Earthquakes and Structures, *Vol. 15, No. 6 (2018) 595-603*
DOI: https://doi.org/10.12989/eas.2018.15.6.595

Statistical reference values for control performance assessment of seismic shake table testing

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(Received May 10, 2018, Revised August 23, 2018, Accepted October 17, 2018)

Abstract. Shake table testing has been regarded as one of the most effective experimental approaches to evaluate seismic response of structural systems subjected to earthquakes. However, reproducing a prescribed acceleration time history precisely over the frequency of interest is challenging because shake table test systems are eventually nonlinear by nature. In addition, interaction between the table and specimen could affect the control accuracy of shake table testing significantly. Various novel control algorithms have been proposed to improve the control accuracy of shake table testing; however, reference values for control performance assessment remain rare. In this study, reference values for control performance assessment of shake table testing are specified based on the statistical analyses of 1,209 experimental data provided by the Seismic Simulator Laboratory of National Center for Research on Earthquake Engineering in Taiwan. Three individual reference values are considered for the assessment including the root-mean-square error of the achieved acceleration time history; the percentage of the spectral acceleration that exceeds the determined tolerance range over the frequency of interest; and the error-ratio of the achieved peak ground acceleration. Quartiles of the real experimental data in terms of the three objective variables are obtained, providing users with solid and simple references to evaluate the control performance of shake table testing. Finally, a set of experimental data of a newly developed control framework implementation for uni-axial shake tables are used as an application example to demonstrate the significant improvement of control accuracy according to the reference values provided in this study.

Keywords: seismic shake table; performance assessment; root-mean-square error; spectral acceleration; peak ground acceleration

1. Introduction

Seismic shake table testing has been considered as an effective and straightforward method to investigate dynamic responses of buildings or infrastructures subjected to ground motions for earthquake engineering studies. Generally, a seismic shake table test system consists of a variety of mechanical, electrical and hydraulic components or devices. Specimens to be tested are mounted on a rigid platen that is excited by servo-hydraulic actuators to replicate historical or artificially-generated ground accelerations up to six degrees of freedom. The actuators are controlled by a digital controller that integrates transducer conditioners, analog-to-digital converters, input-output pairs, and other electrical devices. Displacement and/or acceleration transducers are installed on the specimen to measure its dynamic responses. In this manner, the specimen behaves as it is directly subjected to a real earthquake ground motion and the corresponding response can be recorded and investigated. Furthermore, dynamic effects and rate-dependent behavior can be realistically represented because the specimen is directly subjected to earthquake ground motions in real time. Seismic responses of various structural systems have been investigated through shake table testing including reinforced concrete buildings (Lee *et al*. 2014), eccentrically braced frames (Lian and Su 2017), and wooden structures (Altunisik 2017).

The major difficulty and challenge to reproduce a desired acceleration time history accurately for a shake table test are to overcome the coupling of dynamics between the shake table and the test structure which is known as the control-structure interaction (CSI) (Dyke *et al*. 1995). Conventionally, the servo-hydraulic actuators used to excite seismic shake tables are displacement-controlled by a proportional-integral-differential (PID) controller which generates the control command by taking a combination of proportional, integral and derivative action on the difference between the desired and achieved

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ISSN: 2092-7614 (Print), 2092-7622 (Online)

