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# Design aspects for minimizing the rotational behavior of setbacks buildings

George K. Georgoussis*

*Department of Civil Engineering, School of Pedagogical and Technological Education (ASPETE), N. Heraklion 14121, Attica, Greece*

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**Abstract.** An approximate analysis is presented for multi-story setback buildings subjected to ground motions. Setback buildings with mass and stiffness discontinuities are common in modern architecture and quite often they are asymmetric in plan. The proposed analysis provides basic dynamic data (frequencies and peak values of base resultant forces) and furthermore an overview of the building response during a ground excitation. The method is based on the concept of the equivalent single story system, which has been introduced by the author in earlier papers for assessing the response of uniform in height buildings. As basic quantities of the dynamic response of elastic setback buildings can be derived by analyzing simple systems, a structural layout of minimum elastic rotational response can be easily constructed. The behavior of such structural configurations, which is basically translational into the elastic phase, is also examined into the post elastic phase when the strength assignment of the various bents is based on a planar static analysis under a set of lateral forces simulating an equivalent 'seismic loading'. It is demonstrated that the almost concurrent yielding of all resisting elements preserves the translational response, attained at the end of the elastic phase, to the post elastic one.

**Keywords:** setback buildings; modal analysis; eccentric structures; modal centre of rigidity; inelastic analysis

## 1. Introduction

It is generally accepted that non-uniform distribution of mass and stiffness, both in plan and in elevation, is the main cause of the rotational response of building structures during strong ground motions, and in many cases this response has led to partial or total collapse. In recent years a number of investigations have been carried out to demonstrate the seismic vulnerability due to building asymmetry and mass or stiffness irregularity and qualitative reviews have been published from time to time on this issue (e.g., Chandler *et al*. 1996, Rutenberg 1998, De Stefano and Pintucchi 2008, De Stefano *et al*. 2015, Anagnostopoulos *et al*. 2013, 2015a, 2015b, Bosco *et al*. 2015, Meireles *et al*. 2014, Nezhad and Poursha 2015, Roy and Mahato 2013, Stathi *et al*. 2015). It should be noticed here that although the majority of relevant studies concludes that code-designed asymmetric structures are inadequate to provide a sound performance against strong seismic

*Corresponding author, Professor, E-mail: ggeorgo@tee.gr


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actions, a detailed literature review reveals that there are a few other investigations indicating the opposite results (Athanassiadou 2008, Das and Nau 2003).

In-plan stiffness irregularity is usually created by the eccentric location of the lateral force resisting elements, while an in-elevation irregularity is formed when an architectural setback (related to functional or aesthetical requirements) causes a sudden change of the size of the floor plans above a certain lever. In such cases the building planning is followed by the curtailment of some of the resisting elements and the drastic discontinuity of the floor mass. Note here, that modern building codes (e.g., EC8-2004) specify a full 3-dimensional dynamic analysis for structures which do not satisfy the regularity criteria. There are not recommendations of how the practicing engineer can assess the fundamental frequency by a simple formula or methodology and there are not provisions which allow the structural detailing by a pseudo-static structural design against an equivalent lateral load. In a recent publication (Sarkar *et al*. 2010) an empirical formula is proposed to assess the fundamental period of a setback building in relation to that of the corresponding regular structure. The proposed expression is based on a single 'regularity index', and differs from an earlier approach (Karavasilis *et al*. 2008) where two indices are used to represent the regularity of a stepped building. The need of an accurate assessment of the fundamental period is crucial in a practical application, especially when it falls in the velocity sensitive region, where the spectral acceleration is sensitive to the fundamental period.

The recognition of the seismic vulnerability of asymmetric or irregular building structures has also raised the issue of mitigating the torsional effects during a strong ground motion. To control and further to minimize these phenomena in multi-storey structures designed to withstand ground motions into the inelastic region, different element strength distributions were studied by Aziminejad *et al*. (2008) and Aziminejad and Moghadam (2009). They concluded that models with a smaller strength eccentricity perform better, but, in general, the optimum eccentricity is shown to be a function of the selected damage index. In their investigations the problem of element strength distribution on the rotational response of the structure is studied by using a proper configuration of the centers of mass, strength and stiffness according to the findings obtained from single story systems with elements having strength dependant stiffness (Myslimaj and Tso 2002, 2004). This approach, which is recently extended to include the soil-structure interaction (e.g. Shakib and Atefatdoost 2014), is based on Paulay's studies (1998, 2001) in individual concrete shear walls, where it is sufficient to assume that the stiffness is strength dependant. However, in multi-story lateral force resisting bents composed by a number of columns and beams, or further, by a combination of walls and frames, any analysis should be based on predetermined flexural stiffnesses, as it is specified by all building codes, on the grounds of the lateral dimensions of beams and columns.

