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Analysis of wave propagation and free vibration of functionally graded porous material beam with a novel four variable refined theory

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Abstract. A free vibration analysis and wave propagation of functionally graded porous beams has been presented in this work using a high order hyperbolic shear deformation theory. Unlike other conventional shear deformation theories, a new displacement field that introduces indeterminate integral variables has been used to minimize the number of unknowns. The constituent materials of the beam are assumed gradually variable along the direction of height according to a simple power law distribution in terms of the volume fractions of the constituents. The variation of the pores in the direction of the thickness influences the mechanical properties. It is therefore necessary to predict the effect of porosity on vibratory behavior and wave velocity of FG beams in this study. A new function of the porosity factor has been developed. Hamilton's principle is used for the development of wave propagation equations in the functionally graded beam. The analytical dispersion relationship of the FG beam is obtained by solving an eigenvalue problem. Illustrative numerical examples are given to show the effects of volume fraction distributions, beam height, wave number, and porosity on free vibration and wave propagation in a functionally graded beam.

Keywords: higher-order beam theory; functionally graded beam; free vibration; wave propagation; porosity

1. Introduction

Composite materials have been used successfully in the civil engineering, aeronautics and other technological applications industries. However, traditional composite materials are unusable under a high temperature environment. Metals have been generally used in the field of technology for many years due to their excellent mechanical strength and hardness. However, under high temperature conditions, the mechanical strength of the metal becomes low as for traditional composite materials. Ceramic materials have excellent characteristics in resistance, thermal. However, the applications of ceramics are usually limited because of their low hardness. The development of composite materials has made it possible to associate specific properties to different materials within the same room. The local optimization of these properties by combining a high hardness material on the surface of the same tenacious material poses the problem of the interface, the abrupt transition in the properties of materials through the interface between discrete materials can cause a large inter-laminar stress or a high concentration of stresses leading to plastic deformation or cracking. A technique to overcome these adverse effects and to use a graded material (FUNCTIONALLY GRADED MATERIALS (FGM)). Recently, a new class of composite materials known as Functionally Graded Material (FGM), or gradient materials properties, has attracted special attention (Bouderba *et al.* 2013, Tounsi *et al.* 2013, Kar and Panda 2014, Ahmed 2014, Zidi *et al.* 2014, Zemri *et al.* 2015, Akavci 2016, Ahouel *et al.* 2016, Bounouara *et al.* 2016, Bellifa *et al.* 2016, Sekkal *et al.* 2017a, Bellifa *et al.* 2017a, Khetir *et al.* 2017, Abdelaziz *et al.* 2017, Besseghier *et al.* 2017, Meksi *et al.* 2018). Several studies have been carried out to analyze the behavior of plates and beams in FGM material. For example, Reddy (2000) has analyzed the static behavior of FGM rectangular plates based on his third-order shear deformation plate theory. Reddy and Cheng (2001) have presented a three-dimensional model for an FGM plate subjected to mechanical and thermal loads, both applied at the top of the plate. Vel and Batra (2004) has come closer to real behavior of structure by studying free vibration of FGM rectangular plates with three-dimensional solution. Zenkour (2005) presented the sinusoidal shear deformation plate theory to study buckling and free vibration of simply supported FG plates. Zenkour (2006) presented a generalized shear deformation theory in which the membrane displacements are expanded as trigonometric function across the thickness. Kadoli *et al.* (2008) investigated the bending response of FG beams by utilizing