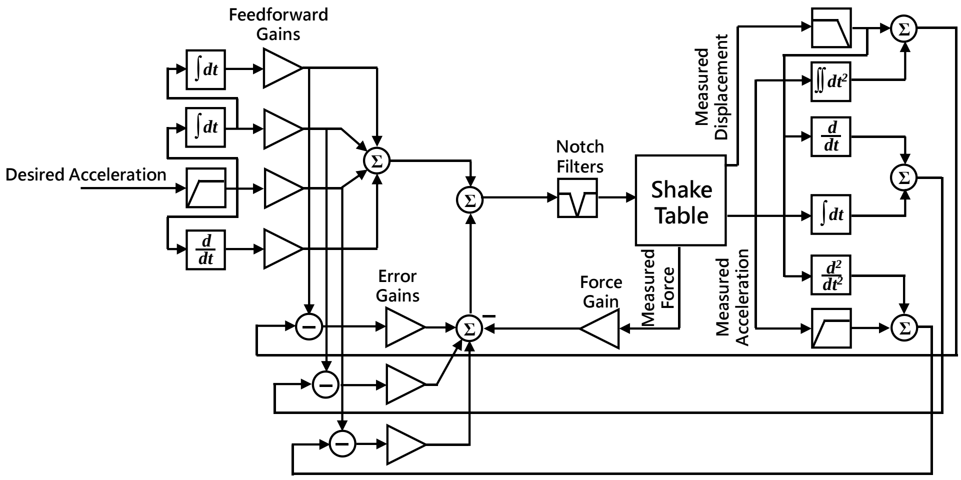


Fig. 1 Block diagram of a TVC control loop

displacements. The PID controller used for displacement control provides reasonable performance in the low frequency range; however, the accuracy of acceleration reproduction is not guaranteed over the frequency of interest. On the other hand, velocity and acceleration feedback can be taken and added to the displacement control loop, leading to a wider system frequency bandwidth and improvement of system stability (Tagawa and Kajiwara 2007, Yao *et al*. 2011). One of the most well-known applications is the three-variable controller (TVC), which is also known as state variable control and has been widely implemented by MTS Systems Corporation (Nowak *et al*. 2000). The three variables in a TVC control loop are displacement, velocity, and acceleration. Both feedforward and feedback loops utilize the three variables with corresponding control gains as shown in Fig. 1. It is noted that in a TVC control loop, only one primary state variable (can be displacement, velocity, or acceleration) is treated as the control target while the other variables are used for tuning to improve the system stability. For seismic shake table testing, acceleration is mostly regarded as the control target. In a TVC control loop, the desired acceleration is highpass-filtered to form the acceleration reference and integrated to obtain the displacement and velocity references. Meanwhile, the acceleration reference is differentiated to form the jerk reference. For the feedback signals, the velocity is estimated from the measured displacement and acceleration through a crossover filter. The tracking performance can be improved by tuning gains multiplied by the errors between the references and feedback signals. Notch filters and force feedback can be further used to compensate for CSI and suppress the oil column resonance, respectively. The TVC control loop has been recognized as one of the state-of-the-art control methods for seismic shake tables.

In addition to these extensively-used control approaches, various control schemes have been developed in the past decade to improve the tracking performance of seismic shake tables. Spencer and Yang (1998) proposed the transfer function iteration method in which the errors between the desired and achieved accelerations need to be iteratively modified offline until acceptable performance is obtained. Stoten and Gomez (2001) presented the minimal control synthesis algorithm for shake tables that allows online tuning the controller without any prior knowledge of system dynamics. Twitchell and Symans (2003) used an inverse of the identified transfer function from the reference to the measured displacements to generate the control command without iteration. Nakata (2010) presented a combined control strategy which includes command shaping, an acceleration feedforward controller, a displacement feedback controller, and a Kalman filter for measured displacements. Shen *et al*. (2011) combined a feedforward inverse model and an adaptive inverse controller to reduce the displacement and acceleration replica error for shake table testing. Phillips *et al*. (2014) proposed a model-based multi-metric control strategy that takes both displacement and acceleration measurements for control calculation. Yang *et al*. (2015) implemented a sliding mode controller to improve the shake table performance when the specimen contained serious nonlinearity. Chen *et al*. (2017) developed a control framework which incorporates a feedforward controller for improving the tracking performance as well as a feedback controller for strengthening the system robustness. It is undoubted that these aforementioned controllers improved tracking accuracy of seismic shake tables. Consequently, it is crucial to have reference values for comparing the control performance of newly-developed control schemes with that of commercial state-of-the-art control methods thoroughly and objectively. However, these reference values are still rare nowadays.

In this study, statistical reference values for control performance assessment of seismic shake table testing are specified. A total number of 1,209 experimental data are collected and collated from the seismic shake table tests completed in the Seismic Simulator Laboratory of National Center for Research on Earthquake Engineering (NCREE) in Taiwan from 2012 to 2014. Three individual objective variables are selected and analyzed to obtain the quartiles for reference values. Accordingly, the performance of a seismic shake table test can be evaluated by considering the three individual reference values. Finally, the given reference values are used to investigate the control performance of a uniaxial shake table before and after applying an outer-loop controller as an application example.

2. Objective variables

The existing methods that have been regularly used to evaluate the performance of shake tables, either can be quantized or illustrated schematically, can be divided into time-domain and frequency-domain aspects. It is recognized that the purpose of a shake table test is to duplicate a predetermined acceleration time history so that the corresponding dynamic response of a structural model fixed on the table can be investigated. In other words, shake table control essentially deals with a trajectory tracking problem up to a maximum of six degrees of freedom. Time-domain investigation is straightforward because it compares the time histories of desired and achieved accelerations. On the other hand, frequency-domain representation includes the information of magnitude and phase, giving a thorough and clear vision of the achieved acceleration. Without loss of generality, three common objective variables obtained from time-domain and frequency-domain analyses are selected. The advantages of the three objective variables are clarified in the section.

2.1 Normalized root-mean-square error

Generally, the tracking performance of a seismic shake table test considers the difference between the desired and achieved accelerations. The normalized root-mean-square (RMS) error provides a unit-free number and has been used as an index to evaluate the tracking performance of shake tables as it is simple and straightforward. The normalized RMS error of the achieved acceleration is defined as

$$\mathrm{RMS}_{error}(\%) = \sqrt{\frac{\sum_{k=1}^{N}\left(a_r[k]-a_m[k]\right)^2}{\sum_{k=1}^{N}a_r[k]^2}} \times 100\% \tag{1}$$

where N represents the number of data; $a_r[k]$ and $a_m[k]$ are the desired and measured accelerations at the k^{th} step, respectively. Less difference between the desired and measured accelerations leads to a smaller normalized RMS error; therefore, a lower normalized RMS error indicates better tracking performance. Besides, normalized RMS error is not affected by the intensity of ground motions to be reproduced as the square of error is divided by the square of reference. Therefore, it is considered appropriate as the first objective variable. For simplicity purposes, the normalized RMS error is named RMS error in this paper.

