Earthquakes and Structures, *Vol. 16, No. 1 (2019) 109-118*
DOI: https://doi.org/10.12989/eas.2019.16.1.109

Harmonic seismic waves response of 3D rigid surface foundation on layer soil

Salah Messioud*[1], Badredine Sbartai[2a] and Daniel Dias[3,4a]

[1]*LGCE, University of Jijel, BP 98 Ouled Aissa, 18000 Jijel, Algeria*
[2]*Department of Civil Engineering, University of Badji Mokhtar-Annaba, Algeria*
[3]*School of Automotive and Transportation Engineering, Hefei University of Technology, Hefei, China*
[4]*Geotechnical Expert, Antea Group, 92160 Antony, France*

(Received September 11, 2017, Revised December 1, 2018, Accepted December 15, 2018)

Abstract. This study, analyses the seismic response for a rigid massless square foundation resting on a viscoelastic soil layer limited by rigid bedrock. The foundation is subjected either to externally applied forces or to obliquely incident seismic body or surface harmonic seismic waves P, SV and SH. A 3-D frequency domain BEM formulation in conjunction with the thin layer method (TLM) is adapted here for the solution of elastodynamic problems and used for obtained the seismic response. The mathematical approach is based on the method of integral equations in the frequency domain using the formalism of Green's functions (Kausel and Peck 1982) for layered soil, the impedance functions are calculated by the compatibility condition. In this study, The key step is the characterization of the soil-foundation interaction with the input motion matrix. For each frequency the impedance matrix connects the applied forces to the resulting displacement, and the input motion matrix connects the displacement vector of the foundation to amplitudes of the free field motion. This approach has been applied to analyze the effect of soil-structure interaction on the seismic response of the foundation resting on a viscoelastic soil layer limited by rigid bedrock.

Keywords: BEM-TLM; soil structure interaction; soil layer; harmonic waves

1. Introduction

The steady-state dynamic response of rigid surface foundations in viscoelastic homogeneous soils has been the subject of much research, in both the kinematic and the forced vibration analyses. For the study of rigid surface foundations and embedded foundations in a homogeneous half-space, several methods have been used to solve the soil-structure interaction problem. To simplify the problem linear-analysis techniques have been developed. One of the most commonly used approaches is the sub-structuring method that allows the problem to be analyzed in two parts (Kausel *et al.* 1978, Aubry *et al.* 1992, Pecker 1984). In this approach the analysis of foundation systems can be reduced to the study of the dynamic stiffness at the soil-foundation interface (known as impedance function) and driving forces from incident waves.

The determination of impedance functions and forces of movement related to the incident waves is a complex process. Several studies have been conducted on the dynamic response of foundation resting on viscoelastic homogeneous soils using the finite-element and boundary-element methods. Wong and Luco (1978) have shown the importance of the effect of non-verticality of SV, SH harmonics on the response of a foundation.

*Corresponding author, Ph.D.
E-mail: smessioud@yahoo.fr
[a]Professor

Apsel and Luco (1987) used an integral-equation approach based on Green's functions for multilayered soils determined to calculate the impedance functions of foundation. Using this approach, Wong and Luco (1986) studied the dynamic interaction between rigid foundations resting on a half-space. The finite-element method was applied by Kausel *et al.* (1978), Kausel and Roesset (1981) and Lin and Tassoulas (1987) to determine the behavior of rigid foundations placed on or embedded in soil layer limited by a rigid substratum. A formulation of the boundary-element method in the frequency domain has been developed to address wave-propagation problems of soil-structure interaction and structure-soil-structure which limits the discretization at the interface soil-foundation. In this approach, the field of displacement is formulated as integrals equation in terms of Green's functions Beskos (1987), Aubry *et al.* (1992), Qian and Beskos (1996), Karabalis and Mohammadi (1992), Mohammadi (1991). Celebi *et al.* (2006) used the boundary-element method with integral formulation (BIEM) to compute the dynamic impedance of foundations. The Betti-Rayleigh Dynamic Reciprocal Theorem is used by Cao *et al.* (2010), for déterminante the Green's functions of the soil layers mdeium. The finite element method having transmitting boundaries is considered for layered system considering soil-rock and rock-rock combinations for analysis the dynamic response of the foundation resting on homogeneous soil and rocks Kumar *et al.* (2013). A new scheme to calculate impedance matrices for axial-symmetric foundations embedded in halfspace medium was

http://www.techno-press.com/journals/eas&subpage=7 ISSN: 2092-7614 (Print), 2092-7622 (Online)

developed by Chung and Liou (2013), in this work the half-space medium can be approximated by increasing thickness of one layer stratum on rigid bedrock.

McKay (2009) used the reciprocity theorem based on the BIEM to analyze the influence of soil-structure interaction on the seismic response of foundations. However, Suarez *et al.* (applied the BIEM to determine the seismic response of an L-shaped foundation. In addition, experimental work has been carried out by researchers in Japan to determine the effect of soil-structure interaction on the response of real structure Fujimori *et al.* (1992), Akino *et al.* (1996), Mizuhata *et al.* (1988), Watakabe *et al.* (1992). The finite element method combined with the boundary element method (FEBEM) was used by Auersch (2013) to obtain the dynamic response of the infinitely long plates rested on a layer soil limited by a substratum.

The soil in natural state is rarely homogeneous and it can exist in a state with a hard rock at shallow depth, consisting of different strata with different properties. However the effects of no-homogeneity in the soil medium on the dynamic response of the foundations have not been addressed adequately in the past because of the obvious difficulty in analyzing this case. In this context, the analytical solutions of 3-D wave equations in cylindrical coordinates in layered medium with satisfying the necessary boundary conditions are employed by Liou (1989, 2013), Liou and Chung (2009). Gazetas (1980) is developed an analytical-numerical formulation for study the foundations on an elastic isotropic medium consisting of heterogeneous layers. Mandal and Baidya (2004), Mandal *et al*. (2013) used an experimental investigation for study the effect of layer in rigid base on the dynamic behavior of foundation under a vertical mod of vibration**.** Ahmad and Rupani (1999) presents an extensive investigation into the influence of key mechanical and geometrical parameters on horizontal impedance of square foundations resting on or embedded in a two-layer soil deposit. Using the cone model, the vertical dynamic response of foundation resting on a soil layer over rigid rock is studied by Pradhan *et al.* (2004). The results are presented in the term of vertical dynamic impedance. Using the BEM-TLM method, Sbartai and Boumekik (2008) have studied the dynamic response of two square foundations placed or embedded in soil layered limited by a substratum.

Recently, Messioud *et al.* (2016) studied the effects of soil structure interaction on the seismic response of foundation resting or embedded in the soil layered limited by a rigid bedrock under a harmonic seismic waves. In this study, the method has been applied to analyze the effect of some parameters on the dynamic response of the foundations (depth of the substratum, embedding, masses and shape of the foundation and frequency)..