The first objective of this work is to present an approximate method for assessing basic dynamic data (periods, base resultant forces) of multistory eccentric setback buildings. It is based on author's earlier papers (Georgoussis 2009, 2010, 2012) on uniform multi-story systems, where the aforementioned data can be found with reasonable accuracy by analyzing two equivalent single story modal systems. This methodology is now extended to irregular setback buildings with a mass and stiffness discontinuity. As this method is based on the element frequencies, which for the full-height bents are evaluated from the corresponding individual bents when they are assumed to carry, as planar frames, the mass of the complete structure, the main issue is the assessment of the corresponding frequencies of the bents which are curtailed at the lever of the setback. An approximate formula is proposed and the results of the proposed analysis are presented and compared with the accurate data provided the SAP2000 computer program for the case of 8-story buildings composed by frames, shear walls and coupled walls. The second objective of this work originates from the findings in uniform over the height building systems, where a structural configuration of minimum torsion implies that its practically translational elastic response during a ground excitation is preserved into the inelastic region when the strength assignment of its resisting bents is ‘stiffness proportional’. In other words, when a medium or low height building structure, in the linear phase, is practically responding in a translational mode, the effective seismic forces developed are basically proportional to the first translational mode of vibration. Therefore, a strength assignment obtained from a planar static analysis under a set of lateral loads simulating the aforesaid mode of vibration, represents a system in which all potential plastic hinges at the critical sections (at the ends of the beams and the foot of the ground floor columns and walls, according to the strong column-weak beam concept) are formed at the same instant. This static analysis can be further simplified by assuming that the lateral design load has the shape of the code ‘seismic loading’ (e.g., the shape of inverted triangle) and the moments developed at the critical sections may be taken as the yield moments at the potential plastic hinges. It has been demonstrated in uniform buildings (Georgoussis 2014, Georgoussis *et al*. 2013b, Georgoussis 2015) that the almost concurrent yielding of all resisting elements preserves the translational response, attained at the end of the elastic phase, to the post elastic one. Note that such a response has already been observed in single story systems and it is reminded here a statement by Lucchini *et al*. (2009): the nonlinear response depends on how the building enters the nonlinear range. At present this behavior is investigated in asymmetric multistory structures with a mass and stiffness irregularity. The aforementioned 8-story setback buildings are examined under the characteristic ground motion of Kobe-1995 (component KJM000), selected from the strong ground motion database of the Pacific Earthquake Engineering Research (PEER) Center (hppt://peer. berkely.edu).

## 2. Constructing equivalent single story systems

A typical mono-symmetric multi-storey building with a setback is shown in Fig. 1. The building is uniform over the height $H_b$, which defines the base structure and consists of an $N_b$ number of stories with a uniformly distributed mass, equal to $m_b$ per floor, and a radius of gyration equal to $r_b$ (this is the radius of gyration of the floor mass with respect to the center of mass). Above this level, it has a setback forming a uniform tower structure consisting of $N_t$ stories of a reduced floor plan with a mass per floor equal to $m_t$ and a radius of gyration equal to $r_t$. The height of the tower structure is assumed equal to $H_t$ and the overall number of stories is equal to $N(=N_b+N_t)$. Each floor consists of a rigid slab (deck) and at present the centers of mass (CM) at each floor are assumed to lie on the same vertical line (mass axis) which is passing through the centroids of all decks. All bents within the perimeter of the tower structure (rigid frames, shear walls, coupled wall systems, etc.) extend up to the top of the building, while those outside this area are assumed to be curtailed at the level of the setback.

The methodology to analyze elastic setback buildings like that of Fig. 1 is outlined in author’s earlier papers (Georgoussis 2011a, 2011b). The backbone of this method is based on Southwell’s formula (Newman and Rosenblueth 1972) and it is similar to that applied to uniform over the height systems (Georgoussis 2009, 2010, 2012), which are analyzed by two equivalent single story systems. For a ground excitation along, say, the $y$-direction, each of the equivalent systems has a mass equal to the n-mode effective mass, $M_n^*$ ($n$=1,2) of the uncoupled multi-story structure in the same direction, and it is supported by elements with a stiffness equal to the product of $M_n^*$ with the

