Earthquakes and Structures, *Vol. 13, No. 3 (2017) 279-288*
DOI: https://doi.org/10.12989/eas.2017.13.3.279

A novel two sub-stepping implicit time integration algorithm for structural dynamics

K. Yasamani[1a] and S. Mohammadzadeh[*2,3]

[1]*Malek-ashtar University of Technology, Tehran, Iran*
[2]*College of Engineering, School of Civil Engineering, University of Tehran, Tehran, Iran*
[3]*Payam-Nour University of Urmia, West Azerbaijan, Iran*

(Received August 5, 2017, Revised October 11, 2017, Accepted October 12, 2017)

Abstract. Having the ability to keep on yielding stable solutions in problems involving high potential of instability, composite time integration methods have become very popular among scientists. These methods try to split a time step into multiple sub-steps so that each sub-step can be solved using different time integration methods with different behaviors. This paper proposes a new composite time integration in which a time step is divided into two sub-steps; the first sub-step is solved using the well-known Newmark method and the second sub-step is solved using Simpson's Rule of integration. An unconditional stability region is determined for the constant parameters to be chosen from. Also accuracy analysis is perform on the proposed method and proved that minor period elongation as well as a reasonable amount of numerical dissipation is produced in the responses obtained by the proposed method. Finally, in order to provide a practical assessment of the method, several benchmark problems are solved using the proposed method.

Keywords: simpson rule; newmark method; composite time integration; unconditional stability; numerical damping; period elongation

1. Introduction

Nowadays with great improvement of science, computer simulations need to be even more realistic. One important simulation, which involves many research fields, is dynamic analysis of structures subjected to time-dependent loadings. A quick look at the history of numerical analysis of such structures reveals that numerous methods exist in this field; each having different characteristics. For example, Modal analysis is highly accurate for dynamic analysis of structures with linear behavior but has the disadvantage of finding eigenvalues and eigenvectors of the system which becomes very hard when the number of degrees of freedom increases remarkably. Another disadvantage of this method is its disability of solving nonlinear problems. This goes with Frequency domain analysis methods as well.

Through various existing methods, time stepping methods have become most popular among scientists to the point that most of them exist in commercial computer programs. These methods have different characteristics and can be categorized in many different ways. One categorization has something to do with the information needed to reach an equilibration on a time step. From this point of view, these methods are categorized into two classes being explicit and implicit. The methods which use the differential equation at time t to predict a solution at time $t+\Delta t$ are called explicit (Bathe and Wilson 1976, Dokainish and Subbaraj 1989, Belytschko and Lu 1993, Pezeshk and Camp 1995, Chang 2007, Chang 2009, Noh and Bathe 2013, Chang 2014, 2016). For most real structures, a very small time step is required to obtain a stable solution using explicit methods. Of course recently unconditionally stable explicit methods have also been proposed (Chang and Liao 2005, Rezaiee-Pajand and Hashemian 2016). Implicit methods try to reach equilibrium at time $t+\Delta t$ after the solution at time t is found (Bathe and Wilson 1976, Pezeshk and Camp 1995, Gholampour and Ghassemieh 2013, Bathe 2014, Soares 2016, Tornabene *et al.* 2016). There is also another class in this categorization called predictor-corrector which utilizes both formulations of explicit and implicit methods (Howe 1991, ZHAI 1996, Lourderaj *et al.* 2007, Rezaiee-Pajand and Alamatian 2008).

In another classification, time stepping methods are divided into conditionally and unconditionally stable methods. A method is called stable if the numerical solution, under any initial conditions, does not grow without bound; and is called unconditionally stable if the convergence of the solution is independent of the size of the time step Δt. The Newmark's family of methods, depending on the values of the constants used, stand in this category. In the Newmark integration method, the acceleration varies linearly or remains constant within two instances of time (Newmark 1959). Wilson-θ (Wilson 1962) is another example of such methods. A very important factor in numerical methods is their percentage period elongation and percentage amplitude decay by which

∗Corresponding author, Adjunct Professor
E-mail: Mohamadzade_71@ut.ac.ir
[a]MSc
E-mail: k_yasamani@mut.ac.ir

http://www.techno-press.com/journals/eas&subpage=7
ISSN: 2092-7614 (Print), 2092-7622 (Online)

the accuracy of these methods are assessed.

It has been less than a decade that a new kind of time stepping methods have been introduced to the world of dynamic analysis. These methods, called composite methods, try to split the time step into multiple steps each to be solved using different time stepping method algorithms (Bathe and Baig 2005, Bathe 2007, Silva and Bezerra 2008, Leontyev 2010, Bathe and Noh 2012, Chang 2014, Gautam and Sauer 2014, Chandra *et al.* 2015, Zhang *et al.* 2015, Wen *et al.* 2017, Wen *et al.* 2017, Zhang *et al.* 2017). The recent papers on composite schemes are mostly inspired by the method developed by Bathe and Baig (2005). Using the sub-step strategy similar to Bathe scheme, a generalized robust composite time integration scheme is proposed by Dong (Dong 2010) for nonlinear elastodynamics with the purpose of overcoming the stability problem of existing time integration schemes in problems with nonlinear behavior. In order to improve the dissipation control in composite time integration methods and as an extension to the proposed methods by Bathe and Baig (2005) and Dong (2010), a new composite time integration in which three sub-steps were utilized where the Trapezoidal rule is applied on the first and second sub-steps and the backward difference formula is adopted to perform the third sub-step analysis is proposed by Chandra *et al*. (Chandra *et al.* 2015). Similar to this work, Wen *et al*. (Wen *et al.* 2017) proposed a three sub-step method in which Trapezoidal rule is applied on the first sub-step and the Euler backward method is applied on the second sub-step, as for the third sub-step, the Houbolt method is utilized. The last three mentioned methods assessed numerically for their performances and different features by Wen *et al*. (Wen *et al.* 2017). Composite methods have proved their ability to overcome highly instable conditions. These conditions are very common in nonlinear problems as well as flexible structures which have vast range of natural frequencies (Chang 2002, 2015).

