_2 B'\right)^2\, d_{22} \theta + A_{55}^s\ \left(k_1\ A'\right)^2\, d_{11} \theta \\
&- (Y_{13}^s + Y_{23}^s) \varphi_z - (A_{44}^s + A_{55}^s) \varphi_z = -J_1 \left(k_1\ A' d_1 \ddot{u}_0 + k_2\ B' d_2 \ddot{v}_0\right) + \\
&J_2 \left(k_1\ A' d_{11} \ddot{w}_0 + k_2\ B' d_{22} \ddot{w}_0\right) - K_2 \left(\left(k_1 A'\right)^2 d_{11} \ddot{\theta} + \left(k_2\ B'\right)^2 d_{22} \ddot{\theta}\right)
\end{aligned} \tag{25d}$$

$$\begin{aligned}
&- X_{13} d_1 u_0 - X_{23} d_2 v_0 + (Y_{13} d_{11} + Y_{23} d_{22}) w_0 + (A_{44}^s k_2 B' d_{22} + A_{55}^s k_1 A' d_{11} \\
&- Y_{13}^s k_1 - Y_{23}^s k_2) \theta + (A_{55}^s d_{11} + A_{44}^s d_{22} - Z_{33}) \varphi_z = J_0 \ddot{w}_0 + K_0 + \ddot{\varphi}_z
\end{aligned} \tag{25e}$$

Where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$\begin{aligned}
&d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m} \\
&and \qquad d_i = \frac{\partial}{\partial x_i}, (i, j, l, m = 1, 2).
\end{aligned} \tag{26}$$

3.3 Analytical solution for simply- supported FG plates

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0; v_0; w_0; θ and φ_z can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \\ \varphi \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} e^{i\omega t} \cos(\alpha\, x) \sin(\beta\, y) \\ V_{mn} e^{i\omega t} \sin(\alpha\, x) \cos(\beta\, y) \\ W_{mn} e^{i\omega t} \sin(\alpha\, x) \sin(\beta\, y) \\ X_{mn} e^{i\omega t} \sin(\alpha\, x) \sin(\beta\, y) \\ \phi_{mn} e^{i\omega t} \sin(\alpha\, x) \sin(\beta\, y) \end{Bmatrix} \tag{27}$$

Where $\sqrt{i} = -1$, ω is the natural frequency, and ($\boldsymbol{U_{mn}}$, $\boldsymbol{V_{mn}}$, $\boldsymbol{W_{mn}}$, $\boldsymbol{X_{mn}}$ and $\boldsymbol{\varphi_{mn}}$) are the unknown maximum displacement coefficients.

With

$$\alpha = m\pi / a \quad \text{and} \quad \beta = n\pi / b \tag{28}$$

Substituting Eq. (23) into Eq. (22), the following problem is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{13} & S_{12} & S_{12} & S_{12} & S_{12} \\ S_{14} & S_{12} & S_{43} & S_{44} & S_{45} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} & m_{15} \\ 0 & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{13} & m_{12} & m_{12} & m_{12} & m_{12} \\ m_{14} & m_{12} & m_{43} & m_{44} & m_{45} \\ m_{15} & m_{25} & m_{35} & m_{45} & m_{55} \end{bmatrix}\right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ \phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{Bmatrix} \tag{29}$$

where

$$S_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right),$$
$$S_{12} = -\alpha\beta\left(A_{12} + A_{66}\right),$$
$$S_{13} = \alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2\right),$$
$$S_{14} = \alpha\left(k_1 B_{11}^s + k_2 B_{12}^s - \left(k_1 A' + k_2 B'\right) B_{66}^s \beta^2\right),$$
$$S_{15} = \alpha X_{13},$$
$$S_{22} = -\left(A_{66}\alpha^2 + A_{22}\beta^2\right),$$
$$S_{23} = \beta\left(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right),$$
$$S_{24} = \beta\left(k_2 B_{22}^s + k_1 B_{12}^s - \left(k_1 A' + k_2 B'\right) B_{66}^s \alpha^2\right),$$
$$S_{25} = \beta X_{23},$$
$$S_{33} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right),$$
$$S_{34} = -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1 A' + k_2 B')\, D_{66}^s \alpha^2\beta^2 - k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right)$$