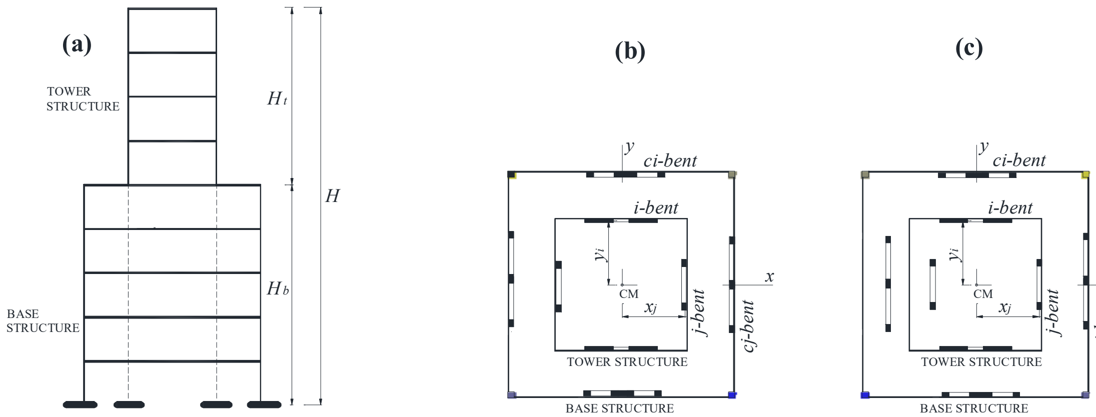


Fig. 1 (a) Multistory setback building with (b) a symmetrical plan configuration; (c) an asymmetric configuration

first mode (when $n$=1) or second mode (when $n$=2) squared element frequencies of the corresponding real bents of the assumed multi-story structure. The center of stiffness of each of the aforementioned equivalent systems represents the corresponding modal center of rigidity ($m_1$-CR for the first ($n$=1) and $m_2$-CR for the second ($n$=2) equivalent single story system) and the significance of $m_1$-CR in uniform systems is outlined in detail in the aforementioned papers (in brief: when the structural layout of a given multistory structure produces an $m_1$-CR point very close to the mass axis, the torsional response is practically negligible). However, the main difference in setback buildings is the calculation of the element frequencies.

These frequencies, for the full height bents, are determined from the corresponding individual bents when they are assumed to carry, as planar frames, the mass of the complete structure. This procedure is not applicable to the curtailed bents, which at present are calculated by means of an indirect method, which requires first the calculation of the first two mode frequencies of the uncoupled multi-story structure (Figs. 1(a) and 1(b)). This procedure is as follows:

The setback building of Fig. 1(a) with the symmetrical plan configuration of Fig. 1(b) (which may be seen as the uncoupled structure of the building with the asymmetric plan layout of Fig. 1(c)) has, for the first two modes of vibration ($n$=1,2) along the y-direction, an effective modal stiffness equal to

$$k_n^* = \omega_n^2 M_n^* \tag{1}$$

where the frequencies $\omega_n$ are given as

$$\omega_n^2 = \frac{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{K}_{\mathrm{ov}}\mathbf{\Phi}_n}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n} = \frac{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{K}_{\mathrm{v}}\mathbf{\Phi}_n}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n} + \frac{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{K}_{\mathrm{cv}}\mathbf{\Phi}_n}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n} = \Sigma\frac{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{k}_{\mathrm{j}}\mathbf{\Phi}_n}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n} + \Delta\omega_n^2 \tag{2}$$

$\mathbf{M}$ is mass matrix of the assumed structure (as defined below), $\mathbf{\Phi}_n$ is the n-mode shape vector ($n$=1,2) in the $y$-direction and $\mathbf{K}_{ov}$ is its lateral stiffness matrix in the same direction, which may be expressed by two parts: $\mathbf{K}_v$ and $\mathbf{K}_{cv}$, representing respectively the stiffness of the full height and the curtailed bents in the y-direction. i.e.

$$\mathbf{K}_{\mathrm{ov}} = \mathbf{K}_{\mathrm{v}} + \mathbf{K}_{\mathrm{cv}} = \Sigma\mathbf{k}_j + \Sigma\mathbf{k}_{cj} \tag{3}$$

($\mathbf{k}_j$ is the $N\times N$ lateral stiffness matrix of the j-full height bent aligned in the $y$-direction and $\mathbf{k}_{cj}$ is the corresponding matrix of the cj-curtailed bent). Note here that the latter matrix has zero elements below (beyond) the $N_b$ row (column), i.e.

$$\mathbf{k}_{cj}=\begin{bmatrix}\mathbf{k}_{cj}^{e} & 0\\ 0 & 0\end{bmatrix} \tag{4}$$

where $\mathbf{k}_{cj}^{e}$ is the effective, $N_b\times N_b$, stiffness matrix of the cj-curtailed bent.

