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A simple quasi-3D sinusoidal shear deformation theory with stretching effect for carbon nanotube-reinforced composite beams resting on elastic foundation

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Abstract. The objective of the present paper is to investigate the bending behavior with stretching effect of carbon nanotube-reinforced composite (CNTRC) beams. The beams resting on the Pasternak elastic foundation, including a shear layer and Winkler spring, are considered. The single-walled carbon nanotubes (SWCNTs) are aligned and distributed in polymeric matrix with different patterns of reinforcement. The material properties of the CNTRC beams are estimated by using the rule of mixture. The significant feature of this model is that, in addition to including the shear deformation effect and stretching effect it deals with only 4 unknowns without including a shear correction factor. The single-walled carbon nanotubes (SWCNTs) are aligned and distributed in polymeric matrix with different patterns of reinforcement. The material properties of the CNTRC beams are assessed by employing the rule of mixture. The equilibrium equations have been obtained using the principle of virtual displacements. The mathematical models provided in this paper are numerically validated by comparison with some available results. New results of bending analyses of CNTRC beams based on the present theory with stretching effect is presented and discussed in details. the effects of different parameters of the beam on the bending responses of CNTRC beam are discussed.

Keywords: bending; stretching effect; CNTRC beams; elastic foundation

1. Introduction

Exceptional electronic and mechanical properties of carbon nanotubes, such as the extremely high elastic modulus, tensile strength, aspect ratio and low density, make them excellent candidate for the reinforcement of polymer composites (Lau *et al.* 2004). The mechanical properties of carbon nanotube reinforced composites (CNTRCs) have been extensively investigated experimentally, analytically and numerically. Han *et al.* (2007) simulated the elastic properties of polymer/carbon nanotube composites. Shen *et al.* (2009) for the first time suggested that the nonlinear bending behavior of CNTRC plates can be considerably improved through the use of a functionally graded distribution of CNTs in the matrix. Based on Timoshenko beam theory, Ke *et al.* (2013) studied the dynamic stability response of functionally graded (FG) nanocomposite beams reinforced by SWCNTs. Bakhti *et al.* (2013) studied the nonlinear cylindrical bending behavior of FG nanocomposite plates reinforced by SWCNTs using an efficient and simple refined theory. Wattanasakulpong and Ungbhakorn (2013) studied the bending, buckling and vibration behaviors of carbon nanotube-reinforced composite (CNTRC) beams where several higher-order shear deformation theories are presented and discussed in details. Lei *et al.* (2013) presented a large deflection analysis of FG-CNT reinforced composite plates by considering different boundary conditions. Alibeigloo (2014) studied the bending behavior of a CNT reinforced composite rectangular host plate attached to thin piezoelectric layers subjected to thermal load and or electric field. Tagrara *et al.* (2015) studied the bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams. Ebrahimi *et al.* (2017) analyze the vibration analysis of embedded size dependent FG nanobeams based on third-order shear deformation beam theory. Akbas (2016) investigated the analytical solutions for static Bending of edge cracked micro beams. Elamary *et al.* (2016) presented the numerical simulation of concrete beams reinforced with composite GFRP-Steel bars under three points bending. Shen *et al.* (2013) investigated the nonlinear analysis of nanotube-reinforced composite beams resting on elastic foundations in thermal environments. Fan *et al.* (2017) studied the effects of matrix cracks on the nonlinear vibration characteristics of shear deformable laminated beams containing carbon nanotube reinforced composite layers. Fan *et al.* (2017) presented the nonlinear low-velocity impact on damped and matrix-cracked hybrid laminated beams containing carbon nanotube reinforced composite layers. Shen *et al.* (2017) analyze the vibration of thermally postbuckled carbon nanotube-reinforced