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higher order shear deformation and numerical method. Malek zadeh (2009) studied the analysis of free vibrations of thick plates in FGM on elastic bases with two-parameter. Later some new shape functions were proposed by Ait Atmane *et al.* (2010), Benachour *et al.* (2011). Shahrjerdi *et al.* (2011) employed the second-order shear deformation theory to analyze vibration of temperature-dependent solar functionally graded plates. Behravan Rad (2012) investigated the static behavior of bi-directional functionally graded (FG) non-uniform thickness circular plate resting on quadratically gradient elastic foundations subjected to axisymmetric transverse and in-plane shear. Ould Larbi *et al*. (2013) presented an efficient shear deformation beam theory based on neutral surface position for bending and free vibration of FG beams. In the same way, Sobhy (2013) studied the vibration and buckling behavior of exponentially graded material sandwich plate resting on elastic foundations under various boundary conditions. Yaghoobi and Torabi (2013a) investigated the post-buckling and nonlinear vibration of imperfect FG beams. Yaghoobi and Torabi (2013b) examined analytically the large amplitude vibration and post-buckling of FG beams resting on non-linear elastic foundations. Ait Amar Meziane *et al*. (2014) presented an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Belabed *et al*. (2014) proposed an efficient and simple higher order shear and normal deformation theory for FG plates. Yaghoobi *et al*. (2014) studied the post-buckling and nonlinear free vibration response of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading using the variation iteration method (VIM). Behravan Rad and Shariyat (2015) obtained analytical solutions for FG porous variable thickness circular plates subjected to non-uniform shear and normal tractions and an external magnetic actuation. Ait Atmane *et al*. (2015) used a variationally consistent shear deformation theory for dynamic behavior of thick FG beams with porosities. Bourada *et al*. (2015) used the concept of the neutral surface position to develop a simple and refined trigonometric higher-order beam theory for bending and vibration behavior of FG beams. Attia *et al*. (2015) examined the dynamic response of FG plates with temperature-dependent properties by employing various four variable refined plate models. Bouguenina *et al*. (2015) presented a numerical analysis of FGM plates with variable thickness subjected to thermal buckling. Beldjelili *et al*. (2015) analyzed the hygro-thermo-mechanical bending response of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Behravan Rad (2015a, 2015b) analyzed the two-dimensional steady-state thermal stresses on a hollow, thick cylinder made of functionally graded materials and obtained a semi-analytical solution. Belkorissat *et al*. (2015) discussed vibration properties of FG nano-plate using a new nonlocal refined four variable model. Kolahchi *et al*. (2015) presented a size-dependent bending analysis of FGM nano-sinusoidal plates resting on orthotropic elastic medium. Mahi *et al*. (2015) developed a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Larbi Chaht *et al*. (2015) studied the bending and buckling behaviors of FG size-dependent nanoscale beams including the thickness stretching effect. Tagrara *et al*. (2015) investigated the bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams. Bousahla *et al*. (2016) examined thermal stability of plates with functionally graded coefficient of thermal expansion. Safari *et al*. (2016) studied the buckling of concrete columns retrofitted with Nano-Fiber Reinforced Polymer (NFRP). In another study, Bennai *et al*. (2015) proposed a novel higher-order shear and normal deformation theory for FG sandwich beams. Zamanian *et al*. (2017) analyzed the Agglomeration effects on the buckling behaviour of embedded concrete columns reinforced with SiO_2 nano-particles. Bouderba *et al*. (2016) discussed thermal stability of FG sandwich plates using a simple shear deformation theory. Draiche *et al*. (2016) used a refined theory with stretching effect for the flexure analysis of laminated composite plates. Bennoun *et al*. (2016) studied the vibration response of FG sandwich plates using a novel five variable refined plate theory. Kolahchi and Moniri Bidgoli (2016a) used a new sinusoidal model of size-dependent beams for the dynamic instability of single-walled carbon nanotubes. A buckling analysis of the armed concrete columns of carbon nanotubes was carried out by Jafarian and Kolahchi (2016b). Madani *et al*. (2016) used a differential cubic method for the analysis of the vibratory behavior of integrated FG-CNT reinforced piezoelectric cylindrical shells subjected to uniform and non-uniform temperature distributions. Kolahchi *et al*. (2016c) presented an analysis of the dynamic stability of visco-reinforced gradient plates based on an orthotropic elastomer medium. Study of the buckling of sandwich plates with FG-CNT reinforced layers based on an orthotropic elastic medium using Reddy's plate theory was carried out by Shokravi (2017). Chikh *et al*. (2017) presented thermal buckling analysis of cross-ply laminated plates using a simplified HSDT. El-Haina *et al*. (2017) proposed a simple analytical approach for thermal buckling of thick FG sandwich plates. Menasria *et al*. (2017) employed a new and simple HSDT for thermal stability analysis of FG sandwich plates. Fahsi *et al*. (2017) developed a four variable refined nth-order shear deformation theory for mechanical and thermal buckling analysis of FG plates. Fourn *et al*. (2018) proposed a novel four variable refined plate theory for wave propagation in FG material plates. Bakhadda *et al*. (2018) presented both dynamic and bending analysis of carbon nanotube-reinforced composite plates with elastic foundation. Recently, Tounsi *et al*. (2016) proposed a new 3-unknowns non-polynomial plate theory for buckling and vibration of FG sandwich plate. The nonlinear steady-state and dynamic behaviors of a functionally graded material plates were studied by Wang and Zu (2017a, 2017b). Behravan *et al*. (2017) presented a static analysis of the variable thickness of Functional Gradient Functional Two-layer Circular Plates (FGPM) based on a Hybrid Gradient Foundation with friction force and composite mechanical loads. Shokravi (2017) developed a sinusoidal shear deformation theory for the analysis of Dynamic pull-in and pull-out of viscoelastic nanoplates under electrostatic and Casimir forces. Bellifa *et al*. (2017b) proposed a nonlocal zeroth-order shear deformation theory for nonlinear post buckling of nano beams. Kolahchi and Cheraghbak (2017a) investigated the effects of agglomeration on stability of viscoelastic microplates reinforced with SWCNTs using the Bolotin method. Behravan Rad (2018) studied the Static analysis of non-uniform 2D functionally graded auxetic-porous circular plates interacting with the gradient elastic foundations involving friction force. For a longitudinally moving plate, the Rayleigh-Ritz method was applied to analyze the vibration under the condition of being immersed in an infinite liquid by Wang and Zu (2017c). A comparative study on the bending, vibration and buckling of viscoelastic sandwich nano-plates based on different nonlocal theories using DC, HDQ and DQ methods was studied by Kolahchi (2017b). Recently, Akbaş (2018) is presented a forced vibratory analysis of the porous deep beams functionally graded. Hadi *et al.* (2017) proposed sinusoidal-visco-piezoelastic theories for the dynamic buckling analysis of functionally gradient and functional gradient carbon nanotube functional layer sensors / actuators. Kolahchi *et al*. (2017c) proposed a dynamic buckling optimization method for sandwich nanocomposite plates with a sensor and actuator layer based on sinusoidal-visco-piezoelastic theories using the Gray Wolf algorithm. The seismic response of concrete underwater pipes carrying SiO2 nanoparticle-reinforced fluids and a fiber-reinforced polymer (FRP) layer is studied by Zarei *et al.* (2017). Shokravi (2017) presented vibration analysis of silica nanoparticles-reinforced concrete beams considering agglomeration effects. Shokravi (2018) presented a buckling analysis of integrated laminated sheets with reinforced composite CNT layers agglomerated using FSDT and DQM. Recently, shear deformation theories with reduced variables are developed by Houari *et al*. (2016), Hachemi *et al*. (2017), Mouffoki *et al*. (2017), Zidi *et al*. (2017), Klouche *et al*. (2017), Belabed *et al*. (2018), Yazid *et al*. (2018), Attia *et al*. (2018), Kaci *et al*. (2018), Youcef *et al*. (2018), Mokhtar *et al*. (2018).

The study of wave propagation in FG structures has also received a lot of attention from several researchers. Han and Liu (2002) investigated SH waves in FG plates, where the material property variation was assumed to be a piecewise quadratic function in the thickness direction. Chen *et al*. (2007) studied the dispersion behavior of waves in a functionally graded plates with material properties varying along the thickness direction. Sun and Luo (2011a) also studied the wave propagation and dynamic response of rectangular functionally graded material plates with completed clamped supports under impulsive load. Considering the thermal effects and temperature-dependent material properties, Sun and Luo (2011b) investigated the wave propagation of an infinite functionally graded plate using the higher-order shear deformation plate theory. Kolahchi *et al*. (2017d) presented a study of the wave propagation of integrated viscoelastic FG-CNT reinforced sandwich plates with sensor and actuator based on the refined zigzag theory. Recently, Benadouda *et al* (2017) used A shear deformation theory for the study of wave propagation in functional beams to gradually take into account the effect of porosity.

Porosities can occur within functionalized materials (FGM) during manufacturing due to technical problems that lead to the creation of micro-voids in these materials. The porous structures FG have many interesting combinations of mechanical properties. In order to deal with this type of problem, some studies on the effect of porosity in FG structures have been published in the literature; Wattanasakulpong *et al*. (2012) gives the discussion on porosities happening inside FGM samples fabricated by a multi-step sequential infiltration technique. Şimşek and Aydın (2012) examined the forced vibration of FG micro plates with porosity effects based on the modified couple stress theory. Wattanasakulpong *et al*. (2014) also give a discussion of the porosities occurring inside the FGMs produced by the sequential infiltration technique. Ebrahimi and Mokhtari (2015) provided DT method for vibration of rotating Timoshenko FG beams with porosities. Moreover, the wave propagation of an infinite FG porous plate based on various simple higher-order shear deformation theories has been studied by Ait Yahia *et al*. (2015). Kolahchi *et al*. (2017e) employed the visco-nonlocal-refined Zigzag and visco-nonlocal-piezoelasticity theories to analyze dynamic buckling of laminated nanoplates. Jahwari and Naguib (2016) investigated FG viscoelastic porous plates with a higher order plate theory and a statistical based model of cellular distribution. Boutahar *et al*. (2016) have presented non-linear free vibrations analysis of FG porous annular plates resting on elastic foundations. They concluded that porosity volume fraction and type of porosity distribution have a significant influence on the geometrically non-linear free vibration response of the FG annular plates at large amplitudes. Moreover, Mouaici *et al*. (2016) proposed an analytical solution for the vibration of FGM plates with porosities. The analysis was based on the deformation theory of shear with taking into account the exact position of the neutral surface. Akbas (2017a) studied the effect of porosity on Post-buckling. Boukhari *et al*. (2016) introduced an efficient shear deformation theory for wave propagation of functionally graded material plates. Recently, Ait Atmane *et al*. (2016) is study the effect of stretching the thickness and porosity on the mechanical response of a FG beam resting on elastic foundations. Akbas SD (2017) studied the thermal effects on the vibratory behavior of FG beams with porosity. A nonlinear static analysis of functional gradient girders with porosity under the thermal effect was carried out by Akbaş (2017b).