RMS error is sensitive to time lag and delay. Even a tiny time delay between the desired and achieved accelerations results in a significant RMS error. A more severe time delay leads to a larger RMS error. It is noted that time lag and delay between the desired and measured accelerations of a shake table test is not important because identical seismic response still can be expected as the specimen is subjected to a ground motion with only time lag and delay. Therefore, time-shifting correction must be completed before RMS error can be used as an objective variable for evaluating the shake table performance. The time-shifting value that leads to a minimum RMS error can be obtained by trying incremental time shifting when calculating the RMS error. Fig. 2 shows an example of the incremental time shifting approach. It is found that the RMS error reduces when the shifted time step increases, and reaches a minimum RMS error at 4 steps of shifting. Then, the RMS error starts to rise when the shifted time step increases. As a result, a 4-step shifting is adopted for the RMS error in this example.

2.2 Spectral acceleration

Spectral acceleration (SA) has been widely used to investigate the seismic responses of structures and nonstructural components in earthquake engineering studies. SA can be used to interpret the seismic response by a value related to the natural frequency of the structural vibration, providing an agreeable approximation to the motion of a structure. When a single-degree-of-freedom structure is subjected to an earthquake ground motion, its peak acceleration response can be obtained. By varying the natural frequency of the structure, the corresponding peak acceleration responses of the structure with various individual natural frequencies subjected to the same earthquake ground motion can be found. The peak acceleration of structure with different natural frequencies can be plotted to form a response spectrum of this specific ground motion. Therefore, SA provides an indirect method to evaluate the performance and accuracy of shake table testing because it depicts the peak acceleration response of a single-degree-of-freedom structure with a constant damping ratio subjected to an earthquake ground motion. The SA of an achieved acceleration is identical to that of a desired acceleration on condition that the shake table is able to reproduce the desired acceleration perfectly. Since SA considers the dynamic responses of a single-degree-of-freedom structure, it is considered much more representative than Fourier amplitude spectrum of the achieved acceleration which merely considers harmonic components of frequencies within the frequency range of interest. It is noted that if the CSI of shake table cannot be suppressed appropriately, the corresponding SA may result in significant error with the frequency range close to the natural frequency of the specimen

SA is not a single value for a specified ground motion as it forms a one-to-one mapping from varying frequencies within the interest of research. It is noted that the test response spectrum (SA of the achieved acceleration) needs to meet the required response spectrum (SA of the desired acceleration) with a tolerance range of 90% to 130% for shake table testing of nonstructural components and systems (AC156 2007). Similar criteria can be applied to seismic shake table testing for structures and bridges. Therefore, a tolerance range in terms of percentage of the SA within the earthquake frequency bandwidth can be determined prior to the test to ensure the test quality. Ideally, it is expected that the SA of a reproduced ground motion should be larger than that of the desired ground motion so that the seismic resistance capacity of a specimen is not overestimated. However, it is difficult to constantly satisfy the requirement for every single SA value within the

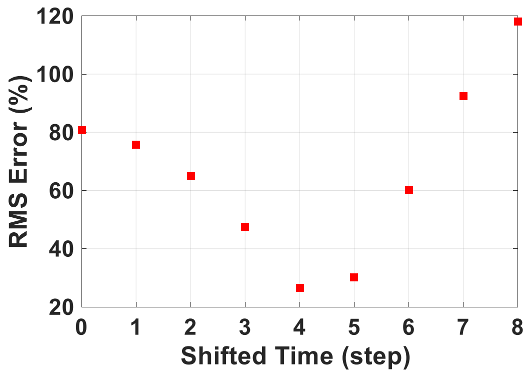


Fig. 2 Illustration of the time shifting approach for RMS error

frequency of interest. As a result, a tolerance range of 90% to 130% is suggested for the SA analysis following the experimental requirements of shake table testing for nonstructural components and systems.

The percentage of the SA value that exceeds the determined tolerance range over the frequency of interest is taken as the second objective variable in this study. It is known that the number of SA value within the frequency of interest depends on the number of the natural frequency adopted for the structural analysis. Different testing types could lead to tremendous different requirements for SA resolution. In addition, the increment between any two natural frequencies may not be constant absolutely. Any arbitrary set of unique natural frequencies can be used for conducting SA analysis. For example, the 1/6 octave method is based on a proportional bandwidth which indicates that each natural frequency of the structure is $2^{1/6}$ times the previous natural frequency. As a result, a percentage of the SA data points that lies outside of the tolerance range is adopted instead of a solid number of SA data points.