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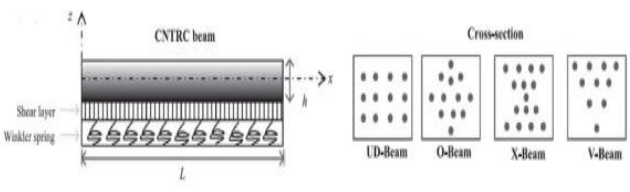
composite beams resting on elastic foundations. Mohammadimehr *et al.* (2017) studied the nonlinear vibration analysis of FG-CNTRC sandwich Timoshenko beam based on modified couple stress theory subjected to longitudinal magnetic field using generalized differential quadrature method. Wattanasakulpong *et al.* (2017) presented the stability and vibration analyses of carbon nanotube-reinforced composite beams with elastic boundary conditions: Chebyshev collocation method. Bourada *et al.* (2015) used a new simple shear and normal deformations theory for functionally graded beams. Hebali *et al.* (2014) studied the static and free vibration analysis of functionally graded plates using a new quasi-3D hyperbolic shear deformation theory. Bennoun *et al.* (2016) analyzed the vibration of functionally graded sandwich plates using a novel five variable refined plate theory. Bousahla *et al*. (2014) investigated a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Draiche *et al.* (2016) used a refined theory with stretching effect for the flexure analysis of laminated composite plates. Hamidi *et al*. (2015) proposed a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Belabed *et al*. (2014) used an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Bessaim *et al*. (2013) investigated a new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets. Bouafia *et al*. (2017), used a nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams. Abualnour *et al*. (2018) analyze the free vibration of advanced composite plates using a novel quasi-3D trigonometric plate theory. Abdelaziz *et al*. (2017) studied the bending, buckling and free vibration of FGM sandwich plates with various boundary conditions using an efficient hyperbolic shear deformation theory. Amar Meziane *et al.* (2014) proposed an efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Bouderba *et al.* (2016) studied the thermal stability of functionally graded sandwich plates using a simple shear deformation theory. Bellifa *et al*. (2016) studied the bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position. Bousahla *et al*. (2016) investigated the thermal stability of plates with functionally graded coefficient of thermal expansion. Beldjelili *et al*. (2016) studied the hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Bouderba *et al*. (2016) analyze the thermal stability of functionally graded sandwich plates using a simple shear deformation theory. Zidi *et al*. (2014) analyse the bending of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory. El-Haina *et al.* (2017) used a simple analytical approach for thermal buckling of thick functionally graded sandwich plates. Menasria *et al.* (2017) analyze the thermal stability of FG sandwich plates using a new and simple HSDT. Chikh *et al.* (2017) studied the thermal buckling analysis of cross-ply laminated plates using a simplified HSDT. Tounsi *et al.* (2013) use a refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates. Mouffoki *et al.* (2017) studied the vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory. Khetir *et al.* (2017) developed a new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates. Hamidi *et al*. (2015) proposed a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Attia *et al.* (2015) developed the free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories. Karami *et al.* (2017) studied the effects of triaxial magnetic field on the anisotropic nanoplates. Zemri *et al*. (2015) proposed an assessment of a refined nonlocal shear deformation theory beam theory for a mechanical response of functionally graded nanoscale beam. Bellifa *et al.* (2017a) used a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Bellifa *et al*. (2017b) used An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates. Bounouara *et al.* (2014) studied the free vibration of functionally graded nanoscale plates resting on elastic foundation using a nonlocal zeroth-order shear deformation theory. Ahouel *et al.* (2016) investigated a size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. Saidi *et al*. (2016) investigated a simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations. Belkorissat *et al.* (2015) developed a new nonlocal refined four variable model for the vibration properties of functionally graded nano-plate. Larbi Chaht *et al.* (2015) studied the bending and buckling of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect. Al-Basyouni *et al.* (2015) investigated size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position. Boukhari *et al*. (2016) used an efficient shear deformation theory for wave propagation of functionally graded material plates. Houari *et al.* (2016) used a new simple three-unknown sinusoidal shear deformation theory for functionally graded plates. Ait Yahia *et al.* (2015) analyzed the wave propagation in functionally graded plates with porosities. Benadouda *et al*.. (2017) developed an efficient shear deformation theory for wave propagation in functionally graded material beams with porosities. Hanifi *et al.* (2017) investigated the size-dependent behavior of functionally graded micro-beams with porosities.

In the present study, the bending of the CNTRC beams with stretching effect is investigated using the Navier solution method. The simply supported CNTRC beams which are placed on the Pasternak elastic foundation, including a shear layer and Winkler spring, are considered.



(a) (b)

Fig. 1 Geometry of a CNTRC beam on elastic foundation (a) and cross sections of different patterns of reinforcement (b)

New solutions of deflections, stresses based on the present shear deformation theory with stretching effect are presented and discussed in details. Several aspects of spring constants, thickness ratios, stretching effect, CNT volume fractions, types of CNT distribution, etc., which have considerable impact on the analytical solutions, are also investigated.

2. CNTRC beams

A straight CNTRC beam made from a mixture of SWCNT and anisotropic polymer matrix is considered. The beam, having length (L) and thickness (h), is placed on the Pasternak elastic foundation, including a shear layer and Winkler spring, as shown in Fig. 1(a). In this study, the beams are assumed to have four different patterns of reinforcement over the cross sections as shown in Fig. 1(b).

The material properties of CNTRC beams can be computed utilizing the rule of mixture which gives the effective Young's modulus and shear modulus of CNTRC beams as (Shen *et al*. 2009, Bakhti *et al*. 2013, Kaci *et al*. 2012, Wattanasakulpong and Ungbhakorn 2013).

