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Effect of the micromechanical models on the bending of FGM beam using a new hyperbolic shear deformation theory

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Abstract. In this paper, a new refined hyperbolic shear deformation beam theory for the bending analysis of functionally graded beam is presented. The theory accounts for hyperbolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the functionally graded beam without using shear correction factors. In addition, the effect of different micromechanical models on the bending response of these beams is studied. Various micromechanical models are used to evaluate the mechanical characteristics of the FG beams whose properties vary continuously across the thickness according to a simple power law. Based on the present theory, the equilibrium equations are derived from the principle of virtual work. Navier type solution method was used to obtain displacement and stresses, and the numerical results are compared with those available in the literature. A detailed parametric study is presented to show the effect of different micromechanical models on the flexural response of a simply supported FG beams.

Keywords: FG beams; micromechanical models; bending; Navier solution

1. Introduction

Functionally graded beams have attracted increasing attention of researchers in recent years. The material properties of functionally graded beams vary along the thickness direction or/and the length direction. For functionally graded beams with thickness wise gradient variation, great progress has been made. For example, Zhong and Yu (2007) formulated an analytical solution of a static cantilever functionally graded beam with the assumption that all the elastic moduli of the material have the same variations along the beam-thickness direction.

Lu *et al.* (2007) presented a semi-analytical elasticity solution for static problems of bidirectional functionally graded beams with exponential gradient distribution within the framework of two-dimensional elasticity theory. Aydogdu and Taskin (2007) investigated free vibration of simply-supported functionally graded beams where Young's modulus vary in the thickness direction according to power law and exponential law. Simsek (2010) employed different higher-order beam theories to compute the fundamental frequencies. Li *et al.* (2010) studied the static and dynamic analyses of functionally graded beams using the a higher-order theory, Ould Larbi Latifa *et al.* (2013) used an efficient shear deformation beam theory based on neutral surface position for bending and free vibration of functionally graded beams. Tounsi *et al.* (2013) used a refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates. Trinh *et al.* (2016) investigated an analytical method for the vibration and buckling of functionally graded beams under mechanical and thermal loads. Mahi *et al.* (2015) used a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Zemri *et al.* (2015) developed a mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory. Ahouel *et al.* (2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. Bousahla *et al.* (2016) investigated the thermal stability of plates with functionally graded coefficient of thermal expansion. Abdelaziz *et al.* (2017) used an efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions. Ayache *et al.* (2018) analysis the wave propagation and free vibration of functionally graded porous material beam with a novel four variable refined theory. Youcef *et al.* (2018), "Dynamic analysis of nanoscale beams including surface stress effects. Zine *et al.* (2018) presented a novel higher-order shear deformation theory for bending and free vibration analysis of isotropic and multilayered plates and shells. Yazid *et al.* (2018) used a novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium. Belabed *et al.* (2018) investigated a new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded

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sandwich plate. Recently, Younsi *et al.* (2018) developed a novel quasi-3D and 2D shear deformation theories for bending and free vibration analysis of FGM plates.

In order to model FGM precisely, it is essential to know the effective or bulk material properties as a function of individual material properties and geometry, in particular at micromechanics level.

In recent years, different models have been proposed to estimate the effective properties of FGMs with respect to reinforcement volume fractions (Shen and Wang 2012, Jha *et al.* 2013). Consequently, several micromechanical models have been used to study and analyze the behavior of FGM structures under different loading conditions. We cite as an example the work of Gasik (1998) in which he proposed a micromechanical model to study FGMs with a random distribution of constituents. Using an appropriate micromechanical model, Yin *et al.* (2004) and Yin *et al.* (2007) have determined the expressions of the linear coefficient of thermal expansion, the Young's moduli and the Poisson's ratio. Mahmoudi *et al.* (2018) studied the effect of the micromechanical models on the free vibration of rectangular FGM plate resting on elastic foundation.

In the present study, static of simply supported FG beams was investigated by using a new hyperbolic shear deformation beam theory. The effect of different micromechanical models on the bending response of these beams is studied. Various micromechanical models are used to evaluate the mechanical characteristics of the FG beams whose properties vary continuously across the thickness according to a simple power law. Then, the present theory together with Hamilton's principle, are employed to extract the motion equations of the functionally graded beams. Analytical solutions for static are obtained. The effects of various variables, such as span-to-depth ratio, gradient index, and micromechanical models on bending of FG beam are all discussed.

