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A simple approach for the fundamental period of MDOF structures

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Abstract. Fundamental period is one of the most critical parameters affecting the seismic design of buildings. In this paper, a very simple approach is presented for estimating the fundamental period of multiple-degree-of-freedom (MDOF) structures. The basic idea behind this approach is to replace the complicated MDOF system with an equivalent single-degree-of-freedom (SDOF) system. To realize this equivalence, a procedure for replacing a two-degree-of-freedom (2-DOF) system with an SDOF system, known as a two-to-single (TTS) procedure, is developed first; then, using the TTS procedure successively, an MDOF system is replaced with an equivalent SDOF system. The proposed approach is expressed in terms of mass, stiffness, and number of stories, without mode shape or any other parameters; thus, it is a very simple method. The accuracy of the proposed method is investigated by estimating the fundamental periods of many MDOF models; it is found that the results obtained by the proposed method agree very well with those obtained by eigenvalue analysis.

Keywords: fundamental-period estimation; seismic design; MDOF structures; TTS procedure; equivalent SDOF system

1. Introduction

The fundamental period is a key parameter for the seismic design of a building structure using the equivalent-lateral-force procedure; in principle, it can be accurately evaluated by means of an eigenvalue analysis (Shibata 2010) on a structural model. In most building-design projects, since the building's period cannot be analytically calculated before it has been designed, accurate computation is generally not possible in the preliminary design stage, and, typically, simple formulae for the fundamental period are used to initiate the design process. These simple formulae also serve as a basis for limiting the period from a finite-element model by applying the upper-bound factor suggested in the 2003 NEHRP Recommended Provisions for Seismic Regulations for New Buildings and subsequently in ASCE 7-05 (ASCE 2005). Therefore, at present, simple formulae for estimating the fundamental period with good accuracy play an important role in structural design (Asteris *et al*. 2015, Young and Adeli 2014).

Many researchers have previously proposed such formulae for this purpose. Generally, there are two kinds of simple formulae for the fundamental period: empirical (Asteris *et al*. 2015a, Asteris *et al*. 2015b, Asteris *et al*. 2016a, Asteris *et al*. 2016b, Asteris *et al*. 2017, Kose 2009, Young and Adeli 2014, Shafei and Alirezaei 2014, Hatzigeorgiou and Kanapitsas 2013, Kwon and Kim 2010, Crowley and Pinho 2004, Balkaya and Kalkan 2003, Goel and Chopra 1997, Goel and Chopra 1998) and analytical (Hsiao 2009, Leng *et al*. 2013, UBC 1997, EC 2007, Eurocode 8 2004). A lot of empirical formulae have been developed. Asteris *et al*. (2015b, 2016a) give an extensive review of these formulae. Empirical formulae adopted in most codes are simply expressed in terms of the height of buildings (Eurocode 8 2004, UBC 1997 *et al*.). Some researchers take into account other parameters apart from the height of building. Kose (2009) takes into account the presence of infill walls and frame type. Hatzigeorgiou and Kanapitsas (2013) proposed an expression considering the soil flexibility, the influence of shear walls, and the external and internal infill wall. Asteris *et al*. (2015b, 2016a) proposed a more accurate formula that takes into account the number of stories, the number of span, the span length, the infill wall panel stiffness and the percentage of openings within the infill wall. Further, Asteris *et al*. (2017) recognized that the vertical geometric irregularity significantly influences the fundamental period, and proposed a reduction factor to quantify this effect.

Analytical formulae also have been adopted in many codes (Eurocode 8 2004, UBC 1997, EC 2007). This study focus on the analytical ones, which have generally been developed based on vibration theory for a multiple-degree-of-freedom (MDOF) system. Among these, Rayleigh's method, Geiger's method, and Dunkerley's method are the three most widely used; the first two of which were specified in the 1997 Uniform Building Code (UBC 1997), the Japanese seismic code (EC 2007), respectively.

In this paper, a new, simpler, and more accurate method for estimating the fundamental period of a MDOF system is proposed. The rest of the paper is organized as follows. Firstly, several most widely used simple formulae for estimating the fundamental period are briefly reviewed in Chapter 2. Then, in Chapter 3, the new method is described. In this method, the fundamental period is estimated by

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replacing the complicated MDOF system with an equivalent single-degree-of-freedom (SDOF) system. Then, to investigate the accuracy of the proposed method, the fundamental periods of numerous MDOF models are estimated by the proposed method in Chapter 4 and compared to their accepted values. Finally, the main results of this study are concluded in Chapter 5.

2. Review of the current methods

Many studies have contributed calculation methods for estimating the fundamental period of MDOF systems. This section reviews several most widely used methods briefly.

The first one, Rayleigh's method, is a simple theoretical technique based on energy principles, which was specified in the 1997 Uniform Building Code (UBC 1997). To introduce the basic consideration of Rayleigh's method for the fundamental period, consider a MDOF system undergoing free harmonic motion with a fundamental frequency ω_1. The displacement vector $\{x(t)\}$ and velocity vector $\{x'(t)\}$ of the MDOF system corresponding to fundamental vibration are given by

$$\{x(t)\} = \{u\}\sin(\omega_1 t + \varphi_1) \tag{1}$$

$$\{x'(t)\} = \{u\}\omega_1 \cos(\omega_1 t + \varphi_1) \tag{2}$$

where $\{u\}$ is a displacement vector representing the fundamental mode shape corresponding to fundamental vibration and φ_1 is the phase angle of the harmonic vibration.