This study, motivated by the method developed by Bathe and Baig (2005), proposes a new composite time integration method which divides a time step into two sub-steps. The first sub-step is solved using the well-known Newmark's method (Newmark 1959) and for the second sub-step the Simpson's Rule (Scherer 2017), which is a double step method, is applied. The stability, accuracy, and overshoot behaviors of the method are analyzed and through numerical examples the method is compared to the existing ones.

2. The proposed algorithm

If a single degree of freedom system is considered with a (non-)linear behavior

$$\mathbf{M}\ddot{U}_{t+\Delta t} + \mathbf{C}\dot{U}_{t+\Delta t} + \mathbf{K}_t \Delta U_{t+\Delta t} + f_{s_t} = P_{t+\Delta t} \tag{1}$$

in which U is the displacement, $\dot{U}$ is the velocity, $\ddot{U}$ is the acceleration, $\mathbf{M}$ is the mass, $\mathbf{C}$ is the damping, $\mathbf{K}_t$ is the tangent stiffness in time t, P is the exciting force, f_{s_t} is the internal force, and Δt is the time step duration. The time step is divided into two equal or unequal sub-steps being $\alpha\Delta t$ and $(1-\alpha)\Delta t$. Imagine the solution is known up to time t and the solution of time $t+\Delta t$ is to be calculated. The first sub-step is solved using Newmark method; as follows

$$\ddot{U}_{t+\alpha\Delta t} = \frac{1}{\beta(\alpha\Delta t)^2}\Big(\Delta U_{t+\alpha\Delta t} - \alpha\Delta t\dot{U}_t - (1/2-\beta)(\alpha\Delta t)^2\ddot{U}_t\Big) \tag{2}$$

$$\dot{U}_{t+\alpha\Delta t} = \frac{\gamma}{\beta(\alpha\Delta t)}\left(\Delta U_{t+\alpha\Delta t}\right) + \left(1-\frac{\gamma}{\beta}\right)\dot{U}_t + \left(1-\frac{\gamma}{2\beta}\right)(\alpha\Delta t)\ddot{U}_t \tag{3}$$

in which β and γ are the constants of Newmark family of methods in which the choice of $\beta = 1/4$ and $\gamma = 1/2$ leads to the Trapezoidal Rule. Substituting Eqs. (2) and (3) into the equation of motion, the incremental equation is obtained; as follows

$$\mathbf{K}_{e1}\Delta U_{t+\alpha\Delta t} = R_1 \tag{4}$$

in which

$$\mathbf{K}_{e1} = \frac{\mathbf{M}}{\beta(\alpha\Delta t)^2} + \frac{\gamma\mathbf{C}}{\beta(\alpha\Delta t)} + \mathbf{K}_t \tag{5}$$

$$R_1 = P_{t+\alpha\Delta t} - f_{s_t} + \left(\frac{1}{\beta(\alpha\Delta t)}\left(\dot{U}_t\right) + \left(\frac{1}{2\beta}-1\right)\ddot{U}_t\right)\mathbf{M} + \left(\left(\frac{\gamma}{\beta}-1\right)\dot{U}_t + \left(\frac{\gamma}{2\beta}-1\right)(\alpha\Delta t)\ddot{U}_t\right)\mathbf{C} \tag{6}$$

$$\Delta U_{t+\alpha\Delta t} = U_{t+\alpha\Delta t} - U_t \tag{7}$$

The calculated values for displacement, velocity, and acceleration at the end of the first sub-step are used as initial values for the second sub-step.

The second sub-step is solved using Simpson's Rule. The following equations are related to the second sub-step

$$\dot{U}_{t+\Delta t} = \dot{U}_t + \frac{(1-\alpha)\Delta t}{6}\left(\ddot{U}_t + 4\ddot{U}_{t+\alpha\Delta t} + \ddot{U}_{t+\Delta t}\right) \tag{8}$$

$$U_{t+\Delta t} = U_t + \frac{(1-\alpha)\Delta t}{6}\left(\dot{U}_t + 4\dot{U}_{t+\alpha\Delta t} + \dot{U}_{t+\Delta t}\right) \tag{9}$$

Substituting Eqs. (8) and (9) into the equation of motion, the following equation is obtained

$$\mathbf{K}_{e2}\Delta U_{t+\Delta t} = R_2 \tag{10}$$

in which

$$\mathbf{K}_{e2} = \frac{36\mathbf{M}}{(1-\alpha)^2(\Delta t)^2} + \frac{6\mathbf{C}}{(1-\alpha)(\Delta t)} + \mathbf{K}_{t+\alpha\Delta t} \tag{11}$$

$$R_1 = P_{t+\Delta t} - f_{s_{t+\alpha\Delta t}} + -\left(\frac{36}{(1-\alpha)^2(\Delta t)^2}\left(\Delta U_{t+\alpha\Delta t}\right) - \frac{12}{(1-\alpha)\Delta t}\dot{U}_t + \right. \tag{12}$$

$$-\frac{24}{(1-\alpha)\Delta t}\dot{U}_{t+\alpha\Delta t} - \ddot{U}_t - 4\ddot{U}_{t+\alpha\Delta t}\Bigg)\mathbf{M} +$$

$$-\left(\frac{6}{(1-\alpha)\Delta t}\Delta U_{t+\alpha\Delta t} - \dot{U}_t - 4\dot{U}_{t+\alpha\Delta t}\right)\mathbf{C}$$

$$\Delta U_{t+\Delta t} = U_{t+\Delta t} - U_{t+\alpha\Delta t} \tag{13}$$

in which α, β and γ are the constant parameters kept into account so that the method could have unconditional stability, which is discussed in detail on the next section. It is notable that incremental equations, presented in Eqs. (4) and (10), can be solved linearly or nonlinearly; in case nonlinear behavior is considered, an equilibrium path tracing algorithm like Newton-Raphson method needs to be applied on the incremental equation.