$$\begin{gathered} S_{35} = -\alpha^2 Y_{13} - \beta^2 Y_{23} \\ S_{44} = -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1 A' + k_2 B')^2 H_{66}^s \alpha^2\beta^2 - k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - \left(k_1 A'\right)^2 A_{55}^s \alpha^2 - \left(k_2 B'\right)^2 A_{44}^s \beta^2 \\ S_{45} = -k_1 Y_{13}^s - k_2 Y_{23}^s - \alpha^2 k_1 A' A_{55}^s - \beta^2 k_2 B' A_{44}^s \\ S_{55} = -\alpha^2 A_{55}^s - \beta^2 A_{44}^s - Z_{33}. \end{gathered} \tag{30}$$

and

$$\begin{gathered} m_{11} = -I_0, \quad m_{13} = \alpha I_1, \quad m_{14} = -J_1 k_1 A' \alpha, \quad m_{22} = -I_0, \\ m_{23} = \beta I_1 \quad m_{24} = -k_2 B' \beta J_1, m_{33} = -I_0 - I_2(\alpha^2 + \beta^2) \\ m_{34} = J_2\left(k_1 A' \alpha^2 + k_2 B' \beta^2\right), \\ m_{44} = -K_2\left(\left(k_1 A'\right)^2 \alpha^2 + \left(k_2 B'\right)^2 \beta^2\right), m_{35} = -J_0, \\ m_{55} = -K_0 \end{gathered} \tag{31}$$

4. Results and discussion

In this part, several numerical examples are presented and discussed to verify the accuracy of the theory presented in this work in the prediction of free vibration responses of simply supported FGM plates by comparing the analytical solution to those of other results available in the literature.

In addition, the influences of the power law index "parameter, thickness ratio" and the stretching of the

Table 1 Comparison of fundamental frequency parameter $\bar{\omega}$ of square plates

a/h	Mode (*m*,*n*)	Theory	Stretching effect	*p*				
				0	**0.5**	**1**	**4**	**10**
5	(1 , 1)	Belabed *et al*. (2014)	$\varepsilon_z \neq 0$	0.2121	0.1819	0.1640	0.1383	0.1306
		Present	$\varepsilon_z \neq 0$	0.2122	0.1825	0.1659	0.1409	0.1318
		Mouaici *et al*. (2016)	$\varepsilon_z = 0$	/	0.1807	0.1631	0.1379	0.1301
		Present	$\varepsilon_z = 0$	0.2113	0.1807	0.1631	0.1378	0.1301
	(1 , 2)	Belabed *et al*. (2014)	$\varepsilon_z \neq 0$	0.4659	0.4041	0.3676	0.3047	0.2811
		Present	$\varepsilon_z \neq 0$	0.4661	0.4042	0.3677	0.3047	0.2812
		Mouaici *et al*. (2016)	$\varepsilon_z = 0$	/	0.3988	0.3606	0.2982	0.2772
		Present	$\varepsilon_z = 0$	0.4623	0.3989	0.3607	0.2980	0.2771
	(2 , 2)	Belabed *et al*. (2014)	$\varepsilon_z \neq 0$	0.6757	0.5890	0.5362	0.4381	0.4008
		Present	$\varepsilon_z \neq 0$	0.6760	0.5893	0.5365	0.4381	0.4009
		Mouaici *et al*. (2016)	$\varepsilon_z = 0$	/	0.5801	0.5253	0.4288	0.3950
		Present	$\varepsilon_z = 0$	0.6688	0.5803	0.0525	0.4284	0.3948
10	(1 , 1)	Belabed *et al.* (2014)	$\varepsilon_z \neq 0$	0.0578	0.0494	0.0449	0.0389	0.0368
		Present	$\varepsilon_z \neq 0$	0.0578	0.0494	0.0449	0.0389	0.0368
		Mouaici *et al*. (2016)	$\varepsilon_z = 0$	/	0.0490	0.0441	0.0380	0.0363
		Present	$\varepsilon_z = 0$	0.0577	0.0490	0.0422	0.0381	0.0364
	(1 , 2)	Belabed *et al*. (2014)	$\varepsilon_z \neq 0$	0.1381	0.1184	0.1077	0.0923	0.0868
		Present	$\varepsilon_z \neq 0$	0.1381	0.1184	0.1077	0.0923	0.0868
		Mouaici *et al*. (2016)	$\varepsilon_z = 0$	/	0.1173	0.1059	0.0902	0.0856
		Present	$\varepsilon_z = 0$	0.1376	0.1174	0.1059	0.0903	0.0856
	(2 , 2)	Belabed *et al*. (2014)	$\varepsilon_z \neq 0$	0.2121	0.1825	0.1659	0.1409	0.1318
		Present	$\varepsilon_z \neq 0$	0.2122	0.1825	0.1659	0.1409	0.1318
		Mouaici *et al*. (2016)	$\varepsilon_z = 0$	/	0.1807	0.1631	0.1379	0.1301
		Present	$\varepsilon_z = 0$	0.2113	0.1807	0.1631	0.1378	0.1301
20	(1 , 1)	Belabed *et al*. (2014)	$\varepsilon_z \neq 0$	0.0148	0.0126	0.0115	0.0100	0.0095
		Present	$\varepsilon_z \neq 0$	0.0148	0.0126	0.0115	0.0100	0.0095
		Present	$\varepsilon_z = 0$	0.0148	0.0125	0.0113	0.0098	0.0094

