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Application of first-order reliability method in seismic loss assessment of structures with Endurance Time analysis

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Abstract. Computational cost is one of the major obstacles for detailed risk analysis of structures. This paper puts forward a methodology for efficient probabilistic seismic loss assessment of structures using the Endurance Time (ET) analysis and the first-order reliability method (FORM). The ET analysis efficiently yields the structural responses for a continuous range of intensities through a single response-history analysis. Taking advantage of this property of ET, FORM is employed to estimate the annual rate of exceedance for the loss components. The proposed approach is an amalgamation of two analysis approaches, ET and FORM, that significantly lower the computational costs. This makes it possible to evaluate the seismic risk of complex systems. The probability distribution of losses due to the structural and non-structural damage as well as injuries and fatalities of a prototype structure are estimated using the proposed methodology. This methodology is an alternative to the prevalent risk analysis framework of the total probability theorem. Hence, the risk estimates of the proposed approach are compared with those from the total probability theorem as a benchmark. The results indicate a satisfactory agreement between the two methods while a significantly lower computational demand for the proposed approach.

Keywords: seismic risk; loss; life-cycle cost; endurance time method; reliability method; FORM

1. Introduction

This paper puts forward a probabilistic seismic risk analysis approach that significantly reduces the computational effort compared to existing approaches. Detailed risk analysis of buildings requires nonlinear dynamic time history analysis of complex finite element models, which is significantly computationally costly. In addition, most existing approaches to computing the loss exceedance probabilities rely on sampling schemes that entail a large number of such analyses, rendering the probabilistic analysis nearly impossible in many cases (Ghosh and Chakraborty 2017). Even practical methodologies, such as the one introduced by FEMA-P-58 (2012), require incremental dynamic analysis to compute the structural responses at a multitude of earthquake intensities and for a suite of ground motion records at each intensity. At the core of the latter approach, the theorem of total probability is employed in the form of a triple integral on a number of probability distributions: the probability of losses given damage measures, the probability of damage measures given engineering demand parameters, the probability of engineering demand parameters given intensity measure, and finally, the probability of earthquake intensity measures. The backdrop of this approach is the pioneering work of Cornell and Krawinkler (2000), commonly known as the framing equation of the Pacific Earthquake Engineering Research (PEER) center. The computation of the probability distribution of engineering demand parameters given the earthquake intensity often involves a high computational cost.

The proposed methodology in this paper battles the problem of computational cost in two fronts: First, in lieu of incremental dynamic analysis, the Endurance Time (ET) analysis (Estekanchi *et al*. 2007) is employed to quantify the structural responses. In the ET method, intensifying acceleration functions are employed to compute structural responses in a continuous range of intensities through a single dynamic time history analysis. Second, the first-order reliability method (FORM) is employed to compute the loss exceedance probabilities. FORM is an efficient and reasonably accurate reliability method that computes the probability of the event of interest, here, exceeding a loss threshold, with only a handful of analyses. For further details on FORM, see book such as Der Kiureghian (2005) and Ditlevsen and Madsen (1996).

A Key component of the proposed approach is the ET method. In this method, specially designed intensifying acceleration functions are used instead of progressively scaled up ground motion records. These acceleration functions are produced using numerical techniques. In this manner, the structural responses can be estimated in a continuous range of intensity levels by a single response-history analysis. The advent of this method dates back to 2004 (Estekanchi *et al*. 2004). Since then, the ET method has evolved in several generations. The ET method considerably reduces the computational demand of

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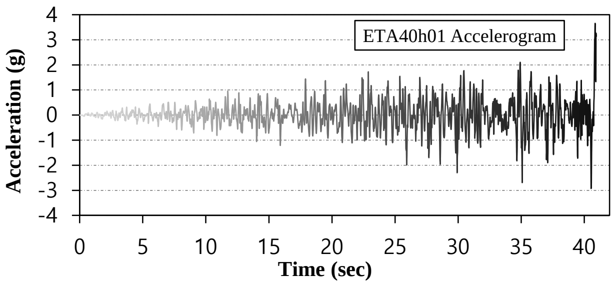

Fig. 1 ETA40h01 acceleration function

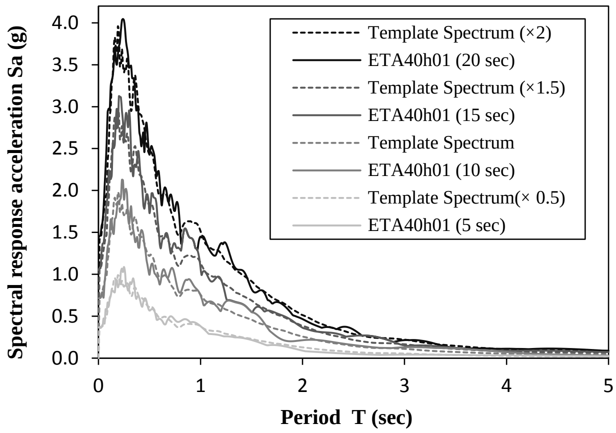

Fig. 2 Acceleration response spectra for ETA40h01 at different excitation times

nonlinear dynamic time history analysis. This unique feature of the ET method was employed by Basim *et al*. (2016) to reduce the computational cost of seismic loss analysis. The framework to calculate deterministic life-cycle cost of a construction using the ET method was presented in the latter work. This framework deterministically estimates the expected seismic losses using the median engineering demand parameters obtained from a single ET analysis. However, neglecting the uncertainties in this framework may lead to unwarrantable estimates of expected losses. To remedy, the present study puts forward an improved methodology that accounts for the prevailing uncertainties. Also, the application of ET method and its merits in seismic loss analysis of structures with the FEMA-P-58 (2012) framework was investigated by Basim and Estekanchi (2015). They used a multi-objective optimization technique to achieve an optimum design with respect to loss measures. However, the methodology used in FEMA-P-58 requires repetitive response estimation analyses at a multitude of hazard levels and a large number of samples in a Monte Carlo analysis. Therefore, it entails a high computational cost rendering it infeasible for analyzing complex or highly detailed systems. In fact, the development of efficient, yet precise loss evaluation methods is currently an active area of research and is targeted in the present paper.

