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Identifying stiffness irregularity in buildings using fundamental lateral mode shape

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Abstract. Soft or extreme soft storeys in multi-storied buildings cause localized damage (and even collapse) during strong earthquake shaking. The presence of such soft or extremely soft storey is identified through provisions of vertical stiffness irregularity in seismic design codes. Identification of the irregularity in a building requires estimation of lateral translational stiffness of each storey. Estimation of lateral translational stiffness can be an arduous task. A simple procedure is presented to estimate storey stiffness using only properties of fundamental lateral translational mode of oscillation (namely natural period and associated mode shape), which are readily available to designers at the end of analysis stage. In addition, simplified analytical expressions are provided towards identifying stiffness irregularity. Results of linear elastic time-history analyses indicate that the proposed procedure captures the irregularity in storey stiffness in both low- and mid-rise buildings.

Keywords: modal analysis; mass participation; open storey; soft storey; storey stiffness

1. Introduction

Vertical stiffness irregularity along the height of multi-storied buildings results from either the choice of building configuration or the distribution of seismic mass (Fig. 1) (Harmankaya and Soyluk 2012, Tabeshpour *et al*. 2012, Saiful *et al*. 2014, Varughese *et al*. 2015). Detrimental effects of stiffness irregularity were identified early, during 1925 Santa Barbara earthquake. To address this issue, a *qualitative* provision was introduced, using configuration of building, in the commentary of the 1975 SEAOC Blue Book (SEAOC 1975). In this, assessment of stiffness irregularity in buildings was left solely to Engineers (FEMA 454 2006). If the building was deemed irregular, 1975 SEAOC Blue Book required the use of *modal analysis* to estimate the magnitude and distribution of design lateral force. *Quantitative* definitions to evaluate presence or absence of *soft storey* and *extreme soft storey* in buildings were later introduced in the 1988 SEAOC Blue Book, to reduce uncertainty associated with the qualitative assessment of stiffness irregularity in buildings (SEAOC 1988, Ambrose and Vergun 1995, Arnold 2001). Till date, seismic design codes (ASCE 7 2010, IBC 2012, IS 1893 (Part 1) 2002, NZ 1170 (Part 5) 2004) use these quantitative definitions to identify stiffness irregularity in buildings. Further, some design codes prohibit buildings with extreme soft storey from being built in high seismic regions (e.g., buildings of category E and F, as in ASCE 7 2010) and, allow buildings with soft storey to be built, provided (a) magnitude and distribution of design lateral force is estimated using modal response spectrum analysis, and (b) building is designed for an increased lateral force. In this regard, FEMA 454 summarizes aptly that "*Code provisions treat the symptoms of irregularity, rather than the cause. The irregularity is still allowed to exist; the hope is that the penalties (requirement of modal analysis and increased lateral force) will be sufficient to cause the designer to eliminate the irregularities. Increasing the design force or improving the analysis to provide better information does not, in itself, solve the problem. The problem must be solved by design*". Hence, it is recommended that stiffness irregularities in buildings are identified during structural analysis stage itself (as part of the design process) and eliminated through proper proportioning of lateral load resisting system.

A building is said to have a *soft storey*, if lateral translational stiffness (K_i) of any storey is (a) less than 70% of that (K_{i+1}) of the storey immediately above it; i.e.

$$\left[\frac{K_i}{K_{i+1}}\right] \le 0.7; or \tag{1}$$

(b) less than 80% of the average of those (K_{i+1}, K_{i+2}, and K_{i+3}) of 3 stories immediately above it, i.e.

$$\left[\frac{3K_i}{K_{i+1}+K_{i+2}+K_{i+3}}\right] \le 0.8 \tag{2}$$

Similarly, a building is said to have an *extreme soft storey*, if

$$\left[\frac{K_i}{K_{i+1}}\right] \le 0.6; or \tag{3}$$

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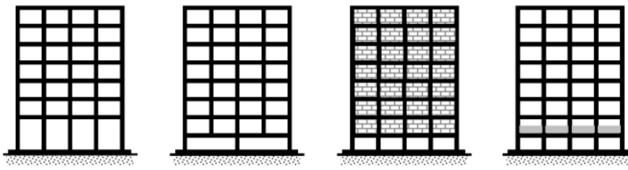


Fig. 1 Building configuration and distribution of seismic mass that may result in vertical stiffness irregularity

$$\left[\frac{3K_i}{K_{i+1}+K_{i+2}+K_{i+3}}\right] \leq 0.7 \quad (4)$$

Assessing presence of soft or extreme soft storey using Eqs. (1) to (4) require estimate of lateral translational stiffness of all storeys in multi-storey building.

2. Storey stiffness

Storey stiffnesses estimated assuming beams to be flexurally rigid and columns to be axially rigid are seldom close to reality. This is because estimate of storey stiffness depends on flexural and axial stiffnesses of *all* columns and beams present in the building, especially those adjoining the storey in focus. Several methods are available to estimate lateral translational stiffness of each storey including flexibilities of beam bounding the storey. In one such method, multi-storey buildings are idealized as a coupled shear-flexure beams (Akkar *et al.* 2005, Tagawa *et al.* 2004) (Fig. 2). In this idealization, *lateral translational stiffness of the shear beam* is estimated as the summation of flexural stiffness of columns in the considered direction of motion (assuming the beams to be flexurally rigid), and *lateral translational stiffness of the flexure beam* is estimated as a factor ρ times the lateral translational stiffness of the shear beam; ρ is assumed to be either constant or varying along the height of the building. The estimate of ρ is determined by comparing the fundamental mode shape of the building or by comparing the distribution of drift along the building height when subjected to a set of ground motions. Thus, lateral translational stiffness of each storey is obtained after static condensation of stiffnessess of coupled shear-flexure beam. This procedure of quantifying ρ is difficult and time consuming.

Alternatively, several closed form expressions are available to estimate lateral translational stiffness of each storey of multi-storey buildings (Muto 1974, Schultz 1992, Hosseini and Imagh-e-Naiini 1999, Caterino *et al.* 2013). But, these expressions have several limitations, including their applicability to buildings with moment resisting frame as sole lateral load resisting system (LLRS) (with the exception of Hosseini and Imagh-e-Naiini, 1999), inability to account for shear deformations, and use of empirical correction factors which may not be applicable for all multi-storey buildings. Further, the use of these expressions poses a challenge, wherein lateral translational stiffness needs to be estimated of each storey in multi-storey buildings with complex geometry. Thus, estimation of storey stiffness using closed form expressions is laborious.

Alternatively, other methods available in literature use results of structural analyses, to estimate storey stiffness in multi-storey buildings. The advantages of such methods include applicability to buildings with all types of LLRS, inherently accounting for shear deformation in members, and not using any empirical correction factor. *Single Storey* method estimates lateral translational stiffness of each storey of multi-storey buildings, as the lateral force required to produce unit relative translational deformation in that storey, with the bottom of the storey restrained from moving laterally (Murty *et al.* 2012). *Unit Load* method estimates lateral translational stiffness of each storey using equivalent stiffness of considered storey with a unit load applied at the top of that storey, with the rest of the building not having any deformational restraint (Tabeshpour and Noorifard 2016). In both these methods, n number of additional analyses is required to evaluate lateral translational stiffness of all stories of an n-storey building. Therefore, these procedures are cumbersome and time consuming; the requirement of n analyses is the main deterrent for engineers to use these methods.