Table 2 Non-dimensional fundamental frequency of different rectangular plates

a/h	Mode (*m*,*n*)	Theory	stretching effect	*p*				
				0	0,5	1	4	10
2	(1 , 1)	Present	$\varepsilon_z \neq 0$	0,6638	0,5785	0,5266	0,4304	0,3940
		Present	$\varepsilon_z = 0$	0,6568	0,5697	0,5158	0,4208	0,3880
	(1 , 2)	Present	$\varepsilon_z \neq 0$	0,9420	0,8253	0,7521	0,6057	0,5497
		Present	$\varepsilon_z = 0$	0,9297	0,8110	0,7356	0,5924	0,5412
	(2 , 2)	Present	$\varepsilon_z \neq 0$	1,7534	1,5515	1,4180	1,1156	0,9970
		Present	$\varepsilon_z = 0$	1,7233	1,5192	1,3844	1,0919	0,9807
5	(1 , 1)	Present	$\varepsilon_z \neq 0$	0,1381	0,1184	0,1077	0,0923	0,0868
		Present	$\varepsilon_z = 0$	0,1376	0,1174	0,1059	0,0903	0,0856
	(1 , 2)	Present	$\varepsilon_z \neq 0$	0,2122	0,1825	0,1659	0,1409	0,1318
		Present	$\varepsilon_z = 0$	0,2113	0,1807	0,1631	0,1378	0,1301
	(2 , 2)	Present	$\varepsilon_z \neq 0$	0,4661	0,4042	0,3677	0,3047	0,2812
		Present	$\varepsilon_z = 0$	0,4623	0,3989	0,3607	0,2980	0,2771
10	(1 , 1)	Present	$\varepsilon_z \neq 0$	0,0365	0,0312	0,0284	0,0247	0,0234
		Present	$\varepsilon_z = 0$	0,0365	0,0310	0,0279	0,0241	0,0231
	(1 , 2)	Present	$\varepsilon_z \neq 0$	0,0578	0,0494	0,0449	0,0389	0,0368
		Present	$\varepsilon_z = 0$	0,0577	0,0490	0,0442	0,0381	0,0364
	(2 , 2)	Present	$\varepsilon_z \neq 0$	0,1381	0,1184	0,1077	0,0923	0,0868
		Present	$\varepsilon_z = 0$	0,1376	0,1174	0,1059	0,0903	0,0856
20	(1 , 1)	Present	$\varepsilon_z \neq 0$	0,0093	0,0079	0,0072	0,0063	0,0060
		Present	$\varepsilon_z = 0$	0,0093	0,0079	0,0071	0,0062	0,0059
	(1 , 2)	Present	$\varepsilon_z \neq 0$	0,0148	0,0126	0,0115	0,0100	0,0095
		Present	$\varepsilon_z = 0$	0,0148	0,0125	0,0113	0,0098	0,0094
	(2 , 2)	Present	$\varepsilon_z \neq 0$	0,0365	0,0312	0,0284	0,0247	0,0234
		Present	$\varepsilon_z = 0$	0,0365	0,0310	0,0279	0,0241	0,0231

