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Evaluation of N2 method for damage estimation of MDOF systems

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Abstract. Methods based on nonlinear static analysis as simple tools could be used for the seismic analysis and assessment of structures. In the present study, capability of the N2 method as a well-known nonlinear analysis procedure examines for the estimation of the damage index of multi-storey reinforced concrete frames. In the implemented framework, equivalent single-degree-of-freedom (SDOF) models are utilized for the global damage estimation of multi-degree-of-freedom (MDOF) systems. This method does not require high computational analysis and subsequently decreases the required time of seismic design and assessment process. To develop the methodology, RC frames with period range from 0.4 to 2.0 s under 40 records are studied. The effectiveness of proposed technique is evaluated through numerical study under near- and far-field earthquake ground motions. Finally, the results of developed models are compared with two other simplified schemes along with nonlinear time history analysis results of multi-storey frames. To improve the accuracy of damage estimation, a modified relation is presented based on the N2 method results for near- and far-field earthquakes.

Keywords: damage index; equivalent SDOF system; MDOF system; N2 method; near and far-field records

1. Introduction

The accurate estimation of structural and non-structural component damages induced by earthquake is of great significance to the seismic design of structures, plays important role in the decision-making process to retrofit existing buildings. Damage level is commonly affected by various parameters such as the ductility and the energy dissipation capacity of structure, amplitude and frequency features of excitation which should be considered properly in the analysis. Nonlinear time history analysis has been known as a comprehensive although complex and time consuming process for the damage estimation of multi degree of freedom (MDOF) systems requiring detailed characterization of input data. Consequently, it is significant to present a simple and efficient method which enables accurate estimations of structural nonlinear responses resulting from number of mechanisms. Simple methods continue to be promoted for seismic design of structures to reduce the process time and more attempts have been done in this field (e.g., Shibata and Sozen 1976, Akkar *et al*. 2005, Yaghmaei-Sabegh *et al*. 2014). Researchers have focused on equivalent single-degree-of-freedom idealization of the MDOF systems as a simple method (Qi and Moehle 1991, Collins *et al*. 1996, Han and Wen 1997, Tataie *et al*. 2012). They have actually tried to answer this query that how the MDOF systems could be simplified when still giving consistent approximations for practical application? Nonlinear static analysis provides a routine-practical technique in this path. The main reason is the simplicity of the process. Main goal of the most of such studies is to estimate the displacement and/or ductility demand of structure based on the time-independent deformation shape (Φ) of the model as a key factor in pushover-based procedures.

In the literature, several methods built upon nonlinear static analyses for the rapid design and evaluation of buildings integrating the nonlinear static analysis of a MDOF model with the response spectrum analysis of an equivalent SDOF model. N2 method is one of the most popular methods for nonlinear static analysis proposed by Fajfar and Fischinger (Fajfar and Fischinger 1987, Fajfar and Fischinger 1988). The N2 method employs a spectral analysis of the equivalent system in combination with the pushover method of the MDOF model. The primary idea of this method was obtained from the Q-model (Saiidi and Sozen 1981). The original and extended version of this method has been vastly used in the past. In 1996, Fajfar and Gašperšič developed N2 method for the seismic evaluation of both existing and newly designed RC buildings implemented in Eurocode 8 (CEN 2004). They compared predictions with the results of the planner MDOF time history analysis to show validity of N2 method. Their results showed that the method provides reliable estimates of global seismic demand for structures vibrating in the fundamental mode. In 1999, Fajfar used the Reinhorn's idea (Reinhorn 1997) and formulated the N2 method in the acceleration-displacement format. In fact, this version of N2 method is another representation of capacity spectrum method based on inelastic spectra. Later, Fajfar (2000) developed the N2 method as a nonlinear analysis method for performance based seismic design of structures. He

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considered firstly a planner MDOF model with specified characteristics and then determined seismic demand by using acceleration-displacement response spectra and performed pushover analysis. Then, the MDOF system was modeled as an equivalent SDOF system, and the seismic demand of equivalent system determined using the graphical or numerical procedure. Finally, the displacement demand of the SDOF system was transformed into the maximum top displacement of the MDOF system in order to find expected seismic demand through a pushover analysis.

Fajfar *et al*. (2005a, b) extended N2 method to plan-asymmetric buildings when torsional effects are considerable. Their parametric studies showed that in the most of asymmetric building structures, represented by a 3D structural model, torsional effects can be estimated by a linear dynamic (spectral) analysis. They determined displacement demand at the mass center by the basic variant of N2 method, which is based on pushover analysis, and then used elastic dynamic analysis and determined the amplification of demand due to torsion. Dolsek and Fajfar (2005) revealed that the N2 method can simply be used for the determination of approximate summarized IDA.

In 2010, Kilar and Koren used N2 method for the inelastic seismic analysis of base-isolated structures. They observed that a triangular distribution, with an additional force at the base, works best in the most of practical cases. In addition, they indicated that the N2 method can generally provide a reasonable accurate estimation of the actual top displacement and expected damage to the superstructure. An effort has been made by Koren and Kilar (2011) for the response estimation in plan-asymmetric base-isolated building structures through the use of N2 method. Similar to the method proposed by Fajfar *et al*. (2005a), Koren and Kilar (2011) obtained results based on pushover analysis of a 3D structural model to combine with the linear dynamic analysis estimates in order to take into account of structural asymmetry. They compared the results obtained from the extended N2 method with the average results of nonlinear dynamic analysis which demonstrated that the extended N2 method can present a reasonable prediction of the torsional effects when plan-asymmetric is not considerable. Later, the extended N2 was proposed to study the higher mode effects in irregular buildings (Kreslin and Fajfar 2011, 2012). Kreslin and Fajfar demonstrated that higher mode effects depend on the intensity of the ground motion. They compared the obtained results with the results of nonlinear time history analysis and the basic N2 analysis (without the consideration of higher mode effects). They concluded that the extended N2 method could improve seismic demand estimation in the upper part of the building and at flexible edges in comparison with the basic N2 method. Magliulo *et al*. (2012) developed the N2 method to plan-irregular buildings. They proposed three methods combining the accidental eccentricity defined by Eurocode 8 (CEN 2004) to construct a procedure extending the N2 method to flexible structures. They observed that the behavior factor assigned by Eurocode 8 (CEN 2004) seems more conservative in the case of torsionally flexible systems.