Lateral Force-Deformation method estimates lateral translational stiffness of each storey of multi-storey buildings, as the ratio of storey shear and corresponding inter-storey lateral displacement of each storey when multi-storey building is subjected to design lateral forces (Vijayanarayanan *et al.* 2015; Tabeshpour and Noorifard, 2016; Caterino *et al.* 2013). Although this method presents a simple way to estimate storey stiffness, estimate of the same depends on the distribution of design lateral force profile along the height of the building. This dependence on the distribution of lateral force is of concern as different codes recommends different lateral force distribution along the height of the building; for example, Indian seismic design code recommends a parabolic distribution of lateral force, whereas the American seismic design code recommends a triangular distribution of the lateral force in addition to a portion of lateral load applied as point load at the roof of the building. Thus, this method *does not provide unique numerical value* of stiffness of a particular storey.

Number of buildings built with stiffness irregularity has increased significantly in many countries (Fig. 3). This increase is in spite of the fact that design codes have quantitative checks to identify the presence of stiffness irregularity along the height of multi-storey buildings (Eqs.(1) to (4)). A possible explanation for this may be attributed to the want of a clear quantitative method to estimate lateral translational stiffness of a storey. Thus, there is a need for a simple procedure to identify stiffness irregularity in multi-storey buildings to help designers identify presence of soft or extreme soft storeys at the preliminary analysis and design stage.

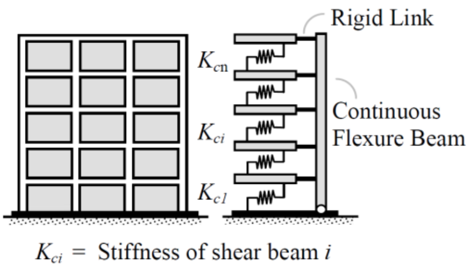


Fig. 2 Frame idealized as coupled shear-flexure beam

Fig. 3 Typical skyline of cities showing reinforced concrete (RC) frame buildings with extreme stiffness irregularity due to open storey at ground level

3. Proposed procedure

When modal analysis of a (3-dimensional or 2-dimensional) numerical model of a building is performed (as part of the analysis and design process), fundamental natural period and associated (lateral translational) mode shape of the building (model) are obtained. This readily available information of fundamental lateral translational mode shape is used in this proposed procedure to assess stiffness irregularity along height of buildings. For the purpose, a shear beam mathematical model of the building (Fig. 4) is conceived that has, (a) same number of stories and same storey heights as the building whose lateral translational stiffness of each storey is being estimated; (b) lumped seismic mass same as the seismic mass present at each storey level of the building; (c) only one translational degree of freedom in the direction along which stiffness irregularity is being estimated; and (d) fundamental lateral translational natural period and associated fundamental lateral translational mode shape same as that of the numerical model of the building.

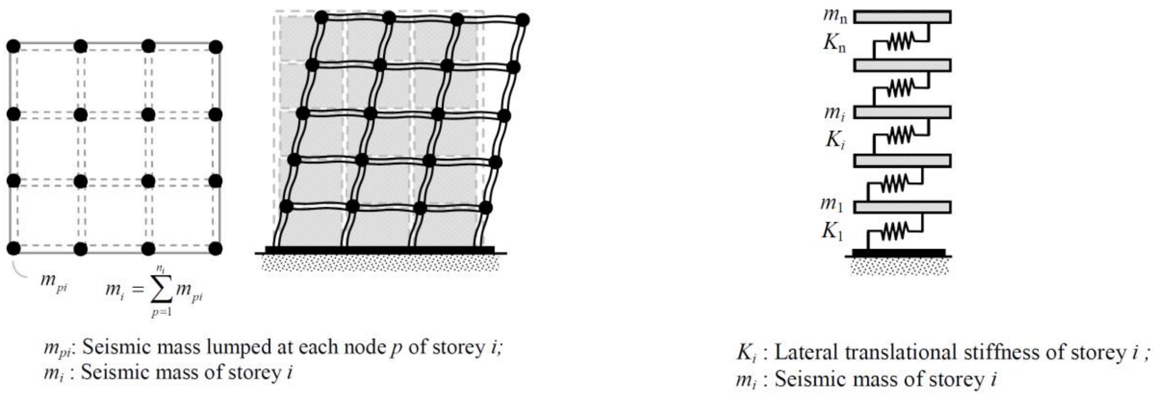
Thus, the characteristic equation of the mathematical model is

$$\left[[K]-\omega^2[M]\right]\{\varphi\}=\{0\} \tag{5}$$

where $[M]$ is the lumped mass matrix of the mathematical model given by Eq. (6), $\{\varphi\}$ and ω are the fundamental lateral translational mode shape and fundamental lateral translational circular frequency, respectively, both obtained from the modal analysis of the numerical model of the building. The fundamental lateral mode shape and fundamental lateral translational frequency of the building obtained inherently account for the flexibility of the beams and columns present in the building. Since both the building and the conceived shear beam model are assumed to have the same fundamental lateral mode shape and fundamental lateral translational frequency, the combined effect of flexibilities of the beams and columns present in the building is implicitly accounted in the lateral stiffness matrix $[K]$ of the mathematical model (Eq. (5)). Thus, in Eq. (5), only the lateral translational stiffness matrix $[K]$ is unknown, represented by Eq. (7)

$$[M]=\begin{pmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{pmatrix} \tag{6}$$

$$[K]=\begin{pmatrix} K_1+K_2 & -K_2 & & & 0 \\ -K_2 & K_2+K_3 & -K_3 & & \\ & -K_3 & \ddots & \ddots & \\ & & \ddots & K_{n-1}+K_n & -K_n \\ 0 & & & -K_n & K_n \end{pmatrix} \tag{7}$$