In our study, the solution is derived from the BEM in the frequency domain with constant quadrilateral elements and the thin-layer method is used to analyze the effect of soil-structure interaction on the seismic response of the foundation resting on a viscoelastic soil layer limited by rigid bedrock. The results are presented as of displacements as a function of dimensionless frequency, angle of incidence (vertically and horizontally) and the effects of rigidity soil layer.

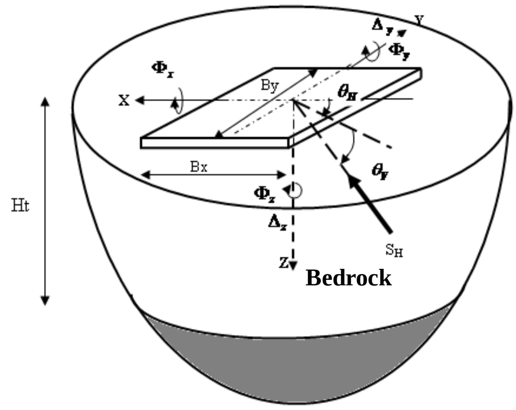

Fig. 1 Geometry of a foundation subjected to harmonic seismic waves

2. Physical model and basic equations

The geometry of the calculation model is shown in Fig 1. Consider 3-D rigid massless surface foundation of arbitrary shape *S* in full contact with a homogeneous, isotropic, and linearly-elastic soil that is limited by bedrock. The soil is characterized by its density ρ, shear modulus G, damping coefficient β and Poisson's ratio v. The foundation is subject to harmonic oblique-incident waves that are time-dependent: *P*, *SV*, *SH* and *R*.

The movement of a not-specified-point "ζ" can be obtained from solving the wave equation

$$\int (C_P^2 - C_S^2)\, u_{j,ij} + C_S^2\, u_{i,jj} - \omega^2 u_i = 0 \qquad (1)$$

Where C_s, and C_p are the velocities of shear and compression waves and ω the angular frequency of excitation.

u_i is the component of the harmonic displacement-vector in the *x*-direction;

$u_{j,ij}$ is the partial derivative of the displacement field with respect to *x* and *y*;

$u_{i,jj}$ is the second partial derivative of the displacement field with respect to *y*.

The solution of Eq. (1) may be expressed by the following integral equation

$$\int u_j(x,\omega) = \int_S G_{ij}(x,\xi,\omega)\ T_i(\xi,\omega)\ ds(\xi) \qquad (2)$$

With G_{ij} denoting the Green's functions at point *i* due to unit-harmonic load (vertical and horizontal) of the ground at point *j* and T_i being a load (traction) distributed over an area of soil.

The medium is continuous so this relationship is very difficult to assess. However, if the soil mass is discretized appropriately, this relationship can be made algebraic and displacement can be calculated. The key step of this study is to determine the impedance matrix linking the harmonic forces applied to the resulting harmonic displacement. Even with a continuous medium the determination of the impedance matrix is still very difficult, if not impossible, due to the propagation problem and its mixed-boundary conditions. However, if the medium is discretized vertically and horizontally then it is possible to making the problem algebraic by considering that the variation of interface displacement is a linear function

2.1 Discretization of the model

We limit ourselves to the general representation of the problem. If a source acts on (or in) a massif multilayered, a multitude of reflection and refraction takes birth to the level of the interfaces. Even if the nature of the primary incidental wave is known, it would be very difficult to know the nature of the resulting waves in the massif. The quantity of energy transmitted of a layer to the other by the refracted waves would be therefore difficult to determine.

Considering the pressure of confinement, the rigidity of the layers generally increases with the depth. The energy transmitted of a layer to the other decreases therefore with the depth. Besides, the Bedrock is (of very elevated rigidity) sufficiently deep, the reflection of the waves would be total to this level. The incidental and reflexive waves form this fact, in the massif, a system of stationary waves, whose energy would propagate itself mainly in the parallel direction to the interfaces (horizontally). It would be therefore reasonable, to simplify the survey of problem, to make the hypothesis of a horizontal propagation of the waves thought in the multilayered massif The principle of horizontal and vertical discretization of soil mass is shown in Fig. 2. The principle of vertical discretization based on the division of every soil layer into a number of sub-layers of height h_j with similar physical characteristics. Each sublayer is assumed to be horizontal, viscoelastic, and isotropic, and characterized by constant Lame λ_j, a shear modulus μ_i and a density ρ_j. The bedrock at depth H_t is considered infinitely rigid and is not discretized. The reflection wave is assumed to be total and the displacements null.

H_t: height of soil mass;

h_1: height of the sublayer 1;

N_x: number of elements in the x-direction for a horizontal plane;

N_y: number of elements in the y-direction for a horizontal plane;

N_z: number of soil layers.

n : number of soil layer

N is the total number of elements in soil-foundation interface and B_x and B_y are the dimensions of the foundation.

Within a given sublayer, the displacement is assumed to be a linear function of interface displacement above and below. This is true when the height of the sublayer is small in relation to the wavelength considered (in the order of λ/10). This method is comparable to the FEM in the sense that the movements within each sublayer are completely defined from the displacements in the middle of the interfaces. The interaction between the elements is done only through the nodes. The degrees of freedom of the soil mass are reduced to the degrees of freedom of the nodes. The stiffness matrix of soil mass is obtained in a similar manner to how it is determined in the FEM.

This technique has been developed by Lysmer and Waas (1972) and is known as the thin-layer method (TLM) and is

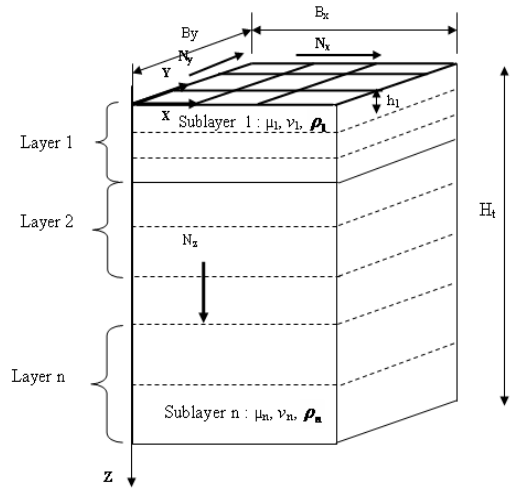

Fig. 1 The model of calculation

used mainly for horizontal soil layers. This method has the advantage to making the problem algebraic and thus obtains the Green’s functions by applying the boundary elements method in the soil-foundation interface. For this reason a horizontal discretization of the interface soil-foundation is established.