In 2014, Krolo *et al*. successfully applied the original N2 method considering semi-rigid joints for steel moment resisting frames. They investigated the joint stiffness effects on the displacement and storey drift demands. The application of the extended N2 method on RC frames with asymmetric setbacks was investigated by Menachery and Manjula (2014) where the obtained results were compared with displacement coefficient method in FEMA-356. It was shown that the extended N2 method estimate displacement and drift demands with reasonable accuracy. Mekki *et al*. (2014) extended the N2 method to integrate soil-structure interaction effects in seismic performance evaluation of concrete structures.

With the aim of providing a simplified tool, there has been a large volume of research devoted to simplified methods for the evaluation of seismic demands; however there have been few attempts at including such models in damage estimation. The effectiveness of equivalent systems have been indicated for the estimation of seismic responses of the MDOF models in the past while limited investigations (e.g., Ghosh *et al*. 2011) have focused on the efficiency of such schemes for damage assessment. In 2011, Ghosh *et al*. proposed three equivalent single-degree idealization schemes based on nonlinear static pushover analyses (NSPA) for the Park-Ang damage index estimation of planner frames.

To the authors' knowledge on the state-of-art damage quantification methodology, there were no significant attempts to use the high capability N2 method in damage estimation of structures. Thus, the main aim of this article is introducing a simple and accurate scheme in order to decrease the high computational analysis and the required time of seismic design and assessment process of the multi-degree-of-freedom systems. To this end, the focus of study is put on an equivalent single-degree idealization based on the N2 method (Qi and Moehle 1991, Collins *et al*. 1996, Gašperšič *et al*. 1992). Then, the potential of this method for the accurate estimates of damage index is examined on concrete frames of 4, 6, 8, 10, 12, 16, 18 and 20 storey under several near- and far-field ground motions. The performance of proposed equivalent system is verified by comparing with the estimates from the rigorous nonlinear dynamic analysis of the MDOF models as benchmark values. This research looks at the damage prediction results of the N2 based procedure along with two another NSPA based schemes. In order to achieve reasonable estimates of damage indices, a simplified modification relation is proposed which help in the better using of proposed method for the accurate damage assessment of real multi-storey buildings subjected to near- or far-field earthquakes.

The rest of this paper is organized as follows. A brief review of damage index is presented in section 2. The properties of the considered MDOF models and the characteristics of records, used in the analysis, are reported in section 3. Section 4 explains theoretical background of the proposed methodology. Obtained results are presented in section 5 and the conclusion is summarized in the final part of paper (section 6).

2. Damage index

Providing adequate resistance for the structures against

Table 1 Park-Ang damage index range for concrete structures (Park and Ang 1985)

Damage index range	Damage state of components
$DI \leq 0.1$	without damage and/or minor local cracking
$0.1 \leq DI \leq 0.25$	ignorable damage with minor cracking
0.25≤EI≤0.4	medium damage with extensive cracking
0.4≤DI≤0.8	severe damage and crushing
$DI \geq 0.8$	collapse of structure

the damage imparted during strong ground motions are the main concepts of seismic design methods and performance-based design. As a result it is required to define an appropriate parameter for the control and the assessment of structural damage amount. To this end, researchers have been introduced and utilized different damage indices (Krawinkler and Zohrei 1983, Park and Ang 1985, Fajfar 1992, Wang *et al*. 2007, Seyedpoor *et al*. 2015). Damage indices can be classified based on different criteria. For example, local, global and storey damage indices or cumulative and non-cumulative damage measures. Most of definitions assign '0' to the un-damaged state and '1.0' to the fully collapsed state.

Comparison and evaluation of damage indices have been debated in many articles. Kunnath and Jenne (1994) studied the damage amount of structures according to different indices and compared them with the empirical observations. Their results showed that the Park and Ang damage index has good agreement with laboratory results. This index was firstly proposed for different damage ranges in concrete structures (Table 1) in 1985 (Park and Ang 1985) and then was developed for the damage assessment of steel structures (Park and Ang 1987). The primary form of the Park and Ang damage index is defined as Eq. (1)

$$D_{PA} = \frac{\delta_m}{\delta_u} + \frac{\beta}{Q_y \, . \delta_u} \int dE \tag{1}$$

where δ_m=maximum deformation under dynamic loading, δ_u=ultimate deformation under monotonic loading, dE= incremental hysteretic energy, Q_y=yield strength, and β= non-dimensional parameter. Later, the Park and Ang damage index has been modified to various forms corresponding to the specific requirements. One of the main modifications was proposed in the third version of the IDARC software (Kunnath *et al*. 1992) as Eq. (2)

$$DI = \frac{\varphi_m - \varphi_y}{\varphi_u - \varphi_y} + \frac{\beta}{M_y \cdot \varphi_u} \int dE \tag{2}$$

where φ_m=maximum rotation under an earthquake, φ_u= ultimate rotation under monotonic loading, φ_y=recoverable rotation during unloading, and M_y=yield moment.

Damage model of Park and Ang has been frequently used in different studies, because of good correspondence with the observed damages for reinforced concrete, steel and wooden structures with different hysteretic properties. Though Park and Ang damage model is an old one, it has been used as the most realistic measure of structural damage in many current studies for different purposes (Pazoki and Tasnimi 2015, Aghagholizadeh and Massumi 2016, Yaghmaei-Sabegh and Makaremi 2017). Then in this article, damage of considered systems was evaluated by the modified Park and Ang damage model, Eq. (2).