(a) Plan and Elevation of Numerical Model (b) Mathematical Model

Fig. 4 Numerical and mathematical model

$$\begin{pmatrix} K_1+K_2 & -K_2 & & & 0 \\ -K_2 & K_2+K_3 & -K_3 & & \\ & -K_3 & \ddots & \ddots & \\ & & \ddots & K_{n-1}+K_n & -K_n \\ 0 & & & -K_n & K_n \end{pmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \dots \\ \phi_{n-1} \\ \phi_n \end{Bmatrix} = \begin{Bmatrix} m_1\phi_1\omega^2 \\ m_2\phi_2\omega^2 \\ \dots \\ m_{n-1}\phi_{n-1}\omega^2 \\ m_n\phi_n\omega^2 \end{Bmatrix} \tag{8}$$

$$\begin{aligned} (K_1+K_2)\phi_1 - K_2\phi_2 &= m_1\phi_1\omega^2 \\ -K_2\phi_1 + (K_2+K_3)\phi_2 - K_3\phi_3 &= m_2\phi_2\omega^2 \\ \dots & \\ -K_{n-1}\phi_{n-2} + (K_{n-1}+K_n)\phi_{n-1} - K_n\phi_n &= m_{n-1}\phi_{n-1}\omega^2 \\ -(K_n)\phi_{n-1} + K_n\phi_n &= m_n\phi_n\omega^2 \end{aligned} \tag{9}$$

where K_i is the lateral translational stiffness of storey i. Thus, using Eqs. (6) and (7), Eq. (5) is written as Eq. (8). To estimate the lateral translational stiffness K_i of each storey of the mathematical model, Eq. (8) is re-written as n simultaneous equations, as Eq. (9). Considering the last equation of Eq. (9), estimate of lateral translational stiffness of storey n of the mathematical model is given by

$$K_n = \left[\frac{\omega^2 m_n\phi_n}{\phi_n - \phi_{n-1}}\right] \tag{10}$$

Further, estimate of lateral translational stiffness of storey n-1 of the mathematical model is obtained from the summation of last two equations of the set of simultaneous equation (Eq. (9)) and is given by

$$K_{n-1} = \left[\frac{\omega^2\left(m_n\phi_n + m_{n-1}\phi_{n-1}\right)}{\phi_{n-1}-\phi_{n-2}}\right] = \left[\frac{\omega^2 \sum_{i=n-1}^{n} m_i\phi_i}{\phi_{n-1}-\phi_{n-2}}\right] \tag{11}$$

Similarly, the lateral translational stiffness K_i of each storey of the mathematical model is obtained from Eq. (9) and is given by

$$\begin{Bmatrix} K_1 \\ \dots \\ K_{n-1} \\ K_n \end{Bmatrix} = \begin{Bmatrix} \left[\dfrac{\omega^2 \sum_{i=1}^{n} m_i\varphi_i}{\varphi_1}\right] \\ \dots \\ \left[\dfrac{\omega^2 \sum_{i=n-1}^{n} m_i\varphi_i}{\varphi_{n-1}-\varphi_{n-2}}\right] \\ \left[\dfrac{\omega^2 m_n\varphi_n}{\varphi_n-\varphi_{n-1}}\right] \end{Bmatrix} \tag{12}$$

Thus, with K_i obtained from Eq. (12), the presence of soft or extreme soft storey as governed by Eqs. (1) to (4) along the height of building can be determined using

$$\begin{aligned} \frac{K_i}{K_{i+1}} &= \frac{\omega^2\left\{\sum_{j=i}^{j=n} m_j\phi_j\right\}}{\left\{\phi_i-\phi_{i-1}\right\}} \frac{\left\{\phi_{i+1}-\phi_i\right\}}{\omega^2\left\{\sum_{j=i+1}^{j=n} m_j\phi_j\right\}} \\ &= \frac{\left\{\sum_{j=i}^{j=n} m_j\phi_j\right\}}{\left\{\sum_{j=i+1}^{j=n} m_j\phi_j\right\}} \frac{\left\{\phi_{i+1}-\phi_i\right\}}{\left\{\phi_i-\phi_{i-1}\right\}} \end{aligned} \tag{13}$$

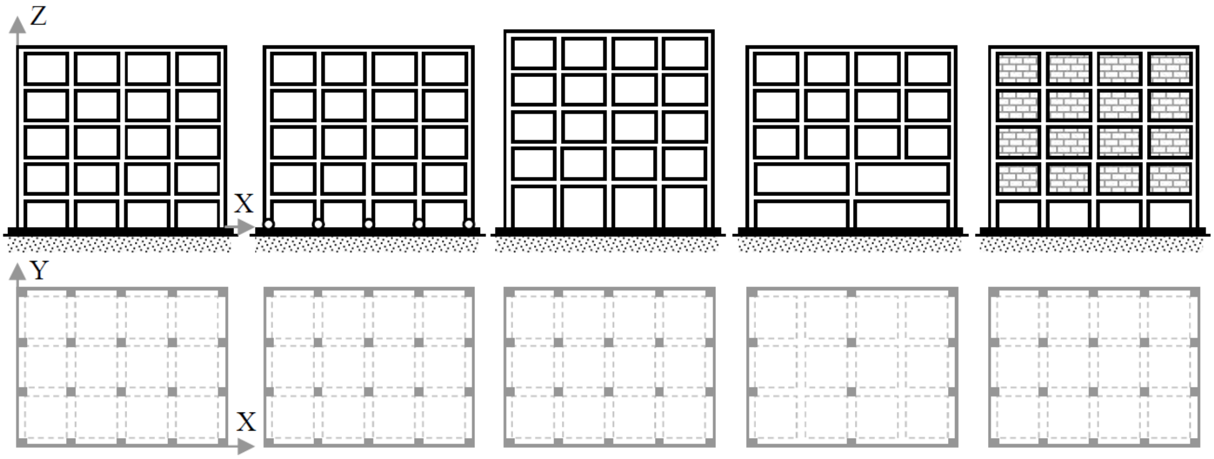
$$\frac{K_i}{\left[\dfrac{K_{i+1}+K_{i+2}+K_{i+3}}{3}\right]} = \frac{3\left(\dfrac{\left\{\sum_{j=i}^{j=n} m_j\phi_j\right\}}{\left\{\phi_i-\phi_{i-1}\right\}}\right)}{\left[\left(\dfrac{\left\{\sum_{j=i+1}^{j=n} m_j\phi_j\right\}}{\left\{\phi_{i+1}-\phi_i\right\}}\right) + \left(\dfrac{\left\{\sum_{j=i+2}^{j=n} m_j\phi_j\right\}}{\left\{\phi_{i+2}-\phi_{i+1}\right\}}\right) + \left(\dfrac{\left\{\sum_{j=i+3}^{j=n} m_j\phi_j\right\}}{\left\{\phi_{i+3}-\phi_{i+2}\right\}}\right)\right]} \tag{14}$$

The proposed procedure uses all properties of members (including effects of axial, shear and flexure deformations) without any simplifying assumptions. Also, estimate of storey stiffness obtained using the proposed procedure is unique as this utilizes the fundamental lateral translational natural circular frequency and its associated mode shape of the building (Eq. (12)).