3. Stability

It is common to assess the stability of a time integration method by considering the equation of motion for a single degree of freedom system with arbitrary initial conditions. The amplification matrix $[A]$ is calculated for such problem and then the spectral radius of this matrix is calculated. The method is stable if spectral radius is less than or equal to unit (Gholampour *et al.* 2011, Bathe and Noh 2012, Bathe 2014, Verma *et al.* 2015, Mohammadzadeh *et al.* 2017). Eq. (14) shows the amplification matrix of Newmark method in a free vibration problem; as follows

$$\begin{Bmatrix} \ddot{U}_{t+\alpha\Delta t} \\ \ddot{U}_t \\ \dot{U}_{t+\alpha\Delta t} \\ U_{t+\alpha\Delta t} \end{Bmatrix} = [A]\begin{Bmatrix} \ddot{U}_t \\ \ddot{U}_{t-\Delta t+\alpha\Delta t} \\ \dot{U}_t \\ U_t \end{Bmatrix};\ [A] = \begin{bmatrix} a_{11} & 0 & a_{12} & a_{13} \\ 1 & 0 & 0 & 0 \\ a_{21} & 0 & a_{22} & a_{23} \\ a_{31} & 0 & a_{32} & a_{33} \end{bmatrix} \tag{14}$$

in which the constants of the amplification matrix $[A]$ are presented in Appendix A. For the second sub-step the following amplification matrix is obtained

$$\begin{Bmatrix} \ddot{U}_{t+\Delta t} \\ \ddot{U}_{t+\alpha\Delta t} \\ \dot{U}_{t+\Delta t} \\ U_{t+\Delta t} \end{Bmatrix} = [B]\begin{Bmatrix} \ddot{U}_{t+\alpha\Delta t} \\ \ddot{U}_t \\ \dot{U}_{t+\alpha\Delta t} \\ U_{t+\alpha\Delta t} \end{Bmatrix};\ [B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \tag{15}$$

in which the constants of the amplification matrix $[B]$ are presented in Appendix A. Finally, the amplification matrix for the proposed composite method is obtained; as follows

$$\begin{Bmatrix} \ddot{U}_{t+\Delta t} \\ \ddot{U}_{t+\alpha\Delta t} \\ \dot{U}_{t+\Delta t} \\ U_{t+\Delta t} \end{Bmatrix} = [B][A]\begin{Bmatrix} \ddot{U}_t \\ \ddot{U}_{t-\Delta t+\alpha\Delta t} \\ \dot{U}_t \\ U_t \end{Bmatrix} \tag{16}$$

If it is to find the stability region, the critical state being zero damping ratio has to be considered. Fig. 1 presents the spectral radius as a function of $\Delta t/T$.

According to Fig. 1, one definite region of unconditional stability is presented in Eq. (17)

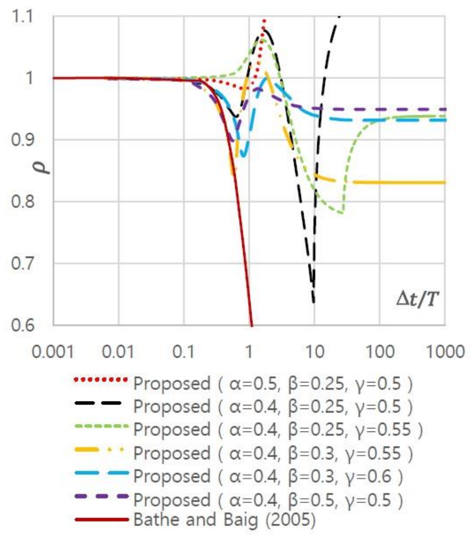


Fig. 1 Spectral Radius as a function of $\Delta t/T$

$$\alpha = 0.4\ \ \&\ \ \gamma \geq 0.5\ \ \&\ \ \beta \geq 0.5 - \gamma/3 \tag{17}$$

Of course there may exist more regions of unconditional stability for the constant parameters to be chosen from; here only one definite region is discussed. Additionally, in order to assess the weak instability presence in the proposed method, consider the free vibration of a 2DOf system where a relatively high frequency mode is included; as presented in Fig. 2(a).

Assuming K_1=10^3, K_1=1, M_1=1, and M_2=1 with unitary initial displacement in both degrees of freedom, the displacement responses at the first and second DOFs are presented in Fig. 2(b) and Fig. 2(c), respectively.

According to Fig. 2(b), although the proposed method shows no weak instability, Bathe method has better performance in high frequency dissipation. Regarding the Fig. 2(c), the differences in responses at the second DOF are very marginal. In addition to this, the displacement response of the first DOF is calculated assuming K_1=10^7 (Fig. 2(d)).

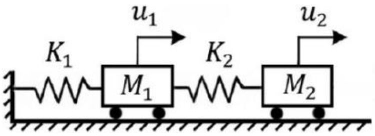
According to the Fig. 2(d), the differences between responses of Bathe method and the proposed method are less tangible and no weak instability is observed in any of these methods. After defining stability regions, the accuracy of method has to be analyzed; as follows in the next section.

4. Accuracy

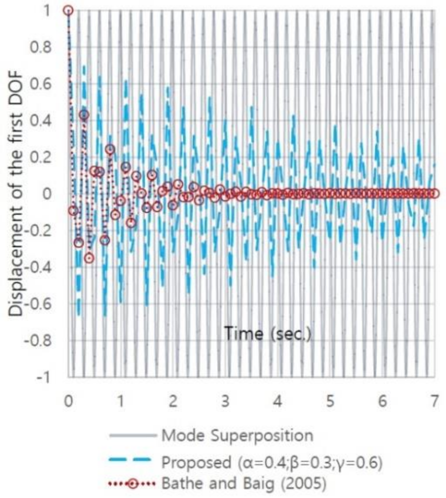
Unconditional stability may provide a remarkable advantage to a method but without having the required accuracy in the responses, a method would not be of interest to any operator. Assessment of accuracy in time stepping methods is usually followed by considering the following equation

$$\ddot{U} + 2\xi\omega\dot{U} + \omega^2 U = 0 \tag{18}$$

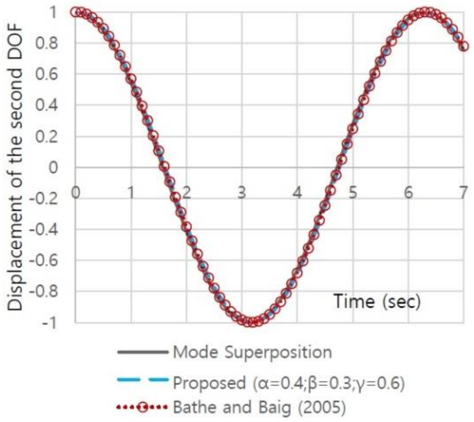
This way, the variables considered in the stability and accuracy analyses are only Δt, ω, and ξ (Bathe 2014).