2. Effective properties of FGMs

Unlike traditional microstructures, in FGMs the material properties are spatially varying, which is not trivial for a micromechanics model (Jaesang and Addis 2014).

A number of micromechanics models have been proposed for the determination of effective properties of FGMs. In what follows, we present some micromechanical models to calculate the effective properties of the FG beam.

2.1 Voigt model

The Voigt model is relatively simple; this model is frequently used in most FGM analyses estimates Young's modulus E of FGMs as (Mishnaevsky 2007)

$$E(z) = E_c \mathrm{V_c} + E_m (1 - V_c) \tag{1}$$

2.2 Reuss model

Reuss assumed the stress uniformity through the material and obtained the effective properties as (Mishnaevsky 2007, Zimmerman 1994)

$$E(z) = \frac{E_c E_m}{E_c (1 - V_c) + E_m V_c} \tag{2}$$

2.3 Tamura model

The Tamura model uses actually a linear rule of mixtures, introducing one empirical fitting parameter known as "stress-to-strain transfer" (Gasik 1995)

$$q = \frac{\sigma_1 - \sigma_2}{\varepsilon_1 - \varepsilon_2} \tag{3}$$

Estimate for q=0 correspond to Reuss rule and with q=100 to the Voigt rule, being invariant to the consideration of with phase is matrix and which is particulate. The effective Young's modulus is found as

$$E(z) = \frac{(1 - V_c) E_m (q - E_c) + V_c E_c (q - E_m)}{(1 - V_c)(q - E_c) + V_c E_c (q - E_m)} \tag{4}$$

2.4 Description by a representative volume element (LRVE)

The local representative volume element (LRVE) is based on a "mesoscopic" length scale which is much larger than the characteristic length scale of particles (inhomogeneities) but smaller than the characteristic length scale of a macroscopic specimen (Ju and Chen 1994). The LRVE is developed based on the assumption that the microstructure of the heterogeneous material is known. The input for the LRVE for the deterministic micromechanical framework is usually volume average or ensemble average of the descriptors of the microstructures.

Young's modulus is expressed as follows by the LRVE method (Akbarzadeh *et al.* 2015)

$$E(z) = E_m \left(1 + \frac{V_c}{FE - \sqrt[3]{V_c}} \right), \quad FE = \frac{1}{1 - \dfrac{E_m}{E_c}} \tag{5}$$

2.5 Mori-Tanaka model

The locally effective material properties can be provided by micromechanical models such as the Mori-Tanaka estimates. This method based on the assumption that a two-phase composite material consisting of matrix reinforced by spherical particles, randomly distributed in the plate. According to Mori-Tanaka homogenization scheme, the Young's modulus is given as

$$E(z) = E_m + (E_c - E_m) \left(\frac{V_c}{1 + (1 - V_c)(E_c / E_m - 1)(1 + \nu)/(3 - 3\nu)} \right) \tag{6}$$

where $V_c = \left(\dfrac{1}{2} + \dfrac{z}{h} \right)^p$ is the volume fraction of the ceramic and where p is the power law index. Since the effects of the variation of Poisson's ratio (ν) on the response of FGM plates are very small (Kitipornchai 2006), this material parameter is assumed to be constant for convenience.

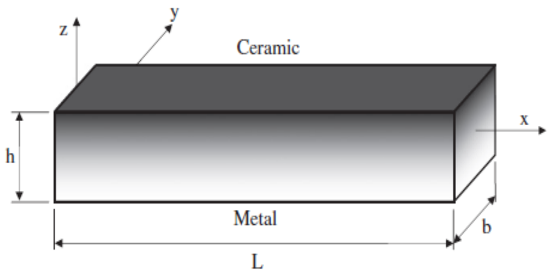

Fig. 1 Geometry and coordinate of a FG beam

3. Problem formulation

Consider a functionally graded beam with length L and rectangular cross section $b \times h$, with b being the width and h being the height as shown in Fig. 1. The beam is subjected to a transverse distributed load $q(x)$.