In the present work, an analytical study of the free vibration and wave propagation in FG porous beams using a new displacement model was presented. The beams are made of an isotropic material with material properties varying in the direction of the thickness. The wave propagation equations in the FG beam are derived using the Hamilton principle, which takes into account the effects of shear deformation and inertia rotation. The analytical relations of dispersion of the beam FG are obtained by solving a problem of eigenvalue. The vibration and phase velocity curves of the wave propagation in a functionally graded beam are plotted. The influences of porosity, volume

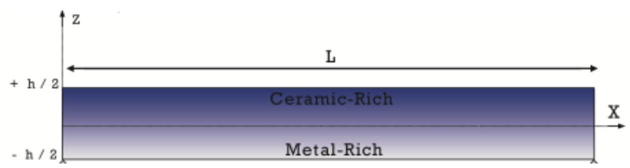


Fig. 1 Coordinates and geometry of functionally graded beam

fraction index and thickness ratio on the vibration and phase velocity of the wave propagation in the FG beam are clearly discussed.

2. Theory and formulation

Consider a porous FGM beam, with total height (h), length (L), and width (b) referred to the Cartesian coordinates (x, y, z) as shown in Fig. 1. The top and bottom faces of the beam are at $z=\pm h/2$, and the horizontal edges of the beam are parallel to axes x and y. The beam is subjected to transverse load of intensity $q(x)$ per unit length.

In this study, the FG beam is composed of a mixture of two types of material, for example a metal and a ceramic. The properties of the materials of the FG beam are assumed to vary continuously through the beam height. The beam is assumed to have porosities extending in thickness due to defect during production. Consider an imperfect FGM with a volume fraction of porosity, λ (λ<<1), distributed equally between metal and ceramic, the modified mixing rule proposed by Ankit Gupta and Mohammad Talha (2017) is used:

Now, the total volume fraction of the metal and ceramic is $V_m+V_c=1$, and the power law of volume fraction of the ceramic is described as

$$P = P_c\left(V_c - \log\left(1+\frac{\lambda}{2}\right)\right) + P_m\left(V_m - \log\left(1+\frac{\lambda}{2}\right)\right) \tag{1}$$

λ is termed as porosity volume fraction (λ<1). λ=0 indicates the non-porous functionally graded beam.

Now, the total volume fraction of the metal and ceramic is $V_m+V_c=1$, and the power law of volume fraction of the ceramic is described as

$$V_f = \left(\frac{1}{2}+\frac{z}{h}\right)^p \tag{2}$$

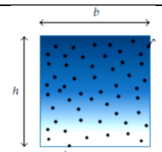
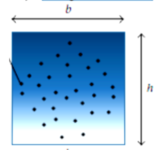
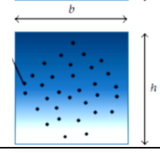
Where 'p' is the volume fraction index. The effective material property of porous FGM beam is given as

$$E(z) = [E_c - E_m]\left(\frac{2z+h}{2z}\right)^p - \xi[E_c + E_m]\left[1-\frac{2|z|}{h}\right] + E_m \tag{3a}$$

$$\rho(z) = [\rho_c - \rho_m]\left(\frac{2z+h}{2z}\right)^p - \xi[\rho_c + \rho_m]\left[1-\frac{2|z|}{h}\right] + \rho_m \tag{3b}$$

Where P denotes the effective material characteristic such as Young's modulus E and mass density ρ subscripts m and c denote the metallic and ceramic components, respectively. ξ it is the factor of the distribution of the porosity according to the thickness of the beam (Table 1). It is noted that the positive real number p ($0\leq p<\infty$) is the power law or volume fraction index, and z is the distance from the mid-plane of the FG beam. When p is sand to zero (p=0) the FG beam become a fully ceramic beam and fully metal beam for large value of p ($p=\infty$). In this study, Poisson's ratio (v) is considered to be constant (Yang *et al.* 2005, Kitipornchai *et al.* 2006, Tounsi *et al.* 2013). The material properties of a perfect FG beam can be evaluated by sand ting α zero.

Table 1 Factor of the distribution of porosity ξ.

	ξ	Geometric Shape
Wattanasakulpong *et al.* (2014)	$\frac{\lambda}{2}$	
Ankit Gupta *et al.* (2017)	$\log\left(1+\frac{\lambda}{2}\right)$	
present	$1-e^{-\frac{\lambda}{2}}$	

2.1 Kinematics relations

A new displacement field has been used in this article to reduce the number of unknowns in the HSDT theory (Bouchafa *et al.* 2015)

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_0}{\partial x} + f(z)\varphi(x,t) \tag{4a}$$

$$w(x,z,t) = w_0(x,t) \tag{4b}$$

where u_0 is the mid-plane displacement of the beam in the x direction, w is the bending and shear components of transverse displacement, respectively; and $f(z)$ is a shape function determining the distribution of the transverse shear are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the beam, thus a shear correction factor is not required. By considering that $\varphi = \int \theta(x)dx$, the displacement field of the present model can be expressed in a simpler form as (Bourada *et al.* 2016)

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_0}{\partial x} + K\, f(z)\int \theta(x,t)\,dx \tag{5a}$$

$$w(x,z,t) = w_0(x,t) \tag{5b}$$

Where

$$f(z) = \frac{\cosh\left(\frac{\pi}{2}\right)}{\cosh\left(\frac{\pi}{2}\right)-1} - \frac{\cosh((\pi/h)*z)}{\cosh\left(\frac{\pi}{2}\right)-1}$$

It can be seen that the displacement field in Eq. (5) introduces only four unknowns (u_0, w_0 and θ).The nonzero strains associated with the displacement field in Eq. (5) are

$$\varepsilon_x = \varepsilon_x^0 + z.k_x^b + f(z).k_x^s \quad , \quad \gamma_{xz} = g(z).\gamma_{xz}^0 \tag{6}$$