2.3 Peak ground acceleration

Peak ground acceleration (PGA) is an important factor for defining earthquake intensity. For shake table testing, PGA has become a critical input parameter for the test structures. A shake table test can be controversial if the PGA of the reproduced ground motion is less or larger than that of the desired acceleration. However, the RMS error and the SA analysis would not be affected significantly by the PGA difference especially when the rest data of the desired and achieved time histories match with each other well. As a result, the difference of PGA between the achieved and desired accelerations must be considered for the performance assessment of shake table testing. In this study, the PGA error ratio is adopted as the third objective variable for the control performance assessment reference. The PGA error ratio between the desired and achieved accelerations is defined as

$$\mathrm{PGA}_{error}(\%) = \left| \frac{P_d - P_a}{P_d} \right| \times 100\% \tag{2}$$

where P_d and P_a represent the PGA of the desired and achieved accelerations, respectively.

3. Seismic shake table of NCREE

The seismic shake table in Taipei Laboratory of NCREE was constructed in 1998 which is able to reproduce major earthquake ground motions that have been measured in the world in three orthogonal directions. The dimensions and weight of the shake table is 5 m×5 m and 270 kN, respectively. Experimental structures with a maximum payload of 500 kN can be accommodated on the table. The shake table is driven by 8 hydraulic actuators in the two orthogonal horizontal directions, and 4 actuators in the vertical direction. A total continuous flow rate of 4,675 liters per minute with a working pressure of 20.68 MPa is supplied by two electrical pumps and three diesel pumps. The entire shake table is mounted on a floating foundation made of 40 MN of reinforced concrete mega block, which is isolated from the outer structure by 96 air springs and 80 viscous dampers in order to suppress the vibration propagation to the research building of NCREE.

The seismic shake table in Taipei Laboratory of NCREE is controlled by a state-of-the-art MTS 469D digital control system, which allows user to perform controller tuning, system operation, and experimental execution in real time. The controller was upgraded from an analog controller to the 469D digital controller in 2007. The control software of the 469D digital controller provides high-level fixed control techniques including: TVC which allows for high fidelity reproduction across a wide frequency bandwidth; degree of freedom control which allows for controlling table motion in the Cartesian coordinate of the table rather than the coordinate of the actuator; force balance compensation which is aimed to compensate the over-constrained effect by adding more degrees-of-freedom controlled to zero; and differential pressure stabilization which is used for dampening oil column resonance to improve the fidelity of system performance.

In addition to the TVC, four adaptive controllers are available for the 469D digital control system, namely amplitude phase control (APC), adaptive harmonic cancellation (AHC), adaptive inverse control (AIC), and online iteration (OLI) (MTS, 2017). For sinusoidal waveforms, APC is used to eliminate amplitude and phase discrepancy between of desired and achieved responses for linear shake table test systems, while AHC is applied to add cancelling signals to the command to reduce harmonic distortion of shake table test systems with significant nonlinearity. For non-sinusoidal waveforms such us earthquake ground motions, AIC can be used to compensate the dynamics of shake table test systems and improve the input-output frequency response of the shake table. OLI adopts the inverse transfer function provided by the AIC and modifies the command to the control system by iteratively correcting the tracking error between the desired and achieved responses. It is noted that the four aforementioned adaptive controllers are implemented as a feedforward controller to shape the desired response as the reference for the TVC control loop so that control fidelity of the shake table can be further improved. Generally, AIC and

Table 1 The ground motions and the corresponding number of experimental data

Ground motion	Number of experimental data
El Centro	356
Kobe KJMA	388
Chi-Chi TCU068	38
Chi-Chi TCU076	138
Chi-Chi TCU078	19
Chi-Chi TCU084	94
Chi-Chi TCU129	176
Total = 1209	

Table 2 Mean value of the three objective variables of different ground motions

Ground motion	RMS error (%)	SA error (%)	PGA error (%)
El Centro	38.78	36.81	6.88
Kobe KJMA	33.90	30.66	4.97
Chi-Chi TCU068	38.57	24.70	4.03
Chi-Chi TCU076	40.61	27.51	5.13
Chi-Chi TCU078	27.45	64.84	5.87
Chi-Chi TCU084	41.43	24.94	9.76
Chi-Chi TCU129	32.80	13.53	4.09

OLI are adopted to generate the drive file of the 469D for seismic testing. Since the state-of-the-art control system is used to drive the shake table in Taipei Laboratory of NCREE, the experimental data collected for statistical analyses are considered representative as a primary benchmark for the proposed performance assessment reference values.

4. Statistical analyses of experimental data

A total number of 1,209 experimental data collected from the year of 2012 to 2014 were provided by the Seismic Simulator Laboratory of NCREE in Taiwan. These data were used to perform the statistical analyses in order to create a primary basis of reference values for evaluating the control performance of seismic shake table testing. Various types of specimens contributed to the 1,209 experimental data including multi-story steel structures, reinforced concrete shear buildings, base-isolated structures, high-tech equipment, and so on. It is known that the requirement of shake table testing varies from test to test. For simplicity purposes, only the longitudinal motion of the six degrees-of-freedom seismic shake table was considered in the statistical analyses. These experimental data were all sampled with a sampling rate of 200 Hz including the 1940 El Centro earthquake in California, the 1995 Kobe earthquake measured at the KJMA station, and the 1999 Chi-Chi earthquake measured at five different stations located in Taichung, namely TCU068, TCU076, TCU078, TCU084, and TCU129. Among the seven records, TCU078 has been recognized as near-source ground motions because the epicentral distance of the stations was only 8 km. Table 1 shows the number of experimental data of the seven different recorded ground motions. All the collected data were lowpass-filtered with a cutoff frequency of 50 Hz before conducting the statistical analyses. It is noted that the experiments that consider scaling of the historical acceleration records were not selected for the statistical analyses in this study for simplicity purposes. Furthermore, the repeatability of shake table tests is not discussed in detail in the study as it is implicitly involved in the statistical analysis results.