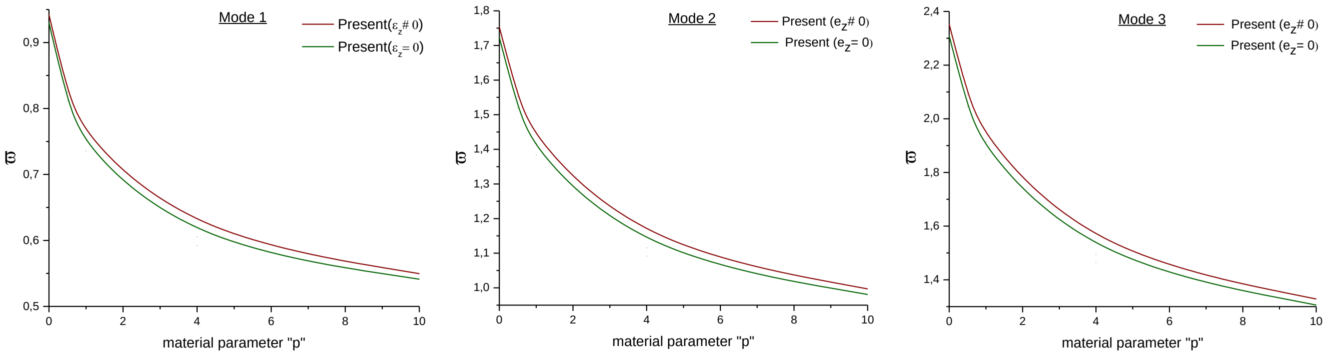


Fig. 2 Variation Non-dimensional fundamental frequency of square perfect FGM plate according to the material power index (*a/h*=2)

thickness on the vibratory behavior of the plates FGM are studied.

The FG plate is taken to be made of Metal and Ceramic with the following material properties:

Ceramic (Alumina, Al2O3) E_c=380 GPa, v=0.3, and ρ_c=3800 kg/m^3.

Metal (Aluminium, Al) E_m=70 GPa, v=0.3, and ρ_m=2702 kg/ m^3.

For simplicity, the following non dimensional natural frequency parameter is used in the numerical examples.

$$\bar{w} = wh\sqrt{\frac{\rho_m}{E_m}}$$

First, we try to verify the accuracy of the present theory by comparing the results of the non-dimensional frequencies obtained with those of the literature. For this, various numerical examples are described, discussed and compared with other existing theories such as the theory of hyperbolic shear deformation presented by Mouaici *et al*. (2016), and the theory proposed by Belabed *et al*. (2014).

The non-dimensional fundamental frequencies for square plates simply supported with different thickness values (2; 5; 10 and 20) and a material parameter p varied from 0 to 10 are presented in Table 1.

The results obtained by the model object of this study are compared by those predicted by Belabed *et al*. (2014) and by Mouaici *et al*. (2016) for both cases (with the effect of stretching) and (without the effect of stretching).