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E_p \tag{1a}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E_p} \tag{1b}$$

$$\frac{\eta_3}{G_{22}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{E_p} \tag{1c}$$

where $E_{11}^{cnt}; E_{22}^{cnt}$ and G_{12}^{cnt} are the Young's modulus and shear modulus of SWCNT, respectively and E_p and G_p are the corresponding material properties of the polymer matrix. Also, V_{cnt} and V_p are the volume fractions for carbon nanotube and the polymer matrix, respectively, with the relation of $V_{cnt}+V_p=1$. To introduce the size-dependent material properties of SWCNT, the CNT efficiency parameters, η_i (i=1, 2, 3), are considered. They can be obtained from matching the elastic moduli of CNTRCs estimated by the MD simulation with the numerical results determined by the rule of mixture (Han and Elliott 2007). By employing the same rule, Poisson's ratio (ν) and mass density (ρ) of the CNTRC beams are expressed as

$$\nu = V_{cnt}\nu^{cnt} + V_p \nu^p, \quad \rho = V_{cnt}\rho^{cnt} + V_p \rho^p \tag{2}$$

where ν^{cnt}, ν^p and ρ^{cnt}, ρ^p are the Poisson's ratios and densities of the CNT and polymer matrix respectively. For different patterns of carbon nanotube reinforcement distributed within the cross sections of the beams as shown in Fig. 1(b), the continuous mathematical functions employing for introducing the distributions of material constituents are expressed below

UD-Beam

$$V_{cnt} = V_{cnt}^* \tag{3a}$$

$$V_{cnt} = 2\left(1 - 2\frac{|z|}{h}\right)V_{cnt}^* \tag{3b}$$

$$V_{cnt} = 4\frac{|z|}{h}V_{cnt}^* \tag{3c}$$

$$V_{cnt} = \left(1 + 2\frac{z}{h}\right)V_{cnt}^* \tag{3d}$$

where V_{cnt}^* is the considered volume fraction of CNTs, which can be determined from the following equation

$$V_{cnt}^* = \frac{W_{cnt}}{W_{cnt} + \left(\rho^{cnt}/\rho^m\right)\left(1 - W_{cnt}\right)} \tag{4}$$

where W_{cnt} is the mass fraction of CNTs. From Eq. (3), it can be seen that the *O*-, *X*-and *V*-Beams are some types of functionally graded beams in which their material constituents are varied continuously within their thicknesses; while, the UD-Beam has uniformly distributed CNT reinforcement. In this work, the CNT efficiency parameters (η_i) associated with the considered volume fraction V_{cnt}^* are : η_i=1.2833 and $\eta_2=\eta_3$=1.0556 for the case of $V_{cnt}^* = 0.12$; η_1=1.3414 and $\eta_2=\eta_3$=1.7101 for the case of $V_{cnt}^* = 0.17$; η_1=1.3228 and $\eta_2=\eta_3$=1.7380 for the case of $V_{cnt}^* = 0.28$ (Yas and Samadi 2012).

3. Theory and formulations

3.1 Kinematics and constitutive equations

Consider a shear deformation beam theory, the displacement field consisting of the axial displacement, u, and the transverse displacement, w, can be written in the following forms

$$u(x,z) = u_0(x) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{5a}$$

$$w(x,z) = w_b(x) + w_s(x) + g(z)\varphi_z(x) \tag{5b}$$

where u_0 is the axial displacement, w_b and w_s are the bending and shear components of transverse displacement along the mid-plane of the beam. The additional displacement φ_z accounts for the effect of normal stress is included and $g(z)$ is given as follows

$$g(z) = 1 - f'(z) \tag{6a}$$

In this work, the shape function $f(z)$ is chosen based on a trigonometric function as (Tounsi *et al*. 2013)

$$f(z)=z-\frac{h}{\pi}\sin\left(\frac{\pi}{h}z\right) \tag{6b}$$

Clearly, the displacement field in Eq. (5) contains only four unknowns (u_0, w_b, w_s, φ_z). The strains associated with the displacements in Eq. (5) are

$$\varepsilon_x=\frac{\partial u_0}{\partial x}-z\frac{\partial^2 w_b}{\partial x^2}-f(z)\frac{\partial^2 w_s}{\partial x^2} \tag{7a}$$

$$\varepsilon_z=g^{'}(z)\varphi_z \tag{7b}$$

$$\gamma_{xz}=g(z)\left(\frac{\partial w_s}{\partial x}+\frac{\partial \varphi_z}{\partial x}\right) \tag{7c}$$

It can be seen from Eq. (7c) that the transverse shears strain γ_{xz} is equal to zero at the top ($z=h/2$) and bottom ($z=-h/2$) surfaces of the beam, thus satisfying the zero transverse shear stress conditions.