The ratios $\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{k}_{\mathrm{j}}\mathbf{\Phi}_n/\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n$ ($j$=1,2,..) of the last term of Eq. (2) may be approximated by the squared element frequencies of the j-full height bents, which are calculated under the assumption that each of these bents carries, as a planar frame, the mass of the complete structure. In fact, this approximation is based on the potential of the Rayleigh's quotients. These quotients, which are traditionally used to determine a close estimate of the first mode frequency of a given structure when an approximate first mode shape is used, are particularly useful in the case of building structures, which belong to the same family of shear-flexure cantilever systems and have similar not only the first mode, but also the higher mode shapes. Therefore, replacing the shape vector $\mathbf{\Phi}_n$ with the corresponding vector $\mathbf{\Phi}_{jn}$ of the j-bent when it is assumed to carry the complete mass of the assumed building, each of the aforementioned ratios may be taken as the element squared frequency of the j-bent, i.e.

$$\omega_{jn}^2=\frac{\mathbf{\Phi}_{jn}^{\mathrm{T}}\mathbf{k}_{\mathrm{j}}\mathbf{\Phi}_{jn}}{\mathbf{\Phi}_{jn}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_{jn}}\quad (n=1,2) \tag{5}$$

It is worth mentioning here that in the case of buildings composed by very dissimilar bents, a better estimate of the aforementioned first mode ratios ($n$=1) is the effective element frequencies, which are given as (Georgoussis *et al*. 2013a, Georgoussis 2014)

$$\overline{\omega}_{j1}^2=\omega_{j1}^2(M_{j1}^*/M_n^*) \tag{6}$$

where $M_{j1}^*$ is the effective first mode mass of the j-full height bent. For the second mode of vibration, the corresponding effective element frequencies, $\overline{\omega}_{j2}$, can be simply taken equal to $\omega_{j2}$.

Using the expressions of Eq. (5) (or those of Eq. (6) if appropriate) into the Eq. (2), the overall contribution of the curtailed bents to the modal stiffness of the uncoupled structure (Eq. (1)) is determined as

$$\Delta\omega_n^2=\omega_n^2-\Sigma\omega_{jn}^2=\frac{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{K}_{\mathrm{cv}}\mathbf{\Phi}_n}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n}=\Sigma\frac{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{k}_{\mathrm{cj}}\mathbf{\Phi}_n}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n} \tag{7}$$

The contribution of each of the curtailed bents may now be evaluated by interpreting the ratios $\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{k}_{\mathrm{cj}}\mathbf{\Phi}_n/\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n$ in the expression above. Defining this ratio as the effective element square frequency of the cj-curtailed bent, i.e.

$$\omega_{cjn}^2=\frac{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{k}_{cj}\mathbf{\Phi}_n}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n}=\frac{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{k}_{cj}\mathbf{\Phi}_n}{\mathbf{\Phi}_{bn}^{\mathrm{T}}\mathbf{M}_b\mathbf{\Phi}_{bn}}\frac{\mathbf{\Phi}_{bn}^{\mathrm{T}}\mathbf{M}_b\mathbf{\Phi}_{bn}}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n}=\frac{\mathbf{\Phi}_{bn}^{\mathrm{T}}\mathbf{k}_{cj}^{e}\mathbf{\Phi}_{bn}}{\mathbf{\Phi}_{bn}^{\mathrm{T}}\mathbf{M}_b\mathbf{\Phi}_{bn}}\frac{\mathbf{\Phi}_{bn}^{\mathrm{T}}\mathbf{M}_b\mathbf{\Phi}_{bn}}{\mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n}=\rho_{cjn}^2\lambda_n^2 \tag{8a}$$

where

$$\rho_{cjn}^2 = \frac{\mathbf{\Phi}_{bn}^{\mathrm{T}} \mathbf{k}_{cj}^{e} \mathbf{\Phi}_{bn}}{\mathbf{\Phi}_{bn}^{\mathrm{T}} \mathbf{M}_b \mathbf{\Phi}_{bn}} \tag{8b}$$

$$\lambda_n^2 = \frac{\mathbf{\Phi}_{bn}^{\mathrm{T}} \mathbf{M}_{\mathrm{b}} \mathbf{\Phi}_{bn}}{\mathbf{\Phi}_n^{\mathrm{T}} \mathbf{M} \mathbf{\Phi}_n} \tag{8c}$$