4. Kinematics and constitutive equations

The displacement field satisfying the conditions of transverse shear stresses (and hence strains) vanishing at ($z=\pm h/2$) on this outer (top) and inner (bottom) surfaces of the beam, is given as follows (Bousahla *et al.* 2016)

$$u(x,z) = u_0(x) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{7a}$$

$$w(x,z) = w_b(x) + w_s(x) \tag{7b}$$

In this work, the shape function $f(z)$ is chosen based on a hyperbolic function as (Ould Larbi *et al.* 2013)

$$f(z) = z\left[1+\frac{3\pi}{2}\sec h^2\left(\frac{1}{2}\right)\right] - \frac{3\pi}{2}h\tanh\left(\frac{z}{h}\right) \tag{8a}$$

and

$$g(z) = 1 - f'(z) \tag{8b}$$

Where $u_0(x)$, $w_b(x)$ and $w_s(x)$ are the three unknown displacement functions of the middle surface of the beam.

The kinematic relations can be obtained as follows

The strains associated with the displacements in Eq. (8) are

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2} \tag{9a}$$

$$\gamma_{xz} = g(z)\gamma_{xz}^s \tag{9b}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x},\ k_x^b = -\frac{\partial^2 w_b}{\partial x^2},\ k_x^s = -\frac{\partial^2 w_s}{\partial x^2},\ \gamma_{xz}^s = \frac{\partial w_s}{\partial x} \tag{10}$$

The state of stress in the beam is given by the generalized Hooke's law as follows

$$\sigma_x = Q_{11}(z)\,\varepsilon_x \quad \text{and} \quad \tau_{xz} = Q_{55}(z)\,\gamma_{xz} \tag{11a}$$

where

$$Q_{11}(z) = E(z) \quad \text{and} \quad Q_{55}(z) = \frac{E(z)}{2(1+\nu)} \tag{11b}$$

5. Equations of motion

Considering the static version of the principle of virtual work, the following expressions can be obtained

$$\int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz})\,dz\,dx - \int_0^L q\,\delta w\,dx = 0 \tag{12}$$

Substituting Eqs. (9) and (11) into Eq. (12) and integrating through the thickness of the beam, we can obtain

$$\int_0^L \left(N_x \frac{d\delta u_0}{dx} - M_b \frac{d^2\delta w_b}{dx^2} - M_s \frac{d^2 \delta w_s}{dx^2} + Q_{xz}\frac{d\delta w_s}{dx} \right) dx - \int_0^L q(\delta w_b + \delta w_s)\,dx = 0 \tag{13}$$

where N_x, M_b, M_s and Q_{xz} are the stress resultants defined as

$$(N_x, M_b, M_s) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, f)\,\sigma_x\,dz \tag{14a}$$

$$Q_{xz} = \int_{-\frac{h}{2}}^{\frac{h}{2}} g\,\tau_{xz}\,dz \tag{14b}$$

The governing equations of equilibrium can be derived from Eq. (13) by integrating the displacement gradients by parts and setting the coefficients where δu_0, δw_b, δw_s, zero.

Thus, one can obtain the equilibrium equations associated with the present shear deformation theory

$$\delta u_0 :\ \frac{dN_x}{dx} = 0 \tag{15a}$$

$$\delta w_b :\ \frac{d^2 M_b}{dx^2} + q = 0 \tag{15b}$$

$$\delta w_s :\ \frac{d^2 M_s}{dx^2} + \frac{dQ_{xz}}{dx} + q = 0 \tag{15c}$$

Eq. (15) can be expressed in terms of displacements (u_0, w_b, w_s) by using Eqs. (11) and (14) as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} - B_{11}\frac{\partial^3 w_b}{\partial x^3} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} = 0 \tag{16a}$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3} - D_{11}\frac{\partial^4 w_b}{\partial x^4} - D_{11}^s \frac{\partial^4 w_s}{\partial x^4} + q = 0 \tag{16b}$$

$$B_{11}^s\frac{\partial^3 u_0}{\partial x^3} - D_{11}^s\frac{\partial^4 w_b}{\partial x^4} - H_{11}^s \frac{\partial^4 w_s}{\partial x^4} + A_{55}^s\frac{\partial^2 w_s}{\partial x^2} + q = 0 \tag{16c}$$

where A_{11}, D_{11}, etc., are the beam stiffness, defined by

$$\left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}\left(1, z, z^2, f(z), z\,f(z), f^2(z)\right) dz \tag{17a}$$

Table 1 Non-dimensional displacements and stresses of functionally graded beams (p=2 and L=5h)