The horizontal discretization permits to subdivide any horizontal interface of soil-foundation by elements of square-sections S_k. These elements where the constant-moving average is replaced by the movement of the center, assumes that stress distribution is uniform and is shown in Fig. 2. Seeking simplicity of the integration calculation and economy of computing time the square elements are approximated disc elements. If the units loads (along the direction x, y, z) are applied to disc j, the Green’s functions at the center of disc i can be determined. By successively applying these loads on all discs, the flexibility matrix of soil complex at a frequency of a given ω can be formed. The discretized model to calculate the impedance functions of the foundation is also presented in Fig. 2. In the discrete model, Eq. (2) is expressed in algebraic form as follows

$$u_j = \sum_{i=1}^{N} \int_S G_{ij}\ T_i\ ds \tag{3}$$

The Green’s function G_{ij} for a layered stratum is obtained by an inversion of the thin-layer stiffness matrix using a spectral-decomposition procedure of Kausel and Peek (1982), Messioud *et al.* (2016). The advantage of the thin-layer-stiffness matrix technique over the classical transfer-matrix technique for finite layers and the finite-layer-stiffness matrix technique of Kausel and Roesset (1981) is that the transcendental functions in the layered-stiffness matrix are linearized.

3. Calculation model

The total displacement of the soil matrix is obtained by successive application of unit loads on the constituents of the discretized solid ground. The displacements in the soil are then expressed by

$$\{u\}=[G]\{t\} \tag{4}$$

The vectors $\{u\}$ and $\{t\}$ are the nodal values of the amplitudes of displacements and tractions respectively at the interface soil-foundation. $[G]$ is the flexibility matrix of the soil.

When the foundation is in place, it requires different components of soil displacement consistent with rigid body motions. Compatibility of displacements at the contact area S between the soil and the rigid foundation leads to the matrix equation

$$\{u\}=[R]\{\Delta\} \tag{5}$$

[R] is the transformation matrix

$$[R]=\begin{bmatrix} 1 & 0 & 0 & 0 & z & y \\ 0 & 0 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix} \tag{6}$$

$\{\Delta\}=\{\Delta_x, \Delta_y, \Delta_z, \Phi_x, \Phi_y, \Phi_z\}$ is the displacement vector; $\Delta_i(i=x,y,z)$ represents translations and $\Phi_i(i=x,y,z)$ rotations (Fig. 1).

If we denote $\{P\}$ the vector of load applied to the foundation, the equilibrium between the vector of loads applied and the forces (tractions) distributed over the elements discretizing the volume of the foundation is expressed by the following equation

$$\{P\}=[R]^t\{t\} \tag{7}$$

Combining Eqs. (4), (5) and (7) we obtain the following equation

$$\{P\}=\left([R]^t[G]^{-1}[R]\right)\{\Delta\}=[K(\omega)]\{\Delta\} \tag{8}$$

with ω is the circular frequency of vibration and $[K(\omega)]$ the impedance or dynamic-stiffness matrix of the rigid foundation.

Considering an incident plane, *SH*, *P*, *SV* and *R* harmonic waves are characterized by the vertical and horizontal angles of incidence θ_V and θ_H respectively, as shown in Fig. 1. The motion of the half-space due to these seismic waves can be expressed by the following equation

$$\{u^f\}=\{U^f\}e^{[-i\omega(x.\cos\theta_H+y.\sin\theta_H)/c]} \tag{9}$$

$\{U^f\}=\{U_x^f, U_y^f, U_z^f\}^t$, is known as the vector of amplitudes of the soil, that depends on the z coordinate if we want to study the embedded foundations case. However, in the case of surface foundations (z=0), it is known as the vector of amplitudes of the free field. c is the apparent velocity of the incident waves having the form $c=\frac{c_1}{\cos\theta_V}$ *or* $c=\frac{c_2}{\cos\theta_V}$ for P or S waves, respectively, and being equal to the R-wave. The explicit expressions of the vector $\{U^f\}$ of waves SH, P, SV and R may be found in Wong and Luco (1978).

The presence of a rigid foundation on the surface of the half-space results in diffraction of the above waves so that the total displacement field $\{u\}$ is expressed the following equation

$$\{u\}=\{u^f\}+\{u^s\} \tag{10}$$

where $\{u^s\}$ represents the scattered wave field that satisfies the equation of motion (5). Also, the total displacement field in the contact region between the foundation and the half space must be equal to the rigid body motion of the foundation.

$$\{u^s\}=[R]\{\Delta\}-\{u^f\} \tag{11}$$

Substituting Eq. (5) into Eq. (11), written in terms of the traction forces

$$[G]\{t\}=[R]\{\Delta\}-\{u^f\} \tag{12a}$$

hence

$$\{t\}=[G]^{-1}[R]\{\Delta\}-[G]^{-1}\{u^f\} \tag{12b}$$

Multiplying both sides of the Eq. (12b) by the transpose of the transformation matrix and combining with Eqs. (10) and (11) yields the external forces

$$[R]^t\{t\}=[R]^t[G]^{-1}[R]\{\Delta\}-[R]^t[G]^{-1}\{u^f\} \tag{13}$$

The equilibrium between external forces and seismic forces can be as follows

$$\{P\}=[K]\{\Delta\}-[K^*]\{U^f\} \tag{14}$$

with $[K^*]$ is the driving force matrix given by the following formula

$$[K^*]=[R]^t[G^{-1}]e^{[-i\omega(x.\cos\theta_H+y.\sin\theta_H)/c]} \tag{15}$$

Eq. (14) can be replaced by the alternative form

$$\{\Delta\}=[C]\{P\}+[S^*]\{U^f\} \tag{16}$$

where

$[C]=[K]^{-1}$ is the dynamic compliance matrix and $[S^*]$ is the input motion matrix given by the following formula

$$[S^*]=[C][K^*] \tag{17}$$

When the rigid foundation is acted upon by seismic waves only, the external forces are zero (P=0), and the seismic response of the foundation is obtained from Eqs. (15) or (17) by the following expression

$$\{\Delta\}=[S^*]\{U^f\} \tag{18}$$

When the mass of the foundation is not zero, one simply has to replace $[K]$ by $[K]-\omega^2[M]$ in the above equations, where $[M]$ is mass matrix of the foundation.