The mean value of each objective variable provides critical information of shake table testing control performance. Table 2 shows the mean value of the 1,209 earthquakes reproduced by the shake table in Taipei Laboratory of NCREE in terms of the three objective variables. The maximum and minimum mean value of the RMS error are 40.61% and 27.45%, respectively which is larger than expectation by instinct. This is because only one or two pre-executions of the ground motion reproduction in small intensity can be allowed for tuning the gains and parameters of the controllers before the real experiment. However, the dynamics of the shake table, by nature, is highly nonlinear. The gains suitable to reproducing the ground motion in small intensity are not necessarily appropriate to that in larger intensity. Furthermore, the dynamics of the specimen would vary during a shake table test due to the material and structural nonlinearity which could also affect the acceleration tracking accuracy. Definitely, the RMS error can be further reduced by employing an iterative process to reproduce the accelerations. However, it could potentially damage the specimen on the shake table during the iteration. It is noted that the acceleration RMS errors obtained from the real experimental data as shown in Table 2 are considered reasonable. Even under the application of advanced control techniques such as specimen dynamics compensation (Thoen 2016) and a control framework considering tracking performance and system robustness (Chen *et al*. 2017), the acceleration RMS errors are still larger than 20%. Fig. 3 shows the frequency distribution histogam of RMS error of the 1,209 experimental data which is very close to a normal distribution. It indicates that only few experiments have a RMS error less than 20% or larger than 50% among the 1,209 shake table testing data. Most of the RMS errors lie between 20% to 50%, providing a direct statistical sense on general tracking performance of a shake table test in terms of acceleration RMS error.

Spectral acceleration is the most demanding objective variable to be achieved because it evaluates the accuracy of reproduced ground motions through structural dynamic analyses over a broad range of frequency. Generally, a seismic shake table has its own frequency of operation depending on the design, fabrication, and calibration. For example, the nominal operational frequency of the seismic shake table in Taipei Laboratory of NCREE ranges from 1 Hz to 50 Hz. It appears that it is difficult to reproduce a ground motion which is dominated by the signals less than 1 Hz. In order to have the quartiles of the experimental data in terms of SA, a total of 200 natural frequencies of a single degree-of-freedom structure from 0.1 Hz to 20 Hz with 2%

Table 3 Quartiles of the three objective variables of the experimental data

Quartiles	RMS error (%)	SA error (%)	PGA error (%)
1st	30.94	15.00	2.02
2nd	36.21	28.50	4.37
3rd	42.16	42.00	7.51

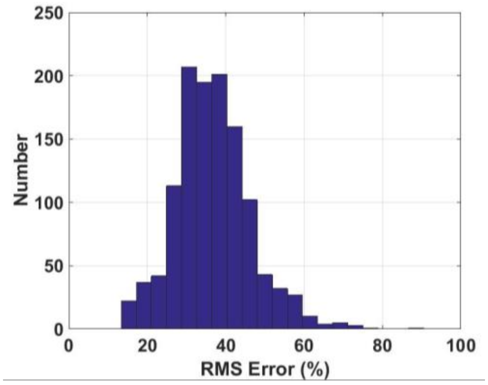


Fig. 3 Frequency distribution histogram of RMS error

damping ratio were used to conduct the structural analyses. In other words, the natural frequency of the structure varies from 0.1 Hz to 20 Hz with an increment of 0.1 Hz. Accordingly, the percentage of the 1,209 experimental data in terms of SA that exceed the tolerance range can be obtained. Table 2 indicates that the mean value in terms of the percentage of the SA data points of each ground motion that lies outside a tolerance range from 90% to 130% is mostly less than 37% except the one of TCU078. This is because TCU078 is a near-source ground motion record which contains much energy in the frequencies less than 1 Hz that is out of the operation frequency of the shake table. It can be demonstrated by surveying the Fourier Spectrum of the acceleration, velocity, and displacement as shown in Fig. 4. It is noted that the capability of a seismic shake table is dominated by displacement in low frequency range. The frequency components of TCU078 in displacement are mostly lied within the frequency range less than 1 Hz as shown in Fig. 4(c). Therefore, it is difficult to achieve acceptable SA error of TCU078. Fig. 5 shows the frequency distribution histogram of SA error of the experimental data. Unlike the distribution of RMS error, SA error of each test is almost equally distributed between 0% to 40% which shows that most of the SA errors of shake table testing lie in this region. The distribution histogram offers users a direct statistical sense on structural dynamic response of a shake table test in terms of SA error.

