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Seismic structural demands and inelastic deformation ratios: a theoretical approach

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Abstract. To estimate the structural seismic demand, some methods are based on an equivalent linear system such as the Capacity Spectrum Method, the N2 method and the Equivalent Linearization method. Another category, widely investigated, is based on displacement correction such as the Displacement Coefficient Method and the Coefficient Method. Its basic concept consists in converting the elastic linear displacement of an equivalent Single Degree of Freedom system (SDOF) into a corresponding inelastic displacement. It relies on adequate modifying or reduction coefficient such as the inelastic deformation ratio which is usually developed for systems with known ductility factors (C_μ) and (C_R) for known yield-strength reduction factor.

The present paper proposes a rational approach which estimates this inelastic deformation ratio for SDOF bilinear systems by rigorous nonlinear analysis. It proposes a new inelastic deformation ratio which unifies and combines both C_μ and C_R effects. It is defined by the ratio between the inelastic and elastic maximum lateral displacement demands. Three options are investigated in order to express the inelastic response spectra in terms of: ductility demand, yield strength reduction factor, and inelastic deformation ratio which depends on the period, the post-to-preyield stiffness ratio, the yield strength and the peak ground acceleration.

This new inelastic deformation ratio (C_η) is describes the response spectra and is related to the capacity curve (pushover curve): normalized yield strength coefficient (η), post-to-preyield stiffness ratio (α), natural period (T), peak ductility factor (μ), and the yield strength reduction factor (R_y). For illustrative purposes, instantaneous ductility demand and yield strength reduction factor for a SDOF system subject to various recorded motions (El-Centro 1940 (N/S), Boumerdes: Algeria 2003). The method accuracy is investigated and compared to classical formulations, for various hysteretic models and values of the normalized yield strength coefficient (η), post-to-preyield stiffness ratio (α), and natural period (T). Though the ductility demand and yield strength reduction factor differ greatly for some given T and η ranges, they remain take close when $\eta > 1$, whereas they are equal to 1 for periods $T \geq 1s$.

Keywords: deformation ratio; yield strength; reduction factors; ductility; inelastic spectra; Pushover; normalized yield strength coefficient; seismic design

1. Introduction

For Nonlinear Time History Analysis (NL-THA) of structures, several simplified methods have been proposed. They estimate the seismic response by considering the structural capacity as well as the seismic intensity. Due to nonlinear materials behavior, the dynamic structural characteristics change with time during major earthquakes. Therefore, ductility plays an important role in structural response and earthquake engineering design. To study the nonlinear response, one may use real time history analysis, by analytical or numerical procedure considering elastic and inelastic response, or a simplified approach such as Pushover analysis. The structural capacity can actually be studied by running inelastic static analysis such as Pushover analysis. Thus, it has been extensively investigated and discussed for seismic performance evaluation.

Among such methods, the most widely used are the Capacity Spectrum Method (CSM) developed by Freeman *et al*. (1975) and adopted by the Applied Technology Council ATC-40 (1996), the N2 method developed by Fajfar (1996, 1999) and the Displacement Coefficient

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Method (DCM) adopted also by the Federal Emergency Management Agency FEMA-273 (1997) and FEMA-356 (2000). More recently, the FEMA-440 (2005) was developed as improvement of nonlinear static seismic analysis procedures of FEMA-356 (2000) and the ATC-40(1996) for seismic rehabilitation of buildings.

The CSM method (ATC-401996) gives an overview of the inelastic behavior of structures and requires structural capacity curves derived from the nonlinear dynamic behavior analyses. It compares the capacity of a structure to resist lateral forces to the demands of earthquake response spectra. Many response spectra have been proposed to replace the conventional elastic spectra in order to achieve accurate evaluation of the inelastic response of structures (Sheng and Biggs 1980, Iwan 1980, Newmark and Hall 1982, Kowalsky 1994, Benazouz *et al*. 2012, Sang *et al*. 2014, Yazdani and Salimi 2015, Guohuan *et al*. 2016).

For instance, FEMA-273 (1997) and FEMA-356 (2000) adopt the DCM method. They derive the maximum structural inelastic deformation from the maximum linear elastic deformation by using a modifying factor. In fact, the DCM method requires various correction factors in order to adjust the linear displacement by an equivalent nonlinear displacement. The most influent correction factor is represented by the modifying factor C_1which concerns the inelastic deformation ratio.

More recently, FEMA-440 (2005), several researchers have investigated the accuracy of the inelastic displacement demands obtained from DCM. Furthermore, drawbacks of the method rise from the fact that it does not take into account accurately neither the P-delta effect nor the soils effect (FEMA-440 2005). For better accuracy, Miranda (2001) developed two methods able to estimate the maximum inelastic displacements through the use of parameters that relate SDOF systems elastic response to their inelastic response. Miranda and Ruiz-Garcia (2002) have also proposed a simplified displacement modification factor which shows that neither earthquake magnitude nor distance to the fault have strong influence on the idealized peak inelastic displacement demands. Same results have also been observed by Chopra and Chintanapakdee (2004). Further studies on DCM method sensitivity analysis have been performed recently (Miranda and Akkar 2003, Matamoros *et al*. 2003, Lin *et al*. 2004, Pankaj and Lin 2005, Bardakis and Dritsos 2007, Akkar and Metin 2007, Goel 2008, Benavent and Escolano 2012, Massumi and Monavari 2013).

The inelastic deformation estimated by the CSM method is based on an equivalent linear method, whereas the DCM method provides displacement correction by the use of modification factors. In FEMA-440 (2005), it is stated that the coefficients used in DCM, FEMA-356 (2000), may lead to excessive under- or over-estimates. Therefore, improved series of coefficients for DCM have been presented in FEMA-440 (2005). Miranda (2001) studied the similarities and differences between the use of inelastic demand spectra in order to estimate inelastic peak deformations as implemented by (Reinhorn 1997, Fajfar 1999, 2000, Chopra and Goel 1999, Chikh *et al*. 2014, Zerbin and Aprile 2015, Kazaz 2016, Chikh *et al*. 2016) and the use of inelastic displacement ratios in order to estimate maximum inelastic deformations as suggested by Miranda (1991, 2000) as well as Seneviratna and Krawinkler (1997).