4. Validation of the method

The accuracy of the BEM-TLM method used in this paper to study the 3D-response of foundations subject to plane-harmonic waves with variable angles of incidence and dimensionless frequency a_o is validated through comparisons with results obtained by Luco and Wong (1977), Qian and Beskos (1996) for a semi-infinite ground. A parametric study was conducted to define the parameters of the calculation model. The influence of the discretization of the soil-foundation interface was studied. The thickness of a sub-layer "h" must be small enough that the discrete model can transmit waves in an appropriate manner and

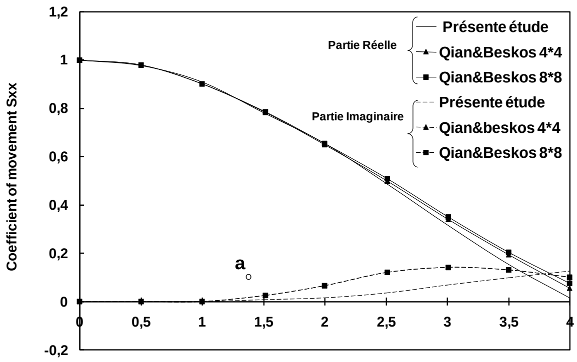

Fig. 4 The coefficient of movement S_{xx} a square foundation (θ_H=0°, θ_V=45° and c_s/c=0.70711)

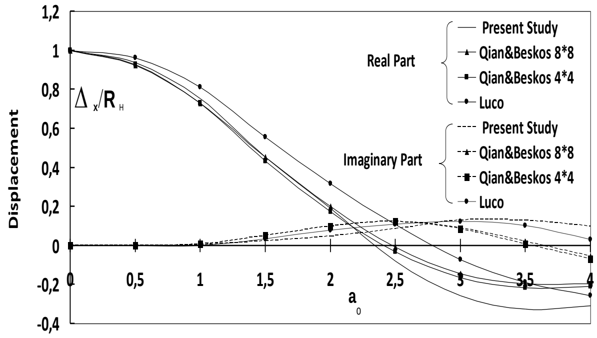

Fig. 5 The response a square foundation under the Rayleigh wave c_R/c=0.9325

without numerical distortion. This size depends on the frequencies involved and the velocity of wave propagation. The frequency of loading and velocity of wave propagation affect the precision of the numerical solution. Kausel and Peek (1982) showed that the thickness of sub-layer must be smaller than a quarter of the wavelength λ. Consequently, the maximum dimensionless frequency must not exceed the number of sub-layer N divided by four.

Consider a rigid massless square foundation of side B_x=2a on the surface of the half-space with a Poisson's ratio of v=1/3 and subjected to plane *P*, *SV*, or *SH* harmonic waves (θ_H=90° and θ_V=45°). Fig. 4 shows the variation of the real and the imaginary part of coefficient S_{xx} movement based on the dimensionless frequency $a_0 = \frac{\omega}{C_s}\frac{B}{2}$. The results obtained by the proposed method are in agreement with those obtained by the method used by Qian and Beskos. Considering the same foundation subjected to a Rayleigh wave where the angle of horizontal incidence is (θ_H=0) and the corresponding velocity taken is equal to c_R=0.9325.c for a Poisson's ratio v=1/3.

Fig. 5 shows the real and the imaginary part of dimensionless displacement $\Delta x/H_R$ function of frequency a_o. The results of this study using the BEM-TLM method were compared with those of Qian and Beskos.

The results obtained are in agreement with those of Beskos and Qian (1996) and those of Luco and Wong (1977). However, a difference is present for dimensionless frequencies a_0 higher than 2.5. This difference can be explained in two ways:

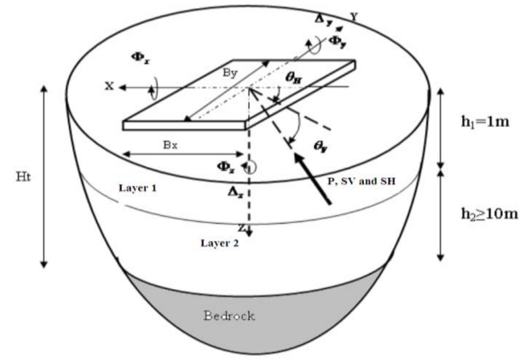

Fig. 6 Geometry of a rigid surface foundation posed in the heterogeneous soil subjected to seismic harmonic waves

Qian and Beskos isoparametric elements were used to determine the soil-foundation interface. These types of elements are more accurate than constant elements used by the method developed in this article.

Qian and Beskos and Luco and Wong use the Green's functions of a semi-infinite soil whereas our method used the Green's functions of soil bounded by bedrock.

5. Response of a massless foundation to incident P, SV and SH-waves traveled the layer medium

A square rigid surface dimension Bx=2a=1 m posed in a two layered soil limited by rigid bedrock and subject to *P*, *SV* and *SH* waves (Fig. 6) is considered. The medium is laminated and consists by two layers soil. Characteristics are the following: The layers soil is characterized by, their Poisson's ratio is v=1/3, the coefficient of the hysteretic damping β=0.05, the shear modulus G=1, and its density ρ=1. The height of the first layer soil is 1m and the height of the second layer soil is greater than 10 m. By varying the stiffness ratio of the first layer on second layer (G_1/G_2 = 0, 0.25, 0.5, 1). The wave velocity is calculated by the following relationship: $Cs = \sqrt{\frac{G}{\rho}}$

Where G and ρ are the characteristics of the first layer.

To present results that are easier to understand visually, the driving-force vectors are converted to input-motion vectors by multiplying by the inverse of the impedance matrix. The response of the massless foundation to incident body waves of type *P*, *SV SH* traveled the heterogeneous soil is considered. The nature of the *P*, *SV* and *SH* waves traveled the medium is determined depending on the characteristics of the first layer. It is assumed that the reflected energy transmitted by the waves propagating is parallel to the layers soil interfaces. The results are presented in terms of displacement and torsion as functions of the dimensionless frequency $a_0 = \frac{\omega}{C_s}\frac{B}{2}$.

5.1 Influence of the flexibility of heterogeneous soil on the seismic response

In this section of results, the apparent velocity of the *P*,

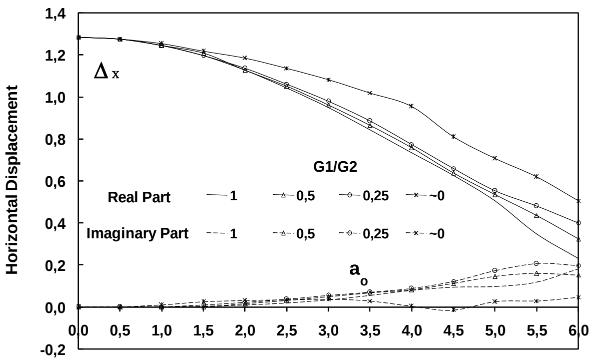

Fig. 7 Horizontal input motion Δ_x due to incident *P*-waves (θ_v=45°, θ_H=0°)

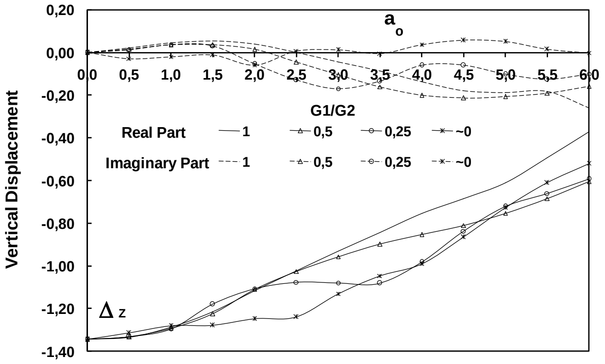

Fig. 8 Horizontal input motion Δ_z due to incident *P*-waves (θ_v=45°, θ_H=0°)