Where

$$\begin{aligned} &\varepsilon_x^0 = \frac{\partial u_0}{\partial x} \quad , \quad k_x^b = -\frac{\partial^2 w_0}{\partial x^2} \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta\, dy \\ k_1 \int \theta\, dx \end{Bmatrix} \\ &k_x^s = k\ \theta \quad , \quad \gamma_{xz}^0 = k \int \theta\, dx \end{aligned} \tag{7a}$$

$$g(z) = \frac{df(z)}{dz} \tag{7b}$$

And the integrals defined in the above equations shall be resolved by a Navier type method and can be written as follows

$$\int \theta\, dx = A' \frac{\partial \theta}{\partial x} \tag{8}$$

Where the coefficients A' is expressed according to the type of solution used, in this case via Navier. Therefore, A', k are expressed as follows

$$A' = -\frac{1}{\kappa^2}, K = \kappa^2 \tag{9}$$

Where κ is the wave number of wave propagation along x-axis direction. By assuming that the material of FG beam obeys Hooke's law, the stresses in the beam become

$$\sigma_x = C_{11}(z)\varepsilon_x \ \ \tau_{xz} = C_{55}(z)\gamma_{xz} \tag{10}$$

where (σ_x, τ_{xz}) and (ε_x, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eq. (1), stiffness coefficients, C_{ij}, can be given as

$$C_{11} = \frac{E(z)}{1-\nu^2}, C_{55} = \frac{E(z)}{2(1+\nu)} \tag{11}$$

2.2 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as

$$0 = \int_0^t (\delta U + \delta V - \delta K)\, dt \tag{12}$$

Where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the beam; and δK is the variation of kinetic energy.

The variation of strain energy of the beam is given by

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x \delta\, \varepsilon_x + \tau_{xz} \delta\, \gamma_{xz}\right] dV \\ &= \int_A \left[N_x \delta\, \varepsilon_x^0 + M_x^b \delta\, k_x^b + M_x^s \delta\, k_x^s + S_{xz}^s \delta\, \gamma_{xz}^0\right] dA = 0 \end{aligned} \tag{13}$$

where A is the top surface and the stress resultants N, M, and S are defined by

$$\left(N_x, M_x^b, M_x^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i\, dz \ , \quad S_{xz}^s = \int_{-h/2}^{h/2} g\, \tau_{xz} dz \tag{14}$$

The variation of the external work can be expressed as

$$\delta V = -\int_A q\delta\, w_0 dA - \int_A \left(N_x^0 \frac{\partial\, w_0}{\partial x} \frac{\partial \delta\, w_0}{\partial x} \right) dA \tag{15}$$

Where q and N_x^0 are transverse and in-plane applied loads, respectively.

For the free vibration and wave propagation problems, the external work is zero. The variation of kinetic energy of the beam can be expressed as

$$\begin{aligned} \delta K &= \int_V \left[\dot{u}\, \delta\, \dot{u} + \dot{w}\, \delta\, \dot{w}\right] \rho(z)\, dV \\ &= \int_A \Bigg\{ I_0\left[\dot{u}_0 \delta \dot{u}_0 + \dot{w}_0 \delta \dot{w}_0\right] - I_1\left(\dot{u}_0 \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial x} \delta\, \dot{u}_0 \right) + \\ &J_1\left((k_1 A')\left(\dot{u}_0 \frac{\partial \delta\, \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \delta\, \dot{u}_0 \right) \right) + I_2\left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta\, \dot{w}_0}{\partial x} \right) + \\ &K_2\left((k_1 A')^2 \left(\frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta\, \dot{\theta}}{\partial x} \right) \right) - J_2\left((k_1 A')\left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta\, \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta\, \dot{w}_0}{\partial x} \right) \right) \Bigg\} dA \end{aligned} \tag{16}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eq. (1); and (I_i, J_i, K_i) are mass inertias expressed by

$$\begin{aligned} (I_0, I_1, I_2) &= \int_{-h/2}^{h/2} (1, z, z^2)\rho(z) dz \\ (J_1, J_2, K_2) &= \int_{-h/2}^{h/2} (f, z\ f, f^2)\rho(z) dz \end{aligned} \tag{17}$$

By substituting Eqs. (13), (15) and (16) into Eq. (12), the following can be derived

$$\begin{aligned} &\delta\, u_0 : \ \frac{\partial N_x}{\partial x} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + k_1 A' J_1 \frac{\partial \ddot{\theta}}{\partial x} \\ &\delta\, w_0 : \ \frac{\partial^2 M_x^b}{\partial x^2} = I_0 \ddot{w}_0 + I_1\left(\frac{\partial \ddot{u}_0}{\partial x} \right) - I_2 \nabla^2 \ddot{w}_0 + J_2\left(k\, A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} \right) \\ &\delta\, \theta : -\mathrm{k}_1 M_x^s + k\, A' \frac{\partial S_{xz}^s}{\partial x} = -J_1\left(k\, A' \frac{\partial \ddot{u}_0}{\partial x} \right) - K_2\left((k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} \right) + \\ &J_2\left(k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} \right) \end{aligned} \tag{18}$$

Substituting Eq. (6) into Eq. (10) and the subsequent results into Eqs. (14), the stress resultants are obtained in terms of strains as following compact form

$$\begin{Bmatrix} N_x \\ M_x{}^b \\ M_x{}^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ k_x{}^b \\ k_x{}^s \end{Bmatrix}, S_{xz}^s = A^s \gamma_{xz} \tag{19}$$

Where

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}$$

$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{20a}$$

$$B^s = \begin{bmatrix} B^s_{11} & B^s_{12} & 0 \\ B^s_{12} & B^s_{22} & 0 \\ 0 & 0 & B^s_{66} \end{bmatrix}, D^s = \begin{bmatrix} D^s_{11} & D^s_{12} & 0 \\ D^s_{12} & D^s_{22} & 0 \\ 0 & 0 & D^s_{66} \end{bmatrix}$$

$$H^s = \begin{bmatrix} H^s_{11} & H^s_{12} & 0 \\ H^s_{12} & H^s_{22} & 0 \\ 0 & 0 & H^s_{66} \end{bmatrix} \tag{20b}$$

$$A^s = \begin{bmatrix} A^s_{44} & 0 \\ 0 & A^s_{55} \end{bmatrix} \tag{20c}$$

and stiffness components are given as:

