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A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate

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Abstract. In this work, a simple but accurate hyperbolic plate theory for the free vibration analysis of functionally graded material (FGM) sandwich plates is developed. The significant feature of this formulation is that, in addition to including the shear deformation effect, it deals with only 3 unknowns as the classical plate theory (CPT), instead of 5 as in the well-known first shear deformation theory (FSDT) and higher-order shear deformation theory (HSDT). A shear correction factor is, therefore, not required. Two common types of FGM sandwich plates are considered, namely, the sandwich with the FGM face sheet and the homogeneous core and the sandwich with the homogeneous face sheet and the FGM core. The equation of motion for the FGM sandwich plates is obtained based on Hamilton's principle. The closed form solutions are obtained by using the Navier technique. The fundamental frequencies are found by solving the eigenvalue problems. Numerical results of the present theory are compared with the CPT, FSDT, order shear deformation theories (HSDTs), and 3D solutions. Verification studies show that the proposed theory is not only accurate and simple in solving the free vibration behaviour of FGM sandwich plates, but also comparable with the higher-order shear deformation theories which contain more number of unknowns.

Keywords: sandwich plate; functionally graded material; a simple 3-unknown theory

1. Introduction

Sandwich structures have been extensively used in aerospace, aeronautic, automotive, naval, underwater, and building structures. Sandwich plates may be used for constructing light-weight structures with high strength or stiffness to weight ratios, noise, vibration, and harshness (NVH) isolation. Generally the conventional sandwich structures are fabricated form three homogeneous layers, two face sheets adhesively bonded to the core. However, due to the mismatch of stiffness properties between the face sheets and the core, sandwich structures are susceptible to face sheet/core debonding, which is a major problem in sandwich construction, especially under impact loading (Abrate 1998). To increase the resistance of sandwich structures to this type of failure, the concept of a functionally graded material (FGM) is being actively explored in sandwich structure design. FGMs are achieved by gradually changing the composition of the constituent materials along one (or more) direction(s), usually in the thickness direction, to obtain smooth variation of material properties and optimum response to externally applied loading (Attia *et al.* 2018, Bakhadda *et al.* 2018, Zine *et al.* 2018, Sekkal *et al.* 2017, Abdelaziz *et al.* 2017, Mouffoki *et al.* 2017, Bellifa *et al.* 2017a, Zidi *et al.* 2017, Benadouda *et al.* 2017, Barati and Shahverdi 2016, Benferhat *et al.* 2016, Ahouel *et al.* 2016, Bounouara *et al.* 2016, Bousahla *et al.* 2016, Belkorissat *et al.* 2015, Zidi *et al.* 2014). The FGM sandwich construction exists in two types: the FGM face sheet-homogeneous core and the homogeneous face sheet-FGM core. For the case of the homogeneous core, the soft-core is commonly employed because of the light weight and high bending stiffness in the structural design. The homogeneous hardcore is also employed in other fields, such as control or thermal environments. The actuators and sensors which are common piezoelectric ceramics are always in the midlayers of the sandwich construction (Shen 2005). Moreover, in the thermal environments, the metal-rich face sheet can reduce the large tensile stress on the surface at the early stage of cooling (Noda 1999).

Studies related to FGM sandwich structures are few in numbers. Etemadi *et al.* (2009) used three-dimensional finite element simulations for investigating low velocity impact response of sandwich panels with a FGM core. A three-dimensional elasticity solution is presented by

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Anderson (2003) for a sandwich composite with a FGM core subjected to transverse loading by a rigid spherical indentor. Shodja *et al.* (2007) developed an exact thermo-elasticity solution for a 2D sandwich structures with FGM coating. Li *et al.* (2008) presented a three-dimensional solution for free vibration of multi-layer FGM system-symmetric and unsymmetric FGM sandwich plates using the Ritz method. Yaghoobi and Yaghoobi (2013) presented analytical solutions for the buckling of symmetric sandwich plates with FGM face sheets resting on an elastic foundation based on the first-order shear deformation plate theory and subjected to mechanical, thermal and also thermo-mechanical loads. Yaghoobi and Fereidoon (2014) studied the mechanical and thermal buckling analysis of FG plates resting on elastic foundation using a simple refined nth-order shear deformation theory. Gulshan Taj *et al.* (2014) studied the bending behaviour of FGM skew sandwich plates by employing a higher shear and normal deformation theory in conjunction with FEM. Ait Amar Meziane *et al.* (2014) proposed an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Attia *et al.* (2015) discussed the free vibration behavior of FG plates with temperature-dependent properties using various four variable refined plate theories. Ait Yahia *et al.* (2015) examined the wave propagation in FG plates with porosities using various higher-order shear deformation plate theories. Akavci (2016) studied the mechanical behavior of FG sandwich plates on elastic foundation. Abdelhak *et al.* (2016) investigated the thermal buckling response of FG sandwich plates with clamped boundary conditions. Bouderba *et al.* (2016) examined the thermal stability of FG sandwich plates using a simple shear deformation theory. Beldjelili *et al.* (2016) presented and studied the hygro-thermo-mechanical bending response of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Barka *et al.* (2016) discussed the thermal post-buckling behavior of imperfect temperature-dependent sandwich FGM plates resting on Pasternak elastic foundation. In a number of recent articles - see (Hadji *et al.* 2011, Bourada *et al.* 2012, Bachir Bouiadjra *et al.* 2012, Tounsi *et al.* 2013, Kettaf *et al.* 2013) a new refined and robust plate theory for bending response and buckling of simply supported FGM sandwich plate with only four unknown functions has been developed.

The increase in FGM applications requires accurate models to predict their responses. Since the shear deformation has significant effects on the responses of functionally graded (FG) plates, shear deformation theories are used to capture such shear deformation effects. Since the first-order shear deformation theory (FSDT) accounts for the transverse shear deformation effect by the way of linear variation of in-plane displacement through the thickness, it violates the equilibrium conditions at the top and bottom surfaces of the plate; hence, shear correction factors are required to correct the unrealistic variation of transverse shear stresses and shear strain through the thickness. The higher-order shear deformation theories (HSDTs) account for the shear deformation effects, and satisfy the zero transverse shear stresses on the top and bottom surfaces of the plate, thus, a shear correction factor is not required. Some of these HSDTs are computational costs because with each additional power of the thickness coordinate, an additional unknown is introduced to the theory (e.g., theories by Pradyumna and Bandyopadhyay 2008) and Neves *et al.* (2012) with nine unknowns, Reddy (2011) with eleven unknowns, Talha and Singh (2010) with thirteen unknowns). Although some well-known HSDTs have five unknowns (e.g., third-order shear deformation theory (Reddy 2000), sinusoidal shear deformation theory (Etemadi *et al.* 2003, Anderson 2003, Shodja *et al.* 2007), hyperbolic shear deformation theory (Atmane *et al.* 2010, Benyoucef *et al*. 2010) or only four unknowns (Hadji *et al.* 2011, Bourada *et al.* 2015, Benachour *et al.* 2011, 2012, Bachir Bouiadjra *et al.* 2012, Tounsi *et al.* 2013, Kettaf *et al.* 2013, Bouderba *et al.* 2013, Boukhari *et al.* 2016, Menasria *et al.* 2017, Chikh *et al.* 2017, Khetir *et al.* 2017, Fahsi *et al.* 2017, El-Haina *et al.* 2017), their equations of motion are much more complicated than those of FSDT. Thus, needs exist for the development of HSDTs which are simple to use.