Then, the maximum kinetic energy, KE, of the system can be expressed as

$$KE = \frac{1}{2}\{u\}^T[M]\{u\}\omega_1^2 \tag{3}$$

where $[M]$ is the mass matrix of the MDOF system, and the maximum strain energy, SE, of the system can be expressed as

$$SE = \frac{1}{2}\{u\}^T[K]\{u\} \tag{4}$$

where $[K]$ is the stiffness matrix of the MDOF system.

It is known that when the kinetic energy of the system is maximal, the strain energy will be zero; on the contrary, when the strain energy of the system is maximal, kinetic energy will be zero. Then, based on the principle of conservation of energy (i.e., total mechanical energy is constant), the KE is equal to the SE. Accordingly, the fundamental frequency ω_1 is given by

$$\omega_1^2 = \frac{\{u\}^T[K]\{u\}}{\{u\}^T[M]\{u\}} \tag{5}$$

As shown in Eq. (5), before calculating the fundamental period using Rayleigh's method, the fundamental mode shape $\{u\}$ should be determined. For simplicity, instead of using an accurate eigenvalue analysis, the fundamental mode shape is always determined based on some assumption. Thus, the accuracy of Rayleigh's method depends entirely upon the assumed fundamental mode shape. A widely used estimate for the fundamental mode shape is the static displacement resulting from subjecting the masses in the system to forces proportional to their weights. Based on this assumption, the fundamental period, T_R, is given by

$$T_R = 2\pi\sqrt{\frac{\sum_{i=1}^{n} G_i u_i^2}{g\sum_{i=1}^{n} G_i u_i}} \tag{6}$$

where $G_i = gm_i$, m_i is the mass of the ith degree of freedom, and n is the number of total degrees of freedom.

The second technique, Geiger's method, is also a widely used approximation method for estimating the fundamental period of a MDOF system. This method was specified in the Japanese seismic code (EC 2007). To introduce the basic consideration of this method, consider an SDOF system with mass m and lateral stiffness k. Then, the fundamental period T_G of the SDOF system can be given by

$$T_G = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mg}{kg}} \tag{7}$$

By defining $\delta = mg/k$ and $C = \sqrt{g}/2\pi$, T_G can be expressed as

$$T_G = \frac{\sqrt{\delta}}{C} \tag{8}$$

where δ represents the top lateral displacement resulting due to the weight of the system.

When Eq. (8) is applied to estimating the fundamental period of an MDOF system, the top displacement (in cm) is estimated by

$$\delta = \sum_{i=1}^{n}\frac{\sum_{j=i}^{n} m_j g}{k_i} \tag{9}$$

In Japanese seismic code (EC 2007), C is determined empirically according to the number of stories n and equals 5.4 when $n = 2$ and 5.7 when $n > 2$.

Eurocode 8 (2004) also uses Eq. (8) to estimate the fundamental period, but C is adopted as 5.

Note that, as with Rayleigh's method, when Eq. (8) is used to calculate the fundamental period, the top displacement should be estimated.

Another method, Dunkerley's method, is based on the flexibility of the system-eigenvalue problem and provides an "upper-bound" estimation of the fundamental period. The basic premise of this method is to reduce the actual system into a number of simple subsystems; then, the square of the fundamental period, T_D^2, equals the sum of that of each subsystem. Dunkerley's equation can be expressed as

$$T_D^2 = T_{11}^2 + T_{22}^2 + ... + T_{nn}^2 \tag{10}$$

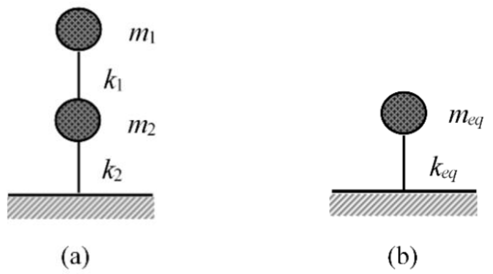


Fig. 1 Illustration of the concept of replacing a 2-DOF system with an equivalent SDOF system

where T_{ii} is the natural period of an SDOF system with mass "m_i" acting alone at state i.

Unlike the previous two methods, Dunkerley's method considers only the mass and stiffness of the analyzed MDOF system, without mode shape or top displacement. However, it has been reported that this method is not as accurate as the others (Leng *et al*. 2013).

A new, simpler, and more accurate method for estimating the fundamental period of MDOF structures is described in the following chapter.

3. The proposed method for estimating the fundamental period

A simple method for estimating the fundamental period of an MDOF system is proposed in this chapter. The basic principle is to replace a complicated MDOF system with an equivalent SDOF system for which the fundamental period can be easily obtained. To realize the SDOF-system equivalence, a procedure to replace a two-degree-of-freedom (2-DOF) system with an SDOF system having the same fundamental period, called the two-to-single (TTS) procedure, is developed firstly; then, using the TTS procedure successively, the MDOF system can be replaced with an equivalent SDOF system having approximately the same fundamental period.

3.1 A procedure to replace a 2-DOF system with an SDOF system

In order to develop the TTS procedure to reduce a 2-DOF system to an SDOF system with the same fundamental period, a 2-DOF system and an equivalent SDOF system are considered, as shown in Fig. 1. In essence, developing the TTS procedure means expressing parameters including mass, m_{eq}, and stiffness, k_{eq}, of the equivalent SDOF system in terms of the parameters of the 2-DOF system. For this purpose, the following two equivalent equations are considered

$$m_{eq} = m_1 + m_2 \tag{11}$$

$$T_{eq} = T_{2-DOF} \tag{12}$$

here, m_i, i = 1, 2, is mass of the ith degree of freedom and $T_{2\text{-}DOF}$ is the fundamental period of the 2-DOF system; T_{eq} is the fundamental period of the equivalent SDOF system. In order to determine the stiffness, k_{eq}, of the equivalent SDOF system using Eq. (12), the fundamental period, $T_{2\text{-}DOF}$, of the 2-DOF system should be derived firstly.