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B^s_{11} & D^s_{11} & H^s_{11} \\ A_{12} & B_{12} & D_{12} & B^s_{12} & D^s_{12} & H^s_{12} \\ A_{66} & B_{66} & D_{66} & B^s_{66} & D^s_{66} & H^s_{66} \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \frac{1-\nu}{2} \end{Bmatrix} dz \tag{21a}$$

$$(A_{22}, B_{22}, D_{22}, B^s_{22}, D^s_{22}, H^s_{22}) = (A_{11}, B_{11}, D_{11}, B^s_{11}, D^s_{11}, H^s_{11}) \tag{21b}$$

$$A^s_{22} = A^s_{55} = \int_{-h/2}^{h/2} C_{44}[g(z)]^2 dz, \tag{21c}$$

Introducing Eq. (19) into Eq. (18), the equations of motion can be expressed in terms of displacements (u_0, w_0, θ) and the appropriate equations take the form

$$\begin{aligned} &A_{11}d_{11}u_{00} - B_{11}d_{111}w_0 + \left(B^s_{11}k\right) d_1\theta = \\ &I_0\ddot{u}_0 - I_1\, d_1\ddot{w}_0 + J_1\, A'k d_1\ddot{\theta}, \end{aligned} \tag{22a}$$

$$\begin{aligned} &B_{11}d_{111}u_0 - D_{11}d_{1111}w_0 + \left(D^s_{11}\, k\right) d_{11}\theta = \\ &I_0\ddot{w}_0 + I_1\left(d_1\ddot{u}_0\right) - I_2\left(d_{11}\ddot{w}_0\right) + J_2\left(k\ \ A'\ d_{11}\ddot{\theta}\right) \end{aligned} \tag{22b}$$

$$\begin{aligned} &-\left(B^s_{11}k\ \right) d_1u_0 + \left(D^s_{11}k\right) d_{11}w_0 - H^s_{11}k^2\ \theta + A^s_{55}\ (k\,A')^2\, d_{11}\theta = \\ &-J_1\left(k\,A' d_1\ddot{u}_0\right) + J_2\left(k\,A' d_{11}\ddot{w}_0\right) - K_2\left((kA')^2\, d_{11}\ddot{\theta}\right) \end{aligned} \tag{22c}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$\begin{aligned} &d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m} \\ &d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, d_i = \frac{\partial}{\partial x_i}, (i, j, l, m = 1,2). \end{aligned} \tag{23}$$

2.3 Dispersion relations

We assume solutions for u_0, w_0 and θ_0 representing propagating waves in the x plane with the form

$$\begin{Bmatrix} u_0(x,t) \\ w_0(x,t) \\ \theta_0(x,t) \end{Bmatrix} = \begin{Bmatrix} U \exp\left[i(\kappa x - \omega t)\right] \\ W \exp\left[i(\kappa x - \omega t)\right] \\ X \exp\left[i(\kappa x - \omega t)\right] \end{Bmatrix} \tag{24}$$

where U; W and X are the coefficients of the wave amplitude, κ is the wave numbers of wave propagation along x-axis direction, ω is the frequency, $\sqrt{i} = -1$ the imaginary unit.

Substituting Eq. (24) into Eq. (22), the following problem is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{13} & m_{23} & m_{33} \end{bmatrix}\right) \begin{Bmatrix} U \\ W \\ X \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{25}$$

where

$$\begin{aligned} &S_{11} = -\left(A_{11}\kappa^2\right), \ S_{12} = i \cdot \left(B_{11}\kappa^3\right), \ S_{13} = i \cdot \kappa\left(kB^s_{11}\right), \\ &S_{21} = -i \cdot \left(B_{11}\kappa^3\right), \ S_{22} = -\left(D_{11}\kappa^4\right), \ S_{23} = -k_1\left(D^s_{11}\kappa_1{}^2\right), \\ &S_{31} = -i \cdot \kappa\left(kB^s_{11}\right), \ S_{32} = -k_1\left(D^s_{11}\kappa_1{}^2\right), \\ &S_{33} = -\left(H^s_{11}k^2\right) - (kA')^2 A^s_{55}\kappa^2 \\ &m_{11} = -I_0, \ m_{12} = i \cdot \kappa I_1, \ m_{13} = -i \cdot J_1 kA'\kappa, \\ &m_{21} = -i \cdot \kappa I_1, m_{22} = -I_0 - (I_2\kappa^2) \\ &m_{23} = J_2\left(k\ \ A'\kappa^2\right), m_{31} = i \cdot J_1 k\ \ A'\kappa, \\ &m_{32} = J_2\left(k_1\, A'\kappa^2\right), m_{33} = -K_2\left(k\ \ A'\right)^2 \kappa^2 \end{aligned} \tag{26}$$

The dispersion relations of wave propagation in the functionally graded beam are given by

$$\left|\,[K] - \omega^2[M]\,\right| = 0 \tag{27}$$

The roots of Eq. (27) can be expressed as

$$\omega_1 = W_1(\kappa), \ \omega_2 = W_2(\kappa) \ \text{and} \ \omega_3 = W_3(\kappa) \tag{28}$$

They correspond to the wave modes M_1, M_2 and M_3 respectively.

The wave modes M_1 correspond to the flexural wave, the wave mode M_2 and M_3 corresponds to the extensional wave.

The phase velocity of wave propagation in the functionally graded beam can be expressed as

$$C_i = \frac{W_i(\kappa)}{\kappa}, \ (i = 1,2,3) \tag{29}$$

3. Numerical results and discussions

In order to analyze the effect of porosity on the vibratory and behavior and phase velocity of the FGM beams, illustrative examples have been presented in this part. A functionally graded beam is made from two Si_3N_4/ $SUS30_4$ materials; whose properties of these are presented

Table 2 Naturel frequencies of a porous FG beam for various thickness ratios, porosity parameters, power law indices and porosity distributions

l/*h*	*p*	λ=0			λ=0,1			λ=0,2		
		Nuttawit 2013	Gupta 2017	présent	Nuttawit 2013	Gupta 2017	présent	Nuttawit 2013	Gupta 2017	présent
5	0	48384,74	48384,74	48384,74	52657,67	50263,29	50262,47	59582,99	52412,18	52404,69
	0,5	32793,32	32793,32	32793,32	33266,79	33104,72	33104,59	33869,00	33424,90	33423,85
	1	28440,78	28440,78	28440,78	28350,97	28494,03	28494,01	28224,53	28537,72	28537,59
	5	22531,39	22531,39	22531,39	21925,12	22298,80	22298,90	21183,54	22037,26	22038,16
	10	21419,08	21419,08	21419,08	20744,85	21154,25	21154,36	19927,92	20858,52	20859,54
10	0	31574,16	31574,16	31574,16	34362,53	32962,52	32961,91	38881,75	34552,15	34546,61
	0,5	21375,69	21375,69	21375,69	21687,89	21684,49	21684,36	22087,94	22006,50	22005,43
	1	18560,14	18560,14	18560,14	18502,21	18691,14	18691,09	18420,97	18821,75	18821,33
	5	14868,36	14868,36	14868,36	14490,04	14822,66	14822,68	14026,84	14765,81	14766,02
	10	14117,28	14117,28	14117,28	13694,34	14050,12	14050,15	13182,73	13971,24	13971,52