Peak ground acceleration of a ground motion has been considered a critical indicator for earthquake engineering studies because it affects how severely a building could be damaged during an earthquake. Generally, shake table testing is required mostly because researchers intend to realize the seismic resistance capacity of a specimen subjected to ground motions with incremental levels of seismic intensity. Accordingly, accurate replication of PGA is extremely important for researchers to assess the seismic response of specimen objectively. Overshoot or undershoot

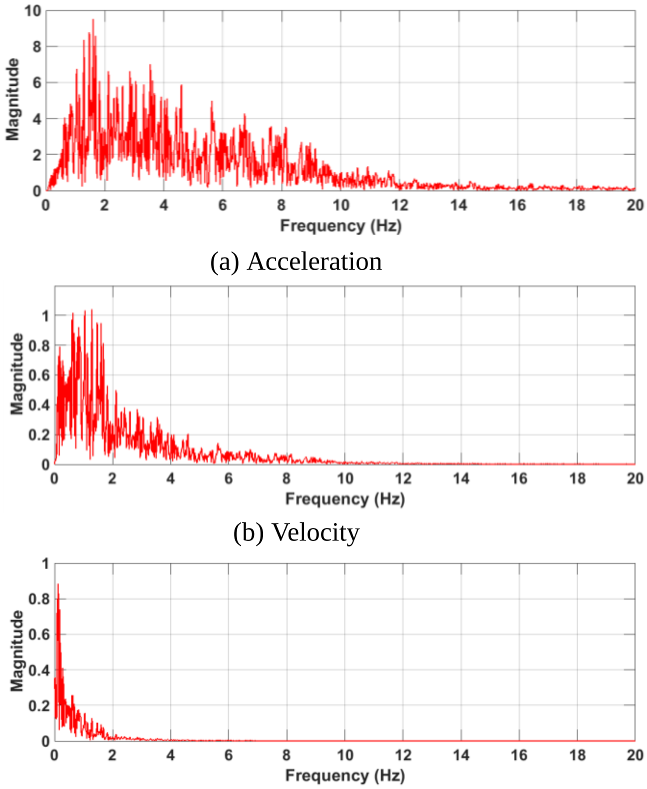


Fig. 4 Fourier spectrum of the Chi-Chi TCU078 record

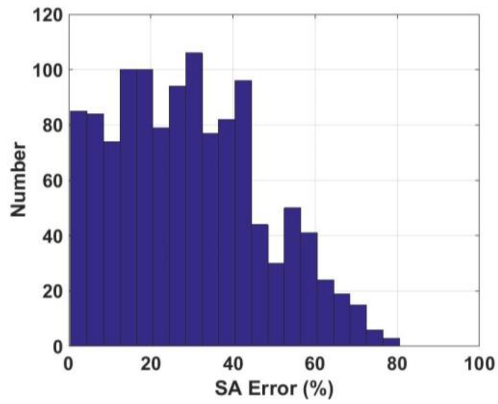


Fig. 5 Frequency distribution histogram of SA error

of PGA value could lead to misjudgment of real seismic resistance capacity of a structural model on the shake table. Therefore, the mean value in terms of PGA error of each ground motion is less than 10%, which is much smaller than the acceleration RMS error as shown in Table 2. This is because the technicians in the laboratory were asked to pay much attention to replicating PGA precisely during the one or two pre-executions of ground motion reproduction. Fig. 6 shows the frequency distribution histogram of PGA error of the experimental data which is similar to a half normal distribution. The maximum number of PGA error lies between 0% to 2%. Barely few shake table tests have a PGA error larger than 12% based on the statistical data. The distribution histogram delivers statistical information to evaluate the PGA replicative performance of a shake table

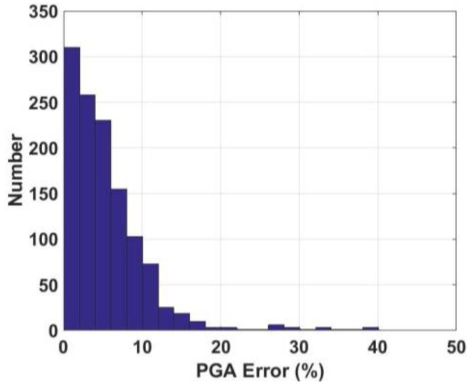
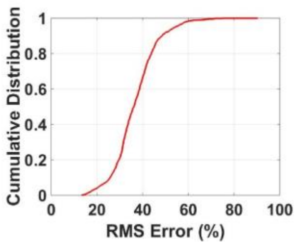
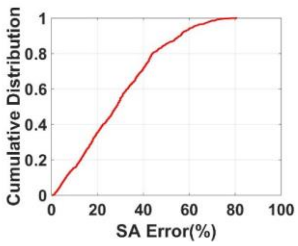


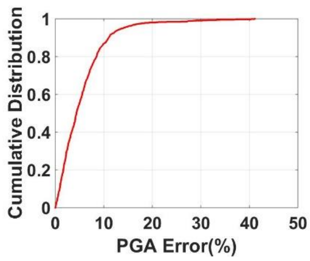
Fig. 6 Frequency distribution histogram of PGA error



(a) RMS error



(b) SA error



(c) PGA error

Fig. 7 Cumulative distribution curve of each objective variable

test in terms of PGA error.