Risk analysis using the structural reliability methods is another relevant field of research. The use of reliability methods in risk analysis is an alternative to the prevalent risk analysis approaches that are based on the theorem of total probability, such as the ones enumerated above. Haukaas (2008) was the first to apply structural reliability methods in risk analysis. Later, Koduru and Haukaas (2010) and Mahsuli and Haukaas (2013a, b) extended this framework. For this purpose, Mahsuli and Haukaas (2013c) developed *Rt*, a computer program for multi-model reliability and risk analysis, which is freely available online. Amongst the structural reliability methods, FORM is particularly efficient at computing small probabilities reasonably accurately with a low computational effort. These small probabilities are particularly important in seismic risk analysis because they represent very severe earthquakes with dramatic consequences. In a FORM analysis, random variables with predefined probability distributions describe the uncertainties, and a limit-state function defines the event for which the probability is sought (Moustafa and Mahadevan 2011). In the classical structural reliability, the limit-state function is defined in terms of the capacity and demand of the structure and hence, yields the probability of failure, i.e., the probability that the demand exceeds the capacity. To extend the usage of FORM to risk analysis, Mahsuli and Haukaas (2013b) defined the limit-state function in terms of the seismic loss, as described later in this paper. FORM requires the computation of the limit-state function and its gradient at a small number of random variable realizations in order to compute the desired probability. Hence, the limit-state function must be continuously differentiable in terms of the underlying random variables. As a result, a suite of naturally recorded ground motions cannot be employed in a FORM-based risk analysis to model the ground motion. In contrast, the results of ET analyses are continuously differentiable, which motivates the developments in the present work.

To demonstrate the proposed method, a probabilistic loss model of a five-story, three-bay steel frame is developed. FORM analysis is used to estimate the annual probability of exceeding loss values based on the median structural response parameters estimated by the ET analysis. The FORM analyses are performed by the computer program *Rt* (Mahsuli and Haukaas 2013c). To validate the results, the same model is used in a total probability framework (the PEER framework) (Cornell and Krawinkler 2000) and loss curves are calculated using rigorous direct integration as a point of comparison. A general framework for the proposed method is presented and can be implemented in loss analysis of other structural systems.

2. Endurance Time method (ET)

Endurance time method is a dynamic response-history analysis method in which predesigned artificial acceleration functions are used as input excitations. In these acceleration functions, the level of excitation increases with time to cover a full range of intensities in each response-history analysis. The increase is calibrated using numerical optimization techniques such that the acceleration response spectrum of the accelerogram from zero to any particular excitation time scales up as the excitation time increases, matching a template spectrum in shape all the time (Nozari and Estekanchi 2011). Various sets of ET accelerograms with desired template linear or nonlinear spectrum have been designed and are available via the ET method web site (Estekanchi 2018). The ETA40h accelerograms that match the average response spectrum of seven records recommended in FEMA-440 for Site Class C as template spectrum are used in this study. Three ET records of each series that are obtained using different start points in the aforementioned optimization analysis are commonly used for a better estimation of structural responses. The ETA40h01 acceleration function is depicted in Fig. 1. The acceleration response spectra of the record in various excitation times are displayed in Fig. 2. It can be seen that the produced spectra match the template spectrum with a scale factor at all times. This characteristic of the ET records provides the prerequisites to define a relationship between excitation time and induced spectral intensity (Mirzaee *et al*. 2012).

As the hazard intensity levels are generally determined by acceleration response spectra, a correlation between excitation time in the ET analysis and the equivalent hazard return period can be found on this basis. ET excitation time corresponding to any intensity level defined by a particular spectrum is calculated by seeking a time at which the response spectrum of the ET record matches the target spectrum at effective periods, e.g., from 0.2 to 1.5 times of fundamental period of the structure. The relationship calculated based on the acceleration response spectra defined for Tehran to characterize ground shaking intensity at different hazard return periods is presented by the authors in Basim *et al*. (2016). This relationship gives the equivalent ET excitation time for any hazard level defined by its return period.

The ET analysis results are normally presented by a curve named “ET Curve” or performance curve with excitation time on the horizontal axis and the desired response parameter on the vertical axis (Estekanchi and Basim 2011). Therefore, the performance of the structure in the desired range of hazard intensities can be quantified using a single response history analysis (Estekanchi *et al*. 2016). To construct an ET curve, the structure is commonly analyzed under three ET acceleration functions. Thereafter, the results of three analyses are averaged and smoothed using the moving average method. Hariri-Ardebili *et al*. (2014) studied the application of ET method in performance-based design. The capability of ET method in estimating structural responses at various excitation intensities with reasonable accuracy is demonstrated in their studies. They used the ET curve to assess the performance of structures by comparing it with the target performance curve that is proposed by performance evaluation guidelines. Similar works have been accomplished to verify the capability of the ET method in performance assessment of various types of structural systems (Estekanchi 2018). These curves may be more perceptible by substituting hazard return period or probability of exceedance for the time on horizontal axis using the relationship mentioned above. This will provide a proper baseline to calculate expected costs due to hazards in a continuous range of intensities (Mirzaee and Estekanchi 2015). In the following sections, ET curve is used to obtain the required engineering demand parameters as a function of spectral intensity.

3. Loss estimation using FORM: the methodology

The objective of the proposed methodology is to estimate the annual probability of loss exceedance, hereafter referred to as loss curve, using FORM and ET analyses. A loss curve displays the probability of exceeding any loss value which may be resulted by structural damages, damaged contents, downtime, injuries, and fatalities. The continuous nature of the response parameters in the ET analysis enables the use of a gradient-based reliability analysis, such as FORM. However, the product of a classical structural reliability technique such as FORM contrasts the loss curve resulted by a risk analysis. In a reliability analysis, the failure event is defined by a limit-state function while the uncertainties are characterized by random variables and the reliability method is used to estimate the failure probability. In a loss analysis, this limit-state function should be defined as exceeding a loss threshold value such as C_0 (Mahsuli and Haukaas 2013b). This will result in an individual point on the loss curve. By repeating the FORM analysis for multiple values of C_0, an estimation of loss curve can be obtained. Each point, as will be demonstrated later, can be obtained by defining a mentioned limit-state function and using FORM to calculate the probability of exceedance. The required computational demand is much decreased with respect to the direct integration method.