To evaluate the inelastic deformation ratio, the present work develops a new theoretical expression for SDOF bilinear systems derived from the bilinear capacity curve (Pushover curve).

2. Inelastic deformation ratio for SDOF bilinear systems

Structural nonlinear time history analysis (NL-THA) is time consuming and not easy to implement as it requires sophisticated nonlinear models for the whole structural components. The results are very sensitive to the model accuracy.

Thus, for engineering purposes, it is still challenging to develop simplified and enough accurate methods. For instance, for an inelastic SDOF system (see Fig. 1), its deformation $x(t)$ under a ground motion effect is expressed as

$$m\ddot{x}(t) + c\dot{x}(t) + f(x,\dot{x}) = -m\ddot{x}_g(t) \tag{1}$$

Where: m, c and f represent the mass, damping and resisting force of the inelastic system, respectively; $\ddot{x}_g(t)$ denotes the ground acceleration; $\ddot{x},\ \dot{x},\ x$ represent respectively the acceleration, velocity and displacement (deformation) of the SDOF.

For inelastic bilinear system, the resisting force $f(t)$ is defined as the sum of the linear part and the hysteretic part, and depends on the history of deformations (see Fig. 1)

$$f(t) = k_p x(t) + Qz(t) \tag{2}$$

Where: k_p is the post-yield stiffness; Q is the yield strength (ordinate at origin of the creeping part) whereas f_y represents the yielding force; and the adimensional variable $z(t)$ which characterizes the Bouc-Wen hysteresis model (Bouc1967, Wen1976, Baber and Noori 1985, Kunnath *et al*. 1992) is the solution of the differential Eq. (1) given by

$$\dot{z}(t) = \frac{\dot{x}(t)}{x_y}\left[A - |z|^{\lambda}(B.sign(\dot{x}z) + \beta)\right] \tag{3}$$

Where: $z(t)$ depends on the yield displacement x_y, as well as A, B, λ, and β that are the parameters that control the shape of the hysteresis loop. The adopted values are: $A = 1,\ B = 0.1,\ \lambda = 0.9$ and $\beta = 6$ for bilinear system

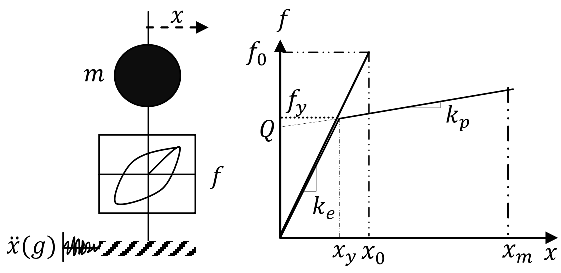


Fig. 1 Behavior of an (SDOF) bilinear system

(Wen 1976); sign(.) is the sign function.

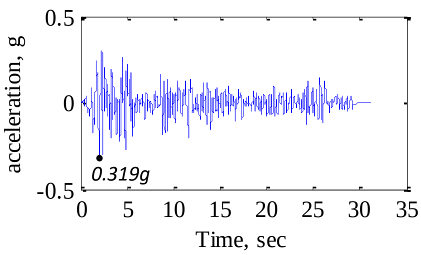
Eqs. (1)-(2) lead to

$$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \alpha\omega^2 x(t) + qgz(t) = -\ddot{x}_g(t) \qquad (4)$$

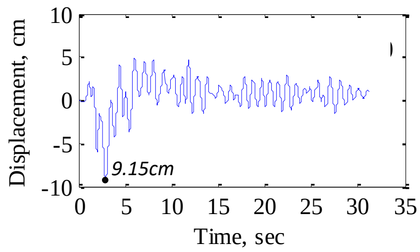
Where: $\xi = \frac{c}{2.\omega.m}$ =damping ratio; ω =circular frequency; α =post-to-preyield stiffness ratio; Q =yield strength; $m =$ mass; g =gravity acceleration; $q = \frac{Q}{mg}$=yield strength ratio.

For illustration purposes, Eq. (4) is solved for El Centro 1940 ground motion (N/S) component ($PGA = 0.32$ g, $PGV = 36.14$ cm/sec, and $PGD = 21.34$ cm) (see Fig. 2(a)). The results for nonlinear analysis are shown in Fig. 2:

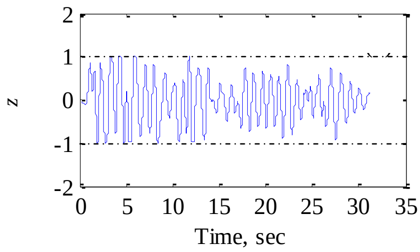
- the displacement history $x(t)$ in Fig. 2(b)
- the yielding history through $z(t)$in Fig. 2(c), and
- the variation of the system force coefficient f/w with respect to displacement in Fig. 2(d).



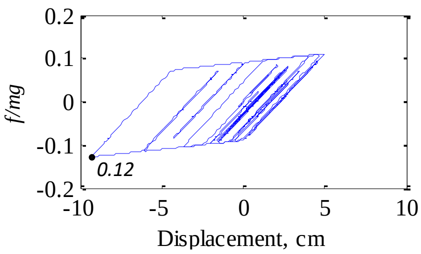
(a) Strong component of El Centro 1940 (N/S) ground motion



(b) System deformation



(c) Yield function $z(t)$



(d) Force-deformation relation

Fig. 2 Results for nonlinear analysis System parameters are: $q = 0.09$, $T = 1.0$ sec, $k_e = 17.5$ kN/cm and $\alpha = 10\%$

In case of elastic linear SDOF system, Eqs. (1), (2) and (4) provide

$$f(t) = k_e x(t) \qquad (5)$$

$$\ddot{x}(t) + 2\xi\omega\dot{x}(t) + \omega^2 x(t) = -\ddot{x}_g(t) \qquad (6)$$

Where: k_e =the elastic stiffness; f=the resisting force.