Consider the 2-DOF system in free harmonic vibration. The basic eigen problem for this system is represented as

$$(\omega_i^2[M]-[K])\{u\}=0 \tag{13}$$

where ω_i, i = 1, 2, are the free-vibration frequencies, [M] and [K] are the mass and stiffness matrices of the 2-DOF system, respectively, and are expressed as

$$[M]=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \qquad [K]=\begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1+k_2 \end{bmatrix}$$

and k_i, i = 1, 2, is the stiffness of the ith degree of freedom.

By eigenvalue analysis, the fundamental frequency ω_1 can be given by

$$\omega_1^2 = \frac{1}{2}\left[\frac{k_1+k_2}{m_2}+\frac{k_1}{m_1}-\sqrt{\left(\frac{k_1+k_2}{m_2}-\frac{k_1}{m_1}\right)^2+4\frac{k_1^2}{m_1 m_2}}\right] \tag{14}$$

As $T_{2\text{-}DOF} = 2\pi/\omega_1$, according to Eq. (12), the stiffness, k_{eq}, of the SDOF system is given by

$$k_{eq} = \frac{(m_1+m_2)}{2}\left[\frac{k_1+k_2}{m_2}+\frac{k_1}{m_1}-\sqrt{\left(\frac{k_1+k_2}{m_2}-\frac{k_1}{m_1}\right)^2+4\frac{k_1^2}{m_1 m_2}}\right] \tag{15}$$

Using Eq. (11) and (15), an equivalent SDOF system having the same fundamental period as the 2-DOF system can be obtained.

3.2 A procedure for estimating the fundamental period of an MDOF system

Successively using the procedure for replacing a 2-DOF system with an equivalent SDOF system as described above, a procedure for finding the fundamental period of an MDOF system can be developed. The concept of this procedure is illustrated in Fig. 2. And, the procedure includes following steps:

1. For the MDOF system shown in Fig. 2(a), the top two masses m_1 and m_2 are assumed to lie on rigid ground and can be considered as a 2-DOF system. Then, based on the TTS procedure(i.e., Eq. (11) and (15)), an equivalent SDOF system having the same fundamental period as the top 2-DOF system can be obtained, forming a new MDOF system as shown in Fig. 2(b).

2. Then, as in step (1), the top two masses of the new MDOF system as shown in Fig. 2(b) are considered as a new 2-DOF system lying on rigid ground and can be replaced with another equivalent SDOF system using Eq. (11) and (15) again, forming another new MDOF system, as shown in Fig. 2(c).

3. By application of the TTS procedure successively to the remaining lower masses, finally, the MDOF system is replaced with an equivalent SDOF system, as shown in Fig. 2(d). Then, the fundamental period can be readily obtained.

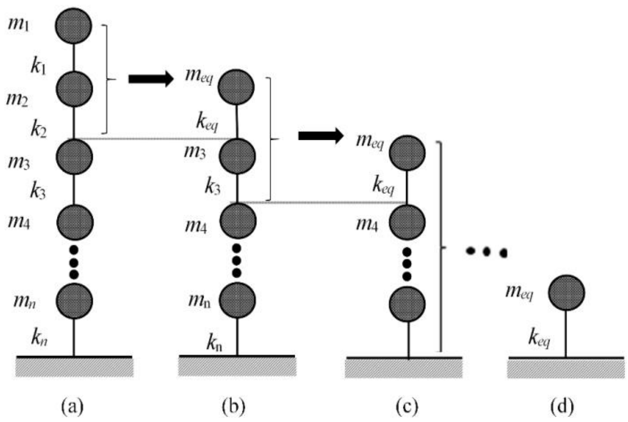


Fig. 2 Illustration of the concept of replacing an MDOF system with an equivalent SDOF system

3.3 Validation of the rigid-ground assumption

In the procedure for replacing an MDOF system with an equivalent SDOF system described in the previous section, at each step of replacement, the top two masses are always considered as a 2-DOF system lying on rigid ground. However, except at the final step, the 2-DOF system lies on a floor with limited stiffness. In order to validate the rigid-ground assumption, the fundamental periods of a large number of MDOF structures are computed using the procedure described in Section 3.2 and compared with those obtained using an eigenvalue analysis.

The analyzed MDOF structures are divided into two major categories: MDOF structures with floor stiffness varying with height and those with only one special floor with different stiffness from the others. As the mass of the actual structure generally varies less significantly as a function of height than does stiffness, the mass, m_0, of the analyzed structures is considered constant.

In the first category, the variation of stiffness with height is expressed as

$$k_i = r^{i-1}k_0 \tag{16}$$

where k_i is the stiffness of the ith mass point, as shown in Fig. 2(a), k_0 is a constant value, and factor r represents the variation degree of stiffness along height. Eq. (16) means that, the stiffness of the top story equals k_0, and stiffness of any lower ith story is r times as large as that of the upper i-1th story. Generally, as the stiffness of the actual structure increases from the top to the bottom, factor r is considered to vary from 1 to 1.5.