The overriding segment of the methodology is development of a probabilistic loss model for the desired structure. The used models should be in a specific form to be implemented in a reliability method. First, uncertainties in the model should be described by random variables with predefined probability distributions and the event for which the probability is sought should be defined by a limit-state function. Also, any model used in FORM analysis as a gradient based reliability analysis should be continuously differentiable with respect to input random variables. The reliability analysis progresses by repeatedly evaluating the limit-state function for new realizations of the random variables and the final result is the probability that the limit-state function takes negative outcomes. Therefore, the models used in this type of reliability analysis must return measurable responses for specific values of random variables. These models contrast with common models used in current seismic risk analysis techniques such as the models used by Wen and Kang (2001b), Mitropoulou *et al*. (2010) or FEMA–NIBS (2003) and ATC-13 (1985). The specifications of probabilistic models intended for use in reliability analysis is addressed in detail in a work by Mahsuli and Haukaas (2013a). New probabilistic models and uncertain parameters involved in expected losses are presented here based on some common damage and cost models in literature and results for an example structure are

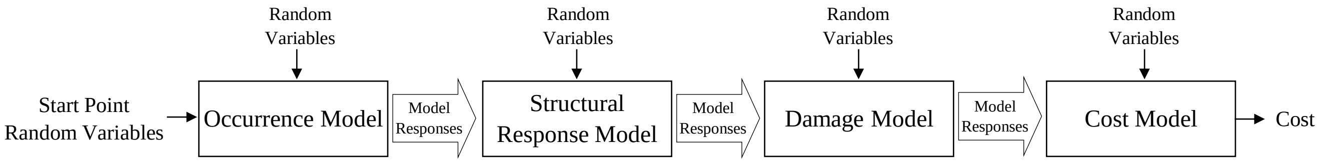

Fig. 3 Overview of a model used in FORM

presented in the following sections.

First, as used in common risk analysis approaches, the defined model from the outset, should account for the uncertainties in nature of the expected hazard. This can be modelled by location and magnitude models used in seismic hazard analysis. Alternatively, the product of a hazard analysis i.e., hazard curve, can be used to characterize the uncertainty in hazard magnitude. A hazard curve contains the probability of exceeding values of a ground shaking intensity, such as *Sa* in the studied site. Therefore, *Sa* with a defined probabilistic distribution can be used as an input random variable to the model. Otherwise, comprehensive magnitude and location models should be used to directly model the occurrence of seismic events (Der Kiureghian and Ang 1977). Using magnitude and location models provides the means to account for numerous sources of seismicity separately and also make importance analyses on each source as an input to the whole loss model.

As the second step, structural responses should be calculated for any realization of previously defined input variables. We recall that the responses should be continuously differentiable with respect to input random variables (*Sa* here). This would be a dramatic problem while using a suite of naturally recorded ground motions to model the seismic hazard. In contrast, the results of ET analyses are continuously differentiable with respect to *Sa* if the smoothing method described in section 2 be used. So, ET analysis is used here to estimate the seismic response parameters. ET curve provides the median engineering demand parameters i.e., drifts and floor accelerations as output vector. This technique has significant effect on the computational cost of the method. An ET analysis provides the required data for all realizations of *Sa*. To account for uncertainties in this section it is required to define random variables which affect the response parameters. The dispersion in structural responses may be resulted by various factors such as modelling uncertainty, construction quality and record to record variability.

The next step is calculating structural damages due to the foregoing random variables i.e., occurrence and response model parameters. Conforming the requirements of a reliability analysis, this part should also be in a form that provide quantifiable measures of damage for any value of structural responses not probabilities. Therefore, many of damage models in current literature cannot be used here directly. Damage fragility models such as the ones presented by FEMA–NIBS (2003) are examples of these intractable models. In section 5 a modified damage model is proposed to be used here based on some common models introduced by ATC-13 (1985) and FEMA–NIBS (2003).

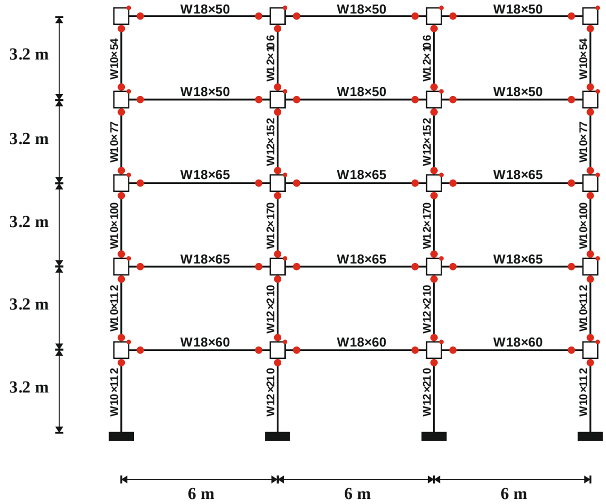

Fig. 4 Frame sections and idealized model of the prototype structure

Uncertainties may be defined as random variables that affect the damage parameters.

The last part of the model is cost model which defines associated costs for values of induced damages. Costs, commonly, are subject to larger uncertainties and may contain various components resulted by damage repair cost, contents cost, costs due to loss of functionality and the cost of injuries and fatalities. One of the well-known cost models is introduced by ATC-13 (1985) restated in FEMA-227 (1992). An example of possible cost models is introduced in section 5. A similar method can be used to define any intended loss model with any intended level of involved details to be used in the proposed method.

Having defined the model, the associated cost with any realization of random parameters of the model would be on hand. Now, FORM analysis can be implemented to calculate the probability that the cost value exceeds any threshold value. This step is explained in details for the studied example structure in section 7. Repeating the FORM analysis for various threshold values and using occurrence random variables for annual probabilities the result would provide an estimate of the loss curve. An overview of the method is displayed in Fig. 3.

4. Structural model

The method is demonstrated by means of a case study on a prototype steel frame building. The structure consists of five stories and three bays with 3.2 m story height and 6 m bay width as depicted in Fig. 4. The frame type is special

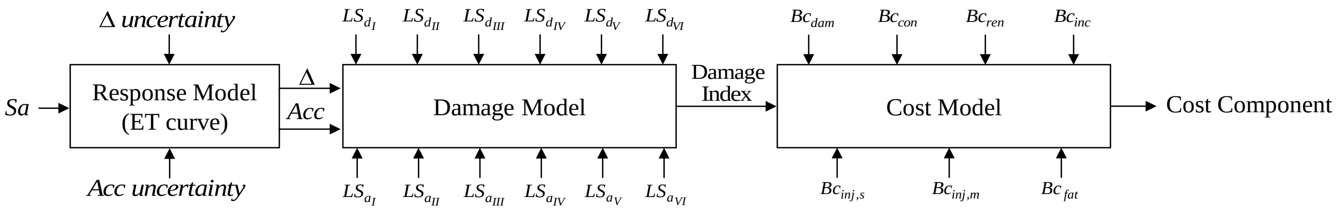

Fig. 5 The cost model and input parameters for prototype structure

moment resisting and all supports are fixed. It is designed according to Iranian National Building Code (INBC) section 6 which is similar to ASCE/SEI7-10 (2010) as a commercial building for loading regulations and INBC section 10 which is almost identical to the ANSI/AISC360 (2010) LRFD design recommendations. The AISC strong column-weak beam requirement is considered in design of the structure. The steel material properties are: yielding stress F_y=235.36 MPa and elastic modulus E=200 GPa. A 2D model of the building with fundamental period of T=1.03 s is studied as the building is assumed symmetrical in plan. The planar area associated with each story of the frame is assumed 140 m^2.