Thus, linear dynamic analysis procedures concern the response spectrum and time history analyses. They provide approximate nonlinear responses and require coupled, second-order, linear differential equations of motion under forced vibration. Therefore, it is unlikely that the displacements are accurately estimated. For instance, the elastic analysis with El Centro 1940 ground motion (N/S) provides the results shown in Fig. 3.

For the same ground motion, one can notice that (see Figs. 2 and 3):

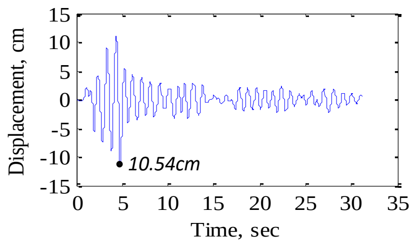
- the linear analysis predicts peak displacements (10.5 cm) slightly larger than nonlinear case (9.15 cm)
- whereas the spectral acceleration coefficient (f/mg) for linear analysis (0.45) is almost four times larger than the nonlinear response (0.12).

2.1 Ductility factor μ, and yield strength reduction factor R_y

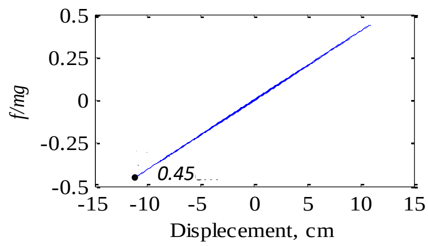
An adequate design is based on yield and peak (maximal) displacements targets according to the ductility demand expected during an earthquake. This ductility demand (or ductility factor) initially introduced through the response spectrum for elastic-perfectly plastic systems (Veletsos and Newmark 1960) for the bilinear system is expressed as (Chopra and Chintanapakdee 2004)

$$\mu = \frac{x_m}{x_y} \qquad (7)$$

Where: x_m is the maximum (or peak) displacement.



(a) System deformation



(b) Force-deformation

Fig. 3 System deformation and force-deformation results of linear dynamic analysis. System period $T = 1.0$ sec

An additional key factor is expressed by the yield strength reduction factor R_y defined as (Chopra and Chintanapakdee 2004)

$$R_y = \frac{f_0}{f_y} = \frac{x_0}{x_y} \tag{8}$$

Where: f_0 and x_0 are respectively peak force and deformation of the elastic system.

Despite numerous researches on experimental and theoretical evaluation of this reduction factor R_y, it is still challenging to find explicit or simplified relationships between this reduction factor and the structural (or material) parameters such as the structural natural period and the post to pre-yield stiffness ratio (Lai and Biggs 1980, Riddell and Newmark 1979, Elghadamsi and Moraz 1987, Riddell *et al.* 1989, Nassar and Krawinkler 1991, Miranda 1993, Vidic *et al.* 1994, Ordaz and Pérez-Rocha 1998, Borzi and Elnashai 2000, Tiwari and Gupta 2000, Riddell *et al.* 2002, Chopra and Chintanapakdee 2004, Farrow and Kurama 2004).

2.2 Equation of motion in terms of ductility factor μ, and yield strength reduction factor R_y

By analogy with the yield strength reduction factor R_y, (see Eq. (8)), it seems worth to associate, for each instantaneous elastic displacement $x_e(t)$, an instantaneous yield strength reduction factor $\Re_y(t)$ defined as

$$\begin{cases} \Re_y(t) = x_e(t)/x_y \\ \dot{\Re}_y(t) = \dot{x}_e(t)/x_y \\ \ddot{\Re}_y(t) = \ddot{x}_e(t)/x_y \end{cases} \tag{9}$$

Where: $x(.)$ is the instantaneous displacement of the system; $\Re_y(.)$, $\dot{\Re}_y(.), \ddot{\Re}_y(.)$ are instantaneous yield strength reduction factor $\Re_y(.)$ and first as well as second derivatives; t is any instant time during the motion.

Also by analogy with the ductility demand for inelastic system μ, (see Eq. (7)), it is also worth to associate, for each instantaneous inelastic displacement $x(t)$, an instantaneous ductility demand $\mathcal{M}(t)$ defined as

$$\begin{cases} \mathcal{M}(t) = x(t)/x_y \\ \dot{\mathcal{M}}(t) = \dot{x}(t)/x_y \\ \ddot{\mathcal{M}}(t) = \ddot{x}(t)/x_y \end{cases} \tag{10}$$

Where: $\mathcal{M}(.)$, $\dot{\mathcal{M}}(.), \ddot{\mathcal{M}}(.)$ are, respectively, instantaneous ductility demand $\mathcal{M}(.)$, its first and second derivatives.

From Eqs. (4), (6), (9) and (10) and after division by x_y, the new motion equations become

$$\begin{cases} \ddot{\mathcal{M}}(t) + 2\xi\omega\dot{\mathcal{M}}(t) + \alpha\omega^2\mathcal{M}(t) + \omega^2(1-\alpha)z(t) \\ \qquad = -\dfrac{1}{x_y}\ddot{x}_g(t) \\ \ddot{\Re}_y(t) + 2\xi\omega\dot{\Re}_y(t) + \omega^2\Re_y(t) \\ \qquad = -\dfrac{1}{x_y}\ddot{x}_g(t) \end{cases} \tag{11}$$

The yield displacement can be expressed as

$$x_y = \frac{q\,g}{\omega^2(1-\alpha)} = \frac{Q}{mg}\cdot\frac{g}{\omega^2(1-\alpha)} \tag{12}$$

Eqs. (11)-(12) can therefore be expressed as

$$\begin{cases} \ddot{\mathcal{M}}(t) + 2\xi\omega\dot{\mathcal{M}}(t) + \alpha\omega^2\mathcal{M}(t) + \omega^2(1-\alpha)z(t) \\ \qquad = -\dfrac{\omega^2(1-\alpha)}{qg}\ddot{x}_g(t) \\ \ddot{\Re}_y(t) + 2\xi\omega\dot{\Re}_y(t) + \omega^2\Re_y(t) \\ \qquad = -\dfrac{\omega^2(1-\alpha)}{qg}\ddot{x}_g(t) \end{cases} \tag{13}$$

By using this instantaneous ductility demand $\mathcal{M}(t)$, yield strength reduction factor $\Re_y(t)$ and the yield strength ratio q, the bilinear system behavior described by Fig. 1 is transformed into a bilinear representation of the resisting force (as described by Eq. (13), (see Fig. 4)) which value equals 1 at the yield position.