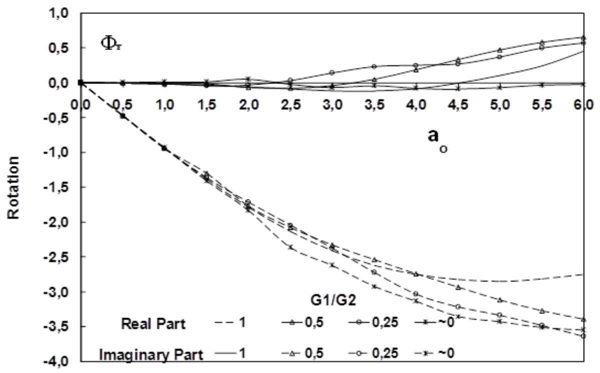

Fig. 9 Rocking input motion ϕ_y due to incident *P*-waves (θ_v=45°, θ_H=0°)

SV and *SH* waves is considered constant. The Flexibility matrix of soil layer is determined by the characteristics of the heterogeneous medium. The dynamic response of the foundation has been obtained for incident *P* and *SH* waves through the first layer soil (Fig. 6) for a vertical angle of incidence equal to 45° the amplitudes of motion in free field wave *SV* equal to zero.

Compression wave (P)

Figs. 12-14 show the response of a massless foundation to an incident *P*-wave. The wave travels in the *x*-direction with its particle motion in the *z* and *x*-directions. Except for the vertical incidence θ_v=90°, the incident *P*-wave causes a conversion mode and reflected *SV*-wave results. One

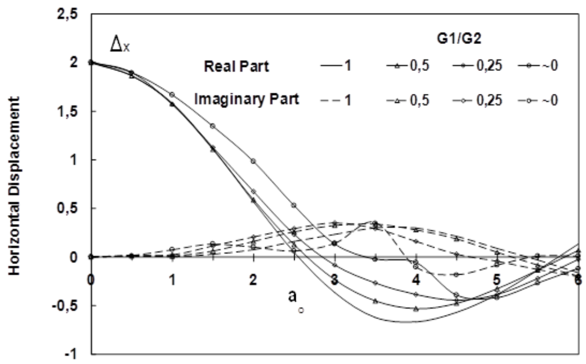

Fig. 10 Horizontal input motion Δ_x due to incident SH-waves (θ_v=45°, θ_H=90°)

considered a vertical angle of incidence of 45° measured with respect to the *x*-axis. The wavelength of the incident *P*-wave is twice as long as that of the incident S-waves; therefore, the kinematic interaction is less prominent. In general, the *P*-wave induces displacement along the *x* and *z*-axes and rotation around the y-axis.

Figs. 7-9 show the variation of displacements and rotation as a function of dimensionless frequency and the influence of the G_1/G_2 ratio on the input motion of foundation.

The displacements (Δ_x, Δ_z) and rotation (ϕ_y) are affected due to the diminution of shear ratio (rigidity) of two soil layers, especially for the high frequencies. In addition, the imaginary part of the vertical displacement Δ_z is strongly affected by the rigidity of first soil layer that the horizontal displacement Δ_x and rotation.

Shear Wave SH

The response of the square massless foundation to a SH-wave is presented in Figs. 10 and 11, with a horizontal angle of incidence θ_H=90°. The incident wave travels in the *y*-direction; therefore, the particle motion of the wave is in the *x*-direction. The shear wave causes displacement and torsion. Figs. 10-11 show the influence of G_1/G_2 ratio on displacement Δ_x and torsion ϕ_z.

Fig. 10 shows the influence of shear ratio G_1/G_2 on the seismic response of a square foundation resting on a layered medium. The real part of Fig. 10 shows, for the ratio G_1/G_2=1 (layered medium is homogeneous) the displacement of the foundation is strongly attenuated compared to displacement given by the low ratios of G_1/G_2. For the ratio G_1/G_2 = 0, the second layer is relatively rigid to the first layer, the figure shows that the displacement is strongly affected by the effect of environment. A significant increasing of the displacement has been marked and the frequency is changed compared to shear ratio (G_1/G_2 =1).

Fig 11 shows the influence of shear ratio G_1/G_2 on the variation of the torsion function of frequency. For high frequencies the imaginary part shows for G_1/G_2 ~0 (the second layer is infinitely rigid), the torsion is strongly affected by the effect of medium. For G_1/G_2=0.25, 0.5, 1 and for lower frequencies a_o=3, the torsion is weakly affected by the effect of medium.

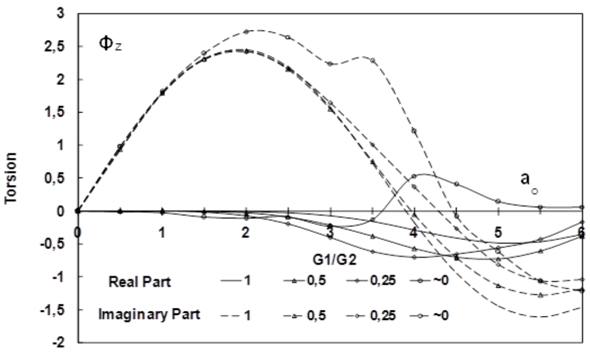

Fig. 11 Torsion-input motion ϕ_z due to incident SH-waves (θ_v=0°, θ_H=90°)

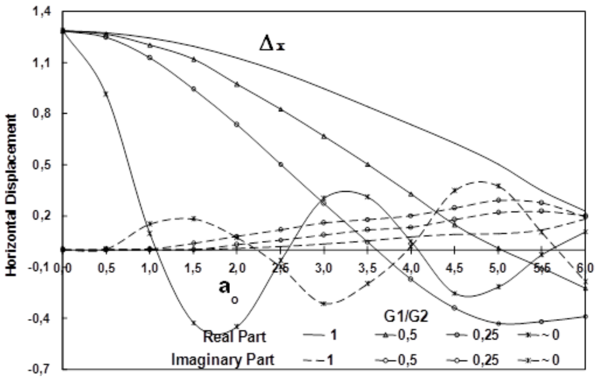

Fig. 12 Horizontal input motion Δ_x due to incident *P*-waves (θ_v=45°, θ_H=0°)

Figs. 10-11 show that the horizontal displacement and the torsion are strongly affected by the rigidity of layers soil. For dimensionless frequencies lower than 3 (a_o <3), the torsion ϕ_z is not affected by increasing the relative rigidity especially for the ratios G_1/G_2=0.25, 0.5, 1. However, the horizontal displacement Δ_x is strongly affected by an increase of the rigidity.