The aim of this work is to develop a new variationally consistent three unknown hyperbolic shear deformation theory for FGM sandwich plates. The present theory has only three unknowns and three governing equations as the classical plate theory (CPT), but it satisfies the stress-free boundary conditions on the top and bottom surfaces of the plate without requiring any shear correction factors. Thus, the number of unknowns and equations of motion of the proposed theory is reduced and hence makes it simple to use. Equations of motion are derived from Hamilton's principle. Two common types of FGM sandwich plates, namely, the sandwich with the FGM face sheet and the homogeneous core and the sandwich with the homogeneous face sheet and the FGM core, are considered. The Navier solution is used to obtain the closed form solutions for simply supported FGM sandwich plates. To illustrate the accuracy of the present theory, the obtained results are compared with three-dimensional elasticity solutions and the results of the first-order and the other higher-order theories.

2. Theoretical formulation

2.1 Geometrical configuration

Consider the case of a uniform thickness, rectangular FGM sandwich plate composed of three microscopically heterogeneous layers referring to a rectangular coordinates (x, y, z) as shown in Fig. 1. The top and bottom faces of the plate are at $z=\pm h/2$, and the edges of the plate are parallel to axes x and y.

The sandwich plate is composed of three elastic layers, namely, "Layer 1", "Layer 2", and "Layer 3" from bottom to top of the plate. The vertical ordinates of the bottom, the two interfaces, and the top are denoted by $h_1=-h/2$, h_2, h_3, $h_4=h/2$, respectively. For the brevity, the ratio of the thickness of each layer from bottom to top is denoted by the

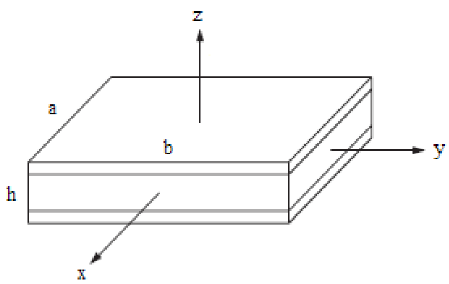

Fig. 1 Geometry of rectangular FGM sandwich plate with uniform thickness in the rectangular Cartesian coordinates

combination of three numbers, i.e., "1-0-1", "2-1-2" and so on. As shown in Fig. 2, two types A and B are considered in the present study:

- Type A: FGM face sheet and homogeneous core
- Type B: Homogeneous face sheet and FGM core

2.2 Material properties

The properties of FGM vary continuously due to gradually changing the volume fraction of the constituent materials (ceramic and metal), usually in the thickness direction only. Power-law function is commonly used to describe these variations of materials properties. The sandwich structures made of two types of power-law FGMs mentioned before are discussed as follows.

2.2.1 Type A: power-law FGM face sheet and homogeneous core

The volume fraction of the FGMs is assumed to obey a power-law function along the thickness direction

$$V^{(1)} = \left(\frac{z-h_1}{h_2-h_1}\right)^k, \qquad z \in [h_1, h_2] \tag{1a}$$

$$V^{(2)} = 1, \qquad z \in [h_2, h_3] \tag{1b}$$

$$V^{(3)} = \left(\frac{z-h_4}{h_3-h_4}\right)^k, \qquad z \in [h_3, h_4] \tag{1c}$$

where $V^{(n)}$, (n=1,2,3) denotes the volume fraction function of layer n; k is the volume fraction index ($0 \le k \le +\infty$), which dictates the material variation profile through the thickness.

2.2.2 Type B: homogeneous face sheet and power-law FGM core

The volume fraction of the FGMs is assumed to obey a power-law function along the thickness direction

$$V^{(1)} = 0, \qquad z \in [h_1, h_2] \tag{2a}$$

$$V^{(2)} = \left(\frac{z-h_2}{h_3-h_2}\right)^k, \quad z \in [h_2, h_3] \tag{2b}$$

$$V^{(3)} = 1, \qquad z \in [h_3, h_4] \tag{2c}$$

in which $V^{(n)}$, and k are as same as defined in Eq. (1).

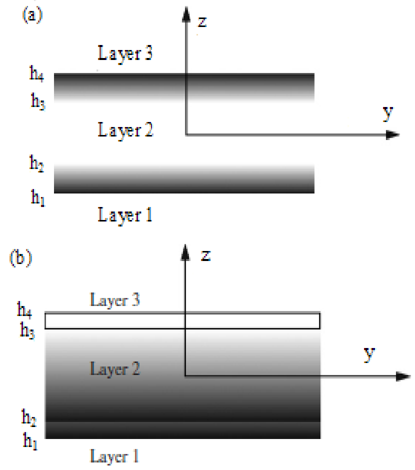

Fig. 2 The material variation along the thickness of the FGM sandwich plate: (a) FGM face sheet and homogeneous core (b) homogeneous face sheet and FGM core

The effective material properties, like Young's modulus E, Poisson's ratio v, and mass density ρ, then can be expressed by the rule of mixture (Marur 1999, Bellifa *et al.* 2016) as

$$P^{(n)}(z) = P_2 + (P_1 - P_2)V^{(n)} \tag{3}$$

where $P^{(n)}$ is the effective material property of FGM of layer n. For type A, P_1 and P_2 are the properties of the top and bottom faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$, (n=1,2,3); For type B, P_1 and P_2 are the properties of layer 3 and layer 1, respectively.

These two types of FGM sandwich plates will be discussed later in the following sections.

2.3 Three-unknown hyperbolic shear deformations theory

The aim of this paper is to develop a simple three-unknown hyperbolic shear deformation theory in which in-plane displacement is expanded as a hyperbolic variation through the thickness. The advantages of the present theory is that the number of variables involved in this theory is same as that in the classical plate theory (CPT), and the stress-free boundary conditions on the top and bottom surfaces of the plate can be guaranteed without use of shear correction factors .