In the second category, the stiffness of only a special floor, k_i, is considered variable, and the others are constant and equal to k_0. The variation of the stiffness of this special floor is expressed as

$$k_i = rk_0 \tag{17}$$

Eq. (17) means that, stiffness of the special story is r times as large as that of other stories equaling k_0. In this case, factor r is considered to vary from 0.5 to 1.5, and i varies from 1 to n, where n the number of stories.

It can be easily shown that, in these designed MDOF structures, the parameters controlling the fundamental period are the factor r, the ratio between stiffness and mass, k_0/m_0, and the number of stories n. Thus, the error in the estimated fundamental period caused by the rigid-ground assumption is also considered to be affected by these three parameters. The variation ranges of the parameter r have been introduced above, for the parameter k_0/m_0, two values, 10,000 (kN/cm)/6 (t) and 10,000 (kN/cm)/60 (t), are considered in the following calculation. The value, 10,000 (kN/cm)/6 (t), is determined according to an actual structure constructed in Japan (Tatsuya *et al.* 2015). To observe the possible effect of the parameter k_0/m_0 on the error clearly, another extreme value, 10,000 (kN/cm)/60 (t), is assumed. The extreme range assumed for the parameter k_0/m_0 is to observe the possible effect clearly instead of representing actual condition. And number of stories n is considered to vary from 3 to 10.

The fundamental periods of these MDOF structures are calculated using the procedure described in Section 3.2 and compared against those obtained using an eigenvalue analysis. The errors are expressed by ratios of the fundamental periods calculated by the procedure in Section 3.2, T_p, with those by an eigenvalue analysis, T_e. Fig. 3(a) shows the results of the first category of MDOF structures. For the second category of MDOF structures, results are very similar regardless of the value of i expressed in Eq. (17); for simplicity, only representative results when $i = n$ are shown in Fig. 3(b). In these figures, the horizontal coordinate is the factor r, representing the variation degree of stiffness, and the longitudinal coordinate represents the error.

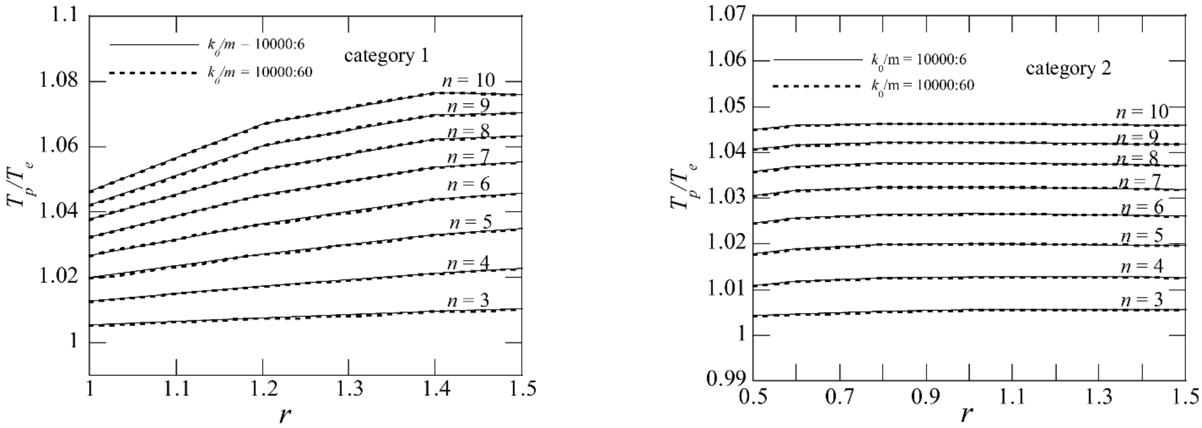
It is observed that, for both subcategories in which there is error in the estimated fundamental period, the maximum relative error is less than 8%. The errors are dependent on the factor r and the number of stories n, but not on the ratio k_0/m_0. The errors increase with increasing r for the first category but do not change noticeably for the second category. For both subcategories, the errors increase with n. Comparing the effects of n and r on the errors, that of n is clearly more prominent.

The reason for the dependence on the number of stories is that, when replacing an MDOF system with an equivalent SDOF system, the top 2-DOF system at each step is assumed to lie on rigid ground, when in fact it lies on a floor with limited stiffness; thus, the more stories the analyzed MDOF system has, the more the assumptions used, resulting in a larger error.

Generally speaking, the rigid-bedrock assumption used in the procedure described in Section 3.2 can cause a calculation error in the fundamental period, but the maximum relative error of the analyzed MDOF structures is below about 8%. The errors are affected by the number of stories n and the variation degree of the stiffness with height, although the former effect is more significant.

3.4 Correction factor

Based on the analysis in the previous subsection, the prediction of the fundamental period using the procedure described in Section 3.2 is improved with the appropriate introduction of a correction factor.



(a) Results for structures in the first category (b) Results for structures in the second category

Fig. 3 Comparison of the fundamental periods obtained by the procedure described in Section 3.2 with those obtained by eigenvalue analysis

The fact that the error in the fundamental period obtained using the procedure in Section 3.2 is affected by the number of stories and the variation degree of the stiffness along height leads us to conclude that the correction factor should be expressed in terms of the number of stories, n, and a factor representing the variation degree of stiffness. However, since the variation degree of the stiffness of an actual building cannot be expressed as a single factor like the idealized one, r, used previously, and since an increase in the number of stories affects the error more significantly than variation of the stiffness, the correction factor is expressed only in terms of n.