By assuming that the material of CNTRC beam obeys Hooke's law, the stresses in the beam become

$$\sigma_x=Q_{11}(z)\,\varepsilon_x+Q_{13}(z)\,\varepsilon_z \tag{8a}$$

$$\tau_{xz}=Q_{55}(z)\,\gamma_{xz} \tag{8b}$$

$$\sigma_z=Q_{13}(z)\,\varepsilon_x+Q_{33}(z)\,\varepsilon_z \tag{8c}$$

where

$$Q_{11}(z)=Q_{33}(z)=\frac{E_{11}(z)}{1-\nu^2}$$
$$Q_{13}(z)=\nu Q_{11}(z) \text{ and } Q_{55}(z)=G_{12}(z) \tag{8d}$$

3.2 Governing equations

The governing equations of equilibrium can be derived by using the principle of virtual displacements. The principle of virtual work in the present case yields

$$\begin{aligned}&\int_0^L\int_{-h/2}^{h/2}\left[\sigma_x\delta\varepsilon_x+\sigma_z\delta\varepsilon_z+\tau_{xz}\gamma_{xz}\right]dzdx\\&+\int_0^L\left[K_w(w_b+w_s)\delta(w_b+w_s)-K_s\frac{\partial^2(w_b+w_s)}{\partial x^2}\delta(w_b+w_s)\right]dx\\&-\int_0^L\left[q\delta(w_b+w_s)\right]dx=0\end{aligned} \tag{9}$$

where K_w and K_s are the Winkler and shearing layer spring constants which can be determined from $K_w=\beta_w A_{110}/L^2$ and $K_s=\beta_s A_{110}$ in wich β_w and β_s are the corresponding spring constant factors. It is also defined that A_{110} is the extension stiffness or the value of A_{11} of a homogeneous beam made of pure matrix material.

Substituting Eqs. (7) and (8) into Eq. (9) and integrating through the thickness of the beam, Eq. (9) can be rewritten as

$$\begin{aligned}&\int_0^L N_x\delta\frac{\partial u_0}{\partial x}-M_x^b\delta\frac{\partial^2 w_b}{\partial x^2}-M_x^s\delta\frac{\partial^2 w_s}{\partial x^2}+R_z\delta\varphi_z\\&+Q_{xz}\delta\left(\frac{\partial w_s}{\partial x}+\frac{\partial\varphi_z}{\partial x}\right)dx\\&+\int_0^L\left[K_w(w_b+w_s)\delta(w_b+w_s)-K_s\frac{\partial^2(w_b+w_s)}{\partial x^2}\delta(w_b+w_s)\right]\\&-\int_0^L q\delta(w_b+w_s+g\varphi_z)dx=0\end{aligned} \tag{10}$$

where N_x, M_x^b, M_x^s and Q_{xz} are the stress resultants defined by

$$\begin{aligned}&(N_x,M_x^b,M_x^s)=\int_{-\frac{h}{2}}^{\frac{h}{2}}(1,z,f)\,\sigma_x dz\\&Q_{xz}=\int_{-\frac{h}{2}}^{\frac{h}{2}}g\,\tau_{xz}dz \text{ and } R_z=\int_{-\frac{h}{2}}^{\frac{h}{2}}\sigma_z g^{'}(z)dz\end{aligned} \tag{11}$$

The governing equations of equilibrium can be derived from Eq. (10) by integrating the displacement gradients by parts and setting the coefficients zero δu_0, δw_b, δw_s and $\delta\varphi_z$ separately. Thus one can obtain the equilibrium equations associated with the present refined shear deformation theory

$$\delta u_0:\frac{\partial N_x}{\partial x}=0 \tag{12a}$$

$$\delta w_b:\frac{\partial^2 M_x^b}{\partial x^2}+q+K_s\left(\frac{\partial^2(w_b+w_s)}{\partial x^2}\right)-K_w(w_b+w_s)=0 \tag{12b}$$

$$\delta w_s:\frac{\partial^2 M_x^s}{\partial x^2}+\frac{\partial Q_{xz}}{\partial x}+q+K_s\left(\frac{\partial^2(w_b+w_s)}{\partial x^2}\right)-K_w(w_b+w_s)=0 \tag{12c}$$

$$\delta\varphi_z:-R_z+\frac{\partial Q_{xz}}{\partial x}+gq=0 \tag{12d}$$

Eq. (12) can be expressed in terms of displacements (u_0, w_b, w_s, φ_z) by using Eqs. (5), (7), (8) and (11) as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}-B_{11}\frac{\partial^3 w_b}{\partial x^3}-B_{11}^s\frac{\partial^3 w_s}{\partial x^3}+X_{13}\frac{\partial\varphi_z}{\partial x}=0 \tag{13a}$$