5.2 Influence of the apparent velocity on the seismic response of the foundation

Now, consider the calculation model presented in the Fig. 6. The apparent velocities of waves *P* and *SH* are calculated as function of characteristics of the first layer. In this section, the results are presented for *P* and *SH* waves for a vertical incidence angle 45° for the ratio G_1/G_2=0, 0.25, 0.5, 1

Compression wave (P)

Figs. 12-13 show the influence of G_1/G_2 ratios on the seismic response of a square foundation posed on stratified medium. Fig. 12 shows that horizontal displacement is strongly affected by the compressible layer soil (layer1). For shear ratio G_1/G_2 ~0, the appearance of resonance peaks in the real and imaginary parts has been marked, and the evolution of the displacement is does more controlled between negative and positive values. However, for shear ratios G_1/G_2=0.25 and 0.5, the variation of displacements is uniform with the real part of displacement Δ_x is strongly

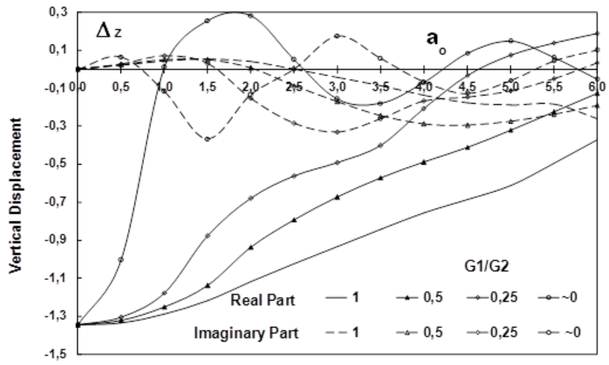

Fig. 13 Vertical input motion Δ_z due to incident *P*-waves (θ_v=45°, θ_H=0°)

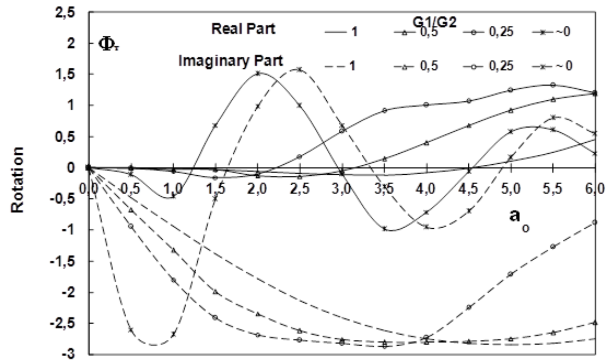

Fig. 14 Rocking input motion ϕ_y due to incident P-waves (θ_v=45°, θ_H=0°)

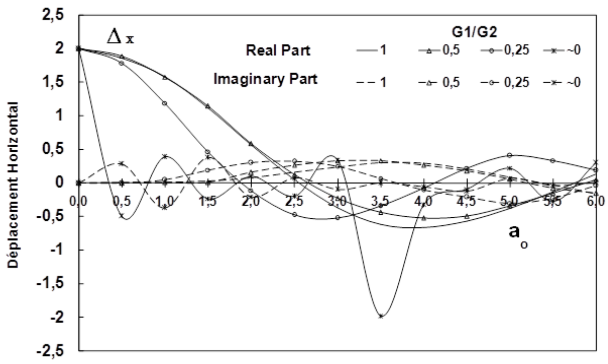

Fig. 15 Horizontal input motion Δ_x due to incident SH-waves (θ_v=45°, θ_H=90°)

attenuated that the displacement of the G_1/G_2=1. For high frequencies the imaginary part is increased according to the diminution of G_1/G_2 ratio. The same remarks are noted for displacement Δz except that the sign of displacement is changed.

Fig. 13 shows that the rotation is strongly affected by the effect of the stiffness of the first layer. For shear ratio G_1/G_2~0 the appearance of resonance peaks for low and high frequencies is noted, and the low frequencies are amplify and the rotation becomes considerable Fig. 14.

Shear Wave SH

Fig. 15 shows the influence of G_1/G_2 ratio on the

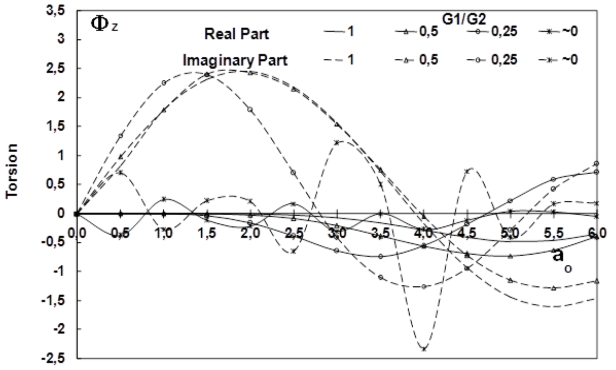

Fig. 16 Torsion-input motion ϕ_z due to incident SH-waves (θ_v=45°, θ_H=90°)

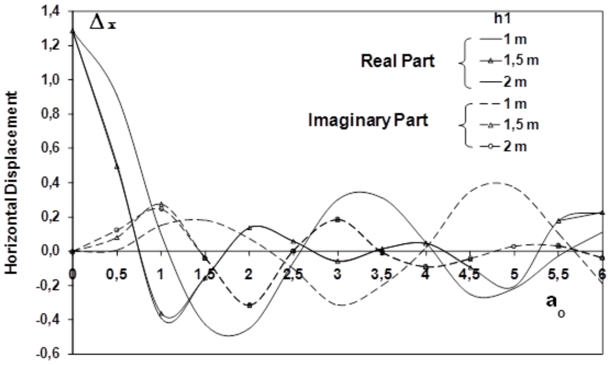

Fig. 18 Horizontal input motion Δ_x due to incident *P*-waves (θ_v=45°, θ_H=0°) G_1/G_2~0.0

seismic response of a square foundation resting on a layered medium. For the G_1/G_2~0, the horizontal displacement is strongly affected by the effect of medium.

A significant diminution of displacement has been marked, the lower frequencies are amplified and the evolution of displacement is not controlled. For the ratio G_1/G_2 =0.25, the real part and imaginary part for horizontal displacement are strongly affected by the rigidity of the first layer soil. The horizontal displacement is strongly attenuated that the displacement for homogeneous medium G_1/G_2 =0.25,1.

Fig. 16 shows the influence of G_1/G_2 ratio on the variation of the torsion versus frequency. For high frequencies and for the G_1/G_2~0 the imaginary part of torsion is strongly affected by the effect of medium and becomes important for low frequencies. For the ratios G_1/G_2=0.25, 0.5, 1 and for lower frequencies (a_o< 3), the torsion is weakly affected by the effect of medium. In contrast, for a report G_1/G_2~0, the torsion is strongly affected.

5.3 Influence of the depth of the first soil layer

Considered the calculation model presented in the Fig. 6. The apparent velocities of waves *P* and *SH* are calculated as function of characteristics of the first layer. The results are presented for *P* and *SH* waves for a vertical incidence angle 45° for the depth *h*1=1, 1.5 and 2 m.