4. Numerical study

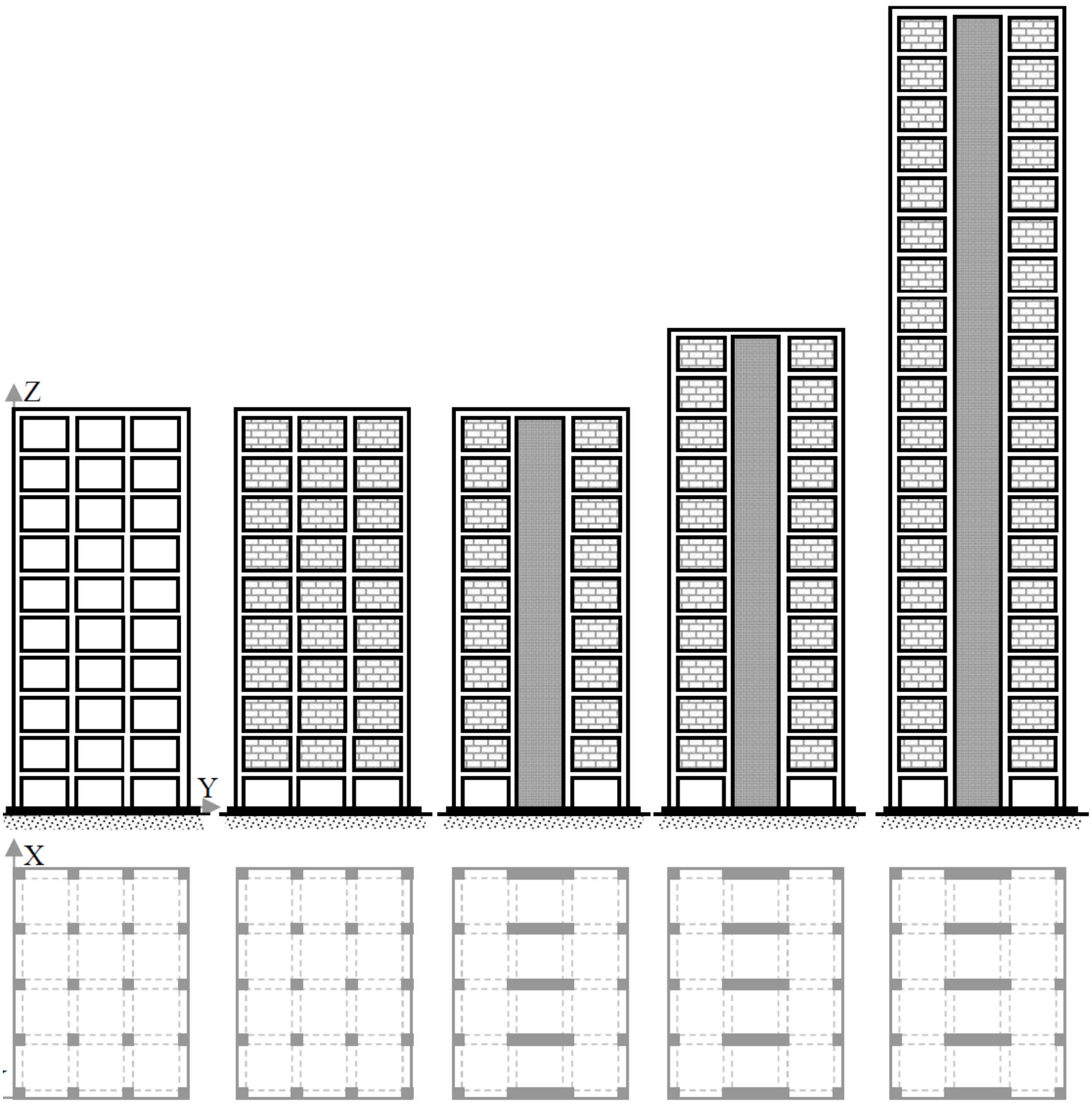
4.1 Specification of Model and Identification of Stiffness Irregularity

Ten buildings are considered in this study; Set A comprises of five 5-storey buildings (Fig. 5(a)) and Set B comprises of one 20-storey, one 12-storey and three 10-storey buildings (Fig. 5(b)). Set A buildings have uniform storey height of 3.5 m (centerline dimension) and Set B 4 m, except the first storey of Building 3 (in Set A) which is 5.5 m tall. All buildings have four and three 6m long bays along each principal plan (X- and Y-) directions, respectively, except Building 4 (in Set A), where intermediate columns in first and second storeys are removed (which results in 12 m long bays in X direction (Fig. 5(b))). All members are made of reinforced concrete; in Buildings 1 to 5, beams are of size 0.3×0.4 m and columns 0.4×0.4 m, and in Buildings 6 to 10, beams are 0.4×0.4 m and columns 0.6×0.6 m. In all buildings,



Building **1** [#]Building **2** Building **3** *Building **4** Building **5**

Fig. 5 (a) Elevation and plan of buildings in Set A; * Plan of second storey; [#] Column bases are pinned



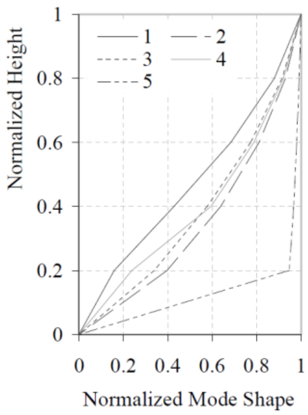
Building **6** Building **7** Building **8** Building **9** Building **10**

Fig. 5. (b) Elevation and plan of buildings in Set B

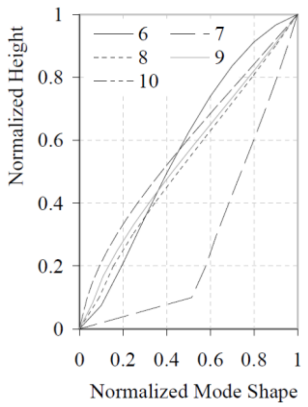
columns and structural walls (when present) are taken to be fixed at their bases, except Building 2, where columns are taken to be hinged at their base. Effective second moment of area (I_{eff}) of beams, columns, structural walls and masonry infills are taken as $0.4I_{gross}$, $0.7I_{gross}$, $0.7I_{gross}$ and I_{gross}, respectively (as per Paulay and Priestely 1992). Moduli of elasticity of masonry infill, concrete and reinforcing steel bars are taken as 4.5 GPa, 25 GPa and 200 GPa, respectively (as per IITK-GSDMA 2005). All beam-column joints are considered to be rigid.

Seismic weights of 100 kN and 200 kN (inclusive of self weight of members and lumped at each beam-column joints) are considered at all stories of buildings in Set A and Set B, respectively. Bare frame is considered in the structural analysis of Buildings 1 to 4 and 6. Masonry infills are considered in structural analysis of Buildings 5, 7, 8, 9 and 10; infill walls are modeled as equivalent diagonal struts, with thickness and width equal to wall thickness and 30% of diagonal dimension of panel in which it is present, respectively. In Buildings 7 to 10 with structural walls, RC walls are modeled as equivalent frame elements with length 6 m and thickness of 0.2 m. Structural analyses of all buildings are performed using structural analysis program SAP 2000 (CSI 2014); analyses are carried out to determine response of buildings in Set A along X-direction and of buildings in Set B along Y-direction. Normalized fundamental lateral translational mode shapes of buildings in Set A and Set B are shown in Figs. 6(a) and 6(b), respectively, and the corresponding fundamental natural periods and associated Mass Participation Factors (MPF) are listed in Table 1. Using these, lateral translational stiffness of each storey (of all buildings) is estimated using Eq. (12) and listed in Table 2.