$$\begin{aligned}&B_{11}\frac{\partial^3 u_0}{\partial x^3}-D_{11}\frac{\partial^4 w_b}{\partial x^4}-D_{11}^s\frac{\partial^4 w_s}{\partial x^4}+Y_{13}\frac{\partial^2\varphi_z}{\partial x^2}+q\\&+K_s\left(\frac{\partial^2(w_b+w_s)}{\partial x^2}\right)-K_w(w_b+w_s)=0\end{aligned} \tag{13b}$$

$$\begin{aligned}&B_{11}^s\frac{\partial^3 u_0}{\partial x^3}-D_{11}^s\frac{\partial^4 w_b}{\partial x^4}-H_{11}\frac{\partial^4 w_s}{\partial x^4}+A_{55}^s\frac{\partial^2 w_s}{\partial x^2}+\left(Y_{13}^s+A_{55}^s\right)\frac{\partial^2\varphi_z}{\partial x^2}+q\\&+K_s\left(\frac{\partial^2(w_b+w_s)}{\partial x^2}\right)-K_w(w_b+w_s)=0\end{aligned} \tag{13c}$$

$$-X_{13}\frac{\partial u_0}{\partial x}+Y_{13}\frac{\partial^2 w_b}{\partial x^2}+\left(Y_{13}^s+A_{55}^s\right)\frac{\partial^2 w_s}{\partial x^2}+A_{55}^s\frac{\partial^2 \varphi_z}{\partial x^2}-Z_{33}\varphi_z+gq=0 \quad (13d)$$

where A_{11}, D_{11}, etc., are the beam stiffness, defined by

$$A_{11}=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}dz,\ B_{11}=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}zdz,$$

$$B_{11}^s=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}fdz,\quad X_{13}=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{13}g'dz, \quad (14a)$$

$$D_{11}=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}z^2dz,\ D_{11}^s=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}z.fdz,$$

$$Y_{13}=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{13}z.g'dz,\ H_{11}^s=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}f^2dz, \quad (14b)$$

$$Y_{13}^s=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{13}.f.g'.dz,\ Z_{33}=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{13}\left[g'\right]^2dz,$$

$$A_{55}^s=\int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{55}g^2dz, \quad (14c)$$

4. Analytical solution

The equilibrium equations admit the Navier solutions for simply supported beams. The variables u_0, w_b, w_s, φ_z, can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \\ \varphi_z \end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix} U_m\cos(\lambda x) \\ w_{bm}\sin(\lambda x) \\ w_{sm}\sin(\lambda x) \\ \phi_{zm}\sin(\lambda x) \end{Bmatrix} \quad (15)$$

Table 1 Dimensionless displacements and stress of UD-Beam with and without elastic foundation under uniform loads

V_{cnt}^*	L/h	Theory	β_w=0, β_s=0			β_w=0.1, β_s=0.02		
			$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$
0.12	10	Wattanasakulpong *et al*. (2013)	0.704	8.399	0.701	0.594	7.053	0.602
		Tagrara *et al*. (2015)	0.703	8.458	0.718	0.593	7.103	0.617
		Present $\varepsilon_z\neq0$	0.734	8.5650	0.716	0.615	7.143	0.612
	15	Wattanasakulpong *et al*. (2013)	0.524	11.849	0.716	0.400	9.556	0.568
		Tagrara *et al*. (2015)	0.524	11.888	0.736	0.400	9.019	0.584
		Present $\varepsilon_z\neq0$	0.556	12.057	0.735	0.418	9.013	0.576
	20	Wattanasakulpong *et al*. (2013)	0.461	15.448	0.725	0.311	10.316	0.520
		Tagrara *et al*. (2015)	0.460	15.479	0.746	0.311	10.336	0.536
		Present $\varepsilon_z\neq0$	0.493	15.708	0.744	0.325	10.245	0.525
0.17	10	Wattanasakulpong *et al*. (2013)	0.449	8.268	0.704	0.403	7.374	0.638
		Tagrara *et al*. (2015)	0.448	8.319	0.722	0.401	7.419	0.654
		Present $\varepsilon_z\neq0$	0.469	8.423	0.721	0.417	7.475	0.650
	15	Wattanasakulpong *et al*. (2013)	0.344	11.762	0.719	0.286	9.737	0.614
		Tagrara *et al*. (2015)	0.344	11.796	0.739	0.286	9.764	0.631
		Present $\varepsilon_z\neq0$	0.365	11.959	0.738	0.300	9.795	0.624
	20	Wattanasakulpong *et al*. (2013)	0.307	15.384	0.726	0.232	11.568	0.575
		Tagrara *et al*. (2015)	0.307	15.410	0.748	0.232	11.587	0.592
		Present $\varepsilon_z\neq0$	0.328	15.630	0.746	0.244	11.551	0.583
0.28	10	Wattanasakulpong *et al*. (2013)	0.325	8.562	0.697	0.299	7.869	0.647
		Tagrara *et al*. (2015)	0.324	8.631	0.713	0.299	7.933	0.662
		Present $\varepsilon_z\neq0$	0.336	8.725	0.712	0.308	7.997	0.660
	15	Wattanasakulpong *et al*. (2013)	0.235	11.959	0.714	0.206	10.469	0.638
		Tagrara *et al*. (2015)	0.234	12.004	0.733	0.206	10.511	0.655
		Present $\varepsilon_z\neq0$	0.246	12.152	0.732	0.215	10.573	0.651
	20	Wattanasakulpong *et al*. (2013)	0.203	15.530	0.723	0.167	12.751	0.613
		Tagrara *et al*. (2015)	0.203	15.566	0.743	0.167	12.781	0.631
		Present $\varepsilon_z\neq0$	0.214	15.766	0.743	0.175	12.808	0.625