2.3.1 Kinematics

The in-plane displacements u and v, and the transverse displacement w of a material point located at (x,y,z) in the plate are assumed according to the following refined shear deformation plate theory

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} - f(z)\frac{\partial^3 w_0}{\partial x^3} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} - f(z)\frac{\partial^3 w_0}{\partial y^3} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{4}$$

where u_0, v_0, and w_0 are three unknown displacement functions of midplane of the plate. In this work, $f(z)$ is a new shape function proposed for representing the distribution of the transverse shear strains and shear stresses through the thickness of the plate and is given as

$$f(z)=\frac{\cosh\left(\frac{\pi}{2}\right)h^2\left(z\pi\cosh\left(\frac{\pi}{2}\right)-h\sinh\left(\frac{\pi z}{h}\right)\right)}{\frac{\pi}{2}\left(\cosh\left(\frac{\pi}{2}\right)-1\right)} \tag{5}$$

The nonzero strains associated with the displacement field in Eq. (4) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix}+f(z)\begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix} \tag{6a}$$

$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{6b}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u_0}{\partial x}\\ \frac{\partial v_0}{\partial x}\\ \frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial^2 w_0}{\partial x^2}\\ -\frac{\partial^2 w_0}{\partial y^2}\\ -2\frac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix} \tag{7a}$$

$$\begin{Bmatrix}\eta_x\\ \eta_y\\ \eta_{xy}\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial^4 w_0}{\partial x^4}\\ -\frac{\partial^4 w_0}{\partial y^4}\\ -\frac{\partial^2(\nabla^2 w_0)}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial^3 w_0}{\partial y^3}\\ -\frac{\partial^3 w_0}{\partial x^3}\end{Bmatrix} \tag{7b}$$

and

$$g(z)=f'(z),\quad \nabla^2 w_0=\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2} \tag{8}$$

2.3.2 Constitutive relations

The linear constitutive relations of a FGM sandwich plate can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{Bmatrix}^{(n)}=\begin{bmatrix}C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{22} & 0 & 0 & 0\\ 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}^{(n)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}^{(n)} \tag{9}$$

where (σ_x, σ_y, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively. The stiffness coefficients, C_{ij}, can be expressed as

$$C_{11}=C_{22}=\frac{E(z)}{1-\nu^2},\quad C_{12}=\nu\, C_{11} \tag{10a}$$

$$C_{44}=C_{55}=C_{66}=G(z)=\frac{E(z)}{2(1+\nu)}, \tag{10b}$$

2.3.3 Equations of motion

Hamilton's principle is employed herein to obtain equations of motion. The principle can be expressed in an analytical form as follows (Meksi *et al.* 2018, Besseghier *et al.* 2017, Klouche *et al.* 2017, Hachemi *et al.* 2017, Bellifa *et al.* 2017b, Houari *et al.* 2016, Mahi *et al.* 2015, Taibi *et al.* 2015, Zemri *et al.* 2015, Al-Basyouni *et al.* 2015)

$$0=\int_0^T(\delta U-\delta K)dt \tag{11}$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is calculated by

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\delta\,\varepsilon_x+\sigma_y\delta\,\varepsilon_y+\tau_{xy}\delta\,\gamma_{xy}+\tau_{yz}\delta\,\gamma_{yz}+\tau_{xz}\delta\,\gamma_{xz}\right]dV\\ &=\int_A\left[N_x\delta\,\varepsilon_x^0+N_y\delta\,\varepsilon_y^0+N_{xy}\delta\,\gamma_{xy}^0+M_x\delta\,k_x+M_y\delta\,k_y\right.\\ &\quad+M_{xy}\delta\,k_{xy}+S_x\delta\,\eta_x+S_y\delta\,\eta_y+S_{xy}\delta\,\eta_{xy}+Q_{yz}\delta\,\gamma_{yz}^0\\ &\quad\left.+Q_{xz}\delta\,\gamma_{xz}^0\right]dA=0\end{aligned} \tag{12}$$

where A is the top surface and the stress resultants N, M, S and Q are defined by

$$(N_i,M_i,S_i)=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}(1,z,f)(\sigma_i)^{(n)}dz,\ (i=x,y,xy) \tag{13a}$$

and

$$Q_i=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}(\tau_i)^{(n)}g(z)dz.,\quad (i=xz,yz) \tag{13b}$$

where h_{n+1} and h_n are the top and bottom z-coordinates of the nth layer.

The variation of kinetic energy of the plate can be written in the form

$$\begin{aligned}\delta K&=\int_V\left[\dot u\delta\,\dot u+\dot v\delta\,\dot v+\dot w\delta\,\dot w\right]\rho(z)dV\\ &=\int_A\left\{I_0\left[\dot u_0\delta\dot u_0+\dot v_0\delta\dot v_0+\dot w_0\delta\dot w_0\right]\right.\\ &\quad-I_1\left(\dot u_0\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial x}\delta\,\dot u_0+\dot v_0\frac{\partial\delta\dot w_0}{\partial y}+\frac{\partial\dot w_0}{\partial y}\delta\,\dot v_0\right)\\ &\quad-J_1\left(\dot u_0\frac{\partial^3\delta\dot w_0}{\partial x^3}+\frac{\partial^3\dot w_0}{\partial x^3}\delta\,\dot u_0+\dot v_0\frac{\partial^3\delta\dot w_0}{\partial y^3}+\frac{\partial^3\dot w_0}{\partial y^3}\delta\,\dot v_0\right)\\ &\quad+I_2\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\,\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\,\dot w_0}{\partial y}\right)\\ &\quad+K_2\left(\frac{\partial^3\dot w_0}{\partial x^3}\frac{\partial^3\delta\,\dot w_0}{\partial x^3}+\frac{\partial^3\dot w_0}{\partial y^3}\frac{\partial^3\delta\,\dot w_0}{\partial y^3}\right)\end{aligned}$$

$$+J_2\left(\frac{\partial \dot{w}_0}{\partial x}\frac{\partial^3 \delta\,\dot{w}_0}{\partial x^3}+\frac{\partial^3 \dot{w}_0}{\partial x^3}\frac{\partial \delta\,\dot{w}_0}{\partial x}+\frac{\partial \dot{w}_0}{\partial y}\frac{\partial^3 \delta\,\dot{w}_0}{\partial y^3}+\frac{\partial^3 \dot{w}_0}{\partial y^3}\frac{\partial \delta\,\dot{w}_0}{\partial y}\right)\Bigg\}dA \quad (14)$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$\left(I_0,I_1,J_1,I_2,J_2,K_2\right)=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}\left(1,z,f,z^2,z\,f,f^2\right)\rho(z)dz \quad (15)$$

Substituting the expressions for δU, and δK from Eqs. (12), and (14) into Eq. (11) and integrating by parts, and collecting the coefficients of δu_0, δv_0 and δw_0, the following equations of motion of the plate are obtained

$$\begin{aligned}
\delta u_0:&\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial \ddot{w}_0}{\partial x}-J_1\frac{\partial^3 \ddot{w}_0}{\partial x^3}\\
\delta v_0:&\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial \ddot{w}_0}{\partial y}-J_1\frac{\partial^3 \ddot{w}_0}{\partial y^3}\\
\delta w_0:&\ \frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+\frac{\partial^4 S_x}{\partial x^4}+\frac{\partial^4 S_{xy}}{\partial x^3\partial y}+\frac{\partial^4 S_{xy}}{\partial y^3\partial x}+\frac{\partial^4 S_y}{\partial y^4}\\
&-\frac{\partial^3 Q_{xz}}{\partial x^3}-\frac{\partial^3 Q_{yz}}{\partial y^3}=I_0\ddot{w}_0+I_1\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)+J_1\left(\frac{\partial^3 \ddot{u}_0}{\partial x^3}+\frac{\partial^3 \ddot{v}_0}{\partial y^3}\right)\\
&\qquad -I_2\left(\frac{\partial^2 \ddot{w}_0}{\partial x^2}+\frac{\partial^2 \ddot{w}_0}{\partial y^2}\right)-2J_2\left(\frac{\partial^4 \ddot{w}_0}{\partial x^4}+\frac{\partial^4 \ddot{w}_0}{\partial y^4}\right)\\
&\qquad -K_2\left(\frac{\partial^6 \ddot{w}_0}{\partial x^6}+\frac{\partial^6 \ddot{w}_0}{\partial y^6}\right)
\end{aligned} \quad (16)$$