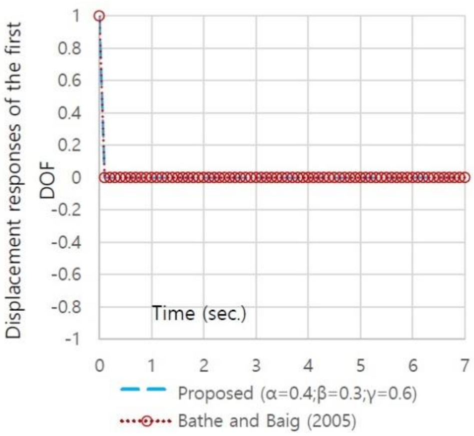
(a) Highly flexible 2DOF system



(b) Displacement response of the first DOF



(c) Displacement response of the second DOF



(d) Displacement response of the first DOF(K_1=10^7)

Fig. 2 Weak instability assessment of the proposed method through solving a highly flexible system

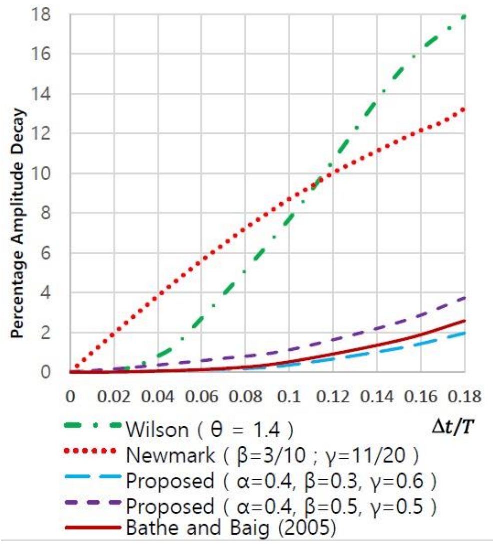


Fig. 3 Percentage numerical damping versus $\Delta t / T$

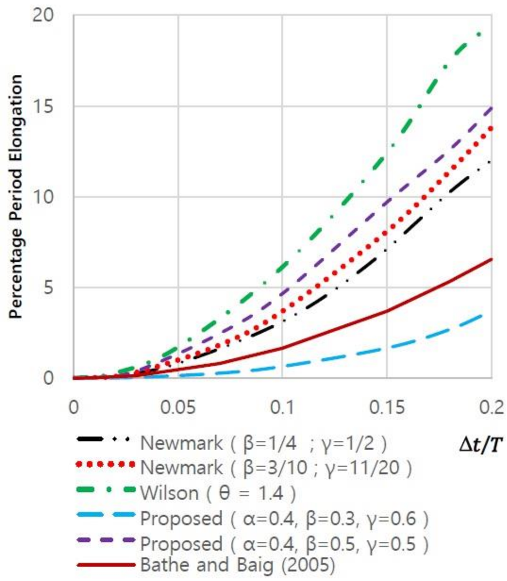
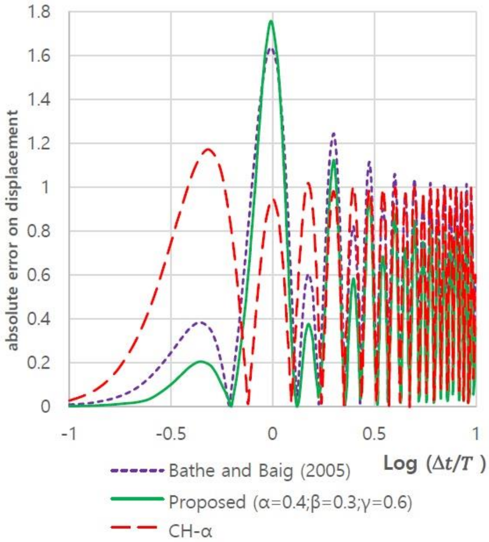


Fig. 4 Percentage period elongation as a function of $\Delta t / T$

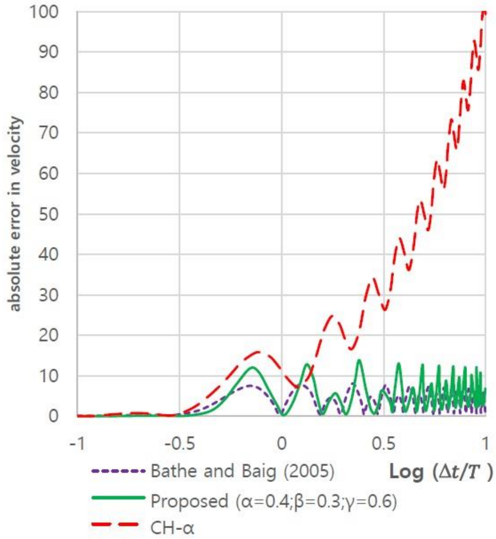
Amplitude decay, also known as numerical damping, is the percentage distortion of amplitude in the responses compared to the exact solution. Fig. 3 presents the percentage amplitude decay of the proposed method along with the other known methods where damping is equal to zero (critical case) and unit displacement initial condition is assumed.

The advantage of having two reasonable levels of numerical damping in the proposed method is easily perceptible from Fig. 3. As stated in (Bathe and Noh 2012), a reasonable amount of numerical damping sometimes helps a method to provide more accurate responses as well as keep its stability during analysis. Of course it should be noted that, it is possible to provide the method with different levels of numerical damping; but according to Fig. 1, the unconditional stability will be lost; so, in this case, it is required to consider the stability region of the proposed method in the analysis.