a convenient interpretation may be given through the shapes of the mode vectors shown in Fig. 2. As $\mathbf{\Phi}_n$ ($n$=1,2) indicates the n-mode shape vector (of order $N\times1$) of the assumed uncoupled building, the sub-vector $\mathbf{\Phi}_{bn}$ (of order $N_b\times1$) may be seen as its part which represents the deflections of the base structure. This sub-vector is diagrammatically shown by solid lines in Fig. 2, together with the sub-vector of the tower structure $\mathbf{\Phi}_{tn}$, of order $N_t\times1$, shown by dotted lines ($\mathbf{\Phi}_n^{\mathrm{T}} = \left\langle \mathbf{\Phi}_{bn}^{\mathrm{T}}, \mathbf{\Phi}_{tn}^{\mathrm{T}} \right\rangle$). It can be seen from the shape of $\mathbf{\Phi}_{b1}$ that the ratio $\rho_{cj1}^2$ for the first mode of vibration ($n$=1) may be approximated by the first frequency of the cj-curtailed bent when it is assumed to carry the mass of the base structure. Therefore, as the generalized coefficient $\lambda_n$ is a common factor for all the curtailed bents, it is evident that the frequency difference $\Delta\omega_1^2$ of Eq. (7), when $n$=1, which represents the total contribution of all the curtailed bents, should be distributed among them in proportion to the squares of their first frequencies, $\omega_{cj1}^2$. That is, the effective first mode frequencies of the cj-curtailed bents, which define their contribution to the modal stiffness of the corresponding equivalent single story system, are taken as

$$\overline{\omega}_{cj1}^2 = \Delta\omega_1^2 \frac{\omega_{cj1}^2}{\Sigma\omega_{cj1}^2} \tag{9}$$

This concept cannot be easily extended to the higher modes of vibration. The shape of the second mode of vibration ($\mathbf{\Phi}_{b2}$ in Fig. 2) does not lead to similar interpretations. In this case ($n$=2), $\rho_{cj2}^2$ cannot be taken as a particular square frequency. In setback buildings with a small tower structure, $\rho_{cj2}^2$ may be considered as the second mode square frequency of the cj-bent, $\omega_{cj2}^2$, computed again on the grounds of the assumption that it carries the mass of the base structure. On the other hand, in buildings with a short and stiff base structure, $\rho_{cj2}^2$ may be taken as the first mode frequency, $\omega_{cj1}^2$, of the cj-bent. At present, the overall frequency difference $\Delta\omega_2^2$ for the second mode of vibration is distributed among the curtailed bents by the following formula, which expresses their effective second mode frequencies and provides their contribution to the second mode stiffness of the equivalent single story system, i.e.

$$\overline{\omega}_{cj2}^2 = (1 - \frac{H_b}{H})\Delta\omega_2^2 \frac{\omega_{cj1}^2}{\Sigma\omega_{cj1}^2} + \frac{H_b}{H}\Delta\omega_2^2 \frac{\omega_{cj2}^2}{\Sigma\omega_{cj2}^2} \tag{10}$$

As stated above, for an asymmetrical setback building, as that in Figs. 1(a) and 1(c), which is excited by a ground motion in the $y$-direction, the n-mode equivalent single story system ($n$=1, 2) is composed by a mass equal to $M_n^*$ and supported by elements in the $y$-direction having stiffnesses equal to the product of $M_n^*$ with the effective element frequencies as given by the Eqs. (6) and (9) (or Eq. (10) if appropriate). These elements are located at the positions of the real bents and the effective stiffnesses of the elements aligned in the $x$-direction are computed in a similar way. The

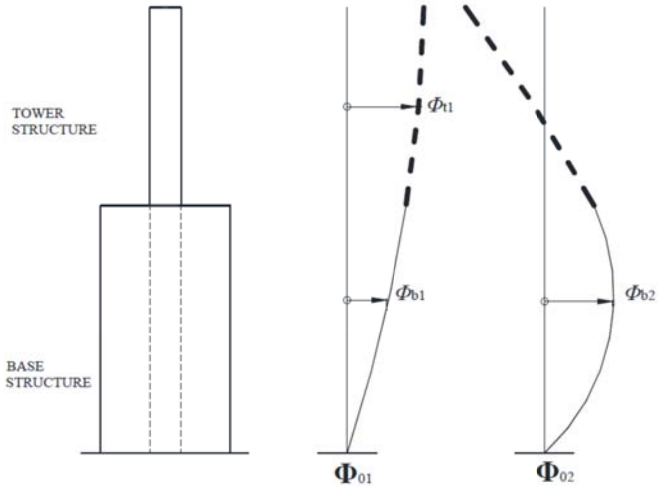


Fig. 2 The first two vibration shapes of a symmetrical setback building

radius of gyration of the aforementioned equivalent single story system $r_{en}$ is taken equal to $\bar{r}_{en} r_b$, where $\bar{r}_{en}$ represents a Rayleigh's quotient, given as (Georgoussis 2011a)