To isolate the effects of variations of stiffness and mass, MDOF structures with constant mass and stiffness with height are used to conduct the correction. MDOF structures composed of 3-20 stories are used for the correction. Then, a correction factor R is introduced, defined as the ratio between the fundamental periods obtained by an eigenvalue analysis and by the procedure described in Section 3.2. To determine the correction factor R, the fundamental ratios of the exact and predicted periods of all analyzed MDOF structures are computed, and the results are shown in Fig. 4. By trial-and-error analysis of a large number of functional forms, a very simple function is adopted for the correction factor R, given by

$$R = (0.4n)^{-1/30} \tag{18}$$

The accuracy of this function can also be found very well from Fig. 4. The standard deviation of residuals expressing the random variability of results by Eq. (18) is almost equal to 0.001.

Finally, considering the correction factor, the fundamental period of an MDOF structure can be estimated as

$$T_{\mathrm{Pr}} = 2\pi R\sqrt{\frac{m_{eq}}{k_{eq}}} \tag{19}$$

where m_{eq} and k_{eq} are the mass and stiffness, respectively, of the final equivalent SDOF system obtained by the procedure in Section 3.2.

The proposed method is composed of three equations (i.e., Eqs. (11), (15), and (18)), of which the second equation seems more complicated than the current methods introduced in Section 2 at first glance. In Rayleigh's method, the mode shape should be determined first; and, in Geiger's method, the top displacement should be estimated. As Eq. (15) is expressed in terms of only mass and stiffness without any other additional parameters, the proposed method is considered simpler and more direct than any presented in Section 2.

It should be noted that, the proposed method is developed for estimation of the fundamental period of the widely used MDOF structural model. This means that, for an actual structure, it must be simplified as an MDOF model before applying the proposed method. During the simplification, besides the structural elements, the infill walls also should be properly considered in the model, since contribution of the infill walls to the fundamental period may be also crucial (Asteris *et al.* 2015b, Asteris *et al.* 2016a).

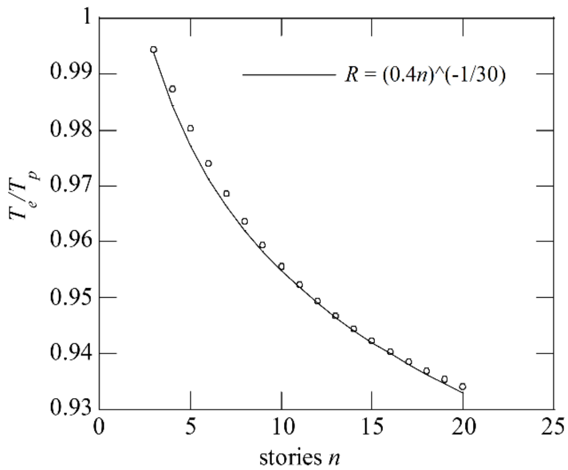
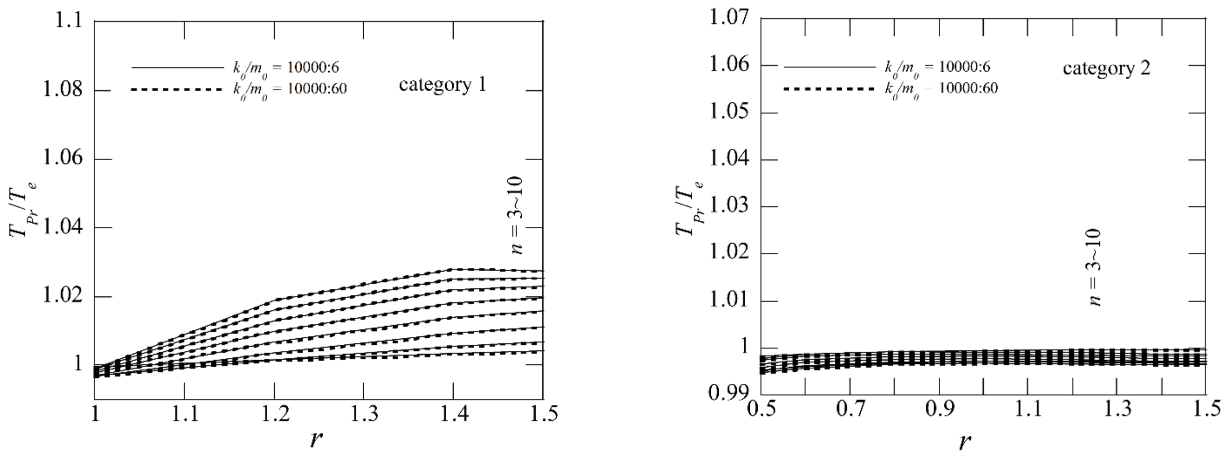


Fig. 4 Ratios between the fundamental periods obtained by eigenvalue analysis and by the method described in Section 3.2



(a) Results for structures in the first category (b) Results for structures in the second category

Fig. 5 Comparison between fundamental periods obtained by the proposed method and by eigenvalue analysis

Table 1 Average relative error in the estimation results

--	Fig. 6(a) (%)	Fig. 6(b) (%)	Fig. 6(c) (%)	Fig. 6(d) (%)
Proposed method	0.617	1.542	0.616	0.624
UBC 1997	0.643	4.298	0.623	0.655
Japanese code	3.032	3.890	3.084	3.000
Dunkerley's method	10.935	18.615	10.876	10.967
Eurocode 8	10.543	18.196	10.484	10.575

Table 2 Parameters of the analyzed actual structures

Model No.	Structure Location	Structure Stories	Direction
01	Wakayama-Ken of Japan	3	X
02			Y
03	Tochigi-ken of Japan	4	X
04			Y
05	Ibaraki-ken of Japan	5	X
06			Y
07	Ibaraki-ken of Japan	7	X
08			Y
09	Tokyo of Japan	23	X
10			Y
11	Tokyo of Japan	36	X
12			Y
13	--	5	X
14			Y
15	--	5	--
16	Tokyo of Japan	3	X
17			Y
18	--	8	NS
19			EW

In addition, as the proposed method considers variations of mass and stiffness with height, thus the method is available for structures with vertical irregularity. For structures with plan irregularities, torsion may be caused to the building, thus torsional stiffness should be considered in the model of the structure. However, during the derivation of the proposed method, only lateral stiffness is considered. Thus, the proposed method is only available for the shear-type MDOF system. Improving the proposed method to analyze structures with plan irregularities is necessary in the further study.