By substituting Eq. (6) into Eq. (9) and the subsequent results into Eq. (13), the stress resultants are obtained as

$$\begin{Bmatrix} N\\ M\\ S\end{Bmatrix}=\begin{bmatrix} A & B & B^s\\ B & D & D^s\\ B^s & D^s & H^s\end{bmatrix}\begin{Bmatrix}\varepsilon\\ k\\ \eta\end{Bmatrix},\quad Q=A^s\gamma, \quad (17)$$

in which

$$N=\left\{N_x,N_y,N_{xy}\right\}^t,\ M^b=\left\{M_x^b,M_y^b,M_{xy}^b\right\}^t,\quad S=\left\{S_x,S_y,S_{xy}\right\}^t \quad (18a)$$

$$\varepsilon=\left\{\varepsilon_x^0,\varepsilon_y^0,\gamma_{xy}^0\right\}^t,\ k=\left\{k_x,k_y,k_{xy}\right\}^t,\ \ \eta=\left\{\eta_x,\eta_y,\eta_{xy}\right\}^t \quad (18b)$$

$$A=\begin{bmatrix} A_{11} & A_{12} & 0\\ A_{12} & A_{22} & 0\\ 0 & 0 & A_{66}\end{bmatrix},B=\begin{bmatrix} B_{11} & B_{12} & 0\\ B_{12} & B_{22} & 0\\ 0 & 0 & B_{66}\end{bmatrix},\ D=\begin{bmatrix} D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{66}\end{bmatrix} \quad (18c)$$

$$B^s=\begin{bmatrix} B_{11}^s & B_{12}^s & 0\\ B_{12}^s & B_{22}^s & 0\\ 0 & 0 & B_{66}^s\end{bmatrix},\ D^s=\begin{bmatrix} D_{11}^s & D_{12}^s & 0\\ D_{12}^s & D_{22}^s & 0\\ 0 & 0 & D_{66}^s\end{bmatrix},\ H^s=\begin{bmatrix} H_{11}^s & H_{12}^s & 0\\ H_{12}^s & H_{22}^s & 0\\ 0 & 0 & H_{66}^s\end{bmatrix} \quad (18d)$$

$$Q=\left\{Q_{xz},Q_{yz}\right\}^t,\gamma=\left\{\gamma_{xz}^0,\gamma_{yz}^0\right\}^t,\ \ A^s=\begin{bmatrix} A_{44}^s & 0\\ 0 & A_{55}^s\end{bmatrix} \quad (18e)$$

and stiffness components are given as:

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11}\\ A_{12} & B_{12} & D_{12}\\ A_{66} & B_{66} & D_{66}\end{Bmatrix}=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}C_{11}^{(n)}\left(1,z,z^2\right)\begin{Bmatrix}1\\ \nu^{(n)}\\ \dfrac{1-\nu^{(n)}}{2}\end{Bmatrix}dz$$
$$\begin{Bmatrix} B_{11}^s & D_{11}^s & H_{11}^s\\ B_{12}^s & D_{12}^s & H_{12}^s\\ B_{66}^s & D_{66}^s & H_{66}^s\end{Bmatrix}=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}C_{11}^{(n)}\left(f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix}1\\ \nu^{(n)}\\ \dfrac{1-\nu^{(n)}}{2}\end{Bmatrix}dz \quad (19e)$$

$$\left(A_{22},B_{22},D_{22},B_{22}^s,D_{22}^s,H_{22}^s\right)=\left(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s\right) \quad (19b)$$

$$A_{44}^s=A_{55}^s=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}C_{44}^{(n)}\left[g(z)\right]^2dz, \quad (19c)$$

By substituting Eq. (17) into Eq. (16), the equations of motion can be expressed in terms of displacements (u_0, v_0 and w_0) as

$$\begin{aligned}
&A_{11}\frac{\partial^2 u_0}{\partial x^2}+A_{66}\frac{\partial^2 u_0}{\partial y^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2 v_0}{\partial x\partial y}-B_{11}\frac{\partial^3 w_0}{\partial x^3}\\
&-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x\partial y^2}-B_{66}^s\frac{\partial^5 w_0}{\partial x^3\partial y^2}-\left(B_{12}^s+B_{66}^s\right)\frac{\partial^5 w_0}{\partial x\partial y^4}\\
&-B_{11}^s\frac{\partial^5 w_0}{\partial x^5}=I_0\ddot{u}_0-I_1\frac{\partial \ddot{w}_0}{\partial x}-J_1\frac{\partial^3 \ddot{w}_0}{\partial x^3},
\end{aligned} \quad (20a)$$

$$\begin{aligned}
&A_{22}\frac{\partial^2 v_0}{\partial y^2}+A_{66}\frac{\partial^2 v_0}{\partial x^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y}\\
&-B_{22}\frac{\partial^3 w_0}{\partial y^3}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x^2\partial y}-B_{66}^s\frac{\partial^5 w_0}{\partial x^2\partial y^3}\\
&-\left(B_{12}^s+B_{66}^s\right)\frac{\partial^5 w_0}{\partial x^4\partial y}-B_{22}^s\frac{\partial^5 w_0}{\partial y^5}=I_0\ddot{v}_0\\
&\qquad -I_1\frac{\partial \ddot{w}_0}{\partial y}-J_1\frac{\partial^3 \ddot{w}_0}{\partial y^3},
\end{aligned} \quad (20b)$$

$$\begin{aligned}
&B_{11}\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x\partial y^2}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2\partial y}\\
&+B_{22}\frac{\partial^3 v_0}{\partial y^3}-D_{11}\frac{\partial^4 w_0}{\partial x^4}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_0}{\partial x^2\partial y^2}\\
&-D_{22}\frac{\partial^4 w_0}{\partial y^4}+B_{11}^s\frac{\partial^5 u_0}{\partial x^5}+\left(B_{12}^s+B_{66}^s\right)\frac{\partial^5 u_0}{\partial x\partial y^4}
\end{aligned}$$