Table 3 The phase velocities of a porous FG beam for various thickness ratios, porosity parameters, power law indices and porosity distributions

l/*h*	*p*	λ=0			λ=0,1			λ=0,2		
		Nuttawit 2013	Gupta 2017	présent	Nuttawit 2013	Gupta 2017	présent	Nuttawit 2013	Gupta 2017	présent
5	0	4838,47	4838,47	4838,47	5265,77	5026,33	5026,25	5958,30	5241,22	5240,47
	0,5	3279,33	3279,33	3279,33	3326,68	3310,47	3310,46	3386,90	3342,49	3342,38
	1	2844,08	2844,08	2844,08	2835,10	2849,40	2849,40	2822,45	2853,77	2853,76
	5	2253,14	2253,14	2253,14	2192,51	2229,88	2229,89	2118,35	2203,73	2203,82
	10	2141,91	2141,91	2141,91	2074,48	2115,42	2115,44	1992,79	2085,85	2085,95
10	0	3157,42	3157,42	3157,42	3436,25	3296,25	3296,19	3888,17	3455,22	3454,66
	0,5	2137,57	2137,57	2137,57	2168,79	2168,45	2168,44	2208,79	2200,65	2200,54
	1	1856,01	1856,01	1856,01	1850,22	1869,11	1869,11	1842,10	1882,17	1882,13
	5	1486,84	1486,84	1486,84	1449,00	1482,27	1482,27	1402,68	1476,58	1476,60
	10	1411,73	1411,73	1411,73	1369,43	1405,01	1405,01	1318,27	1397,12	1397,15

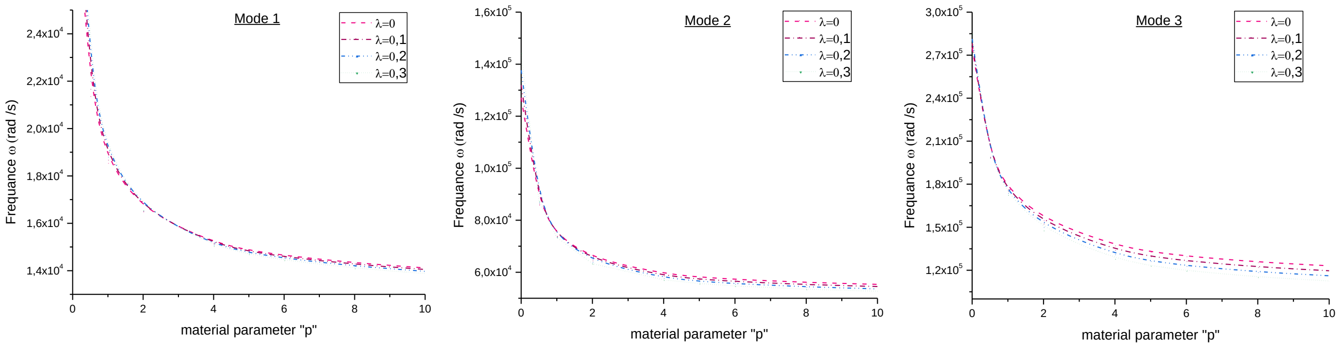


Fig. 2 Variation of the natural frequency of the FGM beam according to the material power index (κ=10 and *L*/*h*=10)

in the following table:

These properties change through the thickness of the beam according to the power law. The upper surface of FGM beam is rich in Si_3N_4 ceramic, while the lower surface of the FGM beam is rich in SUS_30_4 metal. The thickness of the functionally graded beam is taken *h*=0.02 m. various numerical examples are presented and discussed to check the accuracy of present theory in investigating the wave propagation and free vibration of FG beams. The analysis based on the present model is carried out using MAPLE.

Tables 2 and 3 present the frequencies and phase velocities of an FGM beam for the three formulas of the porosity distribution factor. From the results presented in this two tables, we can observe the values of the frequencies and the velocity obtained by the present model are in good agreement with those of the Gupta (Ankit Gupta *et al.* 2017) model for the two cases λ=0,1 and λ=0, 2 regardless of the value of the ratio *L*/*h*.

The variation curves of the natural frequency (ω) and the phase velocity for the first three modes of the various functionally graded beam, as a function of the material power index (p) for different values of the porosity were

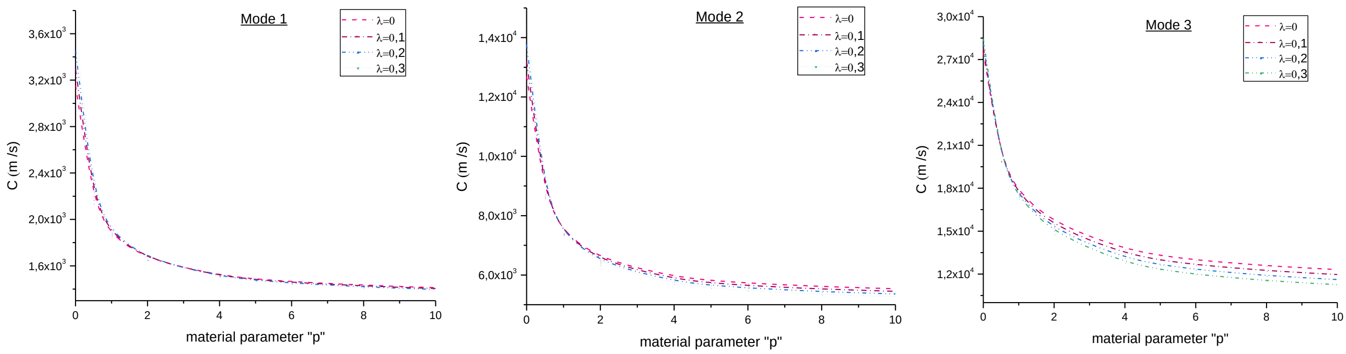


Fig. 3 Variation of the phase velocity of the FGM beam according to the material power index (κ=10 and L/h=10)