Table 2 Dimensionless displacements and stress of UD-Beam with and without elastic foundation under sinusoidal loads

V_{cnt}^*	L/h	Theory	β_w=0, β_s=0			β_w=0.1, β_s=0.02		
			$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$
0.12	10	Wattanasakulpong *et al*. (2013)	0.562	6.970	0.472	0.475	5.890	0.399
		Tagrara *et al*. (2015)	0.560	7.025	0.486	0.474	5.937	0.411
		Present $\varepsilon_z \neq 0$	0.585	7.110	0.486	0.492	5.970	0.408
	15	Wattanasakulpong *et al*. (2013)	0.416	9.716	0.475	0.319	7.439	0.364
		Tagrara *et al*. (2015)	0.416	9.754	0.490	0.318	7.469	0.375
		Present $\varepsilon_z \neq 0$	0.442	9.890	0.489	0.333	7.468	0.369
	20	Wattanasakulpong *et al*. (2013)	0.365	12.608	0.476	0.247	8.535	0.322
		Tagrara *et al*. (2015)	0.365	12.636	0.491	0.247	8.555	0.322
		Present $\varepsilon_z \neq 0$	0.391	12.821	0.491	0.259	8.488	0.325
0.17	10	Wattanasakulpong *et al*. (2013)	0.358	6.842	0.473	0.321	6.126	0.424
		Tagrara *et al*. (2015)	0.357	6.889	0.487	0.320	6.1 69	0.436
		Present $\varepsilon_z \neq 0$	0.374	6.972	0.487	0.346	6.215	0.434
	15	Wattanasakulpong *et al*. (2013)	0.273	9.630	0.476	0.227	8 .0 21	0.396
		Tagrara *et al*. (2015)	0.273	9.662	0.490	0.227	8.048	0.408
		Present $\varepsilon_z \neq 0$	0.289	9.794	0.489	0.254	8.076	0.404
	20	Wattanasakulpong *et al*. (2013)	0.243	12.543	0.476	0.184	9.520	0.362
		Tagrara *et al*. (2015)	0.243	12.567	0.491	0.184	9.539	0.373
		Present $\varepsilon_z \neq 0$	0.259	12.745	0.491	0.212	9.515	0.366
0.28	10	Wattanasakulpong *et al*. (2013)	0.260	7.130	0.472	0.259	7.194	0.485
		Tagrara *et al*. (2015)	0.259	7.194	0.485	0.239	6.632	0.447
		Present $\varepsilon_z \neq 0$	0.268	7.269	0.485	0.247	6.684	0.446
	15	Wattanasakulpong *et al*. (2013)	0.187	9.824	0.475	0.164	8.639	0.418
		Tagrara *et al*. (2015)	0.186	9.868	0.489	0.164	8.679	0.430
		Present $\varepsilon_z \neq 0$	0.196	9.987	0.489	0.171	8.731	0.427
	20	Wattanasakulpong *et al*. (2013)	0.161	12.689	0.476	0.133	10.485	0.393
		Tagrara *et al*. (2015)	0.161	12.722	0.491	0.133	10.514	0.406
		Present $\varepsilon_z \neq 0$	0.170	12.884	0.490	0.139	10.539	0.401

where U_m, W_{bm}, W_{sm} and ϕ_{zm} are arbitrary parameters to be determined, and $\lambda = m\pi/L$. The transverse load q is also expanded in Fourier series as

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\lambda x) \tag{16}$$

where Q_m is the load amplitude calculated from

$$Q_m = \frac{2}{L}\int_0^L q(x)\sin(\lambda x)dx \tag{17}$$

The coefficients Q_m are given below for some typical loads.