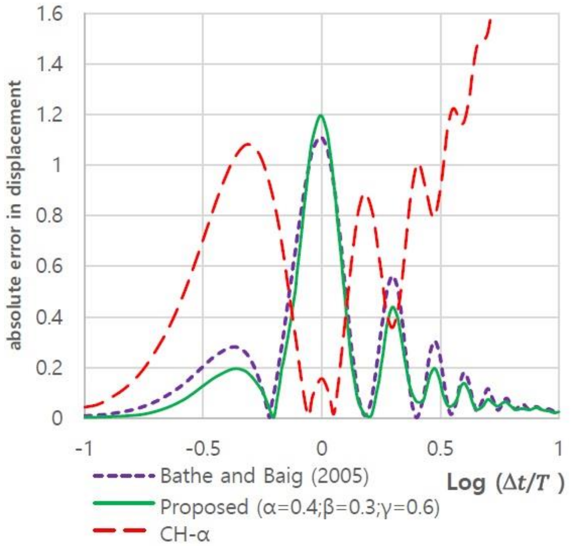
In addition to distortion in amplitude, time marching methods usually yield responses that suffer from distortion in period. The period elongation factor has the duty of measureing such distortion. This error is generally reported



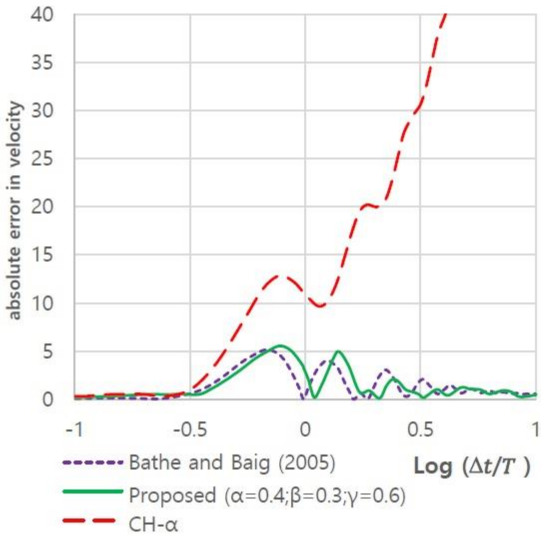
(a) Overshoot results of displacement (0.0 % damping)



(b) Overshoot results of velocity (0.0 % damping)



(c) Overshoot results of displacement (10.0 % damping)



(d) Overshoot results of velocity (10.0 % damping)

Fig. 5 Overshooting assessment of the proposed method through solving a model problem at the first time step using different time step sizes

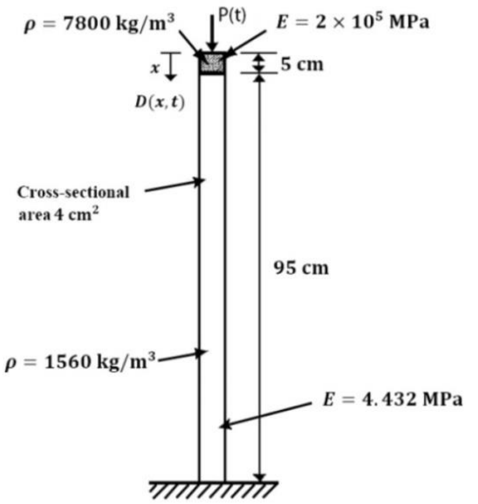
as the percentage difference between the true period and the period of computed solution of undamped free vibration of SDOF systems (Bathe 2014, Shrikhande 2014). Fig. 4 presents the percentage period elongation error for various methods.

Fig. 4 illustrates that the proposed method even with the choice of (α=0.4; β=0.5; γ=0.5), which provides the method with numerical damping, produces slightly higher amount of period elongation compared to the Newmark method with the choice of ($\beta = 3/10$; $\gamma = 11/20$). It is also perceptible from this figure that the proposed method with the choice of (α=0.4; β=0.3; γ=0.6) has the lowest amount of period elongation error among the presented methods.

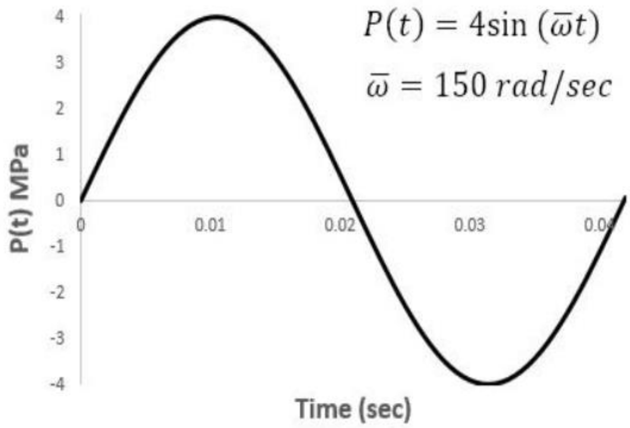
5. Overshoot

This section concerns with the overshoot behavior of time integration methods (Goudreau and Taylor 1972). This behavior of numerical time integration methods is carried out by studying the solution of a model problem at the first time step using different time step sizes. To this end, the absolute errors in displacement and velocity for two cases, being with and without presence of damping, are calculated. Please be noticed that in order to present an overshooting method the results of CH-α (Chung and Hulbert 1993) are also included in the figures. Additionally, the overshooting results of Bathe method and CH-α method can also be found in a recent study by Kadapa *et al*. (Kadapa *et al*. 2017). Fig. 5(a) and Fig. 5(b) present the overshooting analysis results for the displacement and velocity, respectively where damping is assumed to be zero.