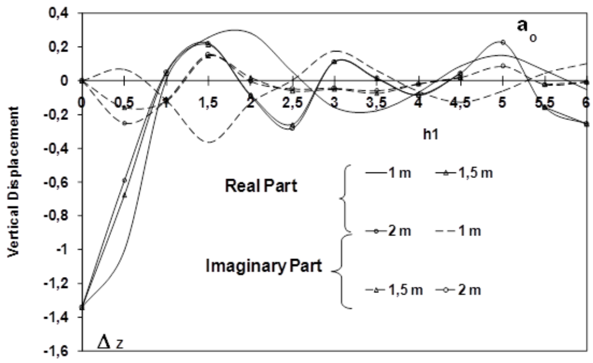

Fig. 19 Vertical input motion Δ_z due to incident *P*-waves (θ_v=45°, θ_H=0°) G_1/G_2~0.0

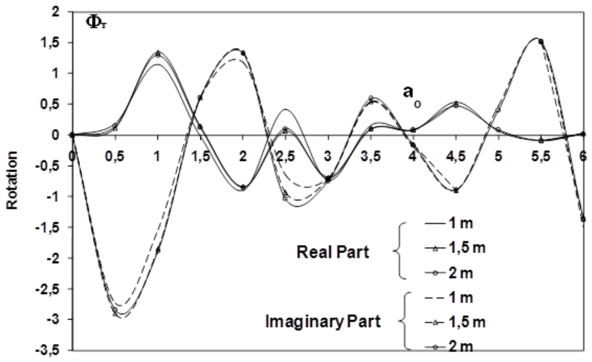

Fig. 20 Rocking input motion ϕ_y due to incident *P*-waves (θ_v=45°, θ_H=0°) G_1/G_2~0.0

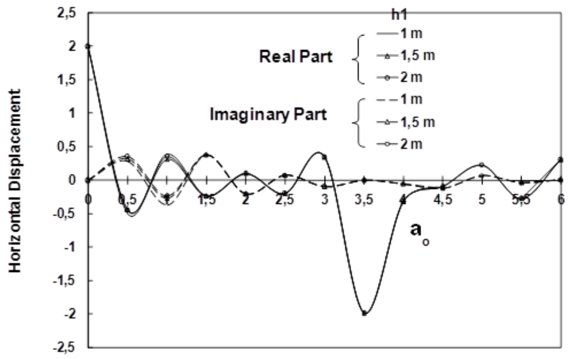

Fig. 21 Horizontal input motion Δ_x due to incident *SH*-waves (θ_v=45°, θ_H=90°) G_1/G_2~0.0

Compression wave (P)

Figs. 18-20 show the influence of the height of the compressible soil layer on the seismic response of the foundation. Figs. 18-19 show that Δ_x and Δ_z displacements are affected by the height of the compressible soil layer especially for height h_r=1.5 m. Beyond this height, the displacements are not affected. Fig. 20 shows that the rotation is not affected by the height of the compressible soil layer. There is a slight difference between the presented results and for some frequencies.

Shear Wave SH

Figs. 21-22 show the influence of the height of the

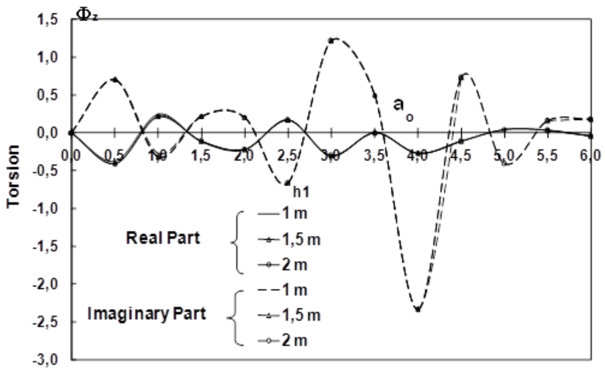

Fig. 22 Torsion-input motion ϕ_z due to incident SH-waves (θ_v=45°, θ_H=90°) G_1/G_2~0.0

compressible layer on the dynamic response of the foundation under the *SH* wave for G_1/G_2=0. These figures show that the displacement and torsion of the foundation is not affected by the height of the compressible soil layer.

6. Conclusions

The interaction of a seismic square-rigid foundation placed in a heterogeneous viscoelastic soil and subjected to obliquely incident harmonic *P*, *SV* and *SH* waves was implemented. A simplified BEM-TLM was developed and used to calculate the foundation-input motion under different travelling seismic waves. The solution was formulated by the boundary-element method in the frequency domain using the formalism of Green's functions. Constant quadrilateral elements were used to study the seismic response of a foundation. The efficiency of this technique was confirmed by comparison with results rigorously obtained by the use of the relationship between the solution of the radiation problem associated with the soil-impedance functions, and the solution of the scattering problem associated with the foundation-input motion. This remarkably simple technique was concluded to be both highly effective and economical to determine input motions for rigid foundations of arbitrary geometry. The originality of the method lies first in the insignificance of the number of elements used in the discretization of the model, and second, in the ability to simulate a medium constituted by several soil layers of which different characteristics.

This study shows the importance of the heterogeneous soil on the behavior of a foundation. The results indicate that:

The heterogeneity of soil affects the terms of displacements, rotations and torsion and therefore the dynamic behavior of the foundation especially for low frequencies. The movement of the foundation is strongly affected by the rigidity of second layer especially at high frequencies.

For the shear wave SH the results have shown that, for the compressible soil layer, the displacements and torsions are amplified, especially for the low frequencies. The resonance peaks are more marked, their amplitudes are weaker because of the reflections of the waves in the rigid soil layer and the movement of the foundation is not controlled.

Contrary to the SH-wave, the amplitudes of the displacement and rotation provoked by the *P*-wave are higher for low and high frequencies. For high frequencies the displacements and rotation are affected due to the diminution of shear ratio G_1/G_2 (rigidity) of soil layers. The imaginary part of the vertical displacement is strongly affected by the rigidity of first soil layer that the horizontal displacement and rotation. In addition, the imaginary part of the horizontal and vertical displacements is increased according to the diminution of G_1/G_2 ratio. As regards the range of motion in translation, that the real part of the displacement is significantly reduced by the presence of the compressible layer soil (for the shear ratio G_1/G_2~0 and 0.25).

For SH-wave, the variation of the depth of the first soil layer has not influence on the movement of the foundation.

Finally the heterogeneity of soil affects the terms of displacements, rotations and torsion and therefore the dynamic behaviour of the foundation especially for low frequencies.

In general, the analysis of the influence of soil heterogeneity is more complex because of the multitude reflexions in the interface of the layers soil and the foundation.

References

Ahmad, S. and Rupani, A.K. (1999), "Horizontal impedance of square foundation in layered soil", *Soil Dyn. Earthq. Eng.*, **18**(1), 59-69.