For illustrative purposes, Eqs. (13) are solved for El Centro 1940 ground motion (N/S) by using the new bilinear model shown in Fig. 4. The corresponding instantaneous ductility demand and yield strength reduction factor for a SDOF system are provided in Fig. 5.

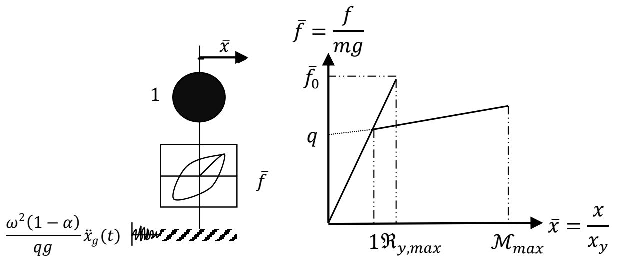
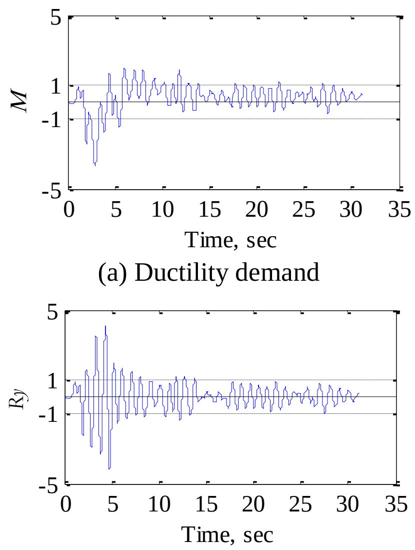


Fig. 4 New SDOF behavior used in Eq. (13) described in terms of new normalized parameters: q, $\mathcal{M}$ and $\Re_y$



(a) Ductility demand

(b) Yield strength reduction factor

Fig. 5 Ductility demand and yield strength reduction factor when $q = 0.09$, $T = 1$ sec and $\alpha = 10\%$

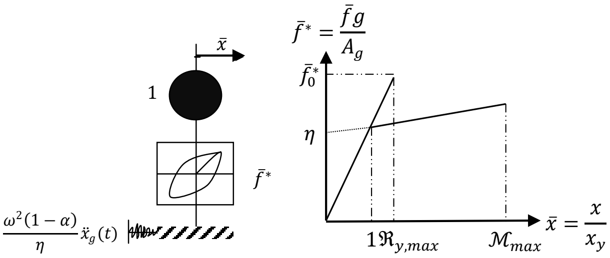


Fig. 6 Bilinear system behavior described in terms of new normalized parameters: η, $\mathcal{M}$ and $\Re_y$

2.3 Normalization and control parameters

The normalized yield strength coefficient η defined as the ratio between the system strength relative and the Peak Ground Acceleration (PGA) value A_g is expressed as (Mahin and Lin 1983, Benazouz *et al*. 2012)

$$\eta = \frac{q.g}{A_g} \tag{14}$$

Incorporating η into Eq. (13) results in:

$$\begin{cases} \ddot{\mathcal{M}}(t) + 2\xi\omega\dot{\mathcal{M}}(t) + \alpha\omega^2\mathcal{M}(t) + \omega^2(1-\alpha)z(t) \\ \quad -\dfrac{\omega^2(1-\alpha)}{\eta}\overline{\ddot{x}_g}(t) \\ \ddot{\Re}_y(t) + 2\xi\omega\dot{\Re}_y(t) + \omega^2\Re_y(t) \\ \quad = -\dfrac{\omega^2(1-\alpha)}{\eta}\overline{\ddot{x}_g}(t) \end{cases} \tag{15}$$

Where: $\overline{\ddot{x}_g}(t) = \frac{\ddot{x}_g(t)}{A_g}$ represents the normalized ground acceleration with respect to the PGA.

The bilinear system behavior described by Fig. 4 is transformed into Fig. 6 when using the new normalized parameters η, $\mathcal{M}(t)$ and $\Re_y(t)$.

From the new representation of the SDOF behavior illustrated in Fig. 6, one can notice that, for a given ground acceleration, the instantaneous ductility $\mathcal{M}(\mathrm{t})$ and the yield strength reduction factor $\Re_y(t)$ depend also on α, η and T or ω (Period or circular frequency of the system).

2.4 Applications and discussion

For illustrative purposes, El-Centro 1940 (N/S) is applied as input for various systems and control parameters, (see Figs. 7 and 8).

- Three inelastic models are used in this investigation in order to verify the influence of the behavior type on the ductility demand and yield strength reduction factor.
- Four values for $\eta = 0.25, 0.5, 0.75$ and 1.0, and four values for $\alpha = 0\%, 3\%, 5\%$ and 10% are used in order to study their influence,
- and the damping ratio $\xi = 5$ for all systems.

The hysteretic models are based on the Wen-Bouc model (Wen 1976) with the set of values given above, i.e., $A = 1$, $B = 0.1, \lambda = 0.9$ whereas β, which controls the rate of transition from the elastic to the yield state, is so that (Lobo 1994):

A large value $\beta = 20$ corresponds to a bilinear hysteretic curve, and

A lower value $\beta = 6$ corresponds to a smoother transition.

The three hysteretic models were adopted with the new theoretical approach. The results show that the whole models are capable of capturing the dynamic response of the inelastic SDOF system, whereas the Bouc-Wen model was reformulated in terms of instantaneous ductility needed for the time history analysis.

The comparison provided in Figs. 7 and 8 shows that the Elastic Perfectly Plastic model gives overestimated results when compared to those obtained by using the bilinear and smooth model. Furthermore, the results of the bilinear behavior are very close to those obtained using the smooth model. Therefore, the post-to-preyield stiffness ratio leads to accurate evaluation of the dynamic response (see Fig. 7).