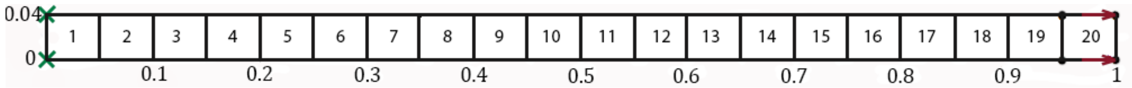
According to the results presented in Fig. 5(a) and Fig. 5(b), no overshooting is observed in the proposed method and Bathe method in displacement and velocity; this is while, significant overshooting is present in CH-α method's velocity responses. Fig. 5(c) and Fig. 5(d) present the overshooting analysis results for the displacement and



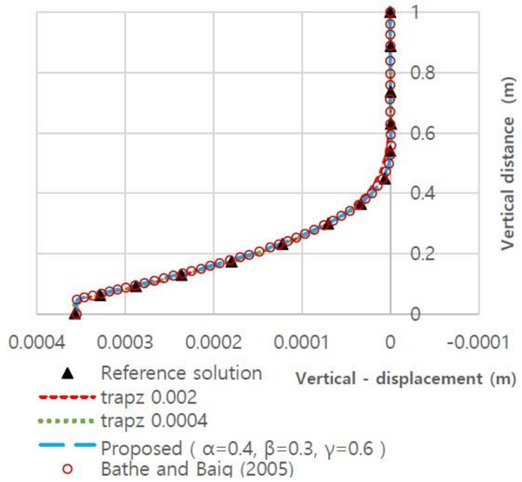
(a) A linearly behaving bar under axial force



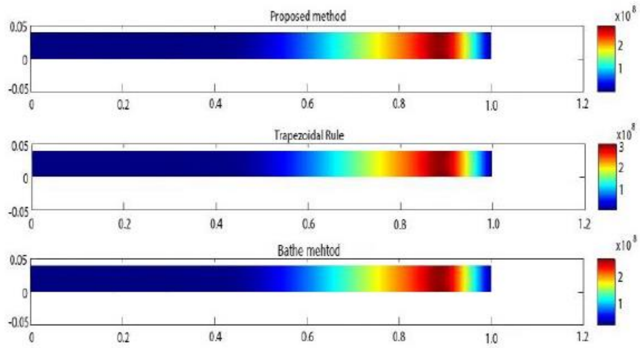
(b) The equation of dynamic axial force



(c) The modeled structure of the bar



(d) Responses to the bar obtained by various method



(e) von Mises stress contour at t=0.01 *sec*.

Fig. 6 First Example: the bar under dynamic axial load

velocity, respectively where damping is assumed to be 10 percent.

As illustrated in Fig. 5(c) and Fig. 5(d), the proposed method and Bathe method do not suffer from overshooting. On the other hand, CH-α method has shown remarkably high overshoot in the results of both displacement and velocity when damping is assumed to be present.

6. Numerical examples

In order to have a practical assessment of the proposed method, in this section the proposed method has been applied on the several benchmark problems.

6.1 Example 1

This example is a benchmark problem chosen from (Bathe 2014) and involves a bar under dynamic axial loading with linear elastic behavior. Fig. 6(a) presents the characteristics of this example in detail. The bar is made of two materials giving it a stiff and flexible sections and the load is acting on the stiff part.

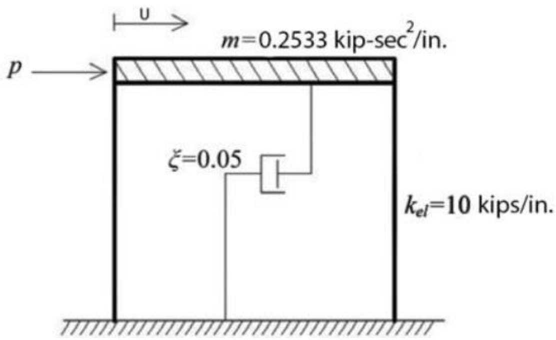


Fig. 7 A SDOF system with nonlinear behavior

The bar is initially at rest and is subjected to a dynamic axial force for which the formula is shown in Fig. 6(b). The

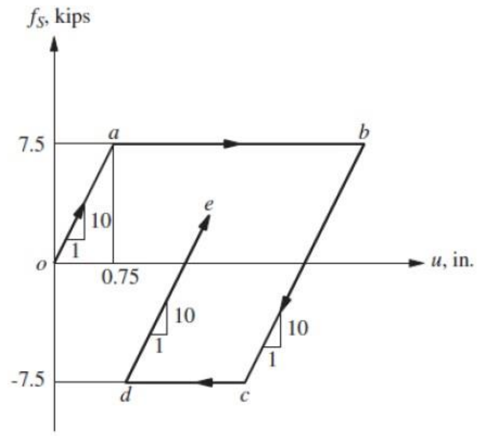


Fig. 8 The elastic-perfect plastic behavior (Chopra 2007)

response of the bar at time 0.01 *sec.* was sought.

As stated in (Bathe 2014), the static correction provided the mode superposition method highly accurate responses. Solution of this problem demands consideration of sufficient number of elements. As illustrated in Fig. 6(c), the bar is modeled with twenty quadrilateral four-noded elements.

The results of displacement of nodes along the bar has been presented in Fig. 6(d) for various methods. Please be noticed that the mode superposition method with static correction has been adopted as reference solution. Also Newmark trapezoidal rule is applied on this problem with two different time step sizes being $\Delta t = 0.0004$ *sec.* and $\Delta t = 0.002$ *sec.* while for the proposed method a time step size of 0.005 *sec.* is adopted.

As it is perceptible from Fig. 6(d), the responses obtained by different methods are very close to each other; the only place in which the responses have perceptible differences is in the middle of the bar. In this region, Trapezoidal rule with a time step size of 0.002 *sec.* has tangible digression from the reference solution; this is while the proposed method and Bathe method with a time step size of 0.005 *sec.* (more than 2 times longer than Trapezoidal-rule's) continue on agreement with the reference solution. The von Mises stress contour is displayed in Fig. 6(e). The mentioned differences are less tangible in this figure.

6.2 Example 2

This example assesses the proposed method in a nonlinear SDOF problem where damping is present. The SDOF system is shown in Fig. 7 and the force-displacement behavior of this system is shown in Fig. 8 in which it is presented that the problem has elastic-perfect plastic behavior (Chopra 2007).