It can be seen that, in general, the results of the current model are in excellent agreement with the other models. We

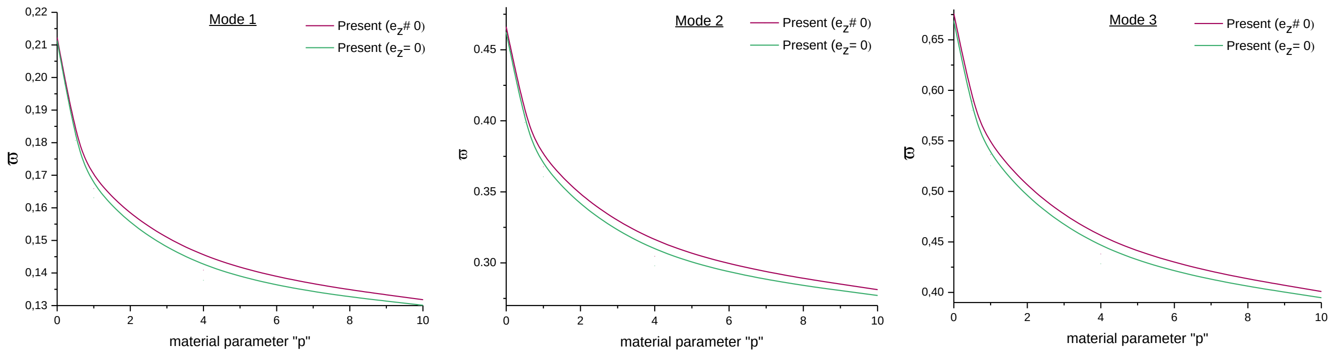


Fig. 3 Variation Non-dimensional fundamental frequency of square perfect FGM plate according to the material power index (*a/h*=5)

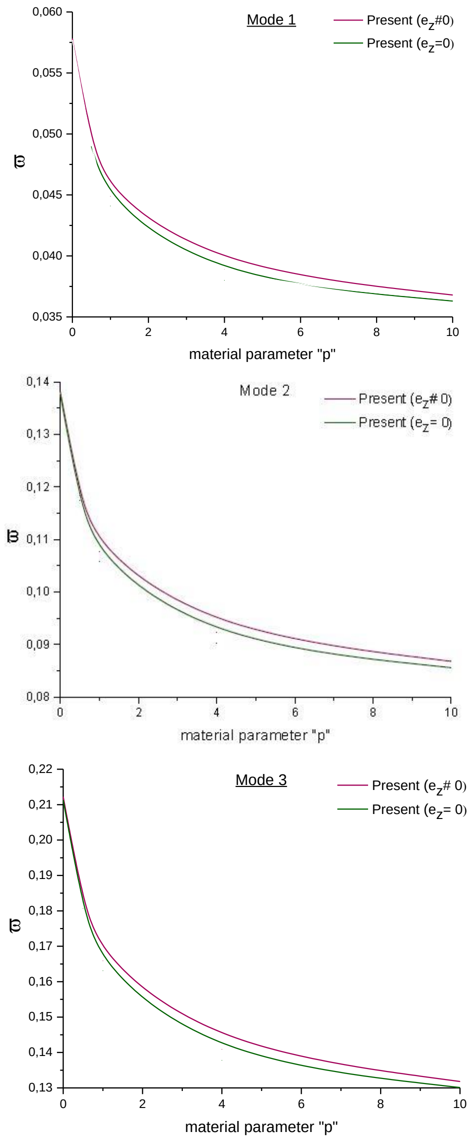


Fig. 4 Variation Non-dimensional fundamental frequency of rectangular perfect FGM plate according to the material power index (*a/h*=10)

can also observe the influence of the effect of the normal strain on the fundamental frequency (the effect of stretching ε_z increases the frequency; this increase becomes more remarkable in the case of a ratio of thickness equal to 5). That is, the effect of normal deformation "ε_z" becomes more sensitive in the case of thick plates, is shear affects thick structures and amplifies the effect of normal deformation.

In Table 2, the results of non-dimensional fundamental frequency obtained for FGM rectangular plates for the two cases: with or without stretching effect thereto, have summers present. The same influence of this effect was noticed from these results except that this latter becomes more remarkable in the case of a thickness ratio equal to 2.

Figs. 2, 3 and 4 show the variation of the non-dimensional fundamental frequency of a square perfect FGM plate as a function of the power index for a thickness ratio equal to 2, 5 and 10, respectively, for the three modes. In addition, the effect of normal deformation is presented in this figure. From these curves it can be seen that increasing the power index values results in a reduction of the frequency. This reduction is quite significant for power values below 6 for the three modes; from this value, the deviation becomes practically constant.