The influences of the normalized yield strength coefficient (η) and the post-to-preyield stiffness ratio (α) are illustrated in Fig. 8. One can notice that, for the considered examples, the ductility demand (μ) and the yield strength reduction factor (R_y) decrease when the normalized yield strength coefficient (η) or the post-to-preyield stiffness ratio (α) increase. The results obtained for R_y indicate that the post-to-preyield stiffness ratio has a small effect on the yield strength reduction factor.

2.5 Inelastic deformation ratio

Structural analyses and designs, such as response spectrum and spectral anaylsis, are usually based on peak response in terms of deformations, accelerations, or shear forces, etc. Furthermore, the inelastic deformation ratio defined as the deformations ratio of inelastic vs. linear system has also been widely investigated (Miranda 2001, Chopra and Chintanapakdee 2003, 2004)

$$C = \frac{x_m}{x_0} = \frac{\mu}{R_y} = \begin{cases} C_\mu \ for\ fixed\ ductility\ factor\ \mu \\ C_R \ for\ fixed\ yield\ strength\ reduction\ factor\ R_y \end{cases} \tag{16}$$

By analogy, solutions of Eq. (15) provide the values of the inelastic deformation ratio defined as

$$C_\eta = \frac{\mu = \mathcal{M}_{max}(t)}{R_y = \Re_{y,max}(t)} \tag{17}$$

The bilinear system behavior described by Fig. 6 is transformed into Fig. 9 when using the new normalized parameters η and C_η.

As shown in Fig. 9, the inelastic deformation ratio C_η is defined as the ratio between the ductility demand μ and the yield strength reduction factor R_y. C_η is then affected by the normalized yield strength coefficient (η) and the post-to-preyield stiffness ratio (α) so that $C_\eta = 1$ when $\mu = R_y$.

Therefore, it appears that the peak ductility factor μ, the peak yield strength reduction factor R_y, and the inelastic deformation ratio C_η correspond to particular values of the

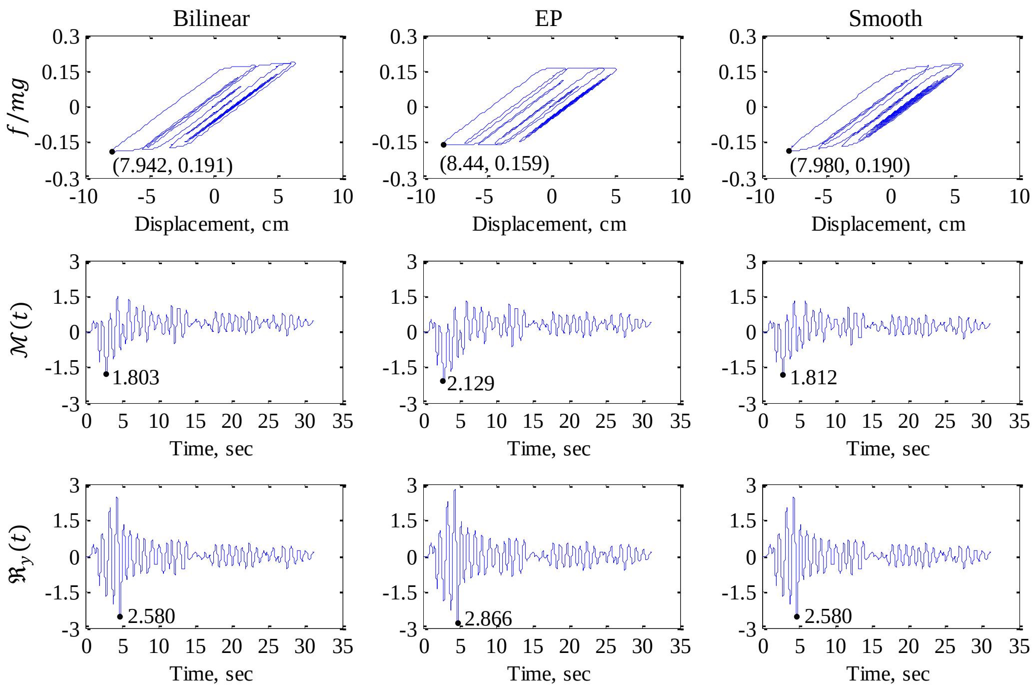


Fig. 7 Hysteretic response f/mg vs. $x(t)$, instantaneous ductility demand $\mathcal{M}(t)$and instantaneous yield st rength reduction factor $\mathfrak{R}_y(t)$ when η = 0.5, α = 10%, T = 1 sec

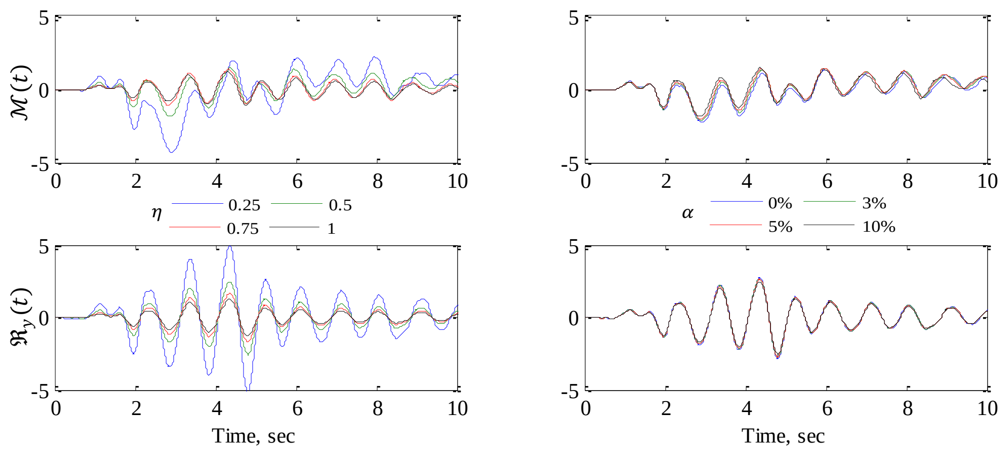


Fig. 8 Ductility demand and yield strength reduction factor time histories: effect of (η) and (α) when $T = 1$ sec