It is suggested that overall performance assessment of shake table testing should consider all the three objective variables. Fig. 7 shows the cumulative distribution curve of each objective variable from the statistical analyses of the experimental data. It is found that the quartiles of each objective variable are nearly linearly allocated along the curve which indicates that the difference between the first and second quartiles and the difference between the second and third quartiles are almost equal. Table 3 lists the quartiles of the three objective variables from the 1,209 sets of experimental data. It is noted that quartiles do not require large information about the distribution; therefore, they can provide straightforward numbers for performance assessment. Accordingly, the performance for each objective variable can be determined. For example, if the RMS error of a shake table test is less than 30.94% as shown in Table 3, it ranks Q1 from the aspect of RMS error. In order to realize the overall control performance of a shake table test, it is straightforward to score the performance in terms of each objective variable. A shake table test scores three points, two points, one point, and zero while it ranks Q1, Q2, Q3, and Q4, respectively according

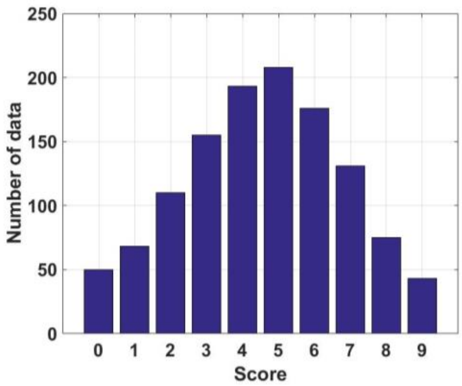


Fig. 8 Frequency distribution histogram of the overall score considering the three objective variables

to the quartiles as shown in Table 3. Finally, the control performance of a shake table test can be judged objectively based on the statistical data. Fig. 8 shows the frequency distribution histogram of the overall score among the 1,209 shake table tests. It shows that the overall control performance considering the three objective variables distributes like a normal distribution. Summarily, the proposed assessment references are based on statistical analyses of the 1,209 experimental data of real shake table testing in the laboratory of NCREE from the aspects of three individual objective variables, providing an objective basis for researchers to evaluate overall control performance of shake table testing.

5. Performance assessment application example

The proposed statistics-based performance assessment reference values are applied to evaluate the control performance of a uniaxial shake table as an application example. This uniaxial shake table was designed, fabricated, and calibrated at NCREE in 2013. A 2500 mm×1200 mm rigid platen was made of steel and driven by a domestically assembled servo-hydraulic actuator. The maximum stroke and force capacity of the actuator were ±250 mm and ±100 kN, respectively. Two individual two-stage servo valves manufactured by Star Hydraulic Ltd. were installed in parallel, providing a maximum flow rate of 454 liters per minute. A portable test controller manufactured by Moog Inc. was adopted to calibrate the displacement transducer and load cell as well as tuning the proportional and integral (PI) gains for the servo-valve. In order to reduce the friction force while the platen was moving, four hydrostatic bearings were used for the sliding mechanism. The allowable payload of the uniaxial shake table was 10 kN due to the pull-resistant capacity of each hydrostatic bearing. A self-balanced reaction frame was mounted on the strong floor in the laboratory through twelve high strength post-tensioned rods. A two-story steel specimen was designed and assembled to evaluate the control performance of shake table considering CSI as shown in Fig. 8. Four acceleration time histories were used including two historical earthquake records and two artificial earthquakes. For historical earthquake records, the

Table 4 Control performance specified by the proposed reference values of the application example

Earthquake	Controller	RMS error (%)	RMS error rank	SA error (%)	SA error rank	PGA error (%)	PGA error rank	Score
El Centro	PI	35.47	Q2	26.50	Q2	3.20	Q2	6
	SC + PI	27.96	Q1	18.50	Q2	3.30	Q2	7
Kobe KJMA	PI	48.75	Q4	28.00	Q2	5.80	Q3	3
	SC + PI	24.28	Q1	16.50	Q2	4.10	Q2	7
GR63-CORE	PI	34.54	Q2	25.00	Q2	8.30	Q4	4
	SC + PI	20.33	Q1	15.50	Q2	7.50	Q3	6
IEEE-693	PI	48.75	Q4	38.00	Q3	10.20	Q4	1
	SC + PI	16.59	Q1	12.50	Q1	9.90	Q4	6