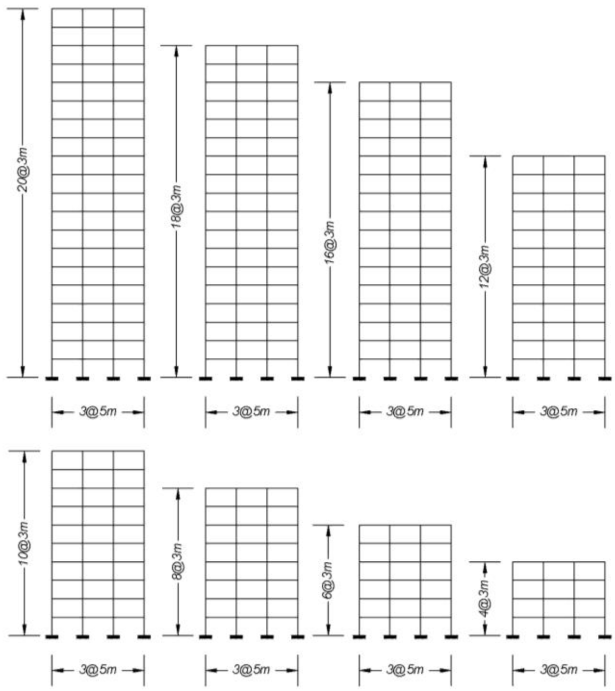


Fig. 1 Elevation view of the frames used in the analysis

3. Models parameters and description dataset

3.1 Structural models

The mathematical models used in the analysis are illustrated in this section of paper. According to Fig. 1, eight 2D regular reinforced concrete frames with 4, 6, 8, 10, 12, 16, 18 and 20 stories and period range from 0.4 to 2.0 s were considered in this study as multi-degree-of-freedom models. The compressive strength of 280 kg/cm^2 and the modulus elasticity of 2.9 kg/cm^2 were used in the analyses. The yield strengths of the longitudinal and shear reinforcements were adopted 4400 and 3000 kg/cm^2, respectively. Square section was selected for columns and the percentage of longitudinal reinforcements of them was considered as 2.5%-3%. The dead load of 600 kg/cm^2 and the live load of 200 kg/cm^2 were assumed to apply over the length of beams. A bilinear hysteric model with unloading stiffness degradation was considered for the structural members of the models. It is worth noting that all considered frames were designed for high seismicity regions according to the Iranian seismic design code (Standard No. 2800). All the analyzed models were designed according to the Iranian concrete code (ABA). The computer program IDARC-2D (Reinhorn *et al*. 2009) was used for the nonlinear dynamic and the pushover analyses of models.

3.2 Earthquake ground motion database

Table 2 Characteristics of near-field records

No.	Earthquake	Year	Station	M	R (km)	PGA (g)	PGV (cm/s)
1	Northridge	1994	Simi Valley	6.7	14.6	0.877	40.9
2	Northridge	1994	Pacomia Dam	6.7	8.0	1.585	55.7
3	Coalinga	1983	Oil City	5.8	8.2	0.866	42.2
4	Loma Prieta	1989	Corralitos	6.9	5.1	0.644	55.2
5	Loma Prieta	1989	Corralitos	6.9	5.1	0.479	45.2
6	Kobe	1995	Kjm	6.9	0.6	0.821	81.3
7	Cape Mendocino	1992	Cape Mendocino	7.1	8.5	1.497	127.4
8	Cape Mendocino	1992	Cape Mendocino	7.1	8.5	1.039	42.0
9	Landers	1992	Lucerne	7.3	1.1	0.785	31.9
10	Nahani,Canada	1985	Site 1	6.8	6.0	1.096	46.1
11	Superstition Hills	1987	SSM	6.7	4.3	0.894	42.2
12	Superstition Hills	1987	SSM	6.7	4.3	0.682	32.5
13	Northridge	1994	Pacomia Dam	6.7	8.0	1.285	103.9
14	Nahani,Canada	1985	Site 1	6.8	6.0	0.978	46.0
15	Morgan Hill	1984	Coyote Lake Dam	6.2	0.1	0.711	51.6
16	Morgan Hill	1984	Coyote Lake Dam	6.2	0.1	1.298	80.0
17	Landers	1992	Lucerne	7.3	1.1	0.721	97.6
18	Coalinga	1983	Transmitter Hill	5.8	9.2	0.840	44.0
19	Coalinga	1983	Transmitter Hill	5.8	9.2	1.083	39.0
20	San Fernando	1971	Pacomia Dam	6.6	2.8	1.226	112.5

Table 3 Characteristics of far-field records

No.	Earthquake	Year	Station	M	R (km)	PGA (g)	PGV (cm/s)
1	San Fernando	1971	Castaic	6.6	24.9	0.324	15.0
2	San Fernando	1971	Castaic	6.6	24.9	0.268	25.9
3	Friuli	1976	Tolmezzo	6.5	37.7	0.351	22.0
4	Friuli	1976	Tolmezzo	6.5	37.7	0.315	30.8
5	Victoria	1980	Cerro Prieto	6.1	34.8	0.621	31.6
6	Northridge	1994	Beverly Hills	6.7	20.8	0.617	40.8
7	Northridge	1994	Beverly Hills	6.7	20.8	0.444	30.2
8	Cape Mendocino	1992	Rio DellOverpass	7.1	18.5	0.385	43.9
9	Cape Mendocino	1992	Rio DellOverpass	7.1	18.5	0.549	42.1
10	Northridge	1994	N Westmoreland	6.7	29.0	0.401	20.9
11	Mammoth Lakes	1980	Long Valley Dam	6.0	20.0	0.921	28.9
12	Mammoth Lakes	1980	Long Valley Dam	6.0	20.0	0.408	33.9
13	Victoria	1980	Cerro Prieto	6.1	34.8	0.587	19.9
14	Cape Mendocino	1992	Shelter Cove Airport	7.1	33.8	0.229	7.1
15	Cape Mendocino	1992	Shelter Cove Airport	7.1	33.8	0.189	6.6
16	Whittier Narrows	1987	Tarzana	6.0	43.0	0.449	20.1
17	Whittier Narrows	1987	Tarzana	6.0	43.0	0.644	22.9
18	Northridge	1994	Glendale	6.7	25.4	0.357	12.3
19	Loma Prieta	1989	UCSC	6.9	18.1	0.309	10.3
20	Loma Prieta	1989	UCSC	6.9	18.1	0.396	13.2

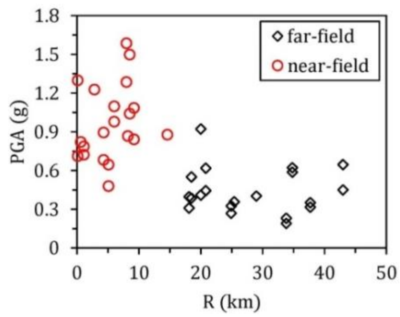


Fig. 2 Distribution of the selected dataset in terms of peak ground acceleration and closest distant to fault rupture