A detailed numerical model of the frame including its plastic deformation and degradation properties is used to reliably capture the structural responses in severe events. Response-history analyses were conducted via OpenSees (Mazzoni *et al*. 2006) where concentrated plastic hinges are modelled with zerolength rotational springs and structural elements are represented by elastic beam-column elements. The plastic hinges in columns are located at the two ends and in beams are located in a distance equal to depth of the beam away from the face of column. Plastic hinges follows a bilinear hysteretic response based on the Modified Ibarra Krawinkler Deterioration Model (Ibarra *et al*. 2005, Lignos and Krawinkler 2011). Readers are encouraged to refer to a work by Basim and Estekanchi (2015) for detailed description of model degradation properties. Second order effects are considered using P-Delta Coordinate Transformation object in the OpenSees platform and shear distortions in panel zones are modelled using Gupta and Krawinkler (1999) method using a rectangle consisting of eight stiff elastic beam-column elements with one zerolength trilinear rotational spring in the corner.

5. Probabilistic loss model

Various risk measures have been introduced by researchers to express the likelihood of losses in constructions to be used in financial analyses by decision making centers, such as the probable maximum loss (PML), probable frequent loss (PFL), expected annualized loss (EAL), and expected seismic life-cycle costs. Expected annualized losses is, in fact, the risk measure that is extensively used in design optimizations and decision making, and represents a risk neutral perspective. Life-Cycle Cost Analysis (LCCA) has been utilized to estimate the expected costs, including the expected losses due to future earthquakes during the design life of constructions (Wen and Kang 2001a, Mitropoulou *et al*. 2011). A detailed assessment of earthquake induced losses is obtained in some research works by taking into account cost components including costs due to structural damages, loss of contents, losses due to downtime, human injuries and fatalities. This will provide a realistic appraisal of the earthquake consequences. These components are quantified based on limit states defined by interstory drift ratio and floor acceleration.

According to works by Wen and Kang (2001b), Mitropoulou *et al*. (2010), damage repair cost, the cost of loss of contents due to interstory drift and also floor acceleration, the loss of rental cost, the loss of income cost, the cost of injuries and the cost of human fatalities are considered here to calculate the life-cycle cost of the structure. In this section, the probabilistic cost model and uncertain parameters involved in expected losses of the prototype structure are introduced. The framework of the model is somehow analogues to the models used by FEMA–NIBS (2003). Some modifications are made to make the model suitable for the proposed methods. Fig. 5 illustrates the overview of the defined model. This model will be utilized to calculate the expected life-cycle cost components of the prototype structure. ET analysis is used to estimate the median seismic response parameters and estimated demand parameters for each story will be used to calculate associated costs.

Analogues to common risk analysis approaches "hazard curve" is used as a starting point. This will provide the annual probability of exceeding values of a site-specific ground shaking intensity which is taken spectral intensity (Sa) here. It is assumed that this random variable completely defines the epistemic and aleatory uncertainties in hazard characteristics. This will be an input for response model (ET curve) and as discussed before, ET curve provides the median engineering demand parameters i.e., drifts and floor accelerations as output vector. No new analysis is required for each realization and an ET analysis provides the whole required data (i.e., structural responses and their gradients with respect to Sa). This will be used as input for damage model. The dispersion in demand parameters to account for modelling uncertainty, construction quality and record to record variability are modelled with drift (Δ) uncertainty and acceleration (Acc) uncertainty parameters as input to response model (Fig. 5). According to the work by Basim and Estekanchi (2015), the total value of dispersion (the standard deviation of the natural logarithms) for story drift in ET analysis method $\beta_{D_{ET}}$=0.38 and for floor acceleration a value of $\beta_{a_{ET}}$=0.43 due to higher record to record dispersion associated with

Table 1 Median drift ratio and floor acceleration limits for damage states

Performance level	Damage states	Drift ratio (%) ATC-13 (1985)	Floor acceleration (g) Elenas and Meskouris (2001)
I	None	$\Delta \le 0.2$	$a_{floor} \le 0.05$
II	Slight	$0.2 < \Delta \le 0.5$	$0.05 < a_{floor} \le 0.10$
III	Light	$0.5 < \Delta \le 0.7$	$0.10 < a_{floor} \le 0.20$
IV	Moderate	$0.7 < \Delta \le 1.5$	$0.20 < a_{floor} \le 0.80$
V	Heavy	$1.5 < \Delta \le 2.5$	$0.80 < a_{floor} \le 0.98$
VI	Major	$2.5 < \Delta \le 5$	$0.98 < a_{floor} \le 1.25$
VII	Destroyed	$5.0 < \Delta$	$1.25 < a_{floor}$

response floor accelerations is used.

In Table 1, limit states defined by ATC-13 (1985) for maximum drift ratios and a work by Elenas and Meskouris (2001) for maximum floor accelerations are presented. Drift ratios (Δ) are used to quantify both structural and non-structural damages and floor accelerations a_{floor} are used to quantify the loss of non-structural contents that are vulnerable to acceleration (Mitropoulou *et al*. 2010). According to HAZUS technical manual (FEMA–NIBS 2003) each of the defined upper bound limits to damage states is assumed to be log-normally distributed random variables. The total variability of each structural damage state limit by the drift ratio (β_{Sds}) is modelled by the combination of uncertainty in the damage-state threshold of the structural system ($\beta_{M(Sds)}$=0.4) and variability in capacity properties of the model building ($\beta_{C(Au)}$=0.25) resulting in the total variability of structural damage state $\beta_{Sds} = \sqrt{\beta_{M(Sds)}^{\;2} + \beta_{C(Au)}^{\;2}} = 0.47$. For the damage states by floor acceleration in non-structural components similarly $\beta_{M(NSAds)}$=0.6 and $\beta_{C(Au)}$=0.25 resulting in the total variability of non-structural damage state $\beta_{NSAds} = \sqrt{\beta_{M(NSAds)}^{\;2} + \beta_{C(Au)}^{\;2}} = 0.65$. Therefore, the lower and upper bounds for limit states are log-normally distributed random variables with median values stated in third and fourth columns of Table 1 and dispersions equal to β_{Sds} and β_{NSAds} calculated above. The defined limits as random variables and demand parameters are inputs for damage block to calculate the damage index vector (Fig. 5). Piecewise linear interpolation is used to set a continuous relation between demand parameter and damage index (Basim *et al*. 2016).