It can also be seen that the fundamental frequencies of a plate without the effect of normal deformation (ε_z=0) are underestimated compared to those of a plate with the effect of normal deformation (ε_z≠0), especially for the frequencies of the third mode.

In Figs. 5 and 6, the variation of the non-dimensional

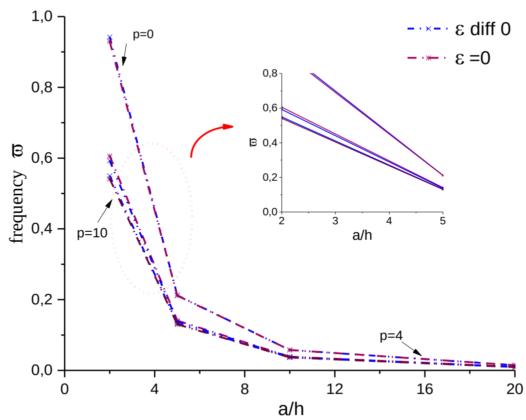


Fig. 5 variation of the non-dimensional fundamental frequency of different square perfect FGM plates according to the length to thickness ratio (a/h)

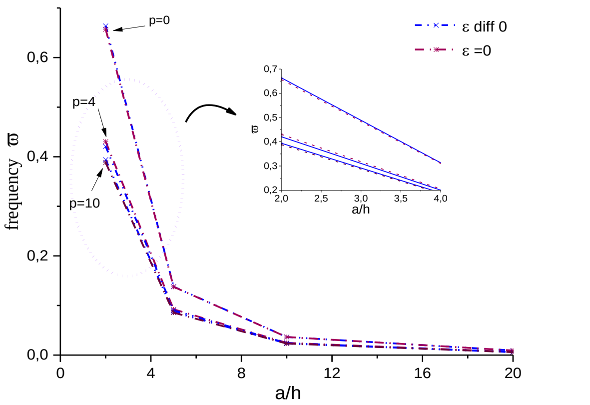
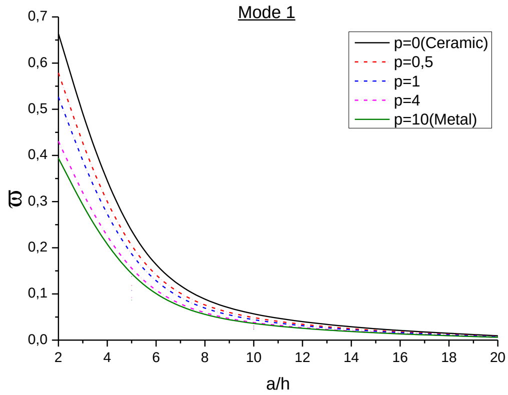
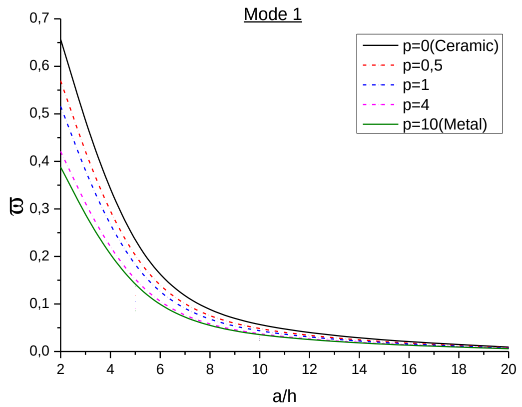


Fig. 6 variation of the non-dimensional fundamental frequency of different rectangular perfect FGM plates according to the length to thickness ratio (*a/h*)



(a)