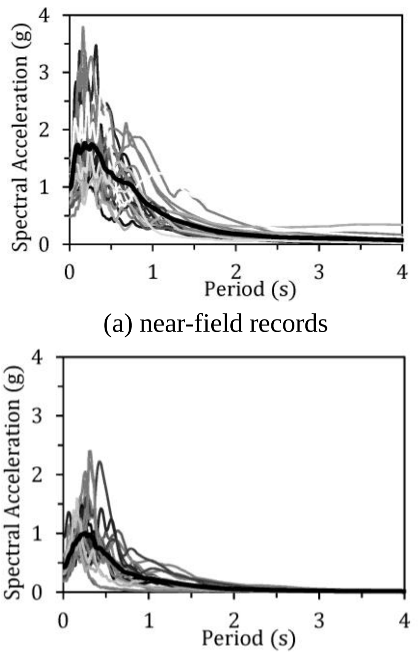
In this study, a total of 40 earthquake records represented by two sets of ground motions were used for the dynamic (or time history) analyses of the MDOF and equivalent SDOF systems. Half of these records were near-field ground motions (Table 2) and the others were belonged to far-field earthquakes (Table 3). Selected records had a surface magnitude range of 5.1 to 7.1.

Fig. 2 shows the distribution of selected ground motions with respect to peak ground acceleration (PGA) and closest distant to fault rupture (R). As can be seen from Fig. 2, records with closest distance shorter than 15 km were considered as near-field records in this study and others were classified as far-field ones. Also, it is obviously seen that all the records selected as near-field earthquakes have often higher peak ground accelerations (PGA) and velocity (PGV). The 5%-damped elastic acceleration response spectra of all ground-motion records have been shown in Fig. 3. More details about the database records involving the earthquake magnitude, peak ground acceleration, peak ground velocity and closest distant to fault rupture have been presented in Tables 2 and 3.

4. Proposed methodology for damage estimation

As previously mentioned, the N2 method is one of the simplest nonlinear methods used for the estimation of the nonlinear response of structures. This method combines the nonlinear pushover analysis of a multi-degree-of-freedom model with the response spectrum analysis of an equivalent single-degree-of-freedom model. Two basic assumption of this method are given as: (i) the response of the structure is determined by one fundamental mode and (ii) Fundamental mode of system is considered constant and time-independent, (i.e., considered fundamental mode does not change under different intensity measures).

In the present study, the N2 method was used for the damage assessment of multi-storey RC frames. With reference to that the all considered frames are regular; the basic version of the N2 method was used in this



(a) near-field records

(b) far-field records

Fig. 3 Acceleration response spectra (5%-damped) of selected records (the mean spectra are shown with thicker lines)

investigation. The framework of the proposed methodology (model 1) is elaborated below in four steps.

1. In the first step, the properties of the MDOF model are determined.
2. The pushover analysis of the MDOF model is performed under a lateral load pattern representing the internal forces which would be experienced by the structure subjected to ground motion. Then, a characteristic nonlinear force-displacement relationship for the MDOF system is provided.

An important step in the pushover analysis is the selection of an appropriate lateral load distribution. In the N2 method, the vector of lateral load **f**, used in the pushover analysis, is defined as

$$\mathbf{f} = \rho \mathbf{M} \mathbf{\Phi} \tag{3}$$

where **M** is the mass matrix of the structure, **Φ** is the time-independent shape vector and ρ controls the magnitude of lateral loads. It is worth noting that if the assumed displacement shape was exact and constant during ground shaking, the distribution of lateral loads would be like to the distribution of effective earthquake loads.

3. The quantities of the MDOF system are transformed to the equivalent SDOF characteristics. In the N2 method, seismic demand is achieved by the use of response spectra. Accordingly, the MDOF system should be modeled as a SDOF system. Different methods have been utilized to characterize the properties of an equivalent SDOF system. The method used in the current version of the N2 method is briefly described herein (more details are available in Fajfar 2002).

The dynamic equation of a MDOF model is written as follows

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{R}^{*} = -\mathbf{M}\iota\ddot{u}_g \tag{4}$$

$$\mathbf{U} = \mathbf{\Phi} D(t) \tag{5}$$

where **U** represents lateral displacement, $\mathbf{R}^*$ represents internal forces, $\ddot{u}_g$ is the ground acceleration as a function of time, t is a vector representing the direction of ground motion, **Φ** is the time-independent displacement shape and $D(t)$ represents the time-dependent roof displacement. It should be noted that the adopted displacement shape **Φ** is normalized to the roof displacement (i.e., the component at the top is equal to 1). For simplicity, the effect of damping would be considered in the design spectrum and is not regarded in Eq. (4).

From the statics, the internal forces, $\mathbf{R}^*$, are equal to the statically applied external loads, **f**, i.e.

$$\mathbf{R}^{*} = \mathbf{f} \tag{6}$$

where the lateral loads vector utilized in the nonlinear static pushover analysis of the N2 method is defined as Eq. (3). Substituting Eqs. (3)-(6) in Eq. (4) and then multiplying with $\mathbf{\Phi}^T$ (from the left side) yields to

$$\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}\ddot{D}(t) + \mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}\rho = -\mathbf{\Phi}^T\mathbf{M}\iota\ddot{u}_g \tag{7}$$

The dynamic equation of the equivalent SDOF system, Eq. (8), is obtained by multiplying and dividing the left hand side of the Eq. (7) with $\mathbf{\Phi}^T\mathbf{M}t$

$$m^{*}\ddot{D}^{*} + F^{*} = -m^{*}\ddot{u}_g \tag{8}$$

where m^*, D^* and F^* represent the mass, displacement and strength of the equivalent SDOF system, respectively and are calculated by the following equations

$$m^{*} = \mathbf{\Phi}^T\mathbf{M}\iota = \sum m_i\Phi_i \tag{9}$$

$$D^{*} = \frac{D(t)}{\Gamma} \tag{10}$$

$$F^{*} = \frac{V}{\Gamma} \tag{11}$$

$$V = \sum \rho m_i \Phi_i = \sum P_i = \mathbf{\Phi}^T\mathbf{M}\iota\rho = pm^{*} \tag{12}$$

$$\Gamma = \frac{\mathbf{\Phi}^T\mathbf{M}\iota}{\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}} = \frac{\sum m_i\Phi_i}{\sum m_i\Phi_i^2} = \frac{m^{*}}{\sum m_i\Phi_i^2} \tag{13}$$

where i denotes the storey level, V is the base shear of the MDOF model and Γ is a constant controlling the transformation from the MDOF into SDOF model and vice-versa. With reference to the same constant value of Γ applying for the transformation of both displacements and forces, the force-displacement relationship obtained for the MDOF system (V–$D(t)$ diagram) also applies to the equivalent SDOF system (the F^*–D^* diagram). Thus, the initial stiffness of the equivalent SDOF system equals with the ones defined by the V–$D(t)$ diagram of the MDOF system.