The total life-cycle cost of the prototype structure according to the defined cost model equals to the sum of considered components (Eqs. (1), (2)).

$$C_{LC} = C_{dam} + C_{con} + C_{ren} + C_{inc} + C_{inj} + C_{fat} \tag{1}$$

$$C_{con} = C_{con}^{\Delta} + C_{con}^{acc} \tag{2}$$

where C_{dam} is the damage repair cost, C_{con}^{Δ} the loss of contents cost due to interstory drift, C_{con}^{acc} the loss of contents cost due to floor acceleration, C_{ren} the loss of rental cost, C_{inc} the cost of income loss, C_{inj} the cost of injuries and C_{fat} is the cost of human fatality (Mitropoulou *et al*. 2011).

Table 2 Formulas for cost components calculation in Dollars (ATC-13 1985, Wen and Kang 2001b, Mitropoulou *et al*. 2011)

Cost Component	Formula	Basic cost
Damage repair (C_{dam})	Replacement cost×floor area ×damage index	LN(500,100) \$/m^2
Loss of contents (C_{con})	Unit contents cost×floor area × damage index	LN(150,30) \$/m^2
Loss of rental (C_{ren})	Rental rate×gross leasable area ×loss of function time	LN(10,2) \$/month/m^2
Loss of income (C_{inc})	Income rate×gross leasable area×down time	LN(300,60) \$/year/m^2
Minor injury ($C_{inj,m}$)	Minor injury cost per person ×floor area×occupancy rate ×expected minor injury rate	LN(2000,400) \$/person
Serious injury ($C_{inj,s}$)	Serious injury cost per person ×floor area×occupancy rate ×expected serious injury rate	LN(20000,4000) \$/person
Human fatality (C_{fat})	Human fatality cost per person ×floor area×occupancy rate ×expected death rate	LN(300000,60000) \$/person

Table 3 Damage state parameters for cost components calculations (ATC-13 1985; FEMA-227 1992)

Damage states	Mean damage index (%)	Expected minor injury rate	Expected serious injury rate	Expected death rate	Loss of function time (days)	Down time (days)
(I)-None	0	0	0	0	0	0
(II)-Slight	0.5	0.00003	0.000004	0.000001	1.1	1.1
(III)-Light	5	0.0003	0.00004	0.00001	16.5	16.5
(IV)-Moderate	20	0.003	0.0004	0.0001	111.8	111.8
(V)-Heavy	45	0.03	0.004	0.001	258.2	258.2
(VI)-Major	80	0.3	0.04	0.01	429.1	429.1
(VII)-Destroyed	100	0.4	0.4	0.2	612	612

Formulas to calculate the associated cost components with damage index values are presented in Table 2. The basic cost (*Bc*) in first term of each formula is given in the last column of this table as a log-normally distributed random variable. The mean and standard deviation of each variable is presented in this column. The assumed dispersion is about 0.2 for all basic costs. Other required parameters for each damage state are provided in Table 3 based on ATC-13 (1985) and FEMA-227 (1992). The data for engineering facility classification 16 and medium rise moment resisting steel frame is selected according to ATC-13 (1985). Occupancy rate is taken 2 persons per 100 m^2. For the prototype structure the replacement cost is assumed \$500 per m^2 over the 700 m^2 total area of the structure.

It is assumed that the owner prefers to replace the building if the repair cost of the structure exceeds 70% of the building total replacement cost and also, if the damage index in a story exceeds damage state VII the building is deemed to be collapsed and should be replaced. In these

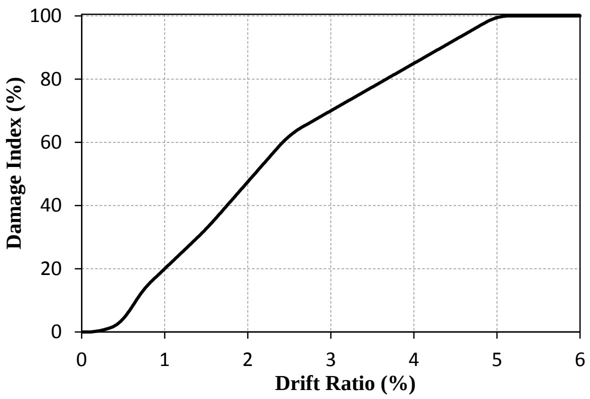

Fig. 6 Smoothed relation between damage index and drift ratio

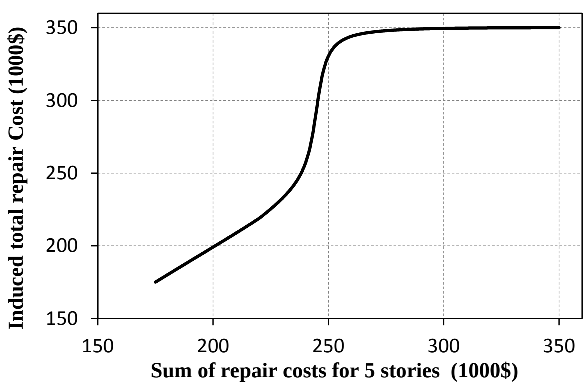

Fig. 7 Smoothed relation between sum of stories repair cost and induced total repair cost

cases, the damage repair cost will rise to the building replacement cost which equals to \$500 per m^2 over the 700 m^2 total area of the structure and the rental and income cost will be imposed for 612 days of reconstruction and the contents cost, minor and serious injuries cost and the fatalities cost will rise to maximum in all stories. One mode of collapse is considered here and it is assumed that extensive lateral displacement at any story will result in total collapse of the building (Zareian *et al*. 2010). The input parameters to the cost block is depicted in Fig. 5.