4. Examples using the proposed method

4.1 Designed MDOF structures

In order to investigate the accuracy of the proposed method, a recalculation of the fundamental periods of the two categories of MDOF structures introduced in Section 3.3 is performed, and fundamental period ratios between the predicted periods, T_{pr}, and the exact ones are shown in Fig. 5. It is observed that errors are very low for both categories, with the maximum relative error below 3%. Although the error increases with the number of stories for the first category of MDOF structures, the error level (3%) is considered acceptable for engineering use.

In addition, in order to compare the accuracy of the proposed method with those of the methods introduced in Chapter 2, the fundamental periods of the two categories of MDOF structures are also estimated by the current methods. MDOF structures with as many as 60 stories are considered for comparison. Representative results are shown in Figs. 6 (a)-(d). In these figures, the horizontal coordinate is n and the longitudinal coordinates are the fundamental periods calculated by different methods.

It can be noted that all results obtained by the proposed method are much more accurate than those obtained by Dunkerley's method, the Eurocode 8 method and Geiger's method adopted in Japanese code. Indeed, the accuracy of the proposed method is nearly equivalent to that of Rayleigh's method adopted in UBC 1997. Further comparisons of the average relative errors of the results estimated by different methods are conducted. The

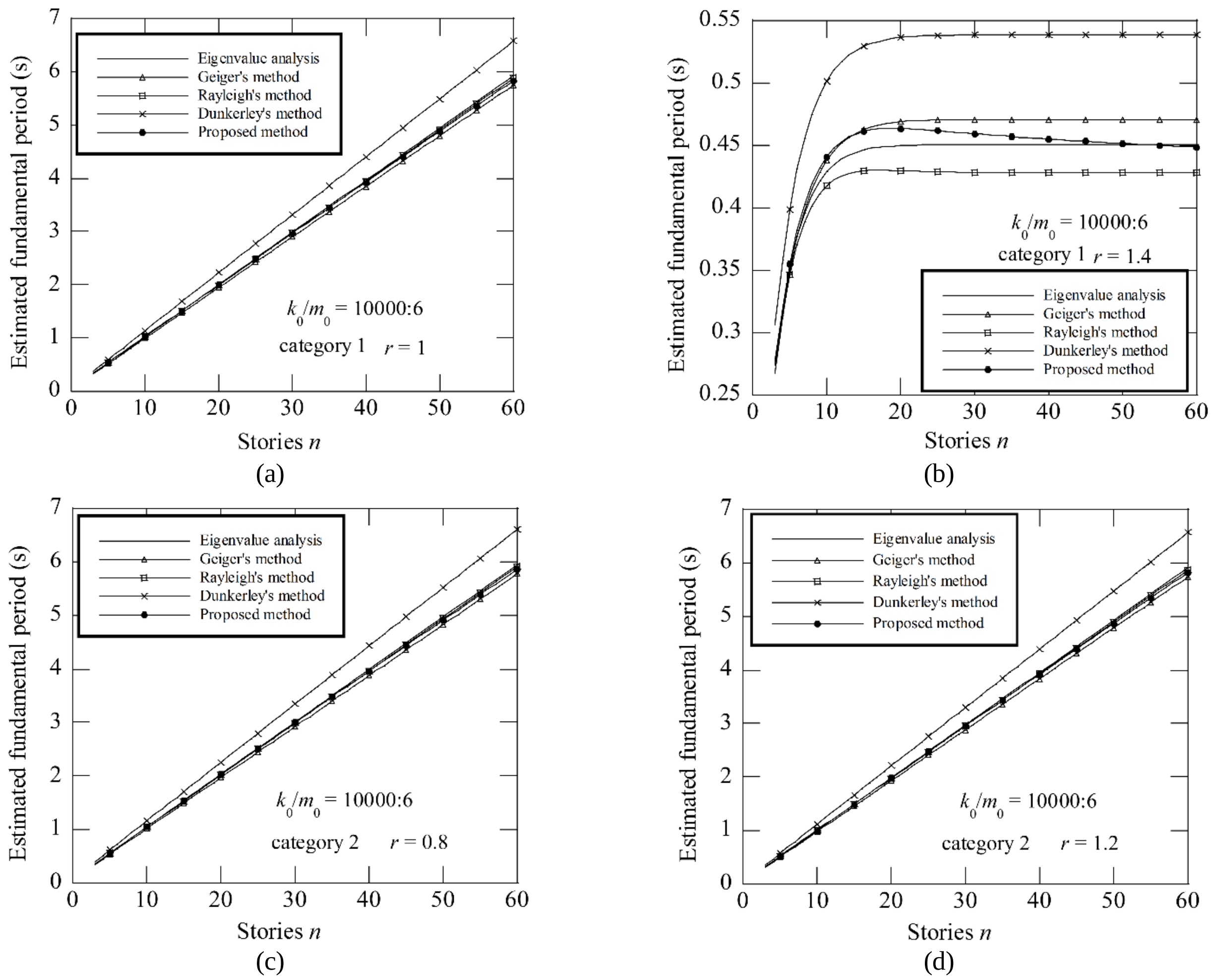


Fig. 6 Fundamental periods calculated by different methods. (a) Results for structures in the first category when r = 1. (b) Results for structures in the first category when r = 1.4. (c) Result for structures in the second category when r = 0.8. (d) Results for structures in the second category when r = 1.2

corresponding results of those MDOF structures used in Fig. 6 are listed in Table 1. The average relative errors by the proposed method are smaller than those of the current methods.