Following transforming the MDOF quantities to the corresponding equivalent SDOF ones, it is necessary to assume an approximate bilinear force-displacement relationship for the equivalent SDOF system that requires engineering judgment. In the graphical procedure used in the N2 method, the post-yield stiffness is considered equal to zero. Thus, the elastic period of the idealized bilinear system T^* is calculated as

$$T^* = 2\pi\sqrt{\frac{m^* D_y^*}{F_y^*}} \tag{14}$$

where F_y^* and D_y^* are the yield strength and displacement, respectively.

4. In the last step of proposed process, damage index of the equivalent SDOF system is determined. For this purpose, the nonlinear time history analysis is performed for the equivalent SDOF system and then the damage index is calculated.

The damage estimates based on proposed scheme are compared with those of time history analysis of the MDOF systems. The results of two other equivalent SDOF schemes used by Ghosh *et al*. (2011) are considered in comparison process as well. Ghosh *et al*. (2011) deliberated these equivalent single-degree idealization approaches based on nonlinear static pushover analyses (NSPA) for the estimation of the Park-Ang damage index of the planner of the three-, nine- and twenty storey frames (these models named models 2, 3 in this paper). The main formulation of these equivalent systems is explained briefly (more details are available in Ghosh *et al*. 2011).

The formulation of these equivalent systems (models 2, 3) is initially established by Eq. (15); where dynamic equilibrium of the MDOF system subjected to horizontal ground acceleration $\ddot{u}_g$ is written as

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{R} = -\mathbf{M}\iota\ddot{u}_g \tag{15}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{U}$ is the lateral displacement vector, ι is the influence vector and $\mathbf{R}$ is the restoring force vector. Using Eq. (5) and multiplying of Eq. (15) with shape vector $\mathbf{\Phi}^T$, yields to the following equation

$$\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}\ddot{D}(t) + \mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}\dot{\mathbf{D}} + \mathbf{\Phi}^T\mathbf{R} = -\mathbf{\Phi}^T\mathbf{M}\iota\ddot{u}_g \tag{16}$$

If the same shape vector is regarded for the nonlinear static pushover analysis of the structure, the restoring force vector could be represented by the base shear (V). With assuming a bilinear idealization of the V versus D pushover curve, the base shear can be expressed as a function of the roof displacement as

$$V = K\,G(D) \tag{17}$$

where K is the initial slope of the pushover curve and $G(.)$is the scalar mathematical function of V which describes the shape of the pushover curve. This process reduces Eq. (16) to the equilibrium equation of an inelastic SDOF system with mass M^*, damping C^* and linear elastic stiffness K^*

$$M^*\ddot{D} + C^*\dot{D} + K^*G(D) = -L^*\ddot{u}_g \tag{18}$$

where $M^*=\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}$, $C^*=\mathbf{\Phi}^T\mathbf{C}\mathbf{\Phi}$, $K^*=\mathbf{\Phi}^T K\mathbf{f}$, $L^*=\mathbf{\Phi}^T\mathbf{M}\iota$ and $\mathbf{f}$ is the force vector utilized in the nonlinear static pushover analysis. Dividing both sides of Eq. (18) by the mass of the equivalent SDOF system, the dynamic equilibrium can be written as Eq. (19)

$$\ddot{D} + 2\zeta\omega^*\dot{D} + (\omega^*)^2 G(D) = -\Gamma^*\ddot{u}_g \tag{19}$$

where ω^* and ζ are the linear elastic frequency and damping ratio of inelastic SDOF system and $\Gamma^*=L^*/M^*$, $(\omega^*)^2=K^*/M^*$, $2\zeta\omega^*=C^*/M^*$.

The main properties of the models 2, 3 depend on the adopted shape vector $\mathbf{\Phi}$ and the assumed lateral force distribution used in the pushover analysis. In the model2, the force-displacement curve of the structure is obtained from the NSPA with the lateral load distribution of $\mathbf{f}$, Eq. (20); whereas in the model 3, the NSPA is performed using the lateral force distribution proposed by IBC 2006 (ICC 2006), Eq. (21a).

$$\mathbf{f} = \mathbf{M}\mathbf{\Phi} \tag{20}$$

$$f_i = \frac{w_i h_i^k}{\sum_{j=1}^{n} w_j h_j^k} V_b \tag{21a}$$

$$k = \begin{cases} 1 & T_1 \le 0.5s \\ 0.5T_1 + 3/4 & 0.5 \le T_1 \le 2.5s \\ 2 & T_1 \ge 2.5s \end{cases} \tag{21b}$$

where V_b is the base shear, T_1 is the fundamental period of structure, k is the period-dependent parameter, n is the number of storey, w_i and h_i are the weight and height of ith storey, respectively.

It is worth noting that, the model 2 is based on the assumption that the fundamental mode dominates the structural response and the contribution of other modes can be ignored. Thus, the model 2 is formed by assuming $\mathbf{\Phi}=\mathbf{\Phi}_1$ in Eq. (20), where $\mathbf{\Phi}_1$ is the fundamental mode shape of the structure normalized to a roof displacement. The shape vector of the model 3 is not defined based on modal shape of structure and it is determined using the deformation shape obtained based on the linear elastic response. The general framework of damage estimation process proposed in this investigation (model 1) and the models 2, 3 have been summarized in Fig. 4.