As it mentioned, it is required that any model used in FORM analysis as a gradient based reliability analysis be continuously differentiable with respect to input parameters. So, the variation of model response is smoothed in vicinity of leaps and if-statements that introduce different model forms. As an example, the smoothed relation between drift ratio and damage index in damage states from Table 1 and Table 3 is presented in Fig. 6. The preference of total replacement over repair, described in previous paragraph, is also modelled by a smooth relation as depicted in Fig. 7. By a similar method, all kinks and leaps in models are replaced by smooth transitions to make the models amenable to gradient-based reliability analysis (Mahsuli and Haukaas 2013a).

6. Total probability theorem

One of the well-defined approaches of risk analysis and probable loss calculation is analytical integration by the total probability theorem (Cornell and Krawinkler 2000). In this section using the defined probabilistic model, loss curves for the cost components are calculated as a point of comparison. The life-cycle cost function used in the present section is based on the work by Wen and Kang (2001a). A similar method with assuming a continuous relation between interstory drift and cost had been utilized in a work by Basim *et al*. (2016) to quantify the damage costs using ET method. Application of the ET analysis in life-cycle cost analysis has been formulated in this reference. Here, the proposed method to account for uncertainties in cost calculations is described.

Expected annual rate of exceeding any value of each cost component due to future earthquakes is calculated here in PEER framework (Cornell and Krawinkler 2000). ET analysis provides a proper baseline to obtain this term as a loss curve in a straightforward procedure. Expected annual component cost of the prototype building can be calculated using the area under such a curve. In this framework a formula known as PEER framework formula is used

$$\lambda(dv) = \int_{dm}\int_{edp}\int_{im} G(dv|dm)\left|dG(dm|edp)\right|\left|dG(edp|im)\right|\left|d\lambda(im)\right| \tag{3}$$

where *im* is an intensity measure, *edp* is an engineering demand parameter, *dm* is a damage measure and *dv* stands for a decision variable. Here, $G\left(x|y\right) = P\left(x < X \mid Y = y\right)$ presents the conditional complementary cumulative distribution function of random variable *X* given *Y*=*y*, and $\lambda(x)$ is the mean rate of $\{x<X\}$ events per year (Kiureghian 2005). This formula is used to estimate the annual rate that a decision variable such as damage repair cost exceeds a specified threshold. All types of uncertainties present in hazard characteristics and induced losses can be properly taken into account by this formula. Making some independence assumptions on aforementioned variables, it would be possible to make a decomposition on earthquake engineering task and $\lambda(im)$, $G(edp|im)$, $G(dm|edp)$ and $G(dv|dm)$ can be studies separately. So, $\lambda(im)$ can be acquired from a seismic hazard analysis and $G(edp|im)$ from response analysis by the ET method. $G(dm|edp)$ and $G(dv|dm)$ is calculated using the defined model. Each cost component can be considered as the decision variable and $\lambda(dv)$ in Eq. (3) gives the annual rate that the cost components values *DV* exceeds any value *dv*. This will result in loss curve with cost component values in horizontal axis and annual rate of exceedance on vertical axis (Yang *et al*. 2009).

Here, using total probability theorem in cost calculation, it is tried to account for the uncertainties in demand parameters estimated by ET analysis in aforementioned loss estimation procedure. The ET curve of a hypothetical structure can be assumed as Fig. 8. The introduced cost model gives the cost associated with any drift ratio (*dr*). In order to account for variabilities on estimated drifts a lognormal distribution using Eq. (4) is assumed for

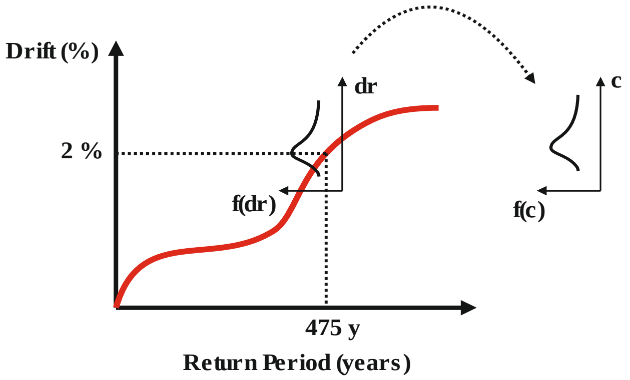

Fig. 8 ET curve for a hypothetical structure and assumed dispersion on drift values

experienced drift at any return period. In Fig. 8 the lognormal distribution is displayed for drift at 475 y return period as an hypothetical example with mean value (μ) equal to the data provided from ET results and a dispersion β. Now, conditioned on occurrence of a hazard with a specified return period and conditioned on having a specified response parameter the expected cost component should be calculated using direct integration on cost model. This procedure will be repeated for any probable EDP value and any hazard return period. This will result in the probability density function of cost values for 475 y return period (RP).

$$f(dr) = \frac{1}{dr\,\beta\sqrt{2\pi}}\, e^{-\frac{(\ln(dr)-\ln(\mu))^2}{2\beta^2}} \quad (4)$$

Now, the expected cost for the considered return period $E[c|RP]$ (and any other return periods) can be calculated by

$$E[c\,|\,RP] = \int_0^{\infty} c\ f(c)\ dc \quad (5)$$

If the above procedure is repeated for a series of return periods, a curve containing the data on expected cost at each return period can be obtained. The results for the prototype five story frame are presented here. The ET curve is derived by averaging results from three ETA40h records and using moving average to smooth ET results for maximum interstory drift envelope. So, the proposed loss estimation method requires only 3 response-history analyses instead of repetitive ground motion analysis scaled at multiple intensities. Estimated demand parameters for each story will be used to calculate associated costs. The probabilistic approach as in Fig. 8 can be used here to calculate expected cost components of each story. At each hazard return period, the demand parameters estimated by the ET method are in the form of a single vector as mean values of 3 ET analyses. At any hazard return period the lognormal distribution is considered for demand parameter at one pilot story and the value for other stories are scaled maintaining the shape of demand parameters vector along building stories. So, it is assumed that the building maintains a same response profile at any given intensity, but with different amplitude (FEMA-P-58 2012). The cost for the whole building will be derived by summing the cost values at each story. Results from total probability method are depicted in Fig. 9. The required computational demand is too large for this method. The CPU time for this simple model is about 48 h and it is obvious that this method will quickly lose its efficiency as the number of random variables increases. As the main focus of this study is on estimation of probability of losses due to rare events and these probabilities are near zero, to enhance the readability of figures, loss curves are depicted in a form that the horizontal axis is the return period and the vertical axis is the associated cost components.