Generally speaking, the accuracy of the proposed method is very good and is much better than that of Dunkerley's method and the Eurocode 8 method. For most of the estimated structures, the accuracy of the proposed method is better than those of Rayleigh's method adopted in UBC 1997 and Geiger's method adopted in Japanese code.

4.2 MDOF models of actual structures

Further accuracy investigations are performed by estimating the fundamental periods of 19 MDOF models of actual structures. Parameters of these MDOF models are listed in Table 2. A wide range of structures with 3-36 stories are considered. Most of these structures are in Japan, and others are found in unknown locations. The fundamental periods of the 19 MDOF models are calculated by the proposed method and the current methods; the obtained results are listed in Table 3, and the corresponding relative errors are also estimated and listed in brackets. It

Table 3 Fundamental periods and corresponding relative errors of the analyzed MDOF models calculated by different methods

Model No.	Theoretical method (s)	Dunkerley's method (s)	UBC 1997 (s)	Japanese code (s)	Proposed method(s)	Eurocode 8 (s)
01	0.213	0.258 (20.069)	0.211 (0.850)	0.205 (3.417)	0.213 (0.236)	0.234 (10.105)
02	0.122	0.148 (20.059)	0.121 (0.833)	0.118 (3.326)	0.122 (0.323)	0.134 (10.209)
03	0.313	0.406 (27.236)	0.307 (1.850)	0.314 (0.295)	0.314 (0.420)	0.358 (14.336)
04	0.392	0.527 (31.768)	0.383 (2.322)	0.394 (0.458)	0.394 (0.448)	0.449 (14.522)
05	0.139	0.207 (43.524)	0.137 (1.603)	0.140 (0.578)	0.141 (1.337)	0.159 (14.659)
06	0.139	0.207 (43.524)	0.137 (1.603)	0.140 (0.578)	0.141 (1.337)	0.159 (14.659)
07	0.713	1.061 (42.435)	0.704 (1.239)	0.704 (1.314)	0.720 (0.911)	0.802 (12.501)
08	0.696	1.038 (41.781)	0.686 (1.422)	0.692 (0.653)	0.707 (1.591)	0.789 (13.256)
09	1.026	1.673 (48.196)	1.012 (1.362)	1.043 (1.682)	1.049 (2.242)	1.189 (15.917)
10	1.047	1.811 (56.246)	1.032 (1.385)	1.074 (2.558)	1.076 (2.828)	1.224 (12.916)
11	2.084	3.081 (33.481)	2.051 (1.557)	2.089 (0.266)	2.112 (1.368)	2.382 (14.303)
12	2.170	3.180 (32.368)	2.138 (1.508)	2.172 (0.067)	2.198 (1.283)	2.467 (14.076)
13	0.150	0.187 (21.487)	0.149 (0.892)	0.147 (1.992)	0.150 (0.044)	0.168 (11.730)
14	0.139	0.206 (43.525)	0.137 (1.604)	0.140 (0.583)	0.141 (1.338)	0.159 (14.665)

Table 3 Continued

Model No.	Theoretical method (s)	Dunkerley's method (s)	UBC 1997 (s)	Japanese code (s)	Proposed method(s)	Eurocode 8 (s)
15	0.691	0.816 (15.463)	0.686 (0.754)	0.669 (3.126)	0.691 (0.013)	0.763 (10.437)
16	5.491	8.094 (43.031)	5.346 (2.638)	5.520 (0.531)	5.625 (2.441)	6.897 (25.625)
17	5.933	8.881 (45.817)	5.760 (2.919)	5.954 (0.352)	6.053 (2.032)	7.469 (25.900)
18	0.342	0.403 (13.273)	0.339 (0.904)	0.335 (2.023)	0.343 (0.287)	0.382 (11.694)
19	0.355	0.429 (15.822)	0.352 (0.973)	0.349 (1.818)	0.356 (0.391)	0.397 (11.927)
Error (Avg.)	--	33.637	1.485	1.348	1.098	14.602
Error (Max.)	--	56.246	2.919	3.417	2.828	25.900

can be seen that the average relative error by the proposed method equals 1.098% and that the maximum relative errorequals 2.828%. For 79% of the estimated MDOF models, relative error is less than 2%. This accuracy is encouraging, and results by the proposed method are considered to agree very well with those obtained by eigenvalue analysis.

Comparing the results of the various methods, it can be seen that, for all estimated models, the relative error of the proposed method is much lower than that of Dunkerley's method and the Eurocode 8 method; for 84% of the estimated models, the proposed method also obtains a smaller relative error than Rayleigh's method adopted in UBC 1997. In addition, the average and maximum relative errors by the proposed method are the lowest, meaning the accuracy of this method is the highest.

Generally speaking, the accuracy of the proposed method is reasonably good, with a maximum relative error below 2.828%. The accuracy is much better than that of Dunkerley's method and the Eurocode 8 method, and is better than that of Geiger's method adopted in Japanese code or Rayleigh's method adopted in UBC 1997 for most of the structures considered.