The percentage differences in Buildings 2, 7 and 10 are shown in Fig. 7(a) and 7(b) in storey stiffnesses obtained using classical method (lateral force deformation method) using triangular and parabolic distributions, respectively, of lateral force (as defined in IS 1893 (Part 1)) with respect to the estimate obtained using Eq. (12). Difference in the percentage differences of storey stiffness obtained with triangular and parabolic distribution of lateral force re-affirm that the estimate of storey stiffness using lateral force deformation method, depends on the distribution of lateral force along the height of building. For all buildings, the percentage differences in the estimate of storey stiffness



(a)

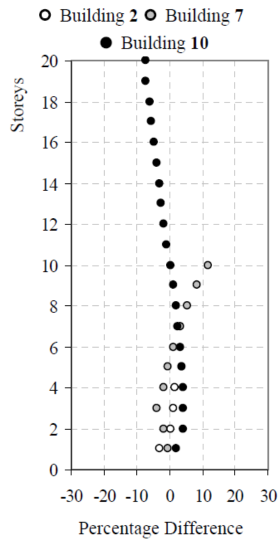


(b)

Fig. 6 Normalized Fundamental Mode shape of building in (a) Set A along X-direction and (b) Set B along Y-direction

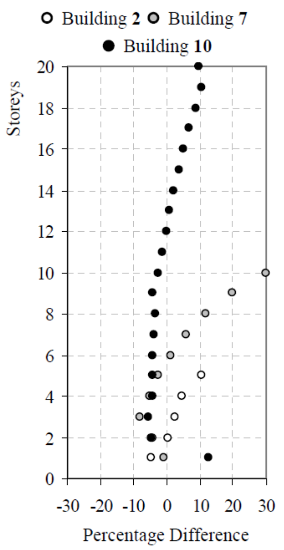
Table 1 Fundamental lateral translational period of buildings in Set A (along X-direction) and Set B (along Y-direction)

Set A			Set B		
Building	Natural Period (s)	Mass Participation of Fundamental Mode (%)	Building	Natural Period (s)	Mass Participation of Fundamental Mode (%)
1	1.48	81	6	2.41	80
2	1.97	92	7	0.72	96
3	1.78	90	8	0.45	75
4	1.85	87	9	0.60	72
5	0.54	99	10	1.45	66



(a)

Fig. 7 Percentage difference in the estimate of storey stiffness obtained using proposed method and classical method (lateral force deformation method) adopting (a) triangular distribution of lateral force and (b) parabolic distribution of lateral force



(b)

Fig. 7 Continued

Table 2 Storey stiffness in buildings present in Set A along X-direction and in Set B along Y-direction

Storey	*Storey Stiffness (kN/mm)*									
	Set A					*Set B*				
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
20	-	-	-	-	-	-	-	-	-	122
19	-	-	-	-	-	-	-	-	-	235
18	-	-	-	-	-	-	-	-	-	338
17	-	-	-	-	-	-	-	-	-	434
16	-	-	-	-	-	-	-	-	-	523
15	-	-	-	-	-	-	-	-	-	608
14	-	-	-	-	-	-	-	-	-	689
13	-	-	-	-	-	-	-	-	-	770
12	-	-	-	-	-	-	-	-	487	851
11	-	-	-	-	-	-	-	-	894	935
10	-	-	-	-	-	83	688	773	1,244	1,025
9	-	-	-	-	-	101	1,236	1,378	1,555	1,126
8	-	-	-	-	-	105	1,687	1,884	1,841	1,243
7	-	-	-	-	-	107	2,084	2,323	2,126	1,384
6	-	-	-	-	-	108	2,462	2,731	2,430	1,563
5	31	30	30	30	3,330	109	2,859	3,175	2,782	1,800
4	36	34	35	34	4,891	110	3,263	3,659	3,221	2,133
3	37	33	35	33	6,405	114	4,095	4,397	3,904	2,677
2	41	29	35	22	5,302	127	3,413	6,162	5,499	3,847
1	74	20	27	35	143	232	460	4,517	4,333	3,744

with triangular distribution of lateral force are lesser than that with parabolic distribution of lateral force.

Stiffness of first storey in buildings is significantly influenced by fixity of columns at their base. First storey stiffnesses in Building 2 and 3 are lower than those of the corresponding storey in Building 1 due to additional flexibility stemming from pinned column bases (in Building 2) and taller first storey (in Building 3). Increased flexibility due to discontinuity of intermediate columns at second storey results in lower first and second storey stiffness in Building 4 as compared to corresponding storeys in Building 1. Presence of masonry infills significantly influences the stiffness of each storey; in Buildings 5 and 7, first storey, which does not have infills, is significantly flexible compared to all other storeys that have infills. Presence of RC structural wall, in Buildings 8 to 10, significantly increases the stiffness of all storeys.

The conditions (Eqs. (13) and (14)) for lateral stiffness irregularity are presented in Table 3 for all the ten buildings. The results of Buildings 1 and 6 (with uniform distribution of both mass and stiffness) do not indicate presence of lateral translational stiffness irregularity. But, first storey in Buildings 2 and 3 are categorized as *extreme soft* and *soft*, respectively, due to lack of fixity at column base (in Building 2) and increase in height of the first storey columns (in Building 3). Likewise, second storey in Building 4 is categorized as *extreme soft* due to discontinuity of columns at the second storey (in Building 4). First storey in Buildings 5 and 7 are categorized as *extreme soft* due to absence of infill walls at the first storey. But, absence of infill walls at first storey along with the presence of structural wall does not indicate the presence of lateral translational stiffness irregularity in Buildings 8 to 10. Thus, it is evident (by comparing results of Buildings 7 and 8) that use of structural wall in buildings with open first storey aids in eliminating vertical irregularity.

4.2 Limits of applicability of the proposed method

Estimates of lateral translational stiffness of each storey in the ten buildings obtained using Eq. (12) is verified using linear elastic time-history analyses. For this purpose, two models of each of the ten buildings are considered, namely a full 3-D model and another 1-D model corresponding to the proposed mathematical model as conceived in Section 3 (Fig. 4(b)). In the 1-D model, the lateral translational stiffness of each storey is same as that estimated and listed in Table 2. Thereafter, both the 3-D and the 1-D models of each building are subjected to a set of earthquake ground motions (as listed in Table 4), and linear elastic time-history analyses are carried out (considering uniform equivalent viscous damping of 5%) of both the models to determine lateral storey displacement and base shear histories. Thus, a total of 140 linear time history analyses are performed. The elastic response spectra of the earthquake ground motions considered are shown in Fig. 8. A typical comparison of lateral storey displacement histories as obtained from 3-D and the corresponding 1-D model of Building 1 is shown in Fig. 9. The general trend of time variation of storey displacement obtained using the 1-D model, with storey stiffness computed using Eq. (12), matches well with that obtained using full 3-D model of the corresponding building. Thus, for the subsequent numerical comparison, only the peak values of lateral storey displacement and base shear are compared to draw conclusions on the limits of applicability of the proposed method.