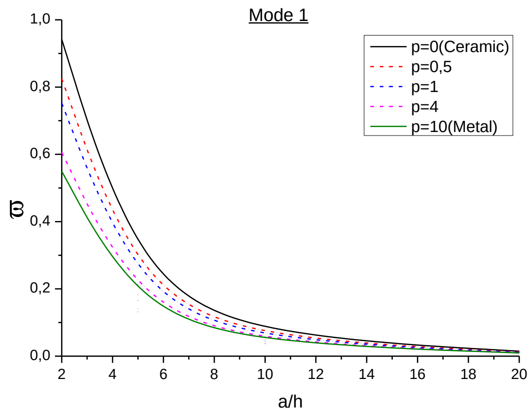


(b)

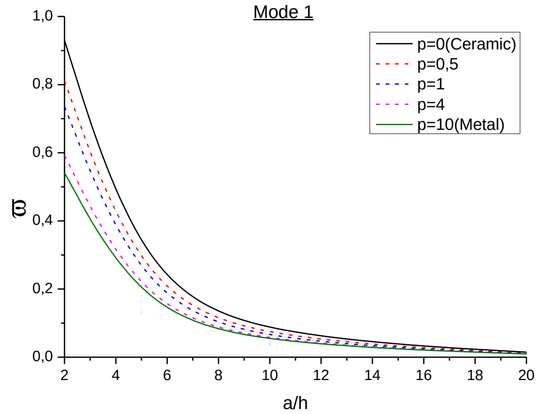
Fig. 7 Variation Non-dimensional fundamental frequency of the rectangular perfect FGM plates (b=2a) according to side-to-thickness ratio a/h. (a) $\varepsilon_z \neq 0$ (b) $\varepsilon_z = 0$

fundamental frequency of different square and rectangular perfect FGM plates, respectively, is presented as a function of the length / thickness ratio (*a/h*).

From the curves shown in these two figures, it can be seen that the frequency decreases with increasing of the thickness ratio (*a/h*). It can also be observed that, more the index of power law decreasing (increase of rigidity), the frequency increases; while when the power law index increase (decrease of rigidity) the frequencies decrease.



(a)



(b)

Fig. 8 Variation Non-dimensional fundamental frequency of square perfect FGM plate according to the material power index (a) $\varepsilon_z \neq 0$ (b) $\varepsilon_z = 0$

Figs. 7 and 8 show the variation of the fundamental frequency of a rectangular and square FGM plate, respectively, as a function of the ratio *a/h*.

In addition, the effect of normal deformation is shown in figures (a) and (b). From these figures, it can be seen that increasing the values of the ratio *a/h* leads to a reduction of the fundamental frequency. This reduction is quite significant for values of *a/h* less than 10. From this value, the fundamental frequency becomes practically constant.

It may also be noted that the introduction of the "ε_z" stretching effect leads to an increase in the fundamental frequency, and that more than the rigidity of the plate increases, the frequency increases; while when the rigidity of the FGM plate decreased, the frequency decreases.

Fig. 9 illustrate the variation of the non-dimensional frequency with respect to the power law index for a thickness ratio and different porosity values. From the curves presented in this figure, we can see that the parameter of the natural frequency of the FGM plates has an inverse relationship with the power law index, and that this parameter and decreased with the increase of the porosity.

In Fig. 10, we have tried to clarify the influence of the porosity index on the non-dimensional fundamental frequency of the different plates in FGM with a thickness ratio of 2 and 10, respectively. The power law index is taken equal to 0, 1, 2, 4 and 10. From these curves, it can be

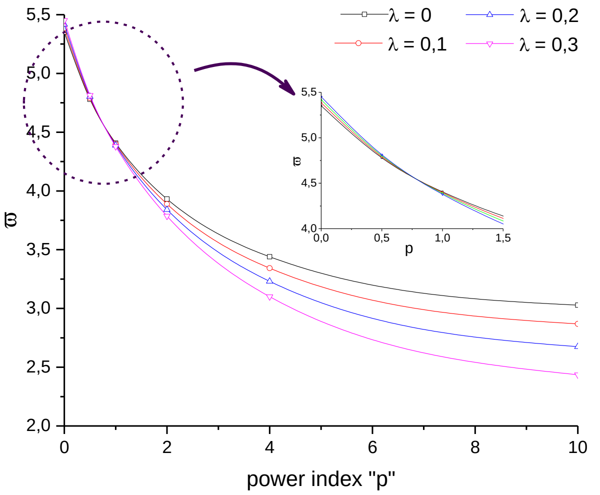


Fig. 9 the effect of porosity on the non-dimensional fundamental frequency of FGM plates ($\varepsilon_z \neq 0$, a/h=2)

observed that there is a diversity in the influence of the porosity on the fundamental frequency of the plates FG for each case of power law index.