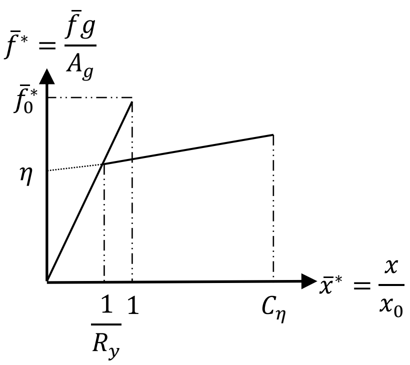


Fig. 9 Normalized bilinear behavior of the systems

corresponding parameters instantaneous values i.e., $\mathcal{M}(t)$, $\mathfrak{R}_y(t)$ and C_η. The SDOF systems response spectra is then entirely described once the governing parameters (η, α) are defined. It becomes then easy to obtain and construct the three specific spectra, i.e.:

- Ductility demand (μ)
- Yield strength reduction factor (R_y), and
- Inelastic deformation ratio C_η.

The required procedure to construct these spectra is thus summarized as follows:

1. Select the ground motion to investigate
2. Select and fix:
- the damping ratio ξ
- the range of natural period T
- the post-to-preyield stiffness ratio α and
- the normalized strength coefficient η.
3. Determine the instantaneous ductility $\mathcal{M}(t)$ and strength reduction factor $\mathfrak{R}_y(t)$ according to the values

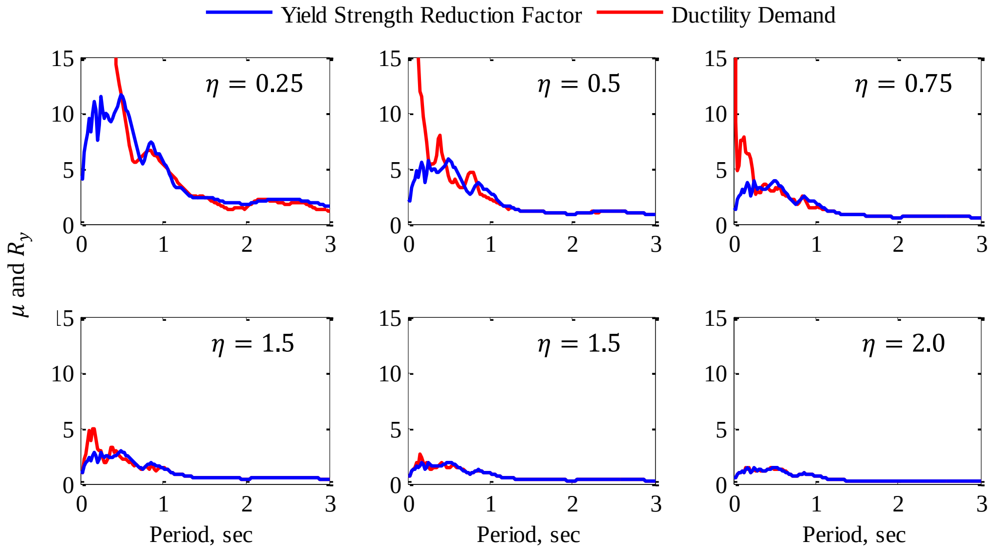


Fig. 10 Ductility demand and yield strength reduction factor spectra for different values of η and $\alpha = 0\%$ subjected to the El-Centro (N/S) components

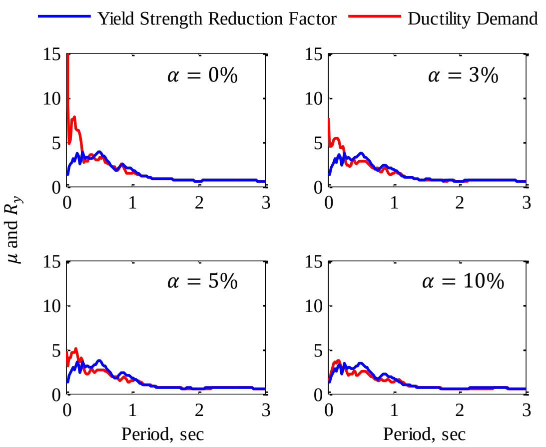


Fig. 11 Ductility demand and yield strength reduction factor spectra for different values of α and $\eta = 0.75$ subjected to the El-Centro (N/S) components

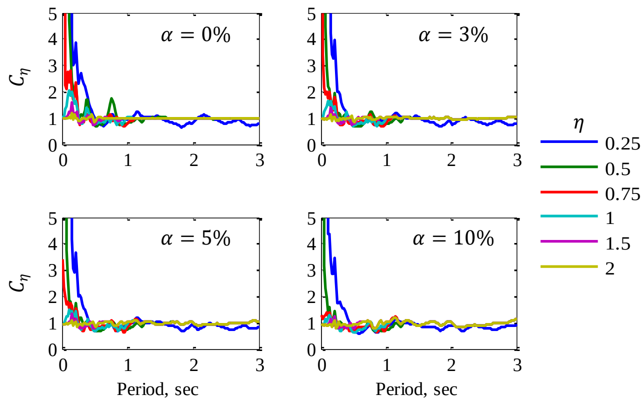


Fig. 12 Inelastic deformation ratio spectra for bilinear SDOF systems subjected to the El-Centro(N/S) components (influence of normalized yield strength coefficient η)

selected for T, ξ, $\eta$and α.

4. Derive from $\mathcal{M}(t)$ and $\Re_y(t)$ the peak ductility demandμ and the yield strength reduction factor R_y.

5. Repeat steps 3 and 4 for the given range of periods T. Calculate the inelastic deformation ratios $C_\eta(\xi,\ T,\ \alpha,\ \eta)$.