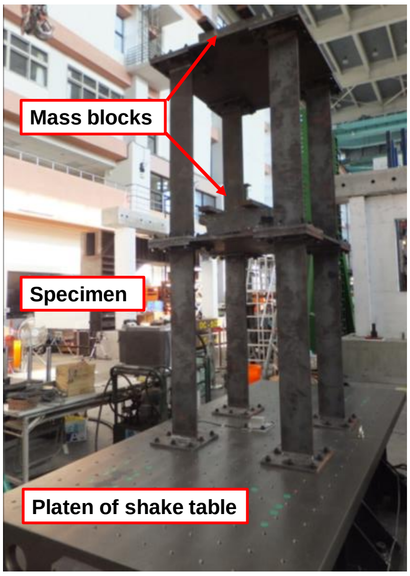


Fig. 9 Experimental setup of the performance assessment application example

far-field 1940 El Centro and near-field 1995 Kobe earthquakes were selected. For artificial earthquakes, the GR-63-CORE (GR-63-CORE 2012) and IEEE693 specifications (IEEE Std 693TM 2006) were adopted to generate the artificial acceleration time histories. The four selected acceleration time histories were normalized to a PGA of 1.0 m/s^2. For simplicity purposes, only two control schemes and their corresponding experimental results were selected to demonstrate the application of the proposed performance assessment reference values including the inner-loop PI controller (denoted as PI), and the PI controller with additional outer-loop weighted shaping controller (denoted as SC+PI). The controller design detail, hardware and software layout, and structural parameters of the specimen have been clearly documented in the reference (Chen *et al*. 2017).

The control performance of shake table testing can be evaluated based on the reference values given in this study. Table 4 shows the performance assessment result of each shake table testing. It is evident that the acceleration RMS errors are significantly reduced after applying the outer-loop controller; therefore, the score regarding the RMS error performance is increased accordingly. Again, 200 natural frequencies of a single degree-of-freedom structure from 0.1 Hz to 20 Hz with 2% damping ratio were used to obtain the SA error for each test. Apparently, the SA errors are significantly decreased after applying the outer-loop controller. However, the score regarding the SA error performance is not increased because the SA errors lie between the first and second quartiles before and after applying the outer-loop controller except the IEEE-693 case. This is because the frequency bandwidth of IEEE693 is wider than that of the other three earthquakes. The outer-loop controller is effective in high frequency range as it contains an acceleration controller to generate the high-frequency signal for the actuator (Chen *et al*. 2017). Therefore, the SA error in high frequency range of the IEEE-693 case is significantly reduced. Unlike the aforementioned two performance objects, the PGA errors are only slightly reduced after applying the outer-loop controller. For the El Centro test case, the PGA error is even increased. This is because the outer-loop weighted shaping controller was not specifically designed for accurately replicating the PGA but for compensating the dynamics of the servo-hydraulic system over the frequency of interest. In other words, PGA error is a single quantized point over the time history of a ground motion while RMS and SA errors are aimed to evaluate the overall performance of a shake table test in time and frequency domains, respectively. Accordingly, the outer-loop weighted shaping controller may not be effective on reducing PGA error. Summarily, it is demonstrated that the performance of shake table testing has been improved after applying the outer-loop weighted shaping controller. The improved performance is compared with the performance of the state-of-the-art shake table control from aspects of the three objective variables. Experimental results demonstrate that the control performance is improved significantly in terms of RMS error from Q4 to Q1 for the cases of Kobe and IEEE-693 based on the performance assessment reference values proposed in this study. The overall control performance is also improved to a score of 6 or 7 for the four ground motion cases.

6. Conclusions

A general-purpose statistical assessment reference values have been proposed to evaluate the control performance of shake table testing in this study. Three objective variables were selected as the reference values including the normalized root-mean-square error of the achieved acceleration time history (RMS error), the percentage of the spectral acceleration that exceeds the determined tolerance range over the frequency of interest (SA error); and the error-ratio of the achieved peak ground acceleration (PGA error). A total number of 1,209 sets of experimental data from real shake table testing completed in NCREE Taipei Laboratory were used to conduct the statistical analyses to obtain the quartiles of the three objective variables. The experimental data of each performance object were ranked and divided into four equal groups. Each group represents a quarter of the experimental data and is given a score of three points to zero from the first to the last group. Finally, overall control performance of a shake table test can be assessed according to the total score of the three objective variables.

The applicability and feasibility of the assessment reference values were examined by using a set of experimental data of a uniaxial shake table as an application example. This uniaxial shake table was designed and installed for the purposes of validating novel methodologies for shake table control. A two-story steel frame was installed on the table, providing control-structure interaction effect on shake table control performance. For simplicity purposes, only two control methods were adopted for the demonstration including conventional inner-loop proportional-integral control with and without a weighted shaping controller in the outer loop. It shows that the performance level of the shake table testing was improved after applying this outer-loop weighted shaping controller in terms of RMS error and SA error. Conclusively, the overall control performance of shake table testing can be evaluated by considering the proposed statistical reference values because these reference values are well grounded in the fact that the 1,209 real experimental data were obtained from a real shake table with a well-known state-of-the-art control system operated by experienced technicians. The statistical data can provide earthquake engineering researchers with simple and solid assessment references for shake table testing quality.

Acknowledgments

This study was supported by the National Center for Research of Earthquake Engineering (NCREE) in Taiwan. The authors would like to thank the technician Mr. Chi-Hung Wu, who have been operating the seismic shake table for more than 15 years for his sharing of the valuable experimental raw data.

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