$$
\begin{aligned}
&+\left(B_{12}^s+B_{66}^s\right)\frac{\partial^5 v_0}{\partial x^4\partial y}+B_{22}^s\frac{\partial^5 v_0}{\partial y^5}+B_{66}^s\frac{\partial^5 v_0}{\partial x^3\partial y^2}\\
&+B_{66}^s\frac{\partial^5 v_0}{\partial x^2\partial y^3}-2D_{11}^s\frac{\partial^6 w_0}{\partial x^6}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^6 w_0}{\partial x^2\partial y^4}\\
&-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^6 w_0}{\partial x^4\partial y^2}-2D_{22}^s\frac{\partial^6 w_0}{\partial y^6}-H_{11}^s\frac{\partial^8 w_0}{\partial x^8}\\
&-2\left(H_{12}^s+H_{66}^s\right)\frac{\partial^8 w_0}{\partial x^4\partial y^4}-H_{66}^s\frac{\partial^8 w_0}{\partial x^6\partial y^2}-H_{66}^s\frac{\partial^8 w_0}{\partial x^2\partial y^6}\\
&-H_{22}^s\frac{\partial^8 w_0}{\partial y^8}+A_{44}^s\frac{\partial^6 w_0}{\partial x^6}+A_{55}^s\frac{\partial^6 w_0}{\partial y^6}=I_0\ddot{w}\\
&+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)+J_1\left(\frac{\partial^3\ddot{u}_0}{\partial x^3}+\frac{\partial^3\ddot{v}_0}{\partial y^3}\right)\\
&-I_2\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)-2J_2\left(\frac{\partial^4\ddot{w}_0}{\partial x^4}+\frac{\partial^4\ddot{w}_0}{\partial y^4}\right)\\
&-K_2\left(\frac{\partial^6\ddot{w}_0}{\partial x^6}+\frac{\partial^6\ddot{w}_0}{\partial y^6}\right)
\end{aligned}
\tag{20c}
$$

3. Analytical solutions

The above equations of motion are analytically solved for free vibration problem of a simply supported rectangular plate. Based on Navier solution procedure, the displacements are assumed as follows

$$
\begin{Bmatrix} u_0(x,y,t)\\ v_0(x,y,t)\\ w_0(x,y,t)\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\cos(\lambda\,x)\sin(\mu\,y)\,e^{i\omega t}\\ V_{mn}\sin(\lambda\,x)\cos(\mu\,y)\,e^{i\omega t}\\ W_{mn}\sin(\lambda\,x)\sin(\mu\,y)\,e^{i\omega t}\end{Bmatrix}
\tag{21}
$$

where $i=\sqrt{-1}$, $\lambda=m\pi/a$, $\mu=n\pi/b$, (U_{mn}, V_{mn}, W_{mn}) are the unknown maximum displacement coefficients, and ω is the angular frequency.

Substituting Eq. (21) into Eq. (20), the analytical solutions can be obtained from

$$
\left(\begin{bmatrix} a_{11} & a_{12} & a_{13}\\ a_{12} & a_{22} & a_{23}\\ a_{13} & a_{23} & a_{33}\end{bmatrix}-\omega^2\begin{bmatrix} m_{11} & 0 & m_{13}\\ 0 & m_{22} & m_{23}\\ m_{13} & m_{23} & m_{33}\end{bmatrix}\right)\begin{Bmatrix} U_{mn}\\ V_{mn}\\ W_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix}
\tag{22}
$$

where

$$
\begin{aligned}
&a_{11}=-\left(A_{11}\lambda^2+A_{66}\mu^2\right)\quad a_{12}=-\lambda\,\mu\left(A_{12}+A_{66}\right)\\
&a_{13}=\lambda\,[B_{11}\lambda^2+(B_{12}+2B_{66})\,\mu^2\\
&\qquad -B_{11}^s\lambda^4-B_{12}^s\mu^4-B_{66}^s\lambda^2\mu^2-B_{66}^s\mu^4]\\
&a_{22}=-\left(A_{66}\lambda^2+A_{22}\mu^2\right)\\
&a_{23}=\mu\,[B_{22}\mu^2+(B_{12}+2B_{66})\,\lambda^2\\
&\qquad -B_{22}^s\mu^4-B_{12}^s\lambda^4-B_{66}^s\lambda^2\mu^2-B_{66}^s\lambda^4]\\
&a_{33}=-D_{11}\lambda^4-2(D_{12}+2D_{66})\lambda^2\mu^2\\
&\qquad -D_{22}\mu^4+2(D_{11}^s\lambda^6+D_{22}^s\mu^6)\\
&\qquad +2\left(\lambda^4\mu^2+\lambda^2\mu^4\right)(D_{12}^s+2D_{66}^s)\\
&\qquad -H_{11}^s\lambda^8-H_{22}^s\mu^8-2\lambda^4\mu^4(H_{12}^s+H_{66}^s)\\
&\qquad -\left(\lambda^6\mu^2+\lambda^2\mu^6\right)H_{66}^s-A_{44}^s\lambda^6-A_{55}^s\mu^6\\
&m_{11}=m_{22}=-I_0\\
&m_{13}=\lambda\left(I_1+J_1\lambda^2\right)\\
&m_{23}=\mu\left(I_1+J_1\mu^2\right),\\
&m_{33}=-\left(I_0+I_2\left(\lambda^2+\mu^2\right)+2J_2\left(\lambda^4+\mu^4\right)\right.\\
&\qquad \left.+K_2\left(\lambda^6+\mu^6\right)\right).
\end{aligned}
\tag{23}
$$

4. Numerical results and discussion

In this section, the natural frequencies of sandwich FGM plates are investigated via the present new 3-unknown hyperbolic shear deformation theory. The material properties used in the present study are:

• Ceramic (P_1: Alumina, Al_2O_3): E_c=380 GPa; v=0.3; ρ_c=3800 kg/m^3 .

• Metal (P_2: Aluminium, Al): E_m=70 GPa; v=0.3; ρ_c=2707 kg/m^3.

The natural frequency parameter is nondimensionalized by the following relation

$$
\bar{\omega}=\frac{\omega\,b^2}{h}\sqrt{\frac{\rho_0}{E_0}}
\tag{24}
$$

where ρ_0=1 kg/m^3, E_0=1 GPa.

To verify the validity of the present new 3-unknown hyperbolic shear deformation theory, the obtained results are compared with other theories existing in the literature such as classical plate theory (CPT), first-order shear deformation theory (FSDT), third-order shear deformation theory (TSDT), sinusoidal shear deformation theory (SSDT), the four variable refined plate theory (RPT) and the three-dimensional linear theory of elasticity.

The non-dimensional natural fundamental frequencies $\bar{\omega}$ of the power-law FGM sandwich plates of Type A, are compared in Table 1 with the results of Hadji *et al.* (2011) based on CPT, FSDT, TSDT, SSDPT, RPT and three-dimensional linear theory of elasticity (Li *et al.* 2008). Table 1 shows a good agreement by comparisons of FGM plates of five different volume fraction indices k=0, 0.5, 1, 5, 10 with other theories. In general, the vibration frequencies computed using the CPT, are much higher than those computed from the other shear deformation theories. This implies the well-known fact that the results estimated by the CPT are grossly in error for a thick plate.