The dynamic load acting on the structure is a half-harmonic force; as presented in Fig. 9. The problem is solved using Trapezoidal rule with $\Delta t = 0.1$ *sec.* and the proposed method with $\Delta t = 0.2$ *sec.* (the values of $\alpha\Delta t$ are recorded as well) Please be noticed that in order to have a reference solution for the methods to be compared with, Trapezoidal rule with $\Delta t = 0.01$ *sec.*, which is a considerably

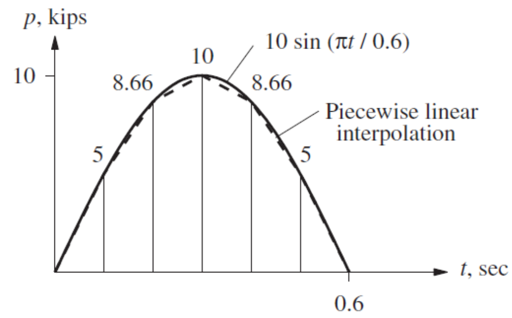


Fig. 9 The harmonic load acting on the system (Chopra 2007)

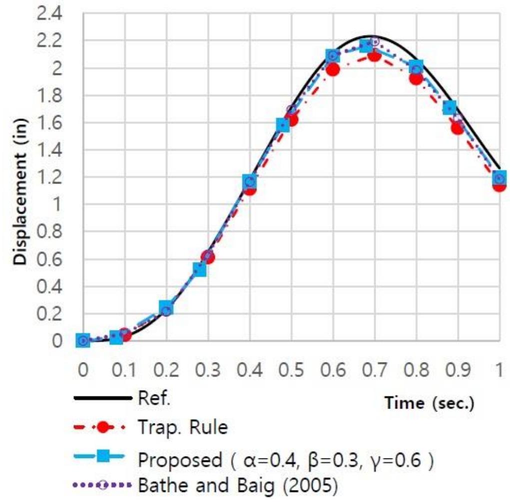


Fig. 10 Displacement responses of SDOF system under dynamic loading

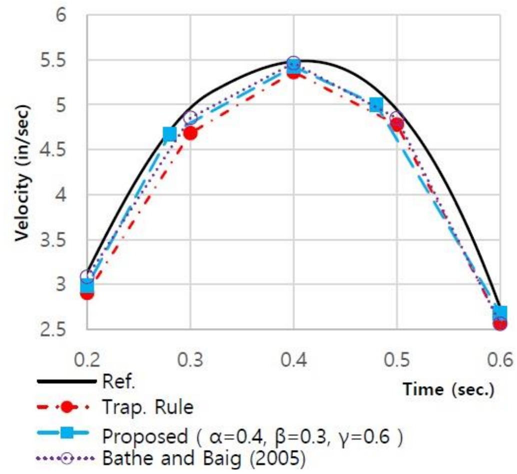


Fig. 11 Velocity responses of SDOF system under dynamic loading

short time step, is referred to as reference solution.

Fig. 10 presents the displacement responses obtained by the mentioned methods as well as Bathe method. It is perceptible from this figure that the proposed method with the same solution effort with Trapezoidal rule, i.e., adoption of 2 times longer time step, has yielded more accurate responses and is in closest agreement with the reference solution. Additionally, the results of proposed method and

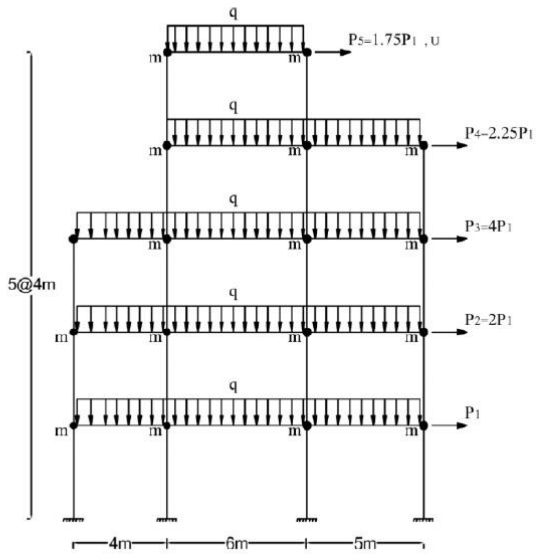


Fig. 12 Five story frame structure

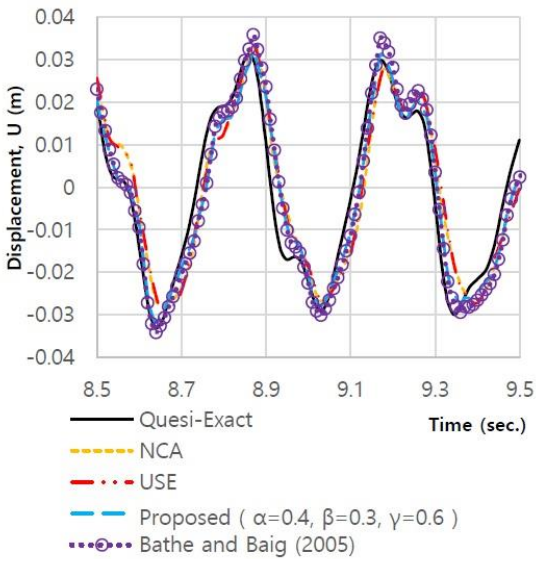


Fig. 13 Roof displacement response

Bathe method have marginal differences.

Fig. 11 presents the velocity responses of the SDOF system in time interval between 0.2 to 0.6 *sec.*. Again as this figure presents, the proposed method, though with a marginal difference, has yielded responses with better accuracy. Regarding Bathe method's responses, in some points Bathe method has yielded responses with higher accuracy while in some other points the proposed method has shown more agreement with the reference solution.

6.3 Example 3

In this example a 5-story building is considered; as illustrated in Fig. 12. The moment resisting frame in this problem is studied by Rezaeei-Pajand and Hashemian (Rezaiee-Pajand and Hashemian 2016). Columns and beams are assumed to be wide flange steel with cross sections W21×50 and W18×35, respectively. The modulus of elasticity is considered to be 2×10^{10} kg/m^2 and the density of material is equal to 800 kg/m^3. The structure is considered to be weightless. A 5000 kgf/m uniform distributed gravity load is acted on all stories. Additionally, a concentrated mass, m=1500 kgs, is considered at each node. According to Fig. 12, the frame is under a set of horizontal dynamic load, $F_i = P_i \sin(20\pi t)$ with P_i being the force acting on each story and P_1=4000 *kgf*.