$$\bar{r}_{en} = \omega_{yn} / \omega_{ryn} \quad (\text{or } \bar{r}_{en} = \omega_{xn} / \omega_{rxn}) \tag{11}$$

where $\omega_{yn}$ (or $\omega_{xn}$) is the n-mode frequency of the uncoupled multistory structure in the y-direction (or $x$-direction) and $\omega_{ryn}$ (or $\omega_{rxn}$) the corresponding frequency of the same structure when the mass in the floors of the tower structure is reduced to $m_{rt} = (r_t / r_b)^2 m_t$. It has been shown (Georgoussis 2011a) that in common setback buildings, the ratio $\bar{r}_{en}$ is very little dependent on the type of the lateral load resisting system (frame, wall, dual system). Therefore, any of the expressions of Eq. (11) may be used for practical applications, but it is advisable to use the mean value of these expressions, since this averaging procedure utilizes the response of the structure in both directions.

## 3. Analyzed systems

To illustrate the application and accuracy of the proposed method, the setback systems (Example buildings 1 and 2) shown in Fig. 3 were analyzed. Both systems are 8-story mono-symmetric buildings, which are divided in two substructures: the base structure represents a uniform building system composed by floors of 22×15 m, while the tower substructure is composed by floors of reduced dimensions 15×10 m. The full height lateral load resisting bents, within the perimeter of the tower section, are two structural walls (Wa and Wb) and a moment resisting frame (FR) which are aligned along the $y$-direction and a pair of coupled-wall bents (CW) which is oriented along the $x$-axis of symmetry. The structural walls Wa and Wb are of cross sections 30×500 cm, the moment resisting frame FR consists of two 75×75 cm columns, 5 m apart, connected by beams of a cross section 40×70 cm, while the CW bents are composed two 30×300 cm walls, 5 m apart, connected by lintel beams of a cross section 25×90 cm at the floor levels. The latter bents are located symmetrically to CM at the edges of the floors of the tower structure, that is at distances equal to ±5

m. The curtailed bents, extending up to the setback level, are a moment resisting frame FRcy and a shear wall Wcy along the $y$-direction and a pair of shear walls Wcx along the $x$-direction, which are located in a symmetrical configuration at the edges of the base floor. The member dimensions of FRcy are the same as those of FR, while the curtailed walls Wcy and Wcx are of a cross-section 30×300 cm. The mass of the base floors is $m_b$=264 t (kNs$^2$/m), the radius of gyration about CM is $r_b$=7.687 m and the corresponding quantities of the tower structure are equal to $m_t$=120 t (kNs$^2$/m) and $r_t$=5.204 m respectively. The story height is 3.5 m and the modulus of elasticity ($E$) is assumed equal to 20×10$^6$ kN/m$^2$, typical for concrete structures. The centers of mass of the floor slabs lie on a same vertical line, which passes through the centroids of all the orthogonal floor plans of the example structures.

In Example building 1 the curtailed frame FRcy lies on the $x$-axis and is located on the right of CM at the edge of the base structure, at $x$=11 m, while the curtailed wall Wcy is located on the other side of CM at $x$=-11 m. In Example building 2 the aforementioned curtailed bents are located in a reversed order as shown in Fig. 3. In both example structures, the wall Wa and frame FR are assumed to be located at fixed positions, the first on the left of CM in a distance equal to 4 m and the second on the right of CM at a distance of 6.5 m, while the second wall Wb is taking all the possible locations along the $x$-axis within the limits of the tower section. Three different models are examined for each of the example buildings described above. In the first model of Example building 1 (T2-B6:m1) the tower structure consists of two floors, in the second model (T4-B4:m1) of four floors and, finally, the third model (T6-B2:m1) has a tower structure consisting of six floors. The same models of Example building 2 (T2-B6:m2, T4-B4:m2, T6-B2:m2) are formed in a similar way.