5. Results and discussions

In this section, damage prediction results of the N2 method along with two simplified models used in Ghosh *et al*. (2011) are presented and discussed. Analysis will be performed for both near and far-field ground motion records. Furthermore, time-history analyses of the same structural systems are carried out for each of input motions to make a realistic damage prediction. Results obtained from the analyses of the MDOF and equivalent SDOF systems are compared in terms of the bias factor, B.F., Eq. (22), which is defined as the ratio of the MDOF model's damage index to the equivalent SDOF model's damage index for a particular ground motion record. This factor could be considered as a useful tool for the measuring of the accuracy and effectiveness of proposed scheme.

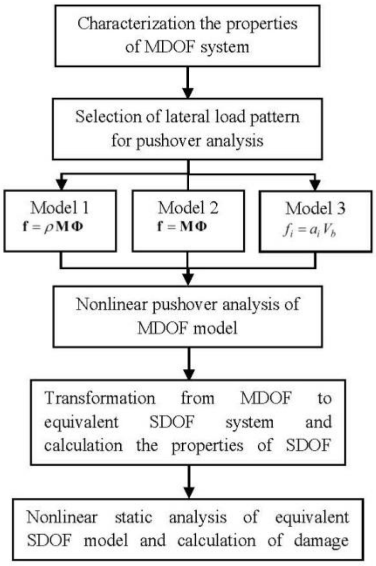
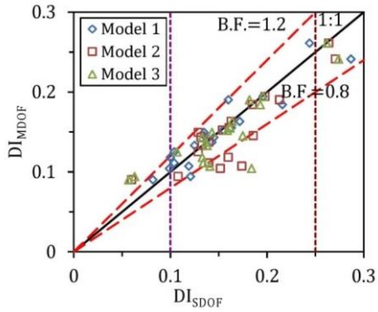
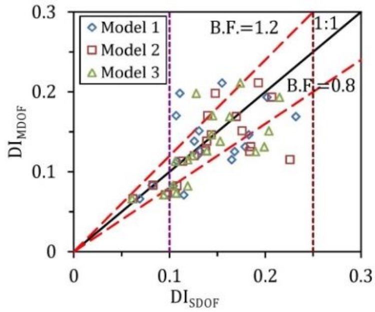


Fig. 4 The procedure of the damage index estimation of MDOF models using equivalent SDOF models



(a) near-field records

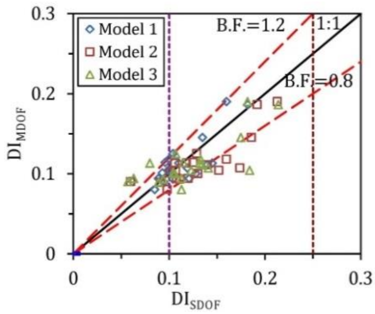


(b) far-field records

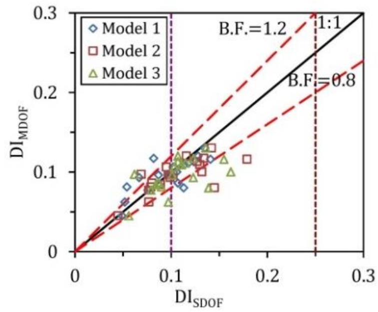
Fig. 5 Scatter plots of damage of the equivalent SDOF and MDOF systems for the 4-storey frame

$$B.F. = \frac{DI_{MDOF}}{DI_{SDOF}} \tag{22}$$

where DI_{MDOF} and DI_{SDOF} represent damage index of the MDOF and equivalent SDOF models, respectively. This factor is computed for all of the MDOF and corresponding equivalent SDOF models when subjected to 40 ground motion records. For a perfect case, this factor should be equal to 1.0 which means that the response of the equivalent SDOF system is exactly same as the MDOF system. The bias factor larger than 1.0 denotes that using the equivalent SDOF system leads to non-conservative damage

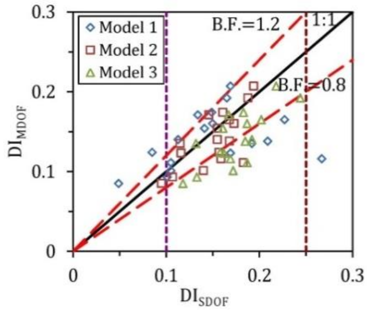


(a) near-field records

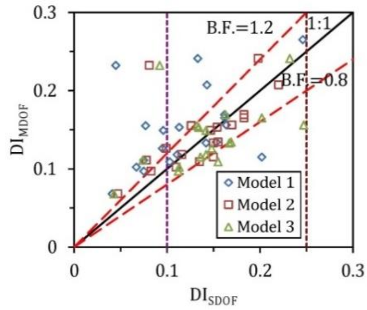


(b) far-field records

Fig. 6 Scatter plots of damage of the equivalent SDOF and MDOF systems for the 6-storey frame



(a) near-field records

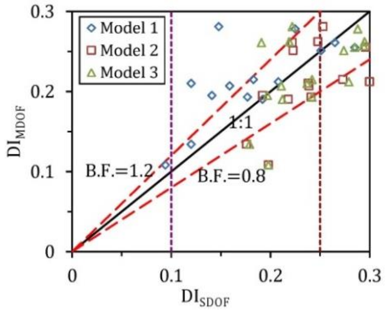


(b) far-field records

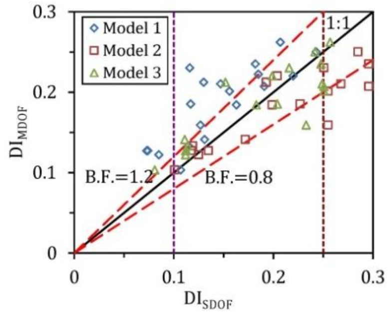
Fig. 7 Scatter plots of damage of the equivalent SDOF and MDOF systems for the 8-storey frame

approximation for the MDOF systems.