5. Conclusions

On the basis of the preceding discussion, one can draw the following conclusions:

• A simple method of evaluating the fundamental period by replacing the complicated MDOF system with an equivalent SDOF system is proposed. The proposed method is available for shear-type MDOF system. As the proposed method is composed of three simple explicit formulae, it can be conveniently implemented in simple spreadsheets. In addition, the application of the proposed method does not require expert knowledge concerning eigenvalue analysis; thus, the proposed method is thought can be used by practicing engineers conveniently. Moreover, as simple formulae are expressed in terms of the mass, stiffness, and number of stories directly without the mode shape or top displacement, the proposed method is a simpler and a more direct method.

• The accuracy of the proposed method is investigated by estimating a series of designed MDOF structures and19 MDOF models of actual structures, and is found to be reasonably good. The accuracy of the proposed method is much better than that of Dunkerley's method and the Eurocode 8 method, and is better than that of Rayleigh's method adopted in UBC 1997 and Geiger's method adopted in Japanese code for most of the analyzed structures.

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References

ASCE (2005), *Minimum Design Loads for Buildings and Other Structures*, American Society of Civil Engineers, U.S.A.

Asteris, P.G., Repapis, C., Cavaleri, L., Sarhosis, V. and Athanasopoulou, A. (2015a), "On the fundamental period of infilled RC frame buildings", *Struct. Eng. Mech.*, **54**(6), 1175-1200.

Asteris, P.G., Repapis, C.C., Tsaris, A.K., Di Trapani, F. and Cavaleri, L. (2015b), "Parameters affecting the fundamental period of infilled RC frame structures", *Earthq. Struct.*, **9**(5), 999-1028.

Asteris, P.G., Repapis, C.C., Repapi, E.V. and Cavaleri, L. (2016a), "Fundamental period of infilled reinforced concrete frame structures", *Struct. Infrastruct. Eng.*, **13**(7), 929-941.

Asteris, P.G., Tsaris, A.K., Cavaleri, L., Repapis, C.C., Papalou, A., Di Trapani, F. and Karypidis, D.F. (2016b), "Prediction of the fundamental period of infilled RC frame structures using artificial neural networks", *Comput. Intell. Neurosci.*, 5104907.

Asteris, P.G., Repapis, C.C., Foskolos, F., Fotos, A. and Tsaris, A.K. (2017), "Fundamental period of infilled RC frame structures with vertical irregularity", *Struct. Eng. Mech.*, **61**(5), 663-674.

Balkaya, C. and Kalkan, E. (2003), "Estimation of fundamental periods of shear-wall dominant building structures", *Earthq. Eng. Struct. Dyn.*, **32**(7), 985-998.

Crowley, H. and Pinho, R. (2004), "Period-height relationship for existing European reinforced concrete buildings", *J. Earthq. Eng.*, **8**(Spec01), 93-119.

European Committee for Standardization CEN (2004), *Eurocode 8: Design of Structures for Earthquake Resistance-Part 1: General Rules, Seismic Actions and Rules for Buildings*, European Standard EN 1998-1:2004.

Editorial Committee of Structure-Related Technical Standard Commentary Book of the Building (EC) (2007), *Structure-Related Technical Standard Commentary Book of the Building*, Tokyo, Japan.

Goel, R.K. and Chopra, A.K. (1997), "Period formulas for moment-resisting frame buildings", *J. Struct. Eng.*, **123**(11), 1454-1461.

Goel, R.K. and Chopra, A.K. (1998), "Period formulas for concrete shear wall buildings", *J. Struct. Eng.*, **124**(4), 426-433.

Hatzigeorgiou, G.D. and Kanapitsas, G. (2013), "Evaluation of fundamental period of low-rise and mid-rise reinforced concrete buildings", *Earthq. Eng. Struct. Dyn.*, **42**(11), 1599-1616.

Hsiao, J.K. (2009), "Computation of fundamental periods for moment frames using a hand-calculated approach", *Electr. J. Struct. Eng.*, **9**, 16-28.

Kwon, O.S. and Kim, E.S. (2010), "Evaluation of building period formulas for seismic design", *Earthq. Eng. Struct. Dyn.*, **39**(14), 1569-1583.

Leng, B., Yan, X. and Lin, K. (2013), "Analysis on natural frequency calculation formula of vibration system", *Shanxi*

Architecture, **39**(21), 20-22.
Kose, M.M. (2009), "Parameters affecting the fundamental period of RC buildings with infill walls", *Eng. Struct.*, **31**(1), 93-102.
Shafei, A. and Alirezaei, M. (2014), "Evaluation of the fundamental period of vibration of irregular steel structures", *J. Eng. Sci. Res. Technol.*, **3**(4), 6083-6090.
Shibata, A. (2010), *Dynamic Analysis of Earthquake Resistant Structures*, Tohoku University, Sendaishi, Miyagi, Japan.
Tatsuya, A., Namihiko, I., Masanori, I., Toshihidel, K., Shin, K., Hiroto, N., Takehiko, T. and Koichi, M. (2015), *Study on Dynamic Soil-Building Structure Interaction Based on Strong Motion Observation*, National Institute for Land and Infrastructure Management, 866.
International Conference of Building Officials (1997), *1997 Uniform Building Code*, Wilier, California, U.S.A.
Young, K. and Adeli, H. (2014), "Fundamental period of irregular moment-resisting steel frame structures", *Struct. Des. Tall Spec. Build.*, **23**(15), 1141-1157.

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