Akino, K., Ohtsuka, Y., Fukuoka, A. and Ishida, K. (1996), "Experimental studies on embedment effects on dynamic soil-structure interaction", *11th World Conference on Earthquake Engineering*, Paper No. 59.

Apsel, R.J. and Luco, J.E. (1987), "Impedance functions for foundations embedded in layered medium: An integral equation approach", *Earthq. Eng. Struct. Dyn.*, **15**, 213-231.

Aubry, D. and Clouteau, D. (1992), "A subdomain approach to dynamic soil-structure interaction", *Earthq. Eng. Struct. Dyn.*, 251-272.

Auersch, L. (2013), "Response to harmonic wave excitation of finite or infinite elastic plates on a homogeneous or layered half-space", *Comput. Geotech.*, **51**, 50-59.

Beskos, D.E. (1987), "Boundary element methods in dynamic analysis", *Appl. Mech. Rev.*, **40**, 1-23.

Cai, Y. and Hu, X. (2010), "Vertical vibrations of a rigid foundation embedded in a poroelastic half space", *J. Eng. Mech.*, **136**(3), 390-398.

Cao, Y.M., Xia, H. and Lombaert, G. (2010), "Solution of moving-load-induced soil vibrations based on the Betti-Rayleigh Dynamic Reciprocal Theorem", *Soil Dyn. Earthq. Eng.*, **30**(6), 470-480.

Celebi, E., Fırat, S. and Cankaya, I. (2006), "The evaluation of impedance functions in the analysis of foundations vibrations using boundary element method", *Appl. Math. Comput.*, **173**, 636-667.

Chung, I.L. and Liou, G.S. (2013), "Calculation of impedances for axial symmetric foundation embedded in half-space medium using solutions for one layer stratum", *Lat. Am. J. Solid. Struct.*, **10**(6), 1225-1241.

Fujimori, T., Tsundoa, T., Izumi, M. and Akino, K. (1992), "Partial embedment effects on soil-structure interaction", *10th World Conference on Earthquake Engineering*, **3**, 1713-1718.

Gazetas, G. (1980), "Static and dynamic displacements of foundations on heterogeneous multilayered soils", *Geotechnique*, **30**(2), 159-177.

Karabalis, D.L. and Mohammadi, M. (1991), "3-D dynamic foundation-soil-foundation interaction on layered Soil", *Soil Dyn. Earthq. Eng.*, **17**, 139-152.

Kausel, E. and Peek, R. (1982), "Dynamic loads in the interior of layered stratum: An explicit solution", *Bull. Seismol. Soc. Am.*, **72**(5), 1459-1481.

Kausel, E. and Roesset, J.M. (1981), "Stiffness matrix for layered soil", *Bull. Seismol. Soc. Am.*, **72**, 1459-1481.

Kausel, E., Whitman, R.V., Morray, J.P. and Elsabee, F. (1978), "The spring method for embedded foundations", *Nucl. Eng. Des.*, **48**, 377-392.

Kumar, A., Manna, B. and Rao, K.S. (2013), "Dynamic response of block foundations resting on soil-rock and rock-rock system under vertical excitation", *Ind. Geotech. J.*, **43**(1), 83-95.

Lin, H.T. and Tassoulas, J.L. (1987), "A hybrid method for three-dimensional problems of dynamics of foundations", *Earthq. Eng. Struct. Dyn.*, **14**, 61-74.

Liou, G.S. (1989), "Analytic solutions for soil-structure interaction in layered media", *Earthq. Eng. Struct. Dyn.*, **18**(5), 667-686.

Liou, G.S. (2013), "Vibrations induced by harmonic loadings applied at circular plate on layered medium", *J. Mech.*, **29**(2), 197-206.

Liou, G.S. and Chung, I.L. (2009), "Impedance matrices for circular foundation embedded in layered medium", *Soil Dyn. Earthq. Eng.*, **29**(4), 677-692.

Luco, J.E. and Wong, H.L. (1977), "Dynamic response of rectangular foundations for Rayleigh wave excitation", *6th World Conference of Earthquake Engineering*, New Delhi, India.

Lysmer, J. and Waas, G. (1972), "Shear waves in plane infinite structures", *J. Eng. Mech. Div.*, ASCE, **98**(1), 85-105.

Mandal, A. and Baidya, D.K. (2004), "Effect of presence of rigid base within the soil on the dynamic response of rigid surface foundation", *Geotech. Test. J.*, **27**(5), 475-482.

Mandal, A., Baidya, D.K. and Roy, D. (2012), "Dynamic response of the foundations resting on a two-layered soil underlain by a rigid layer", *Geotech. Geolog. Eng.*, **30**(4), 775-786.

McKay, K. (2009), "Three applications of the reciprocal theorem in soil-structure interaction", Ph.D. Thesis, University of Southern California.

Messioud, S., Sbartai, B. and Dias, D. (2016), "Seismic response of a rigid foundation embedded in a viscoelastic soil by taking into account the soil-foundation interaction", *Struct. Eng. Mech.*, **58**(5), 887-903.

Mizuhata, K., Kusakabe, K. and Shirase, Y. (1988), "Study on dynamic characteristics of embedded mass and its surrounding ground", *9th World Conference on Earthquake Engineering*, **3**, 679-685.

Mohammadi, M. (1992), "3-D dynamic foundation-soil-foundation interaction by BEM", Ph.D. Thesis, University of South Carolina, Columbia.

Pecker, A. (1984) *Dynamique Des Sols,* Presses de l'Ecole Nationale des Ponts et Chaussées.

Qian, J. and Beskos, D.E. (1996), "Harmonic wave response of two 3-D rigid surface foundations", *Earthq. Eng. Struct. Dyn.*, **15**, 95-110.

Sbartai, B. and Boumekik, A. (2008), "Ground vibration from rigid foundation by BEM-TLM", *ISET J. Technol.*, **45**(3-4), 67-78.

Suarez, M., Aviles, J., Francisco, J. and Sanchez-Sesma, F.J. (2002), "Response of L-shaped rigid foundations embedded in a uniform half-space to travelling seismic waves", *Soil Dyn. Earthq. Eng.*, **22**, 625-637.

Watakabe, M., Matsumoto, H., Ariizumi, K., Fukahori, Y., Shikama, Y., Yamanouchi, K. and Kuniyoshi, H. (1992), "Earthquake observation of deeply embedded building structure", *10th World Conference on Earthquake Engineering*, **3**, 1831-1833.

Wong, H.L. and Luco, J.E. (1978), "Dynamic response of rectangular foundations to obliquely incident seismic waves", *Earthq. Eng. Struct. Dyn.*, **6**, 3-16.

Wong, H.L. and Luco, J.E. (1986), "Dynamic interaction between rigid foundations in a layered half-space", *Soil Dyn. Earthq. Eng.*, **5**(3), 149-158.

CC