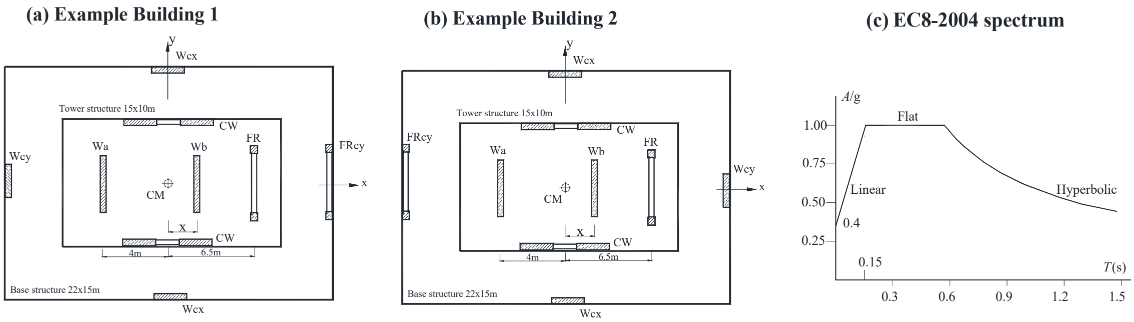


Fig. 3 (a),(b) Plan configurations of analyzed setback buildings; (c) Eurocode 8 acceleration spectrum

Table 1 Modal data of the uncoupled multistory systems

| Model | Dir | 1st mode data | | | 2nd mode data | | |
|---|---|---|---|---|---|---|---|
| | | $\omega_1$ | $\overline{M}_1^*$ | $\overline{r}_{e1}$ | $\omega_2$ | $\overline{M}_2^*$ | $\overline{r}_{e2}$ |
| T2-B6 | $y$ | 6.395 | 0.654 | 0.870 | 29.706 | 0.207 | 0.873 |
| | $x$ | 5.970 | 0.673 | 0.878 | 25.472 | 0.178 | 0.863 |
| T4-B4 | $y$ | 7.079 | 0.581 | 0.734 | 29.945 | 0.264 | 0.877 |
| | $x$ | 6.835 | 0.607 | 0.743 | 24.502 | 0.220 | 0.861 |
| T6-B2 | $y$ | 7.076 | 0.548 | 0.681 | 35.363 | 0.270 | 0.770 |
| | $x$ | 6.938 | 0.561 | 0.681 | 28.726 | 0.235 | 0.758 |

At first the periods/frequencies of the assumed models, for all possible locations of Wb, are examined. The accuracy of the proposed approximate procedure to predict periods of vibrations is investigated by comparison with the results derived from the computer program SAP2000-V11. In the computer analyses, the out of plane stiffness of the bents was neglected and in the wide column analogy used to simulate the CW bents the clear span of the coupling beams was increased by the depth of the beams (Coull and Puri 1968).

To apply the proposed method, the first pair of frequencies, $\omega_n$ ($n$=1,2), and the effective modal masses $M_n^*$ (as shown in Eq. (1)) of the uncoupled multi-storey systems are required together with $\bar{r}_{en}$ (Eq. (11)), which represents the ratio of the radius of gyration of the equivalent single story systems to the radius of gyration of the base structure. They are shown in Table 1 for all example structures, and since they are derived from the translational response of symmetrical systems (basically planar analyses), they are the same for the models of equal number of base and tower floors (e.g., the above quantities are the same for models T2-B6:m1 and T2-B6:m2). Note that in this Table the effective modal masses are shown as ratios, $\overline{M}_n^*$, of the total mass of the assumed models. It is notable that the second mode effective mass of the setback systems with a rather low base structure (models T4-B4 and T6-B2) has an increased participation, while the corresponding first mode effective mass is rather low (in uniform common type buildings, $\overline{M}_1^*$ is higher than 0.62 and $\overline{M}_2^*$ less than 0.19 (Georgoussis 2014). The increased influence of the second and third mode effective mass in setback buildings has first reported by Wong and Tso (1994), and in a parametric form the variation of the first three effective modal masses, in typical setback buildings, is shown in