Another comparative study between different plate theories is carried out on the basis of the homogeneous hardcore and homogeneous soft-core types of FG sandwich plates (Type A). The results are presented in Tables 2 and 3. The results illustrated in Table 2 are obtained for the case of homogeneous hardcore in which the Young's modulus and mass density of layer 1 are E_c=380 GPa and ρ_c=3800 kg/m^3 (P_1, alumina) at the top face and E_m=70 GPa and ρ_m=2707 kg/m^3 (P_2, aluminum) at the bottom face. However, in Table

Table 1 Comparisons of natural fundamental frequency parameters $\overline{\omega}$ of simply supported square power-law FGM plates of Type A with other theories (h/b=0.1)

k	Theories	$\overline{\omega}$					
		1-0-1	2-1-2	2-1-1	1-1-1	2-2-1	1-2-1
0	CPT [(a)]	1.87359	1.87359	1.87359	1.87359	1.87359	1.87359
	FSDT [(a)]	1.82442	1.82442	1.82442	1.82442	1.82442	1.82442
	TSDT [(a)]	1.82445	1.82445	1.82445	1.82445	1.82445	1.82445
	SSDT [(a)]	1.82452	1.82452	1.82452	1.82452	1.82452	1.82452
	RPT [(a)]	1.82445	1.82445	1.82445	1.82445	1.82445	1.82445
	Elasticity [(b)]	-----	-----	-----	-----	-----	-----
	Present	1.83181	1.83181	1.83181	1.83181	1.83181	1.83181
0.5	CPT [(a)]	1.47157	1.51242	1.54264	1.54903	1.58374	1.60722
	FSDT [(a)]	1.44168	1.48159	1.51035	1.51695	1.55001	1.57274
	TSDT [(a)]	1.44424	1.48408	1.51253	1.51922	1.55199	1.57451
	SSDT [(a)]	1.44436	1.48418	1.51258	1.51927	1.55202	1.57450
	RPT [(a)]	1.44424	1.48408	1.50635	1.51921	1.54710	1.57451
	Elasticity [(b)]	1.44614	1.48608	1.50841	1.52131	1.54926	1.57668
	Present	1.44487	1.48472	1.50729	1.52003	1.54828	1.57587
1	CPT [(a)]	1.26238	1.32023	1.37150	1.37521	1.43247	1.46497
	FSDT [(a)]	1.24031	1.29729	1.34637	1.35072	1.40555	1.43722
	TSDT [(a)]	1.24320	1.30011	1.34888	1.35333	1.40789	1.43934
	SSDT [(a)]	1.24335	1.30023	1.34894	1.35339	1.40792	1.43931
	RPT [(a)]	1.24320	1.30011	1.33329	1.35332	1.39557	1.43933
	Elasticity [(b)]	1.24470	1.30181	1.33511	1.35523	1.39763	1.44137
	Present	1.24376	1.30063	1.33379	1.35371	1.39597	1.43969
5	CPT [(a)]	0.95844	0.99190	1.08797	1.05565	1.16195	1.18867
	FSDT [(a)]	0.94256	0.97870	1.07156	1.04183	1.14467	1.17159
	TSDT [(a)]	0.94598	0.98184	1.07432	1.04466	1.14731	1.17397
	SSDT [(a)]	0.94630	0.98207	1.07445	1.04481	1.14741	1.17399
	RPT [(a)]	0.94598	0.98184	1.03043	1.04466	1.10881	1.17397
	Elasticity [(b)]	0.94476	0.98103	1.02942	1.04532	1.10983	1.17567
	Present	0.94709	0.98594	1.03353	1.04905	1.11191	1.17655
10	CPT [(a)]	0.94321	0.95244	1.05185	1.00524	1.11883	1.13614
	FSDT [(a)]	0.92508	0.93962	1.03580	0.99256	1.10261	1.12067
	TSDT [(a)]	0.92839	0.94297	1.03862	0.99551	1.10533	1.12314
	SSDT [(a)]	0.92875	0.94332	1.04558	0.99519	1.04154	1.13460
	RPT [(a)]	0.92839	0.94297	0.99195	0.99550	1.06090	1.12314
	Elasticity [(b)]	0.92727	0.94078	0.98929	0.99523	1.06104	1.12466
	Present	0.92886	0.94689	0.99500	1.00069	1.06468	1.12667

[(a)] Hadji *et al.* (2011) ; [(b)] Li *et al.* (2008)

3 we present results for the case of homogeneous soft-core in which the Young's modulus and mass density of layer 1 are E_m=70 GPa and ρ_m=2707 kg/m^3 (P_1, aluminum) at the top face and E_c=380 GPa and ρ_c=3800 kg/m^3 (P_2, alumina) at the bottom face. Three thickness-side ratios h/b (0.01, 0.1, and 0.2) and five volume fraction indices k (0, 0.5, 1, 5, and 10) are considered. It can be observed from the results presented in Tables 2 and 3, that the dimensionless frequencies computed by the present new plate theory with three unknowns are in good agreement with those obtained by the three-dimensional linear theory of elasticity used by Li *et al.* (2008) and the four variable refined plate theory developed by Hadji *et al.* (2011).

In the next example, the results of 1-8-1 power-law FGM plate of Type B are tabulated in Table 4 where P_1 is referred to the properties of aluminum and P_2 the properties of alumina. In this example, the FGM core is metal-rich at the top face and ceramic-rich at the bottom face. The results are predicted for different values of the thickness-side ratios h/b (0.01, 0.1, and 0.2) and volume fraction indices k (0.5, 1, 2, 5, and 10). From Table 4, it can be shown that the results given by Li *et al.* (2008) and Hadji *et al.* (2011) for sandwich plates with FGM core are in good agreement with the present new 3-unknown hyperbolic shear deformation theory.

Fig. 3 displays the variation of fundamental frequency parameters with respect to the thickness ratio of simply supported power-law FGM sandwich plates with the homogeneous hardcore. Fig. 4 illustrates the curves of the power-law FGM sandwich plates with the homogeneous

Table 2 Comparison of fundamental frequency parameter $\bar{\varpi}$ of simply supported square power-law FGM sandwich plates with homogeneous hardcore