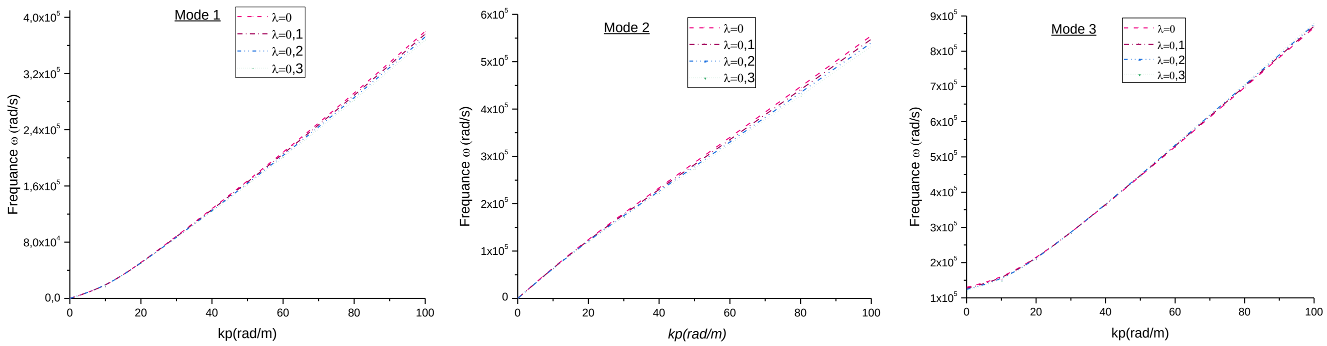


Fig. 4 The natural frequency curves of different functionally graded beam in terms of wave number (p=2 and L/h=10)

Table 4 The property of the materials used

Material	E (GPa)	ρ (kg/m$_3$)	v
Si_3N_4	2370	2370	0.3
SUS_30_4	201.04	8166	0.3

respectively presented in Figs. 2 and 3.

From Figs. 2 and 3, it can be observed that with increasing the power index, the natural frequency and the phase velocity in the FGM beams are decreased, regardless of the number of waves. In addition, it is deduced that the variations of natural frequency and the phase velocity are more sensitive to the porosity factor and especially for high material parameter. Therefore, the maximum frequency is obtained for a ceramic beam (p=0) and a porosity factor. λ=0,3.

The phase velocity and natural frequency in a homogeneous beam is the highest compared to the other FG beams. This is due to the rigidity of the ceramic beam which is the greatest rigidity. Therefore, it is confirmed that the material parameter has a great influence on the phase velocity and the natural frequency in the perfect FG beams.

Fig. 4 shows the frequency curves of the different FG beams as a function of the number of waves kp. From these curves, we notice that the frequency increases with the increase of the number of waves for the same material parameter. We can also observe that the frequency becomes maximal for a perfect beam (λ=0).

Fig. 5 shows the variation of the phase velocity of FG beam as a function of the number of waves with different values of the porosity. The material parameter is taken equal to p=2 and the thickness ratio L/h=10.

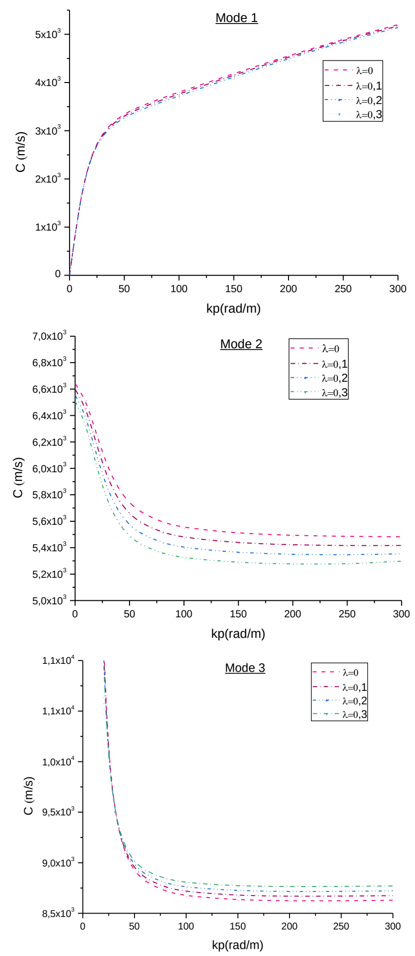
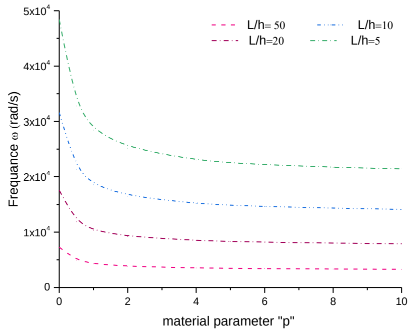
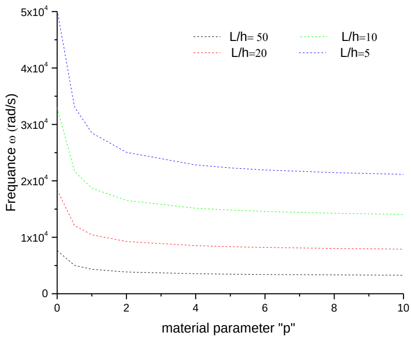


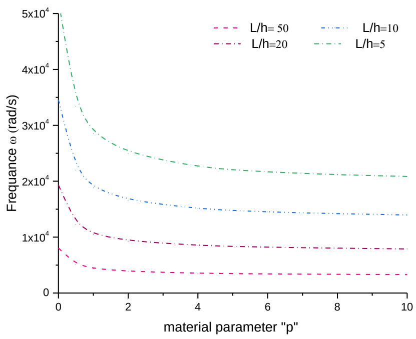
Fig. 5 The phase velocity curves of different functionally graded beam in terms of wave number (p=2 and L/h=10)



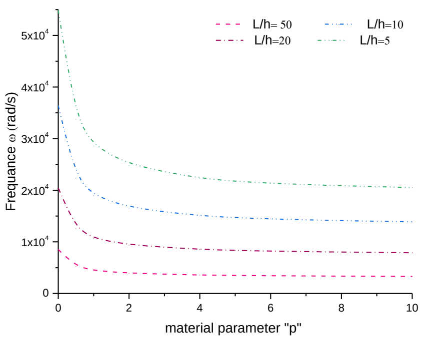
(a) λ=0



(b) λ=0,1

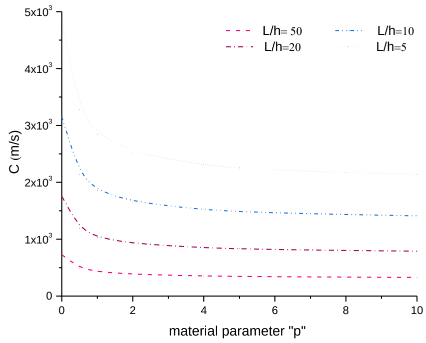


(c) λ=0.2

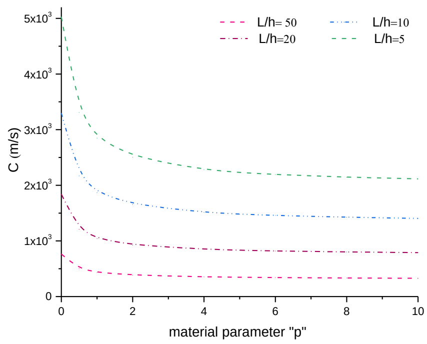


(d) λ=0.3

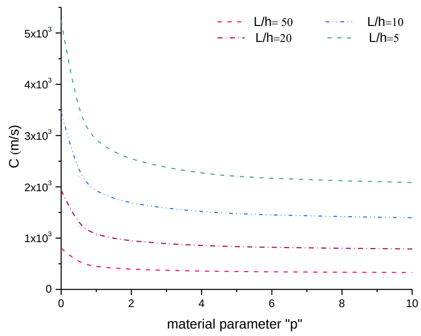
Fig. 6 Influence of thickness ratio on the natural frequency of the FG beam (k_p=10 and L/h=10)