7. FORM analysis

The objective of this section is to estimate the probability that any loss component exceeds a threshold value using FORM analysis. The continuous nature of the response parameters in ET analysis provides a proper baseline to utilize such a gradient based reliability analysis. This analysis will result in individual points on loss curve and by repeating on multiple values of cost component an estimation of loss curve can be obtained. The required computational demand is much decreased with respect to the direct integration method. The soundness of results is investigated comparing with the total probability method as a point of comparison. However, the total probability method is not feasible for large and complex models where the presented method in this section can provide a practical alternative. It is worthy of note that the used cost models are an estimation of cost components to clarify the proposed method and the method has the capability of incorporating detailed calculations on cost components and anticipated consequences.

Input parameters of the probabilistic model which will be implemented in FORM analysis can be collected in the vector $\boldsymbol{x}$ representing all the relevant uncertainties influencing the probability. The limit state function $g(\boldsymbol{x})$ defines the event for which the probability of occurrence is being sought. A reliability analysis such as FORM is used to determine the probability that the limit-state function takes negative outcomes ($P[g(\boldsymbol{x})\leq 0]$). The limit state function in our problem can be taken as $g(\boldsymbol{x})=C_0-C(\boldsymbol{x})$ which yields the probability that the cost component exceeds the threshold value C_0. The defined probabilistic model will be used for evaluation of C as a function of $\mathbf{x}$. In FORM analysis the function g and its gradient $\partial\mathbf{g}/\partial\mathbf{x}$ are evaluated several times for different values of $\mathbf{x}$. In the proposed probabilistic model, as the response values are on hand in a continuous range of hazard probabilities, there is no need to conduct structural response-history analysis for different evaluations of g and the response values and its gradients with respect to hazard intensity can be obtain from the ET curve from a single ET response history analysis (or 3). FORM analysis will also provide valuable information about the relative importance of each random variable (Mahsuli and Haukaas 2013b). In FORM analysis, all the random variables are transformed into their standardized forms and a search for the "design point", which is the most likely realization of $\mathbf{x}$ associated with g_i=0 is accomplished. The result of the

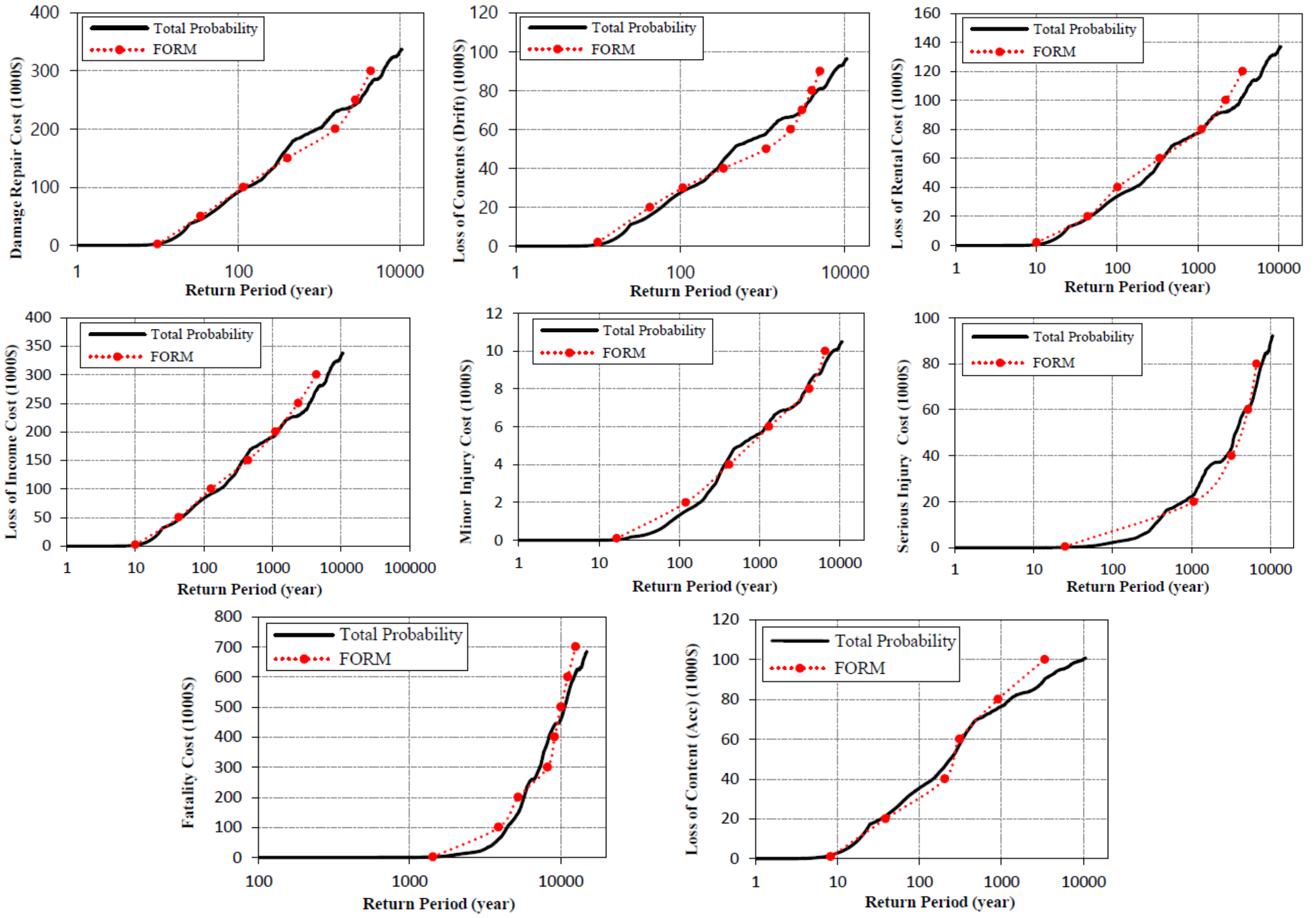

Fig. 9 Estimated cost components for the prototype structure using the total probability method and FORM

search is the reliability index $\overline{\beta}$, which is related to the sought probability by the equation (An over bar is added to make distinct from dispersion β)

$$p = \Phi(-\overline{\beta}) \tag{6}$$

where Φ is the standard normal cumulative distribution function. As only one hazard source is considered in this study, this probability is the probability that the cost component exceeds the threshold value C_0. The reliability analysis is carried out by the computer program *Rt* (Mahsuli and Haukaas 2013c). The defined model is introduced completely to this program and the analysis is repeated for different limit state functions to calculate the probability that any cost component exceeds various thresholds. Results are depicted in Fig. 9. The probability that the cost component exceed several values are calculated to obtain points in loss curve. FORM analysis has provided reasonably accurate probability estimates even for the tail of the loss curve where a jump in induced costs are imposed because of the preference of total replacement over repair which was described before. Results of FORM analysis may be inaccurate if the limit-state function is strongly nonlinear in the space of standard normal variables, but, it is justifiable owing to the reduced computational demand.