Figs. 5-12 display the typical scatter plots of the equivalent SDOF damage predictions versus the damage index values of the MDOF systems for all three considered schemes under near- and far-field records. From these figures, the damage index values of the MDOF systems can be easily compared with the indices of numerous considered equivalent SDOF systems. Since the values of all computed

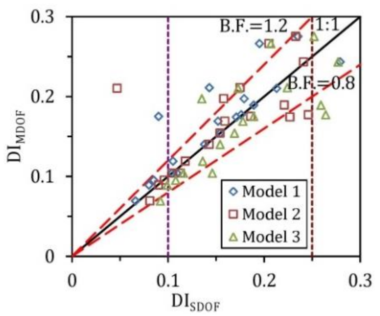


(a) near-field records

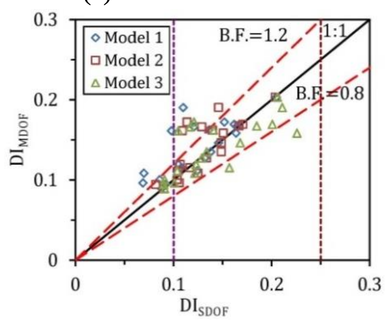


(b) far-field records

Fig. 8 Scatter plots of damage of the equivalent SDOF and MDOF systems for the 10-storey frame

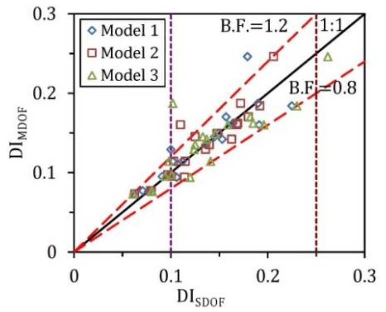


(a) near-field records

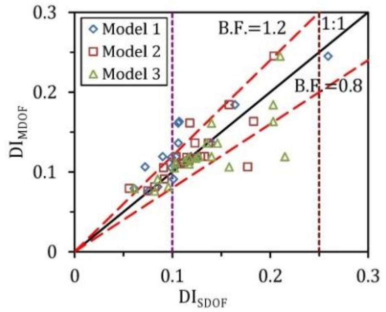


(b) far-field records

Fig. 9 Scatter plots of damage of the equivalent SDOF and MDOF systems for the 12-storey frame

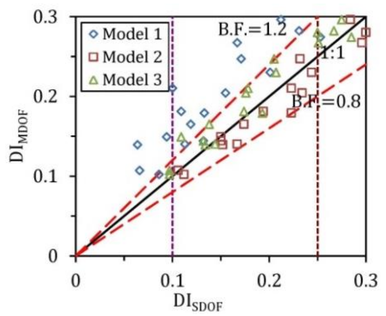


(a) near-field records

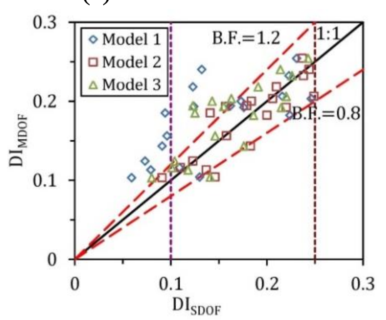


(b) far-field records

Fig. 10 Scatter plots of damage of the equivalent SDOF and MDOF systems for the 16-storey frame



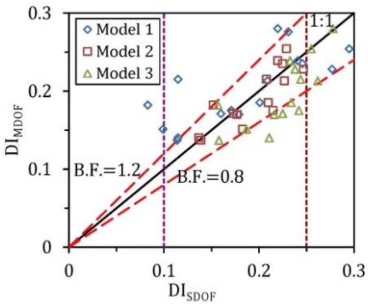
(a) near-field records



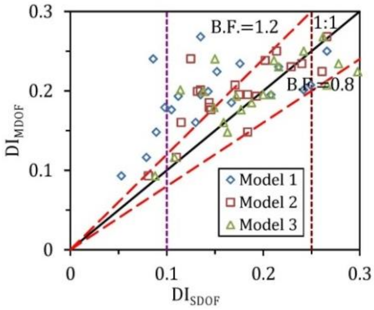
(b) far-field records

Fig. 11 Scatter plots of damage of the equivalent SDOF and MDOF systems for the 18-storey frame

damage indices were less than 0.3, only three first ranges of Table 1 have been identified in these figures: 1) $DI \leq 0.1$, 2) $0.1 \leq DI \leq 0.25$ and 3) $0.25 \leq DI \leq 0.3$. The diagonal line, denoted by 1:1 across scatter plots, is considered as ideal estimate (*B.F.*=1.0). In each plot, the position of points placed in the below and the above of this line denote that using equivalent systems leads to the over- and under-estimation of the damage index of the MDOF systems. It is worth noting that the dispersion of data in each range depends on vibration period, dissipated hysteretic energy by the structure and the characteristics of input motions. In order to measure the accuracy of each considered equivalent SDOF scheme, the *B.F.* range from 0.8 to 1.2 (accurate range) has been defined as a criterion for the evaluation of bias factor values; hence predictions will be considered as accurate results when the values put in this range.

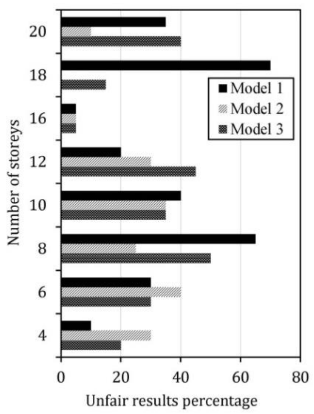


(a) near-field records

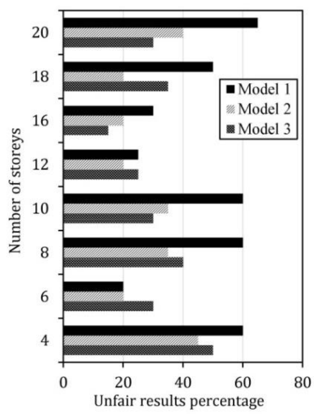


(b) far-field records

Fig. 12 Scatter plots of damage of the equivalent SDOF and MDOF systems for the 20-storey frame



(a) near-field records



(b) far-field records

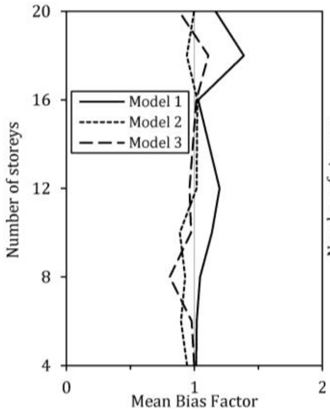
Fig. 13 Unfair results percentage

Figs. 5-12 indicate that the equivalent SDOF scheme of the N2 method are resulted to good predictions for the low-rise frames (4-, 6-storey frames) due to near-field earthquakes at all three considered damage index regions; Whereas, the accuracy of the N2 method decreases with the increase of storey and this model usually produces non-conservative results in the high-rise frames (18-, 20-storey frames).