Table 3 Assessment of stiffness irregularity in study buildings as per Eqs. (13) and (14)

Storey	Set A					Set B				
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	K_i/K_{i+1}									
20	-	-	-	-	-	-	-	-	-	-
19	-	-	-	-	-	-	-	-	-	1.93
18	-	-	-	-	-	-	-	-	-	1.44
17	-	-	-	-	-	-	-	-	-	1.28
16	-	-	-	-	-	-	-	-	-	1.21
15	-	-	-	-	-	-	-	-	-	1.16
14	-	-	-	-	-	-	-	-	-	1.13
13	-	-	-	-	-	-	-	-	-	1.12
12	-	-	-	-	-	-	-	-	-	1.11
11	-	-	-	-	-	-	-	-	1.84	1.10
10	-	-	-	-	-	-	-	-	1.39	1.10
9	-	-	-	-	-	1.22	1.80	1.78	1.25	1.10
8	-	-	-	-	-	1.04	1.36	1.37	1.18	1.10
7	-	-	-	-	-	1.02	1.24	1.23	1.15	1.11
6	-	-	-	-	-	1.01	1.18	1.18	1.14	1.13
5	-	-	-	-	-	1.01	1.16	1.16	1.14	1.15
4	1.16	1.13	1.17	1.13	1.47	1.01	1.14	1.15	1.16	1.19
3	1.03	0.97	1.00	0.97	1.31	1.04	1.25	1.20	1.21	1.26
2	1.11	0.88	1.00	0.67	0.83	1.11	0.83	1.40	1.41	1.44
1	1.80	0.69	0.77	1.59	0.03	1.83	0.13	0.73	0.79	0.97
	$K_i/[(K_{i+1}+K_{i+2}+K_{i+3})/3]$									
20	-	-	-	-	-	-	-	-	-	-
19	-	-	-	-	-	-	-	-	-	-
18	-	-	-	-	-	-	-	-	-	-
17	-	-	-	-	-	-	-	-	-	1.87
16	-	-	-	-	-	-	-	-	-	1.56
15	-	-	-	-	-	-	-	-	-	1.41
14	-	-	-	-	-	-	-	-	-	1.32
13	-	-	-	-	-	-	-	-	-	1.27
12	-	-	-	-	-	-	-	-	-	1.24
11	-	-	-	-	-	-	-	-	-	1.21
10	-	-	-	-	-	-	-	-	-	1.20
9	-	-	-	-	-	-	-	-	1.78	1.20
8	-	-	-	-	-	-	-	-	1.50	1.21
7	-	-	-	-	-	1.11	1.73	1.73	1.37	1.22
6	-	-	-	-	-	1.04	1.48	1.47	1.32	1.25
5	-	-	-	-	-	1.02	1.38	1.37	1.30	1.29
4	-	-	-	-	-	1.02	1.32	1.33	1.32	1.35
3	-	-	-	-	-	1.05	1.43	1.38	1.39	1.46
2	1.18	0.90	1.05	0.68	1.09	1.14	1.00	1.65	1.67	1.75
1	1.95	0.63	0.77	1.18	0.03	1.98	0.13	0.95	1.03	1.30

Table 4 List of Earthquake ground motions considered

Label	Event	Station	Component
GM1	1994 Northridge Earthquake	Santa Monica City Hall Grounds	0°
GM2	1989 Loma Prieta Earthquake	Corralitos Eureka Canyon Rd.	0°
GM3	1989 Loma Prieta Earthquake	Hollister South Street and Pine Drive	0°
GM4	1979 Imperial Valley Earthquake	El Centro Array 6, Hudson Rd	140°
GM5	1994 Northridge Earthquake	Newhall LA County Fire Station	0°
GM6	1992 Lander Earthquake	Yermo, Fire Station	0°
GM7	1994 Northridge Earthquake	Century City LACC North	0°

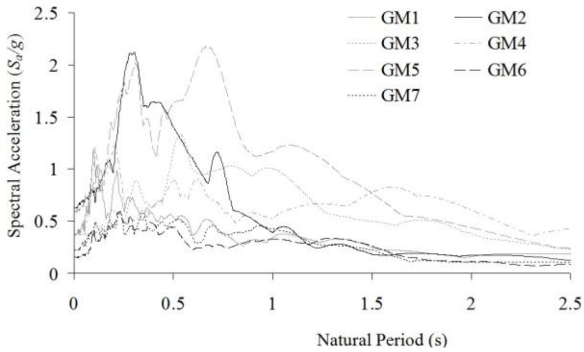


Fig. 8 Elastic response spectra of considered ground motions (at 5% damping)

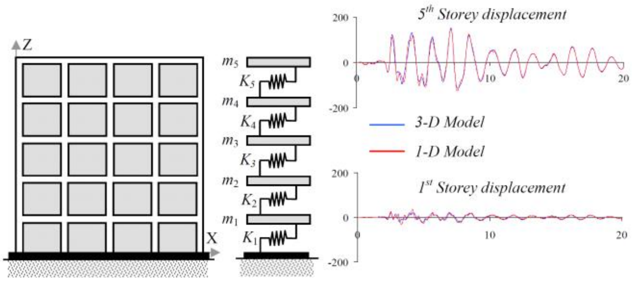


Fig. 9 First 20 second lateral storey displacement response obtained from 3-D model and its corresponding 1-D model of Building 1, for GM2

Estimate of lateral translational stiffness of a storey in a building (as listed in Table 2) is deemed to be 'good' if the differences in the estimates of peak lateral storey displacements and base shear of the building, obtained from linear elastic time-history analysis of its 3-D model and the corresponding 1-D model, are *minimum*. Mean of percent difference in the estimate of peak lateral displacement of all storeys in buildings, analyzed for each of the seven ground motions, is shown in Fig. 10. For all cases other than of Buildings 1, 9 and 10 when subjected to GM2, and Building 1 when subjected to GM7, the mean percent difference in peak lateral storey displacement is less than 10%; this indicates that the proposed procedure (Eq. (12)) helps obtain good estimate of lateral storey stiffness. Larger mean percent difference observed in case of buildings subjected to GM2 is attributed to higher spectral acceleration values

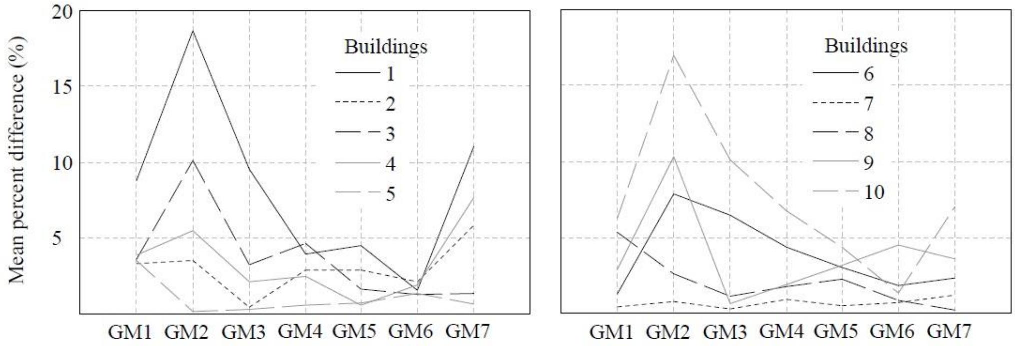


Fig. 10 Mean of percent difference in estimates of peak lateral displacement at each storey obtained using 3-D and corresponding 1-D models of the ten buildings, for each ground motion