h/b	*k*	Theories	$\bar{\varpi}$					
			1-0-1	2-1-2	1-1-1	2-2-1	1-2-1	1-8-1
0.01	0	Present	1.88835	1.88835	1.88835	1.88835	1.88835	1.88835
		RPT [(a)]	1.88825	1.88825	1.88825	1.88825	1.88825	1.88825
		Elasticity [(b)]	1.88829	1.88829	1.88829	1.88829	1.88829	1.88829
	0.5	Present	1.48278	1.52383	1.56068	1.59054	1.61932	1.76368
		RPT [(a)]	1.48241	1.52353	1.56042	1.59030	1.61912	1.76354
		Elasticity [(b)]	1.48244	1.52355	1.56046	1.59031	1.61915	1.76357
	1	Present	1.27205	1.33011	1.38542	1.43022	1.47581	1.69919
		RPT [(a)]	1.27156	1.32972	1.38508	1.42990	1.47554	1.69904
		Elasticity [(b)]	1.27158	1.32974	1.38511	1.42992	1.47558	1.69906
	5	Present	0.96634	0.99961	1.06358	1.13065	1.19736	1.57005
		RPT [(a)]	0.96564	0.99903	1.06309	1.13019	1.19697	1.56985
		Elasticity [(b)]	0.96563	0.99903	1.06309	1.13020	1.19699	1.56988
	10	Present	0.95121	0.95998	1.01289	1.08115	1.14448	1.54182
		RPT [(a)]	0.95044	0.95937	1.01236	1.08065	1.14406	1.54162
		Elasticity [(b)]	0.95042	0.95934	1.01237	1.08065	1.14408	1.54164
0.1	0	Present	1.83181	1.83181	1.83181	1.83181	1.83181	1.83181
		RPT [(a)]	1.82445	1.82445	1.82445	1.82445	1.82445	1.82445
		Elasticity [(b)]	1.82682	1.82682	1.82682	1.82682	1.82682	1.82682
	0.5	Present	1.44487	1.48472	1.52003	1.54828	1.57587	1.71296
		RPT [(a)]	1.44423	1.48408	1.51921	1.54710	1.57450	1.70901
		Elasticity [(b)]	1.44614	1.48608	1.52131	1.54926	1.57668	1.71130
	1	Present	1.24376	1.30063	1.35371	1.39597	1.43969	1.65156
		RPT [(a)]	1.24319	1.30010	1.35332	1.39556	1.43932	1.64892
		Elasticity [(b)]	1.24470	1.30181	1.35523	1.39763	1.44137	1.65113
	5	Present	0.94709	0.98594	1.04905	1.11191	1.17655	1.52885
		RPT [(a)]	0.94598	0.98184	1.04465	1.10881	1.17396	1.52792
		Elasticity [(b)]	0.94476	0.98103	1.04532	1.10983	1.17567	1.52993
	10	Present	0.92886	0.94689	1.00069	1.06468	1.12667	1.50209
		RPT [(a)]	0.92838	0.94296	0.99550	1.06090	1.12313	1.50138
		Elasticity [(b)]	0.92727	0.94078	0.99523	1.06104	1.12466	1.50333
0.2	0	Present	1.68191	1.68191	1.68191	1.68191	1.68191	1.68191
		RPT [(a)]	1.67010	1.67010	1.67010	1.67010	1.67010	1.67010
		Elasticity [(b)]	1.67711	1.67711	1.67711	1.67711	1.67711	1.67711
	0.5	Present	1.34772	1.38437	1.41540	1.43907	1.46329	1.57966
		RPT [(a)]	1.34743	1.38410	1.41508	1.43843	1.46251	1.57476
		Elasticity [(b)]	1.35358	1.39053	1.42178	1.44535	1.46940	1.58186
	1	Present	1.17380	1.22776	1.27468	1.30973	1.34819	1.52710
		RPT [(a)]	1.16976	1.22340	1.27134	1.30753	1.34671	1.52445
		Elasticity [(b)]	1.17485	1.22915	1.27770	1.31434	1.35341	1.53142
	5	Present	0.90078	0.95712	1.01857	1.06975	1.12886	1.42292
		RPT [(a)]	0.89462	0.93594	0.99545	1.05228	1.11318	1.42197
		Elasticity [(b)]	0.89086	0.93362	0.99798	1.05607	1.11900	1.42845
	10	Present	0.87272	0.91923	0.976484	1.02877	1.08745	1.40039
		RPT [(a)]	0.87178	0.89918	0.950331	1.00848	1.06754	1.39932
		Elasticity [(b)]	0.86833	0.89228	0.94984	1.00949	1.07290	1.40568

[(a)] Hadji *et al.* (2011); [(b)] Li *et al.* (2008)

soft-core. In general, the results are the maximum for the ceramic plates and the minimum for the metal plates. It is seen that the results increase smoothly as the amount of ceramic in the sandwich plate increases. It is also shown that the effect of *k* on the 2-1-2 sandwich plate is greater than that of the 1-8-1 sandwich plate.

Table 3 Comparison of fundamental frequency parameter $\overline{\varpi}$ for simply supported square power-law FGM sandwich plates with homogeneous soft-core

h/b	*k*	Theories	$\overline{\varpi}$					
			1-0-1	2-1-2	1-1-1	2-2-1	1-2-1	1-8-1
0.01	0	Present	0.96114	0.96114	0.96114	0.96114	0.96114	0.96114
		RPT [a]	0.96020	0.96020	0.96020	0.96020	0.96020	0.96020
		Elasticity [b]	0.96022	0.96022	0.96022	0.96022	0.96022	0.96022
	0.5	Present	1.66413	1.62442	1.58328	1.52427	1.50814	1.26686
		RPT [a]	1.66283	1.62294	1.58173	1.52279	1.50657	1.26555
		Elasticity [b]	1.66281	1.62291	1.58171	1.52277	1.50658	1.26557
	1	Present	1.82146	1.79317	1.75552	1.68347	1.67669	1.38478
		RPT [a]	1.82034	1.79174	1.75391	1.68194	1.67494	1.38330
		Elasticity [b]	1.82031	1.79163	1.75379	1.68184	1.67490	1.38331
	5	Present	1.92134	1.94423	1.93787	1.86367	1.88733	1.57210
		RPT [a]	1.92089	1.94332	1.93658	1.86239	1.88558	1.57034
		Elasticity [b]	1.92090	1.94313	1.93623	1.86207	1.88530	1.57035
	10	Present	1.91089	1.94776	1.95195	1.88192	1.91365	1.60638
		RPT [a]	1.91061	1.94701	1.95080	1.88076	1.91198	1.60456
		Elasticity [b]	1.91064	1.94687	1.95044	1.88042	1.91162	1.60457
0.1	0	Present	0.93237	0.93237	0.93237	0.93237	0.93237	0.93237
		RPT [a]	0.92776	0.92776	0.92776	0.92776	0.92776	0.92776
		Elasticity [b]	0.92897	0.92897	0.92897	0.92897	0.92897	0.92897
	0.5	Present	1.60763	1.56849	1.52860	1.47205	1.45634	1.22540
		RPT [a]	1.57497	1.52895	1.48666	1.43615	1.41626	1.20477
		Elasticity [b]	1.57352	1.52588	1.48459	1.43419	1.41662	1.20553
	1	Present	1.75948	1.73065	1.69379	1.62479	1.61777	1.33841
		RPT [a]	1.72568	1.68379	1.63966	1.57874	1.56102	1.30766
		Elasticity [b]	1.72227	1.67437	1.63053	1.57037	1.55788	1.30825
	5	Present	1.85944	1.87755	1.86950	1.79864	1.81960	1.51774
		RPT [a]	1.84199	1.84161	1.81730	1.75320	1.74864	1.46600
		Elasticity [b]	1.84198	1.82611	1.78956	1.72726	1.72670	1.46647
	10	Present	1.85109	1.88194	1.88356	1.81670	1.84496	1.55051
		RPT [a]	1.83857	1.85196	1.83665	1.77527	1.77584	1.49439
		Elasticity [b]	1.84020	1.83987	1.80813	1.74779	1.74811	1.49481
0.2	0	Present	0.85607	0.85607	0.85607	0.85607	0.85607	0.85607
		RPT [a]	0.84927	0.84927	0.84927	0.84927	0.84927	0.84927
		Elasticity [b]	0.85286	0.85286	0.85286	0.85286	0.85286	0.85286
	0.5	Present	1.45432	1.41628	1.37957	1.33011	1.31507	1.11299
		RPT [a]	1.38225	1.32772	1.28521	1.24999	1.22481	1.06852
		Elasticity [b]	1.37894	1.32061	1.28053	1.24533	1.22580	1.07016
	1	Present	1.59156	1.56050	1.52541	1.46522	1.45681	1.21218
		RPT [a]	1.51715	1.45515	1.40311	1.36164	1.32828	1.14353
		Elasticity [b]	1.50896	1.43325	1.38242	1.34203	1.32129	1.14451
	5	Present	1.69345	1.69706	1.68355	1.62220	1.63463	1.36919
		RPT [a]	1.65829	1.61777	1.56607	1.52042	1.47440	1.25156
		Elasticity [b]	1.65868	1.58011	1.50284	1.46009	1.42665	1.25210
	10	Present	1.69144	1.70420	1.69785	1.63996	1.65751	1.39777
		RPT [a]	1.66789	1.63913	1.59271	1.54763	1.50143	1.27017
		Elasticity [b]	1.67278	1.60909	1.52671	1.48306	1.44101	1.27065