For illustrative purposes, the response spectra are plotted in Fig. 10 for perfect elasto-plastic behavior $(\alpha = 0\%)$ and for various values of the normalized strength coefficient $(\eta = 0.25, 0.5, 0.75, 1.0, 1.5$ and 2. Fig. 11 is plotted for fixed normalized strength coefficient $(\eta = 0.75)$ and four values of post-to-preyield stiffness ratio $(\alpha = 0, 3, 5$ and $10\%)$, under the effect of El-Centro 1940 ground motion with (N/S) component. From the obtained results it can be drawn that (see Figs. 10-11):

- The effect of η and α is significant. Their increase produces a decrease of μ and R_y values.
- When η increases, the μ spectra approaches the R_y spectra.In the short period region, $\mu > R_y$, and for $T > 1$ sec, $\mu = R_y$ for all values of η (see Fig. 10).
- The (α) ratio reduces μvalues for periods shorter than 0.6 sec, but has a small effect on R_y for all periods (see Fig. 11).

A parametric study is performed in order to study the influence of the normalized yield strength coefficient (η) and the post-to-preyield stiffness ratio (α) on the inelastic deformation ratio (C_η) (see Figs. 12-13). Though they differ greatly for given ranges of α and η, they take close values for systems for any T value when $\eta > 1$. Furthermore, they are equal to 1 for periods $T \geq 1 sec$. This confirms, as shown in Fig. 9, that for periods shorter than 0.6 sec and $\eta < 1$, $(1/C_\eta)$ is proportional to η and α.

The classical methods had up to now to distinguish two conditional parametric ratios C: C_μ when μ is constant and C_R when R_y is constant (Miranda 1993, Whittaker *et al*. 1998, Chopra and Chintanapakdee 2004).

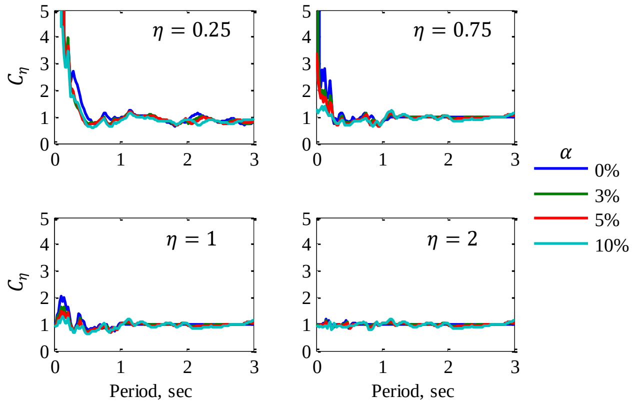


Fig. 13 Inelastic deformation ratio spectra for bilinear SDOF systems subjected to the El-Centro (N/S) components (influence of post-to-preyield stiffness ratio α)

The original method, developed in the present paper, provides similar results whereas it considers a unique parameter, i.e., the newly defined unique inelastic deformation ratio C_η whatever are μ and R_y values.

2.6 Comparison with existing formulations

Based on the fact that the inelastic deformation ratio is defined as a function of α and T, especially η, the new approach results are compared to those obtained by classical methods, Chopra and Chintanapakdee (2004), FEMA-346 (FEMA 2000) and FEMA-440 (FEMA 2005). Chopra and Chintanapakdee (2004) developed the following equation of the inelastic deformation ratio for bilinear systems:

- Deformation of systems with known ductility

$$C_\mu = 1 + \left[(L_\mu - 1)^{-1} + \left(\frac{a}{\mu^b} + c \right) \left(\frac{T}{T_c} \right)^d \right]^{-1} \qquad (18)$$

Where

$$L_\mu = \frac{\mu}{1 + (\mu - 1)\alpha} \qquad (19)$$

With: $a = 105, b = 2.3, c = 1.9$ and $d = 1.7$

- Deformation of systems with known R_y

$$C_R = 1 + \left[(L_R - 1)^{-1} + \left(\frac{a}{R_y^b} + c \right) \left(\frac{T}{T_c} \right)^d \right]^{-1} \qquad (20)$$

Where

$$L_R = \frac{1}{R_y} \left(1 + \frac{R_y - 1}{\alpha} \right) \qquad (21)$$

With: $a = 61, b = 2.4, c = 1.5$ and $d = 2.4$.

Where: L_R =Deformation ratio C_R for zero-period system; L_μ = Deformation ratio C_μ for zero-period system; T = Period of the SDOF system; T_c =Period separating acceleration- and velocity-sensitive regions in elastic response spectra; R_y =Yield strength reduction factor; μ=Ductility factor; and α=Post-to-preyield stiffness ratio.

The inelastic deformation ratio in **FEMA-356** (FEMA 2000) is given for fixed yield strength reduction factor R_y as C_1

$$\begin{aligned} C_1 &= 1 \qquad for\ T_e \geq T_s \\ C_1 &= \frac{\left[1.0 + \frac{(R-1)T_s}{T_e} \right]}{R} \qquad for\ \ T_e < T_s \end{aligned} \qquad (22)$$

Where: T_e=is the effective fundamental period of the SDOF model of the structure; T_s the period associated with the transition from the constant acceleration segment of the spectrum to the constant velocity segment of the spectrum; and R=the strength ratio computed with Eq. (23)

$$R = \frac{S_a}{V_y/W} C_m \qquad (23)$$

With S_a =Spectral acceleration; V_y =Yield strength; W =Effective seismic weight; and C_m =Effective model mass calculated for the fundamental mode.

A simple expression of the inelastic deformation ratio has been proposed in **FEMA-440** (FEMA 2005)

$$\begin{aligned} C_1 &= 1 + \frac{R - 1}{a\, T_e^2} \qquad for\ \ T \leq 1sec \\ C_1 &= 1 \qquad for\ T > 1sec \end{aligned} \qquad (24)$$

Where: a=130, 90 and 60 for site classes B, C and D, respectively.

Earthquake records collected in Algeria during Boumerdes Mw=6.8 earthquake which occurred on May 21, 2003 in Algiers region (Laouami *et al*. 2006), are selected as input motions for sensitivity analysis, see Table 1.