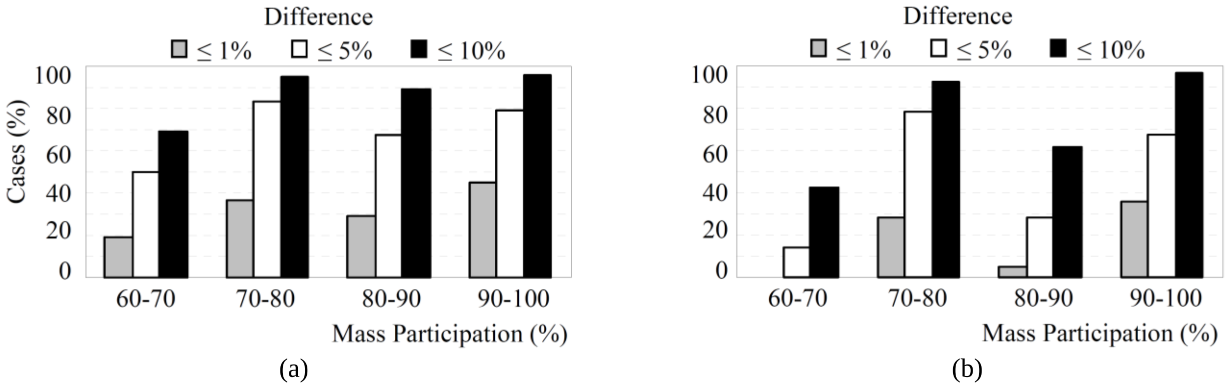


Fig. 11 Distribution of cases with ≤1%, ≤5% and ≤10% difference in estimates of (a) peak lateral storey displacement, and (b) maximum base shear, in buildings with different mass participation of fundamental mode

at natural period corresponding to the second lateral translational mode of oscillation (S_{a2}) than that at natural period of fundamental lateral translational mode of oscillation (S_{a1}). For example, the mean percent difference of 19% and 11% are observed in Building 1 for GM2 (where S_{a2}/S_{a1} is equal to 7.9) and for GM 7 (where S_{a2}/S_{a1} is equal to 2.1), respectively; in comparison, for all other ground motions, S_{a2}/S_{a1} value ranges between 0.9 and 1.9 for Building 1.

Next, the applicability of the proposed procedure is verified for buildings with different mass participation (MP) of the fundamental mode of oscillation, expressed as percent. The MPs of the ten buildings vary from 66% to 99% (Table 1); thus, there are 1 building with MP in the range 60-70%, 2 buildings with MP in the range 70-80%, 3 buildings with MP in the range 80-90%, and 4 buildings with MP in the range 90-100%. Distribution of percent difference (expressed as 1%, less than 5% and less than 10%) in the estimates of peak lateral displacement of all storeys, and maximum base shear, in buildings with MP in the above ranges, are shown in Fig. 11(a) and 11(b), respectively. For the purpose, results of analyses of all the seven ground motions are considered together. Thus, in total 609 cases of peak lateral storey displacement and 70 cases of maximum base shear are considered, for the ten buildings and seven ground motions cases. The results indicate that for buildings with MP in the range 70-80%, 95% cases (146 out of 154 cases) have a difference of 10% or less in estimates of peak lateral storey displacement obtained using 1-D and 3-D models of the buildings (Fig. 11(a)). Similarly, 83% cases and 37% cases offer good estimates if higher accuracy of results with difference less than 5% and 1% are sought. The corresponding values for buildings with MP in the range 90-100% are 95%, 79% and 45% if accuracy of 10%, 5% and 1% are sought.

In other words, only 5% of peak lateral storey displacement estimates obtained from 1-D and 3-D models differ by more than 10%, in buildings with MP in the range 90-100%. Further, relative to the difference in estimates of peak lateral storey displacements, in general, the difference in estimates of maximum base shear is more (Fig. 11(b)). For example, accuracy of 1% in estimate of maximum base shear is obtained for only 29% cases in buildings with MP in the range 70-80% (as against 37% cases for peak lateral storey displacement). Also, there is a relative drop in the accuracy of results obtained for buildings with MP in the range 80-90% for both peak lateral storey displacement and maximum base shear; this can be attributed to the specific choice of the ten buildings and the seven ground motions. In general, there is a clear trend in the results indicating applicability of the proposed procedure for buildings with MP more than 70%. This is evident from the mean percent difference in estimates of peak lateral storey displacements and maximum base shear which are less than 10% and 15%, respectively, for all buildings with MP greater than 70% (Fig. 12). Thus, it is inferred the proposed procedure estimate storey stiffness reasonably well for buildings with MP more than 70%. Therefore, the proposed method is

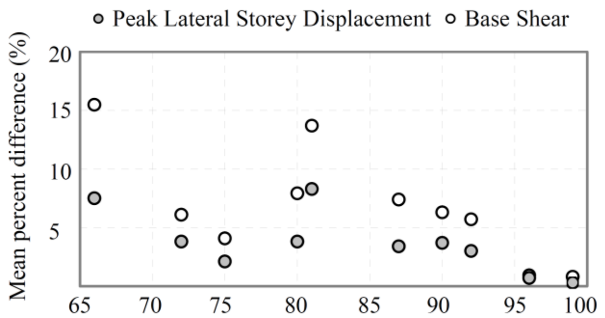


Fig. 12 Mean of percent difference in estimates of peak lateral displacement at each storey obtained using 3-D and corresponding 1-D models of the ten buildings, for each ground motion

efficient in assessing vertical stiffness irregularity in such multi-storey buildings.

4.3 Special cases

MP of fundamental mode of oscillations of buildings depends on location of irregularity along the height of the building. For example, MP of 96% fundamental mode of oscillation of Building 7 reduces to 81%, 68%, 60% and 63% when storeys 3, 5, 7 and 9 are considered open in place of storey 1, respectively (Fig. 13). Therefore, applicability of the proposed procedure is verified for multi-storey buildings with irregularity at an intermediate storey that results in MP of fundamental mode of oscillation to be lower than 70%. For the purpose, one five storey (Building 11) and another ten storey (Building 12) buildings, with open third storey and fifth storey, respectively, are considered (Fig. 14(a)). Geometry, member proportions, material properties and lumped seismic mass of Building 11 and Building 12 are identical to that defined, in section 4.2, for Building 5 and 7, respectively. Normalized fundamental lateral translational mode shapes of Buildings 11 (along X direction) and 12 (along Y direction) are shown in Fig. 14(b). Fundamental lateral translational period of Building 11 (in X direction) and of Building 12 (in Y direction) are 0.49 s and 0.71 s, respectively. MP of Building 11 and Building 12, corresponding to respective fundamental lateral translational period and mode shape, are 62 and 68%, respectively.