For the case of a sinusoidally distributed load, we have

$$m = 1 \quad \text{and} \quad Q_1 = q_0 \tag{18}$$

And for the case of uniform distributed load, we have

$$Q_m = \frac{4Q_0}{m\pi}, \quad (m = 1,3,5\ldots) \tag{19}$$

Substituting the expressions of u_0, w_b, w_s, φ_z and q from Eqs. (15) and (16) into the equilibrium equations of Eq. (13), the analytical solutions can be obtained from the following equations

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \\ \phi_{zm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_m \\ Q_m \\ gQ_m \end{Bmatrix} \tag{20}$$

where

$$\begin{aligned} & a_{11} = A_{11}\lambda^2, a_{12} = -B_{11}\lambda^3, a_{13} = -B_{11}^s\lambda^3, \\ & a_{14} = -X_{13}\lambda, a_{22} = D_{11}\lambda^4 + K_w + K_s\lambda^2, \\ & a_{23} = D_{11}^s\lambda^4 + K_w + K_s\lambda^2, a_{24} = Y_{13}\lambda^2, \\ & a_{33} = H_{11}^s\lambda^4 + A_{55}^s\lambda^2 + K_w + K_s\lambda^2, \\ & a_{34} = Y_{13}^s\lambda^2 + A_{55}^s\lambda^2, a_{44} = A_{55}^s\lambda^2 + Z_{33} \end{aligned} \tag{21}$$

5. Results and discussion

In this section, numerical results of bending, buckling and vibrations behaviors of CNTRC beams are presented and discussed. The effective material characteristics of CNTRC beams at ambient temperature employed throughout this work are given as follows. Poly methyl methacrylate (PMMA) is utilized as the matrix and its material properties are: v^p=0.3 and E^p=2.5 GPa. For reinforcement material, the armchair (10,10) SWCNTs is chosen with the following properties (Tagrara 2015, Yas and Samadi 2012) : v^{cnt}=0.19; $E_{11}^{cnt}=600GPa$; $E_{22}^{cnt}=10GPa$ and $G_{12}^{cnt}=17.2GPa$.

For convenience, the following nondimensionalizations are employed

$$\bar{w}=100\frac{E_p h^3}{q_0 L^4}w;\quad \bar{\sigma}_x=\frac{h}{q_0 L}\sigma_x\left(\frac{L}{2},\frac{h}{2}\right);\quad \bar{\tau}_{xz}=\frac{h}{q_0 L}\tau_{xz}(0,0); \tag{22}$$

5.1 Results for bending analysis of CNTRC beams

For bending analysis of UD beams with and without elastic foundations, the present theory with stretching effect ($\varepsilon_z \neq 0$) are compared with the analytical solutions given by Wattanasakulpong and Ungbhakorn (2013) using third shear deformation theory and the results of Tagrara *et al*. (2015) as shown in Tables 1 and 2. It can be observed that our results with ($\varepsilon_z \neq 0$) are in an excellent agreement to those predicted using the higher order shear deformation theory of Wattanasakulpong and Ungbhakorn (2013), Tagrara *et al*. (2015) with (ε_z=0). However, the small difference found between the results is due to that the theories presented by Wattanasakulpong *et al*. (2013), Tagrara *et al*. (2015) ignore the thickness stretching effect. It can be observed that the beams supported by elastic foundation have lower displacements and stresses compared to those of the beams without elastic foundation. Moreover, increasing amount of CNTs makes the CNTRC beams stiffer.

Fig. 2 illustrates the dimensionless transverse displacement based on the present theory with ($\varepsilon_z \neq 0$) for different types of CNTRC beams under uniform load. Clearly, the maximum displacements are at the middle of each beam with *x/L*=0.5. It can be seen that the strongest beam is the *X*-Beam with the smallest transverse displacement, and followed by the *UD*-, *V*-and *O*-Beams, respectively.

The maximum transverse displacement of the UD-Beam associated with the changes of spring constant factors as illustrated in Fig. 3. The reduction of transverse displacements is almost linear as the spring constant factors are increased.

The dimensionless normal stresses of CNTRC beams subjected to a uniform load are calculated by the present theory with ($\varepsilon_z \neq 0$) as shown in Fig. 4. The unsymmetrical V-Beam has non-zero normal stress at the middle plane. For symmetrical beams, variation of normal stresses across their thicknesses shows different parabolic type distributions.

By using the present theory with ($\varepsilon_z \neq 0$) and the shear deformation theory of Tagrara *et al*. (2015) with (ε_z=0), the dimensionless shear stresses of the UD-Beam on elastic foundation under a uniform load are presented in Fig. 5. In general, the present theory $|\varepsilon_z=0|$ and the shear deformation beam model of Tagrara *et al*. (2015) give almost identical results.