As studied by Rezaeei-Pajand and Hashemian, this problem is solved using Newmark Constant Acceleration (NCA), an Unconditional Stable Explicit (USE) and a quesi exact method as a reference solution (Rezaiee-Pajand and Hashemian 2016). Here, the proposed method and Bathe method with a time step equal to 0.02 *sec.*, which is two times longer than the time step size adopted for the non-composite schemes used in this Example, is applied on the problem; so that a comparison could take place between the mentioned methods. In order to have a better sight on the responses, Fig. 13 shows the response obtained in time interval of 8.5 *sec.* and 9.5 *sec.*.

According to Fig. 13, the proposed method has yielded responses with tangible difference from other two methods and has the closest responses to the Quasi-Exact method's.

7. Conclusions

The Trapezoidal rule is used along with Simpson rule to develop a novel composite time integration method. The properties of the proposed method is carried out with stability and accuracy analysis. The stability analysis determined a definite unconditionally stable region for the constant parameters to be chosen from and accuracy analysis proved that the proposed method produces reasonable amount of amplitude decay which can be used for good in case needed. Also, minor period elongation has been reported after accuracy analysis. Finally, Benchmark problems, all selected from well-known books and papers, have been solved by the proposed method.

As a conclusion, the method has several advantages being: high accuracy, thanks to the reasonable amount of amplitude decay and minor period elongation, unconditional stability, better responses in nonlinear problems, and problems involving multiple degrees of freedom as well as in plane stress/strain problems. Although the proposed method provides only two levels of numerical dissipation with the mentioned options for the constant parameters, one disadvantage of this method can be its less control on numerical dissipation. Please be noticed that solution effort of the proposed method is just like other non-composite methods because the time step size in the proposed method can be adopted 2 times longer than non-composite methods and still expect higher accuracy.

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CC

Appendix A

The constants of the amplification matrix $[A]$ are obtained; as follows

$$
\begin{aligned}
a_{11} &= -(1/2-\beta)m - 2(1-\gamma)k \\
a_{12} &= -\frac{m+2k}{\alpha\Delta t} \quad ; \quad a_{13} = -\frac{m}{(\alpha\Delta t)^2} \\
a_{21} &= (\alpha\Delta t)(1 - m - (1/2-\beta)\gamma m - 2(1-\gamma)\gamma k) \\
a_{22} &= 1 - \frac{m}{2} - k \quad ; \quad a_{23} = -\frac{\gamma m}{(\alpha\Delta t)} \\
a_{31} &= (\alpha\Delta t)^2 (1/2 - \beta - (1/2-\beta)\beta m - 2(1-\gamma)\beta k) \\
a_{32} &= \alpha\Delta t(1 - \beta m - 2\beta k); \quad a_{33} = 1 - \beta m
\end{aligned} \tag{A1}
$$

in which

$$
m = \left(\frac{1}{\omega_n^2 \alpha^2 \Delta t^2} + \frac{2\gamma\xi}{\omega_n^2 \alpha\Delta t} + \beta \right)^{-1} \; ; \; k = \frac{\xi m}{\omega_n \alpha\Delta t} \tag{A2}
$$

Also, the constants of the amplification matrix $[B]$ are obtained; as follows

$$
\begin{aligned}
b_{11} &= \left(\frac{1}{R_1}\right)\left(-\omega_n^2\left(\frac{(1-\alpha)^2\Delta t^2}{9}\right) - 4\xi\omega_n \frac{(1-\alpha)\Delta t}{3}\right) \\
b_{12} &= \left(\frac{1}{R_1}\right)\left(-\omega_n^2\left(\frac{(1-\alpha)^2\Delta t^2}{36}\right) - \xi\omega_n \frac{(1-\alpha)\Delta t}{3}\right) \\
b_{13} &= \left(\frac{1}{R_1}\right)\left(-2\omega_n^2\left(\frac{(1-\alpha)\Delta t}{3}\right)\right) ; b_{14} = \left(\frac{-\omega_n^2}{R_1}\right) \\
b_{21} &= 1 ; b_{22} = 0 ; b_{23} = 0 ; b_{24} = 0 ; b_{31} = \left(\frac{-4}{R_2}\right) \\
b_{32} &= \left(\frac{-1}{R_2}\right); b_{33} = \left(\frac{-2\omega_n^2}{R_2}\right)\left(\frac{(1-\alpha)\Delta t}{3}\right) \\
b_{34} &= \left(\frac{-\omega_n^2}{R_2}\right); b_{41} = \left(\frac{4}{R_2}\right); b_{42} = \left(\frac{1}{R_2}\right) \\
b_{43} &= \left(\frac{1}{R_3}\right)\left(8\xi\omega_n + \frac{24}{(1-\alpha)\Delta t}\right) \\
b_{44} &= \left(\frac{12}{R_3}\right)\left(\frac{\xi\omega_n}{(1-\alpha)\Delta t} + \frac{3}{(1-\alpha)^2\Delta t^2}\right)
\end{aligned} \tag{A3}
$$

in which

$$
\begin{aligned}
R_1 &= \left(1 + \xi\omega_n \frac{(1-\alpha)\Delta t}{3} + \omega_n^2 \frac{(1-\alpha)^2\Delta t^2}{36}\right) \\
R_2 &= \left(\frac{6}{(1-\alpha)\Delta t} + 2\xi\omega_n + \omega_n^2 \frac{(1-\alpha)\Delta t}{6}\right) \\
R_3 &= \left(\frac{36}{(1-\alpha)^2\Delta t^2} + \xi\omega_n \frac{12}{(1-\alpha)\Delta t} + \omega_n^2\right)
\end{aligned} \tag{A4}
$$

In the above equations, ξ represents the damping ratio, and ω_n is the natural frequency of system.