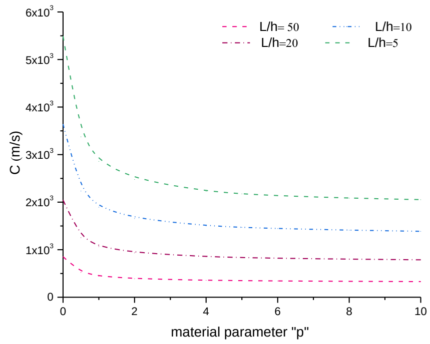
(a) λ=0



(b) λ=0,1



(c) λ=0.2



(d) λ=0.3

Fig. 7 Influence of thickness ratio on the phase velocity of the FG beam (k_p=10 and L/h=10)

Figs. 6 and 7 illustrate the influence of the thickness ratio of the FG beam on the natural frequency and the phase velocity.

From these two figures (Figs. 6 and 7), It can be observed that the thickness ratio (*L/h*) has a considerable effect on the phase velocity and natural frequency of the FG beam. Indeed, the increase of thickness ratio produces a decrease in the frequency and the phase velocity.

Figs. 8 and 9 present respectively the influence of the thickness ratio and the material parameter on the natural frequency and the phase velocity of the beams FG.

From the curves presented in Figs. 8 and 9 it can be seen that there is a clear effect of the thickness ratio on the natural frequency and the phase velocity. It can be also

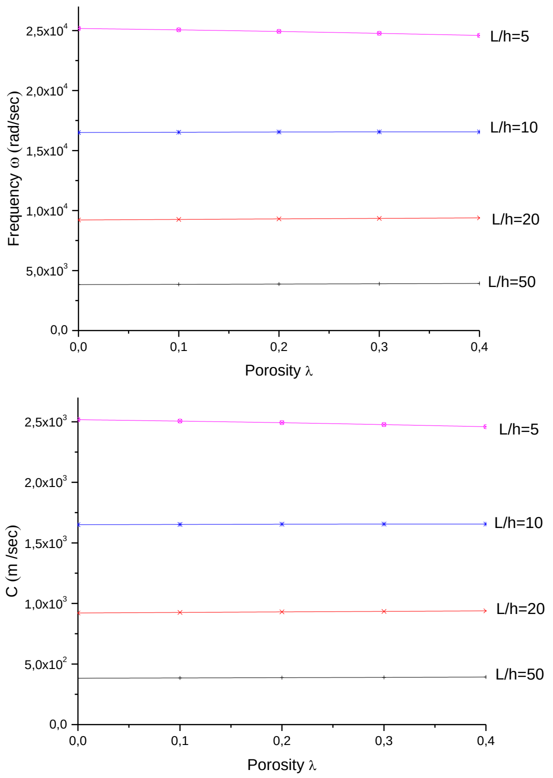


Fig. 8 Influence of the thickness ratio on the natural frequency and the phase velocity (k_p=10 and p=2)

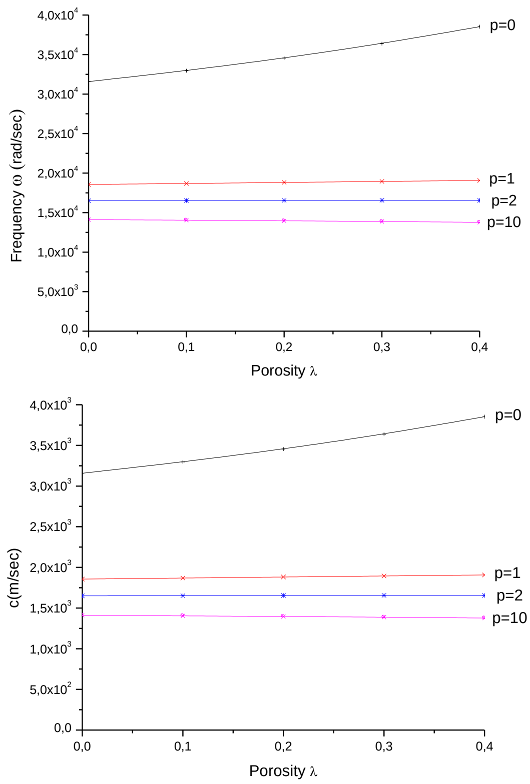


Fig. 9 Influence of the material parameter on natural frequency and phase velocity (k_p=10 and L/h=10)

observed that with increasing the material parameter both the natural frequency and phase velocity decreases.

4. Conclusions

In this research, an analysis of the vibrational characteristic and wave propagation of functionally graded porous beams with a new form of porosity distribution is presented. A theory of hyperbolic shear strain (HSDT) with a new displacement field that introduces undetermined integral variables was used for this study. The properties of the material are considered varied in the direction of thickness as a function of the modified mixing rule. The equations of wave propagation in the functionally graded beam are derived using the Hamilton principle. The analytical dispersion relationship of a porous FG beam is obtained by solving an eigenvalue problem. From the current work, it can be said that various factors such as porosity parameter, porosity distribution, thickness ratio and power law index have a significant effect on natural frequencies and phase velocity of FG beams with porosity. Which emphasizes on the importance of inspected porosity volume fraction effect. Therefore, the porosity effect should be considered in the analysis of vibration behavior of FG structures. Applications of this study for the thicker FG structures can be extended in future with considering new formulations developed by other works (see, e.g., Bessaim *et al*. 2013, Belabed *et al*. 2014, Hebali *et al*. 2014, Bousahla *et al*. 2014, Hamidi *et al*. 2015, Bourada *et al*. 2015, Bennoun *et al*. 2016, Draiche *et al*. 2016, Bouafia *et al*. 2017, Sekkal *et al*. 2017b, Abualnour *et al*. 2018, Benchohra *et al*. 2018, Younsi *et al*. 2018, Karami *et al*. 2018a; Bouhadra *et al*. 2018,; Hebbar *et al*. 2018) and to consider recent development continuum models (Karami *et al*. 2017, 2018b, c, Zine *et al*. 2018) to investigate the mechanical behaviour of FG structures more complex geometrical configurations.

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