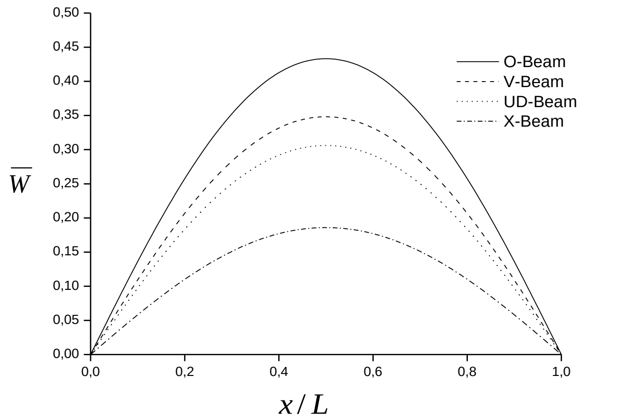


Fig. 2 Dimensionless transverse displacements of CNTRC beams on elastic foundation under uniform load (*L/h*=20; β_w=0.1; β_s=0.02; $V_{cnt}^*=0.12$)

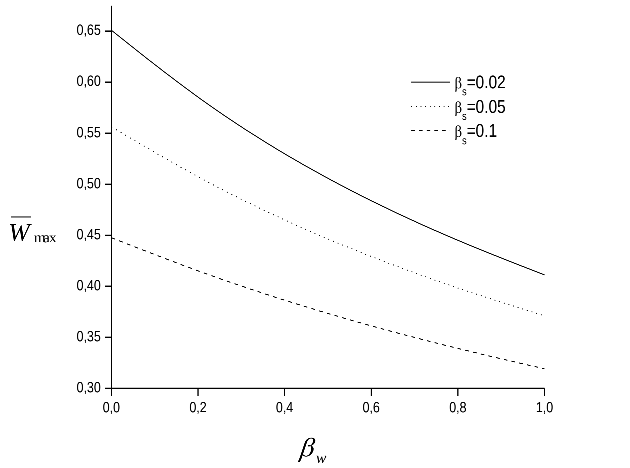


Fig. 3 Dimensionless transverse displacements of UD-Beam beams on elastic foundation under uniform load with variations of spring constant factors (*L/h*=10; $V_{cnt}^*=0.12$)

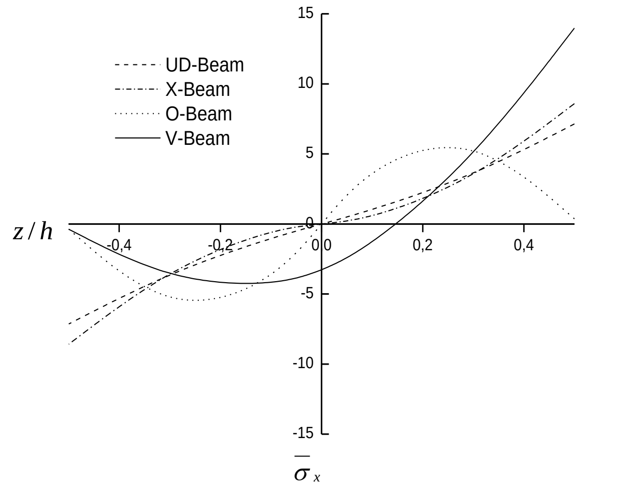


Fig. 4 Dimensionless normal stresses of CNTRC beams under uniform load (*L/h*=10; β_w=0.1; β_s=0.02; $V_{cnt}^*=0.12$)

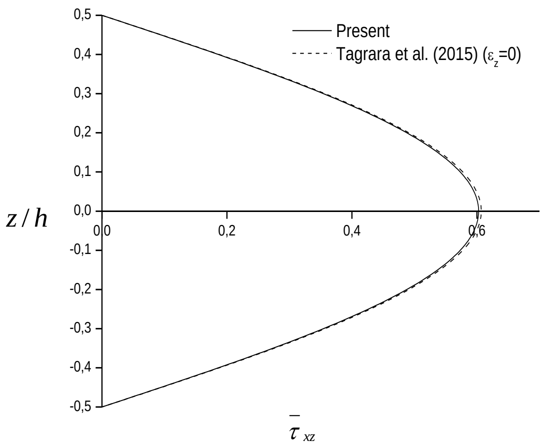


Fig. 5 Dimensionless shear stresses of UD-Beam beams under uniform load (L/h=10; β_w=0.1; β_s=0.02; $V_{cnt}^{*} = 0.12$)

6. Conclusions

In this present study, a refined shear deformation beam theory with stretching effect is employed to investigate the bending problem of simply supported CNTRC beams resting on elastic foundation. The beams are reinforced by different patterns of CNT distributions in the polymeric matrix. The equilibrium equations have been obtained using the principle of virtual displacements. The accuracy of the mathematical models is numerically verified by comparison with some available results. From the numerical results, it is found that the X-Beam is the strongest among different types of CNTRC beams in supporting the flexure, while the O-Beam is the weakest.

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