Lateral translational stiffness of each storey of Buildings 11 and 12, is estimated using Eq. (12) and listed in Table 5. Also, listed in Table 5, are estimates of conditions for lateral stiffness irregularity (Eqs. (13) and (14)). Presence of open storey results in significant reduction in the lateral translational stiffness of third storey and fifth storey in Buildings 11 and 12, respectively. Consequently, third storey and fifth storey of Buildings 11 and 12, respectively, are categorized *extreme soft* storey.

Estimate of lateral translational stiffness of each storey in Buildings 11 and 12 is verified using results of linear elastic time-history analyses. A typical comparison of lateral storey displacement and base shear history as obtained from 3-D and the corresponding 1-D model of Building 12 is shown in Fig. 15. As observed for buildings considered in section 4.2, the general trend of time variation of storey displacement and base shear obtained using the 1-D model, with storey stiffness computed using Eq.(12), matches well with that obtained using full 3-D model of the corresponding building. The mean of percent difference in peak storey lateral displacement of 3-D model and its corresponding 1-D model of Buildings 11 and 12, considering all ground motions, are only 1% and 0.7%, respectively. In addition, mean of percent difference in maximum base shear of the 3-D model and its corresponding 1-D model of Buildings 11 and 12, considering all ground motions, are 2.3% and 3.3%, respectively. In buildings with such extreme soft storeys, the portion of the building above such soft storey responds significantly to earthquake ground shaking (Fig. 13(b)). Thus, MP computed using the entire mass of the building does not reflect the actual dynamic behavior of such buildings. Therefore, notwithstanding low MPs, significantly lower differences in peak lateral storey displacement and maximum base shear are obtained. This indicates the proposed procedure offers good estimate of storey stiffness in buildings with such irregularities in any storey level.

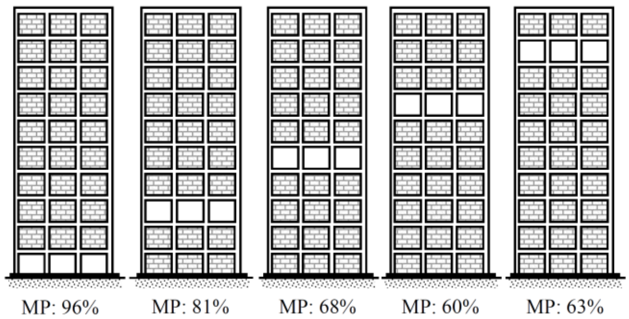
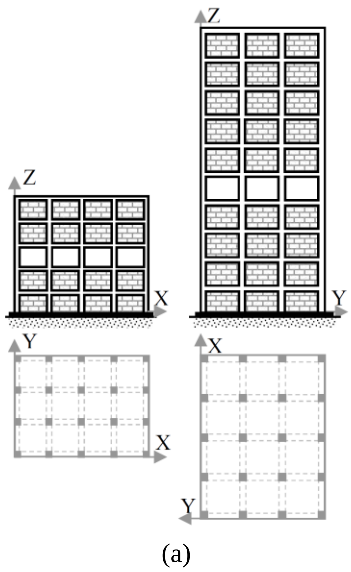
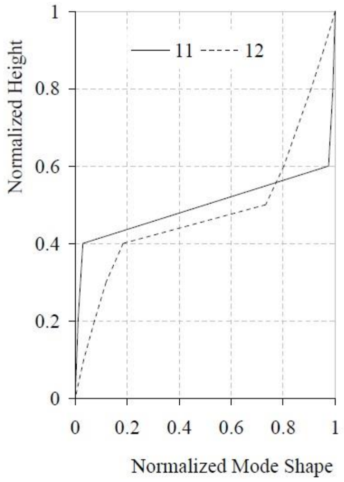


Fig. 13 Mass participation of buildings identical to Building 7 but with different location of open storey



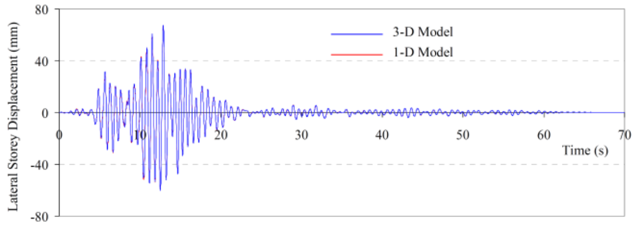
(a)

Fig. 14 (a) Plan and Elevation of Building 11 (five storey) and Building 12 (ten storey) (b) Normalized Mode Shape of Building 11 in X-direction and Building 12 in Y-direction

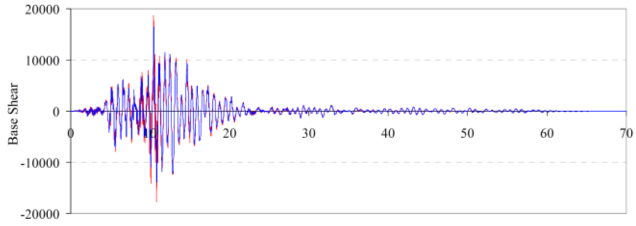


(b)

Fig. 14 Continued



(a)



(b)

Fig. 15 Comparison of (a) lateral storey displacement of storey 1, and (b) base shear, obtained from 3-D model and its corresponding 1-D model of Building 12, for GM1

Table 5 Storey stiffness in Building 11 (along X-direction) and Building 12 (along Y-direction) and assessment of vertical stiffness irregularity

Storey	Storey Stiffness		K_i/K_{i+1}		$K_i/[(K_{i+1}+K_{i+2}+K_{i+3})/3]$	
	Building 11 (X-direction)	Building 12 (Y-direction)	Building 11 (X-direction)	Building 12 (Y-direction)	Building 11 (X-direction)	Building 12 (Y-direction)
10	-	724	-	-	-	-
9	-	1,299	-	1.79	-	-
8	-	1,747	-	1.34	-	-
7	-	2,284	-	1.31	-	1.82
6	-	2,074	-	0.91	-	1.17
5	3,848	308	-	0.15	-	0.15
4	3,943	2,660	1.02	8.64	-	1.71
3	104	3,988	0.03	1.50	-	2.37
2	5,419	4,375	52.11	1.10	2.06	1.89
1	8,800	5,541	1.62	1.27	2.79	1.51

5. Conclusions

A simple procedure is proposed to estimate storey stiffness using only the fundamental lateral translational mode of oscillation of buildings (using their dynamic characteristics, namely natural period and associated mode shape). This provides designers with a method to estimate storey stiffness and identify stiffness irregularity along height of multi-storey buildings, and take necessary action at the analysis stage itself. Also, the method can easily be incorporated in any structural analysis software to highlight presence of stiffness irregularity in buildings since it uses the modal analysis results or knowledge of the fundamental frequency and mode shape which are made available by the software. Further, results of numerical analysis indicate the proposed procedure is suitable for estimation of storey stiffness and thereby identifying stiffness irregularity in both low- and mid-rise buildings that have mass participation factor more than 70%. Further, it is to be noted that the proposed procedure is applicable for buildings with framing systems with or without reinforced concrete or masonry infill walls.

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