Theory			Uniform load				Sinusoidal load			
			$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$
Li *et al.* (2010)			8.0602	3.1134	6.8812	0.6787	-	-	-	-
Ould Larbi *et al.* (2013)			8.0683	3.1146	6.8878	0.6870	-	-	-	-
Present	Voigt		8.0683	3.1145	6.8876	0.6867	6.3759	2.4058	5.6056	0.4482
	Reuss		10.1423	3.7132	8.9445	0.6648	8.0200	2.8664	7.2829	0.4347
	LRVE		9.3322	3.5408	7.8655	0.6461	7.3780	2.7341	6.4049	0.4223
	Tamura	(q=0)	10.1423	3.7132	8.9445	0.6648	8.0200	2.8664	7.2829	0.4347
		(q=100)	9.3004	3.5096	7.9400	0.6583	7.3527	2.7100	6.4650	0.4302
	Mori-Tanaka		9.8656	3.6505	8.5896	0.6613	7.8007	2.8183	6.9941	0.4323

Table 2 Non-dimensional displacements and stresses of functionally graded beams (p=2 and L=20h)

Theory			Uniform load				Sinusoidal load			
			$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$
Li *et al.* (2010)			7.4415	0.7691	27.0989	0.6787	-	-	-	-
Ould Larbi *et al.* (2013)			7.4421	0.7691	27.1005	0.7005	-	-	-	-
Present	Voigt		7.4421	0.7691	27.1002	0.6988	5.8684	0.5952	21.9725	0.4489
	Reuss		9.1348	0.9132	35.1277	0.6777	7.2036	0.7067	28.4820	0.4355
	LRVE		8.4641	0.8724	30.8789	0.6585	6.6746	0.6751	25.0372	0.4231
	Tamura	(q=0)	9.1348	0.9132	35.1277	0.6777	7.2036	0.7067	28.4820	0.4355
		(q=100)	8.4442	0.8646	31.1846	0.6708	6.6589	0.6691	25.2848	0.4310
	Mori-Tanaka		8.9048	0.8983	33.7307	0.6741	7.0222	0.6952	27.3493	0.4331

and

$$A_{55}^{s} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{55}\left[g(z)\right]^2 dz \tag{17b}$$

6. Analytical solution

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, w_0, Q_x can be written by assuming the following variations

$$u_0 = \sum_{m=1}^{\infty} U_m \cos\frac{m\pi x}{x} \tag{18a}$$

$$w_b = \sum_{m=1}^{\infty} W_{bm} \sin\frac{m\pi x}{L} \tag{18b}$$

$$w_s = \sum_{m=1}^{\infty} W_{sm} \sin\frac{m\pi x}{L} \tag{18c}$$

The transverse load q is also expanded in Fourier series as

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\lambda\, x) \tag{19}$$

where $\lambda = m\pi/L$ and Q_m is the load amplitude calculated from

$$Q_m = \frac{2}{L}\int_0^L q(x)\sin(\lambda\, x)dx \tag{20}$$

The coefficients Q_m are given below for some typical loads.

$$m = 1 \text{ and } Q_1 = q_0 \text{ Sinusoidal load} \tag{21a}$$

$$Q_m = \frac{4q_0}{m\pi} \text{ Uniform load} \tag{21b}$$

where q_0 represents the intensity of the load at the beam center.

Substituting the expansions of u_0, w_b, w_s and q from Eqs. (18) and (19) into the equations of motion Eq. (16), the analytical solutions can be obtained from the following equations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_m \\ Q_m \end{Bmatrix} \tag{22}$$

where

$$a_{11} = A_{11}\lambda^2,\ a_{12} = -B_{11}\lambda^3,\ a_{13} = -B_{11}^{s}\lambda^3,\ a_{22} = D_{11}\lambda^4,$$
$$a_{23} = D_{11}^{s}\lambda^4,\ a_{33} = H_{11}^{s}\lambda^4 + A_{55}^{s}\lambda^2 \tag{23}$$

7. Results and discussion

In the present section, the effect of micromechanical models on bending analysis of FG beams using a refined hyperbolic shear deformation theory is presented for investigation. In order to verify the accuracy of the present analysis, the results of this study were verified by comparing them with the various existing beam theories. The material properties used in the present study are:

Ceramic (Alumina, Al_2O_3): E_c=380 GPa; ν=0.3; ρ_c=3960 kg/m^3.

Metal (Aluminium, Al): E_m=70 GPa; ν=0.3; ρ_m=2702 kg/m^3.