The mean response spectrum for inelastic deformation ratio is constructed by selecting a range of values for the period T. For the present study case, the period is considered as varyingwithin the interval 0.02 up to 3 seconds in order to study the inelastic deformation ratio response spectrum for each considered ground motion record, as described in section 2.5. Furthermore, the mean normalized spectra is calculated for the whole ground

Table 1 Peak ground accelerations during Boumerdes earthquake (Laouami *et al*. 2006)

	PGA(g)	
Station	East/West	North/South
Keddara1	0.34	0.26
Keddara2	0.58	0.35
Dar El Beida	0.52	0.46
Hussein Dey	0.27	0.23
Tizi Ouzou	0.2	0.19
Blida	0.05	0.04
Azazga	0.12	0.09
El Afroun	0.16	0.09
Hammam Righa	0.1	0.07
Miliana	0.03	0.026
Ain Defla	0.03	0.02

motions when $\eta = 0.5$, $\alpha = 10\%$ and for each value of T.

The ground motion is defined by the mean spectrum of the whole accelerograms, normalized by the PGA values(peak ground acceleration). The mean spectrum C_η for this set of records with $\xi = 5\%$is shown in Fig. 14.

Fig. 15 illustrates the inelastic deformation ratio calculated for ground motions records from Boumerdes earthquake in 2003 with $\eta = 0.5$, and $\alpha = 10\%$. The values of the inelastic deformation ratioobtained by the proposed approach are closeto those predicted by the most efficient existing methods (Chopra and Chintanapakdee 2004, FEMA 2000, 2005).

3. Conclusions

The new methodology develops a theoretical modeling of the inelastic deformation ratio (C_η) which is expressed as a function of the natural period (T), the post-to-preyield stiffness ratio (α) and the normalized yield strength coefficient (η). Thus, any SDOF bilinear systems motion can be expressed in terms of ductility demand and yield strength reduction factor, the equations of motions for inelastic and elastic systems being then solved separately. It subsequently requires the development of a unique combined effect, i.e., the inelastic deformation ratio (C_η) which is the ratio between the peak ductility demand and the yield strength reduction factor whatever are their individual values. The resolution of the resulting equations, for nonlinear systems, shows efficiency and requires small calculation time duration in order to obtain the maximal response of the structure. To study the efficiency of the method, applications have been performed for SDOF systems under the effect of a selected ground motion, El-Centro ground motion. The results show that the proposed ratio C_η is sensitive to the normalized yield strength coefficient (η) and the post-to-preyield stiffness ratio (α). Furthermore, it can be drawn that.

1. The inelastic deformation ratio (C_η) decreases with any increase of the influencing parameters, i.e., the normalized yield strength coefficient (η), the post-to-preyield stiffness ratio (α) or the period *(T)*.
2. When the normalized yield strength coefficient values is so that ($\eta > 1$), the inelastic deformation ratio (C_η) is not significantly influenced by the post-to-preyield stiffness ratio (α). Actually, when ($\eta > 1$), the system is likely to remain in the elastic range.
3. But, when the normalized yield strength coefficient values is so that ($\eta \leq 1$), (C_η) values decreases significantly with (α) for periods ($T < 0.6$ s), whereas this variation becomes very slight for long periods ($T > 0.6$ s).
4. As limit cases, (C_η) takes very large values for very rigid systems (i.e., natural period T~0 s), whereas it equals 1 for $T > 1$ s.

A sensitivity analysis of the proposed method can therefore be performed in order to investigate with further details individual as well as combined effects of the more influencing parameters such as:

- The effects of earthquakes magnitude as well as the distance from the site to the faults, i.e., usual cases of Large Magnitude Short distance for Rupture (LMSR), Large Magnitude Large distance for Rupture (LMLR), Small Magnitude Short distance for Rupture (SMSR), Small Magnitude Large distance for Rupture (SMLR);
- The soil effects following the National Earthquake Hazard Reduction Program (NHERP) classification for instance;
- The effect of the ductility (μ);
- The effect of the post-to-preyield stiffness ratio (α);
- As well as the damping and the fundamental periods.

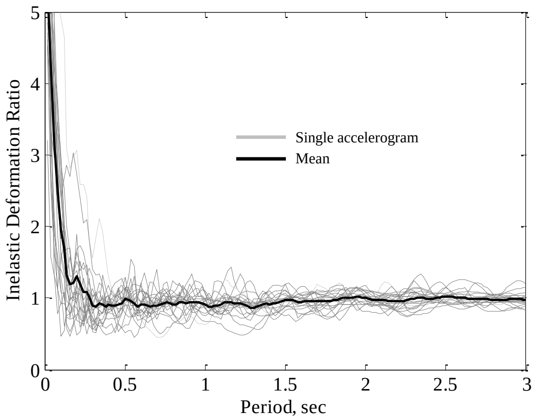


Fig. 14 Inelastic deformation ratio for individual accelerograms (Boumerdes earthquake 2003): set of ground motions and mean spectrum ($\alpha = 10\%, \eta = 0.5$ and $\xi = 5\%$)

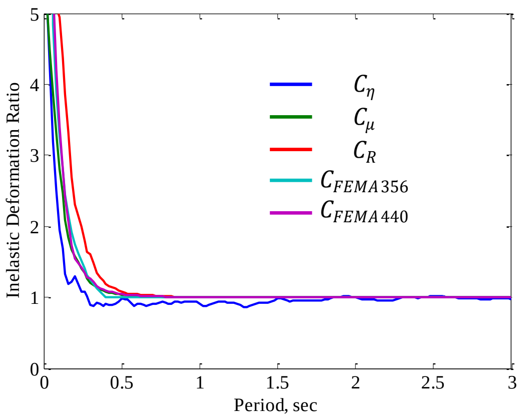


Fig. 15 Inelastic deformation ratio for Boumerdes earthquake 2003 with $\alpha = 10\%$ and $\eta = 0.5$: comparison between the proposed approachand Chopra-Chintanapakdee (2004), FEMA-356 (2000), and FEMA-440 (2005) results

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