According to Eq. (1) the total cost of the building is the sum of cost components. This total value via the two method can be compared in Fig. 10. In this curve the relative error of FORM results with respect to the benchmark method in terms of probability of exceedance is displayed. The error in worst case is 38% where the FORM has resulted in 0.00032 probability of exceedance from 1 million dollars and the total reliability method has resulted in 0.00053.

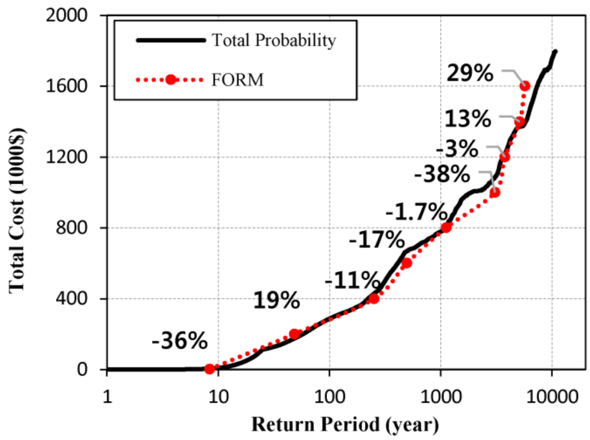

Fig. 10 Loss curve for total cost of the prototype structure using the total probability method and FORM

In each FORM analysis, along with the probability of failure, the ranking of random variables with respect to the

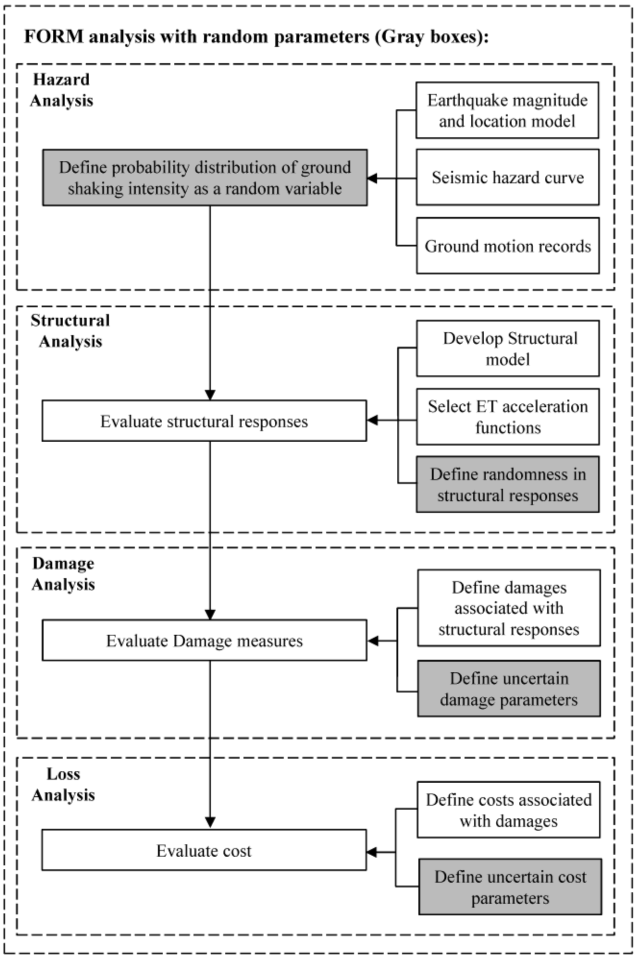

Fig. 11 Framework for loss estimation using FORM and the ET analysis

response sensitivity can be verified. In the studied probabilistic model this ranking vary for each cost component and the considered cost threshold. An overall observation of results reveals that *Sa* is the most important variable in relatively low cost threshold and on the other hand Δ*uncertainty* and Bc_{fat} are the most important parameters for high cost threshold i.e. tail of the loss curve.

8. Conclusions

A practical procedure for taking into account the effect of uncertainties in life-cycle cost assessment by the Endurance Time (ET) method is proposed. The procedure is demonstrated using a probabilistic model of a prototype structure. The model is tailored to conform to the rules requited for risk analysis using gradient-based reliability methods. In such methods, uncertainties are modelled using random variables with predefined probability distributions, and given the realizations of these random variables, the models make probabilistic prediction of physical phenomena. Models are presented here for repair costs due to structural and non-structural damage, and losses due to injuries and fatalities. The potential benefits of ET analyses in reducing the cost of computing structural responses are used in two proposed methods to estimate the loss curve of the building subject to the seismic hazard. Results from ET method can be easily used to calculate loss curve requiring much less computational effort compared to progressively scaled ground motion records. The first proposed method was direct integration based on total probability theorem. Although this method provides accurate calculation of loss curve, but it involves a huge computational cost even for a simple probabilistic model. This method has been used as a benchmark to be compared with an approximate method. In the second method, the first-order reliability method (FORM) as a gradient-based reliability method is used. FORM can use the results of the ET analysis because they are continuously differentiable. The proposed method was used to estimate individual points on loss curves associated with several cost components. The integration of the ET method and FORM drastically reduces the computational cost of risk analysis compared to the prevalent approach of using the theorem of total probability together with the incremental dynamic analysis. The paper shows that the second method provides accurate estimations of the cost exceedance probabilities with a fraction of the computational cost of the competing approaches. Furthermore, FORM provides insightful information on the sensitivity of exceedance probabilities to each random variable, e.g., it reveals the most important sources of uncertainty in the risk analysis.

This method can be implemented in loss analysis of various structural systems following the steps depicted in Fig. 11. Although the method and the probabilistic models used here differ from the ones used in FEMA-P-58 (2012), FEMA–NIBS (2003) or ATC-13 (1985), the data on damage and cost parameters and associated uncertainties provided by these references can be used to build the loss models required in the proposed method.

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