For a *p* equal to 10, 4 and 2, the increase of the porosity for the two cases of the ratio length / thickness (*a/h*=2 and 10) also leads to a decrease of the frequency and which is more visible in the FG plate with *p*=10.

For a *p* equal to 1, the increase in porosity for both cases of the length / thickness ratio (*a/h*=2 and 10) has no influence on the fundamental frequency of the plate FG.

On the contrary, for *p*=0, the increase of the porosity for the two cases of the ratio length / thickness (*a/h*=2 and 10) leads to an increase in the fundamental frequency of the plate FG.

5. Conclusions

The aim of this research was to contribute to the study of the free vibration of functionally graduated porous plates by taking into account the effect of normal deformation. We proposed a new analytical model based on a five-variable high-order theory and a new displacement field that introduces indeterminate integral variables. The properties of the material are assumed to vary in the direction of the thickness of the plate according to the rule of the mixture, which is reformulated to evaluate the characteristics of the material with the porosity. The motion equations governing the porous plate FG were derived using the Hamilton principle. The basic equations are easily solved using Navier solutions. To validate this model, we compared it with others from the literature. All comparative studies demonstrated that fundamental frequencies obtained using this theory and those proposed by Mouaici *et al.* (2016) and Belabed *et al.* (2014) are almost identical. The influence of normal deformation, porosity and power law index on the natural frequency of this plate was examined. In conclusion, it can be said that the proposed theory is accurate and simple to solve problems of free vibration of FG plates.

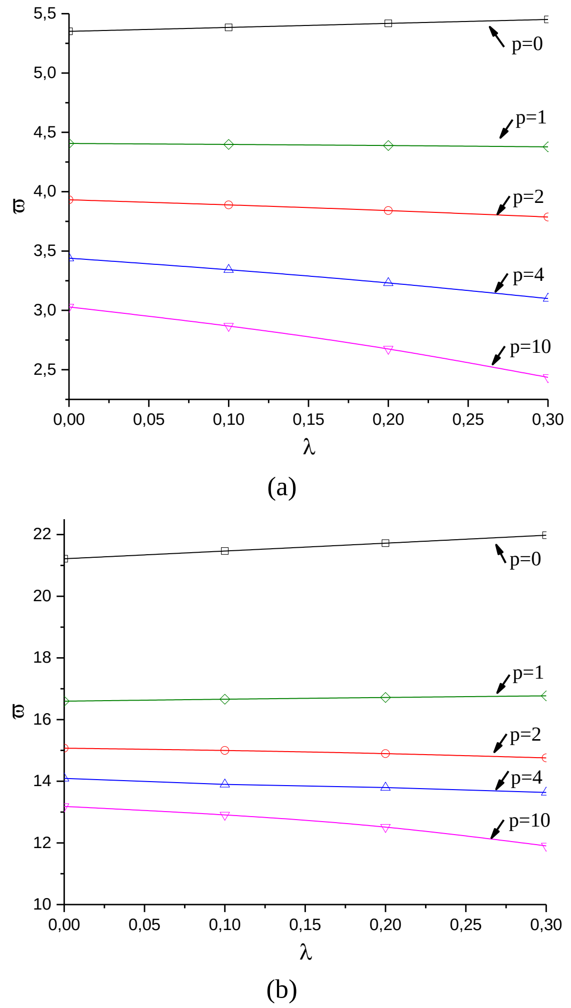


Fig. 10 Variation of the non-dimensional fundamental frequency FGM plates according to the porosity index ($\varepsilon_z \neq 0$), (a) *a/h*=2 and (b) *a/h*=10

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