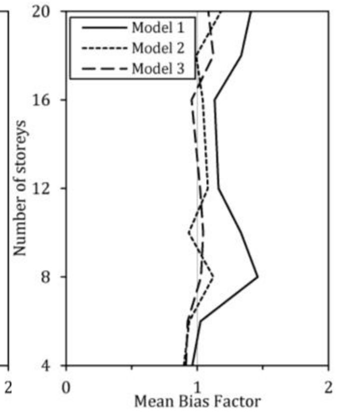
It can also be concluded that using the equivalent systems in all frames under far-field earthquakes generally leads to lower damage indices (in the range of $DI \leq 0.1$) and non-conservative results. In addition, the results in $0.1 \leq DI \leq 0.25$ show that utilizing the equivalent SDOF systems for the frames lower than 10 storey leads to under-estimation of damage index. In the third damage region $0.25 \leq DI \leq 0.3$, the equivalent SDOFs provide conservative results for 4-, 10- and 20-storey frames for far-field records

Table 4 Regression coefficients for the proposed modification relation

Earthquake type	*a*	*b*
Near-field	0.8502	0.0188
Far-field	0.7138	0.0422

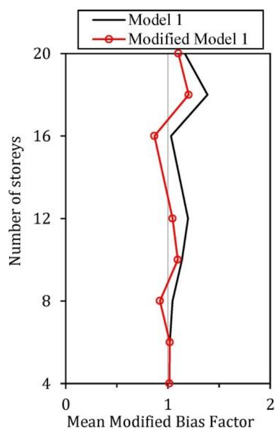


(a) near-field records

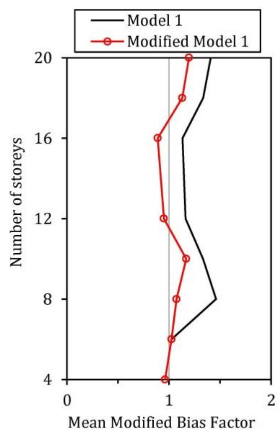


(b) far-field records

Fig. 14 Mean bias factors



(a) near-field records



(b) far-field records

Fig. 15 Mean modified bias factors

where there is no data for other frames in this region.

For the convenience of comparison, the percentage of bias factors placed out of the pre-defined accurate range has been considered as unfair results and presented for all frames. Fig. 13 shows these percentages for all of systems subjected to near- and far-field records. With reference to the Figs. 5-13, it may seem that using of the N2 model in the short period structures ($T \leq 0.6$ s) generally leads to better results for near-field earthquakes. According to Fig. 13, lower percentages of unfair results are frequently observed for near-field earthquakes. Since, it is shown that the proposed method has low percentage of unfair results (smaller than 30%) in the half of the considered frames when subjected to near-field earthquakes. Consequently, it can be concluded that the N2 method is more desire for near-field earthquakes. Obtained results according to Fig. 13 shows that the unfair data percentage of the N2 method is lower than other methods in the low rise structures subjected to near-field records. On the other side, the N2 method is not able to capture damage prediction in the same level of accuracy for the other cases.

For the general comparison of results, the mean of computed bias factors have been plotted in Fig. 14 for all considered equivalent systems. This figure indicates that in the case of near-field records, equivalent systems conducive to good estimations (bias factors are close to 1.0) for the low rise structures. This figure could highlight that utilizing proposed equivalent system is more proper for damage estimation of low rise multi-storey frames (4, 6-storeys) subjected to the near-field earthquakes.

To improve the N2 method results in damage prediction and in order to have an accurate estimation for medium- and high-rise structures (storey number ≥8), a simple modification relation, Eq. (23), has been proposed in this paper. This relation has been obtained based on the actual damage indices of the MDOF systems obtained through nonlinear time history analysis.

$$DI_{MSDOF} = a * \left(DI_{SDOF}\right) + b \tag{23}$$

where DI_{SDOF} and DI_{MSDOF} are the actual and modified damage index of the equivalent SDOF system, respectively and a and b are the linear regression coefficients (Table 4).

Fig. 15 shows the mean bias factors of damage indices computed based on the original N2 method and modified equivalent SDOF for near- and far-field records. As can be seen in this figure, generally the values of bias factors becomes close to 1.0 when the modifying is considered. It is observed that the computed bias factors are limited within ±20% of ideal case (i.e., *B.F.*=1). It can be concluded that by the adopting of a maximum error about 20%, the proposed equivalent SDOF scheme can be used for the damage estimation of the MDOF systems subjected to near- and far-field earthquakes.

6. Conclusions

The results of an investigation aimed at evaluating a methodology based on the simple N2 for the damage estimate was presented in this study. This method prefers the using of equivalent single-degree-of-freedom model for the damage estimation of multi-storey frames based on inelastic demand spectra. Using this method reduces the required time for the damage estimation of the MDOF models. In this study, eight reinforced concrete frames with period range between 0.4-2.0 s, representative of a wide range of generic structures, were subjected to 40 ground motion records. These frames were transformed to the equivalent SDOFs by using of the N2 method. The results of considered scheme were compared with the results from time history analysis of the MDOF systems. The effectiveness of studied models was examined by means of the bias factor, defined as the ratio of the MDOF and the equivalent SDOF damage indices.

Results showed that proposed equivalent single-degree system based on N2 method gives good estimate of damage index for low-rise frames (4-, 6-storey frames) when subjected to near-field ground motions. The damage predictions of the flexible structures (18-, 20-storey frames) by the N2 method are almost in a smaller range than those of nonlinear time history analysis. The simple modification relation proposed in this investigation, enables to use the improved N2 scheme for reliable damage estimation of multi-storey frames subjected to both near- and far-field earthquakes.

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