[a] Hadji *et al.* (2011); [b] Li *et al.* (2008)

5. Conclusions

A new simple and accurate 3-unknowns hyperbolic shear deformation theory is proposed for the free vibration analysis of FG sandwich plates. As the classical plate theory, the present one contains only three unknown displacements. In contrast, the shear deformation effect is included. The accuracy of the present theory is ascertained

Table 4 Comparison of fundamental frequency parameter $\bar{\omega}$ for simply supported square power-law FGM sandwich plates with FGM core

h/b	Theories	k = 0.5	1	2	5	10
0.01	Present	1.34002	1.38723	1.44529	1.53167	1.59124
	RPT [a]	1.33927	1.38665	1.44487	1.53139	1.59103
	Elasticity [b]	1.33931	1.38669	1.44491	1.53143	1.59105
0.1	Present	1.29972	1.34828	1.40695	1.49172	1.54896
	RPT [a]	1.29459	1.34533	1.40514	1.49044	1.54754
	Elasticity [b]	1.29751	1.34847	1.40828	1.49309	1.54980
0.2	Present	1.19476	1.24791	1.30880	1.38931	1.43995
	RPT [a]	1.18682	1.24352	1.30576	1.38736	1.43837
	Elasticity [b]	1.19580	1.25338	1.31569	1.39567	1.44540

[a] Hadji *et al.* (2011); [b] Li *et al.* (2008)

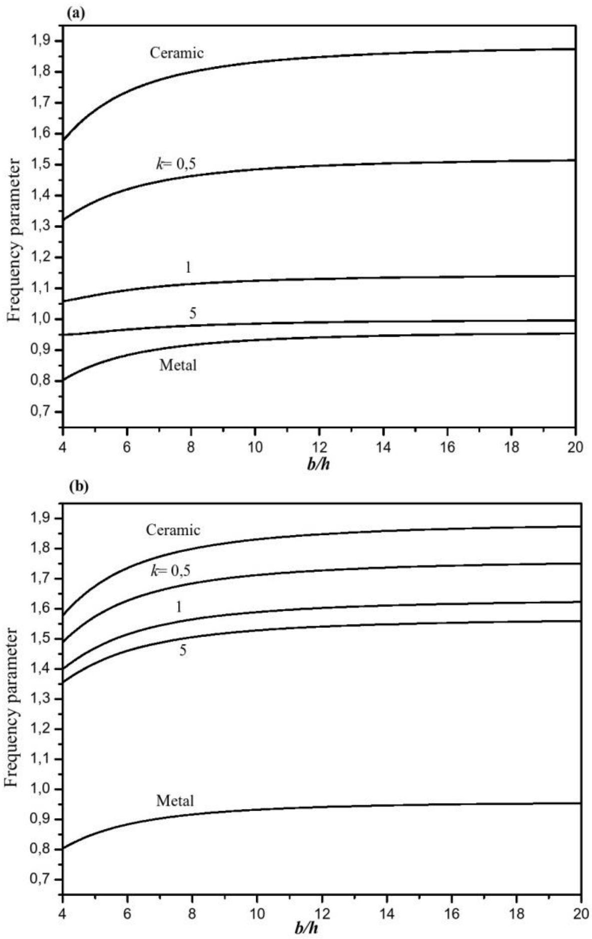

Fig. 3 Fundamental frequencies $\bar{\omega}$ for power-law FGM sandwich plates with homogeneous hardcore: (a) 2-1-2 FGM sandwich plate, (b) 1-8-1 FGM sandwich plate

by comparing it with elasticity solutions and other shear deformation theories having higher number of unknowns and where a good agreement was observed in all cases. Finally, this work will deserve special attention and offer potential for future research. An improvement of present formulation will be considered in the future work to consider the thickness stretching effect by using quasi-3D

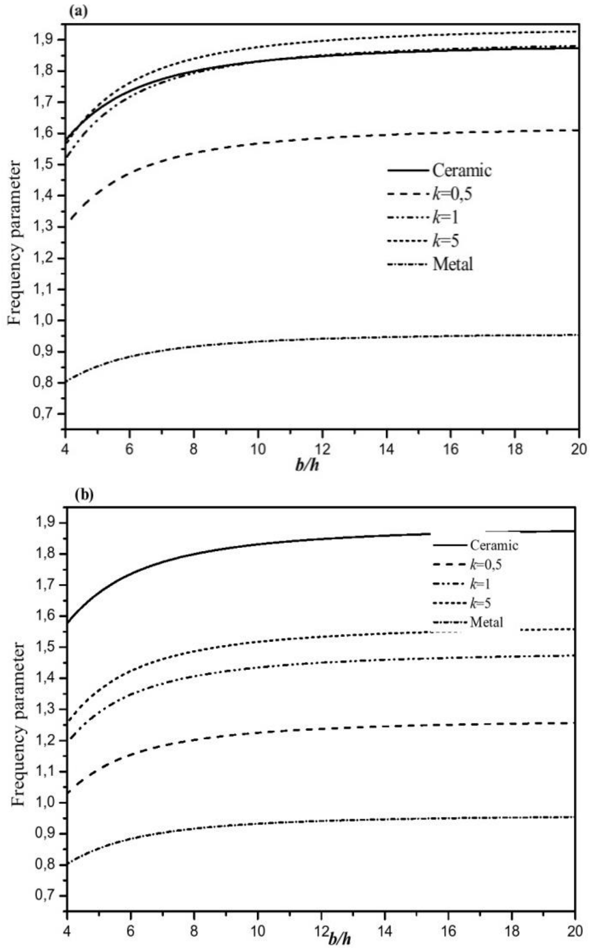

Fig. 4 Fundamental frequencies $\bar{\omega}$ for power-law FGM sandwich plates with homogeneous soft-core: (a) 2-1-2 FGM sandwich plate, (b) 1-8-1 FGM sandwich plate

shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Larbi Chaht *et al.* 2015, Hamidi *et al.* 2015, Bourada *et al.* 2015, Bennoun *et al.* 2016, Draiche *et al.* 2016, Sekkal *et al.* 2017b, Bouafia *et al.* 2017, Abualnour *et al.* 2018, Benchohra *et al.* 2018, Bouhadra *et al.* 2018).

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