For simplicity, the following non-dimensional parameters are used in the numerical examples

$$\bar{w} = 100\frac{E_m h^3}{q_0 L^4} w\left(\frac{L}{2}\right), \quad \bar{u} = 100\frac{E_m h^3}{q_0 L^4} u\left(0, -\frac{h}{2}\right),$$

$$\bar{\sigma}_x = \frac{h}{q_0 L}\sigma_x\left(\frac{L}{2}, \frac{h}{2}\right), \quad \bar{\tau}_{xz} = \frac{h}{q_0 L}\tau_{xz}(0,0)$$

7.1 Comparison studies

Firstly, the example is performed in Tables 1 and 2 for FG beams with power law index p=2 and two span-to-depth ratio L/h. The beam is subjected to uniform and sinusoidal transverse loads in z-direction. Effective Young's modulus is calculated using the aforementioned five micromechanical models. The obtained results are compared with a higher-order theory developed by Li *et al.* (2010) and theory of Ould Larbi *et al.* (2013).

From this table two observations can be made. First, the results obtained from the present hyperbolic shear theory for the Voigt model are very close to those of Li *et al.* (2010) and Ould Larbi *et al.* (2013) and this for the stress or the deflection. Secondly, the results from the present method and calculated with the four other models, namely LRVE, Tamura, Mori-Tanaka and Reuss, are slightly different. This can be explained by the way who the Young's modulus is calculated.

7.2 Parametric studies

In the present paragraph some results and considerations about the effect of the micromechanical models on the bending problem of functionally beams are presented. The analysis has been carried out by means of numerical procedures illustrated above.

In Fig. 2, the variations of the displacement $\bar{w}$ through the thickness direction of FG beam with the power law index p are given for different micromechanical models. It is seen from the figure that the increase of the power law index p produces an increase in the values of the displacement and this whatever the model used. In addition, the Reuss model has the highest displacement values compared to other models. While that of Voigt has the lowest values. The Tamura and Reuss models have the practically same results.

In Fig. 3, the axial stress $\bar{\sigma}_x$ through the thickness is tensile at the top surface and compressive at the bottom surface. The homogeneous ceramic beam p=0 or metal plate p=∞ yields the maximum compressive stresses at the bottom surface and the minimum tensile stresses at the top surface of the FG beams.

In Fig. 4, we present the variation of the axial stress $\bar{\sigma}_x$ through the thickness for different micromechanical models. From this figure, it can be seen that all models give practically the same results in terms of axial stress except that of Voigt, which gives minimum tensile stresses at the top, and minimum compressive stresses at the bottom surface.

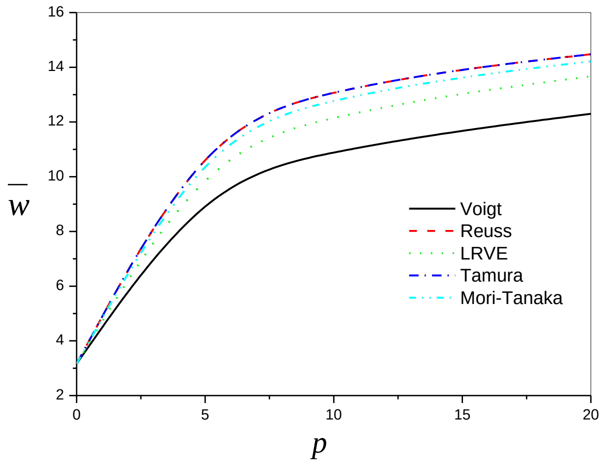

Fig. 2 The transverse displacement $\bar{w}$ versus the power law index p of FG beams for different micromechanical models (L/h=5)

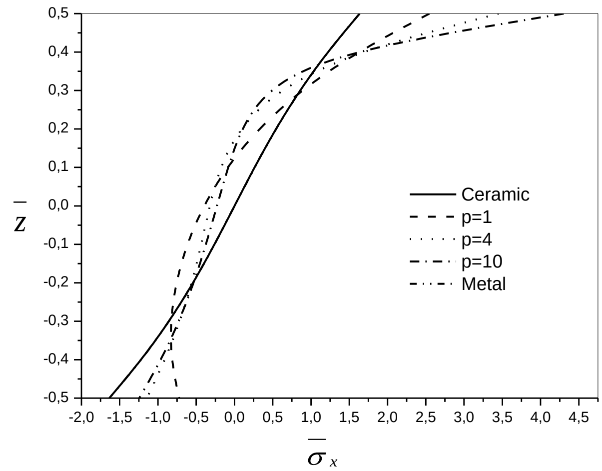

Fig. 3 The variation of the axial stress $\bar{\sigma}_x$ through-the-thickness of a FG beam (L/h=2) -Voigt model-

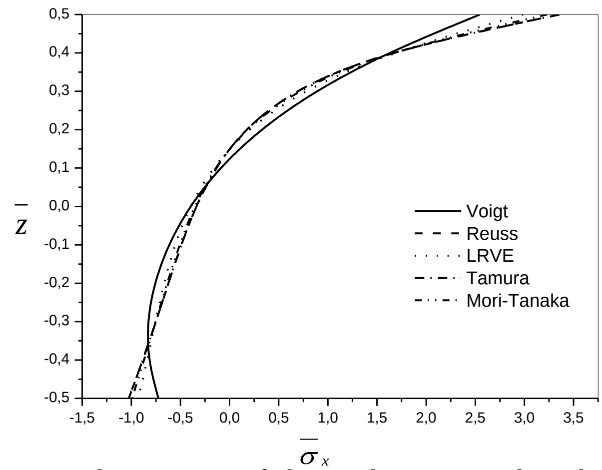

Fig. 4 The variation of the axial stress $\bar{\sigma}_x$ through-the-thickness of a FG beam for different micromechanical models (L/h=2, p=1)

In Fig. 5, we have plotted the variation of the transverse shear stress $\bar{\tau}_{xz}$ through-the-thickness of a FG beam using

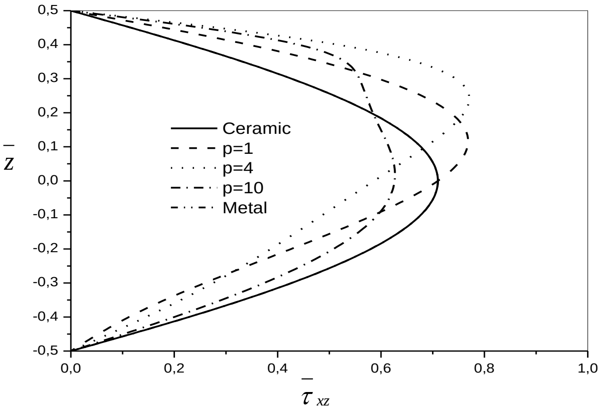

Fig. 5 The variation of the transverse shear stress $\bar{\tau}_{xz}$ through-the-thickness of a FG beam (*L/h*=2) -Voigt model-

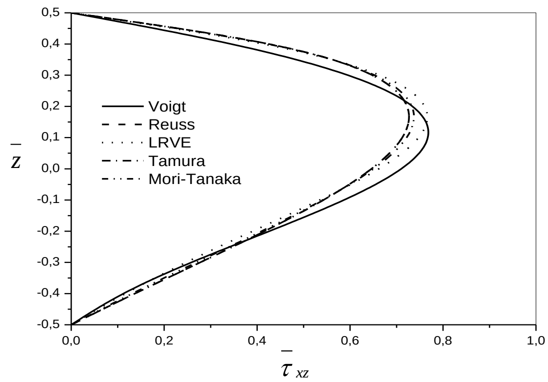

Fig. 6 The variation of the transverse shear stress $\bar{\tau}_{xz}$ through-the-thickness of a FG beam for different micromechanical models (*L/h*=2, *p*=1)

the Voigt model. The through-the-thickness distributions of the transverse shear stresses are not parabolic in the case of non- homogenous plate as in the case of homogeneous beams (ceramic or metal). It can be observed that the homogeneous beams that are either metal or ceramic give the same transverse shear stress.

The effect of the micromechanical models on the variation of the transverse shear stress $\bar{\tau}_{xz}$ across the thickness is shown in Fig. 6. The Voigt model is the one, which gives the highest stresses compared with the others where the difference between the max stresses is minimal. Also, The Tamura and Reuss models have the practically same results.

8. Conclusions

In this paper, we have developed a new refined hyperbolic shear deformation beam theory for the solutions of static bending of FG beam. The theory accounts for hyperbolic distribution of the transverse shear strains and satisfies the zero traction boundary conditions on the surfaces of the functionally graded beam without using shear correction factors. Different micromechanical models were used to determine the effective properties of the FG beams. The Navier method is used for the analytical solutions of the FG beam with simply supported boundary conditions. The results obtained using this new theory, are in a good agreement with reference solutions available in literature.

From these results and comparisons between different micromechanical models, it has been found significant differences between some models. This proves the need for a proper micromechanical modeling of FGMs to accurately estimate the deflection and stress.

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