Table 2 Modal data of the individual bents

| Model | Dir | Bent | Type | 1st mode data | | | 2nd mode data | |
|---|---|---|---|---|---|---|---|---|
| | | | | $\omega_{j1}$ | $\overline{M}_{j1}^*$ | $\overline{\omega}_{j1}$ | $\omega_{j2}$ | $\overline{\omega}_{j2}$ |
| T2-B6 | $y$ | Wa, Wb | Full height | 3.647 | 0.643 | 3.616 | 18.974 | 18.974 |
| | | FR | # | 2.141 | 0.777 | 2.334 | 6.821 | 6.821 |
| | | Wcy | Curtailed | 2.336 | - | 2.064 | 14.007 | 8.763 |
| | | FRcy | # | 2.539 | - | 2.244 | 8.502 | 6.252 |
| | $x$ | CW | Full height | 3.819 | 0.685 | 3.853 | 15.589 | 15.589 |
| | | Wcx | Curtailed | 2.336 | - | 1.725 | 14.007 | 9.022 |
| T4-B4 | $y$ | Wa, Wb | Full height | 4.157 | 0.577 | 4.143 | 19.591 | 19.591 |
| | | FR | # | 2.477 | 0.744 | 2.803 | 6.926 | 6.926 |
| | | Wcy | Curtailed | 4.851 | - | 2.168 | 27.728 | 6.926 |
| | | FRcy | # | 4.019 | - | 1.796 | 14.129 | 4.605 |
| | $x$ | CW | Full height | 4.386 | 0.625 | 4.451 | 15.995 | 15.995 |
| | | Wcx | Curtailed | 2.030 | - | 1.883 | 27.728 | 6.653 |
| T6-B2 | $y$ | Wa, Wb | Full height | 4.314 | 0.550 | 4.322 | 23.500 | 23.500 |
| | | FR | # | 2.669 | 0.688 | 2.991 | 8.063 | 8.063 |
| | | Wcy | Curtailed | 15.349 | - | 1.689 | 71.879 | 7.917 |
| | | FRcy | # | 8.680 | - | 0.955 | 33.573 | 4.284 |
| | $x$ | CW | Full height | 4.597 | 0.587 | 4.701 | 19.082 | 19.082 |
| | | Wcx | Curtailed | 15.349 | - | 1.403 | 71.879 | 6.962 |

Georgoussis (2011a). The element frequencies, and the effective ones, of the full-height bents (Eqs. (5) and (6)) are shown in Table 2, together with the effective first mode masses of the corresponding full height bents. In the same Table are also shown the frequencies of the curtailed bents, when they are assumed to carry the mass of the base structure and their effective frequencies as expressed by Eqs. (9) and (10).

## 4. Model frequencies and observed linear seismic response

The first four periods of vibration of the model setback structures, computed by the proposed method on the grounds of the effective element frequencies (red lines) for different locations of the Wb (indicated by the normalized coordinate ), are shown in Fig. 4, together with the accurate SAP2000 computer values (black lines). In all cases, the error, in absolute values, is less than 2.6% for the first mode of vibration, less than 2.9% for the second mode, less than 4.9% for the third mode and only for the forth mode of vibration this error goes up to 12%. The above estimates are quite satisfactory for practical applications and it is reminded that unsafe spectral acceleration values may be derived by overestimating the periods of a given structure. This is particularly essential for the first mode period, when it falls in the velocity sensitive region of the acceleration spectrum (descending branch of the spectrum shown in Fig. 3(c)).

Normalized base shears (in the $y$-direction), torques and rotations, derived by the proposed approximate procedure for the case of the EC8-2004 acceleration response spectrum (Fig. 3(c)), are shown in Fig. 5 for all models of the example buildings. They are plotted for all possible locations of the wall Wb and the shears have been normalized with respect to the total shear, along the $y$-direction, $V_o$, of the corresponding uncoupled structure, while the base torques are also divided by the radius of gyration of the base structure $r_b$ (i.e., $\overline{V}_{ap} = V_{ap} / V_o$ , $\overline{T}_{ap} = T_{ap} / r_b V_o$ ). All the aforesaid data are shown in red lines and in the same figure are also shown the accurate data $\overline{V}_{com}$ and $\overline{T}_{com}$ (black lines) given by the computer program SAP2000-V11 on the basis of the first 12 peak modal values combined according to the CQC rule (the damping ratio in each mode of vibration was taken equal to 5%). The prediction of the base shears is quite reasonable, but the approximate base torques are not always as close to the accurate ones as in the case of shears. In most of the cases, both methods provide data with a rather flat variation, where there is not a location of Wb which produces a more or less zero torsional response. Only in the case of model T2-B6:m2 it is clear that around the location of $\overline{x} = 0.25$ the system undergoes a practically negligible torsional response. To investigate further the conditions for minimum rotational response, the rotations obtained by the approximate method ($\Theta_{app}$) are also shown in Fig. 5. It should be noted here that these data, represent rotations obtained by the CQC rule from two equivalent single story systems with different characteristics. Not only the masses and the radii of gyration ($\overline{M}_n^*$ and $\overline{r}_{en}$, $n$=1,2) shown in Table 1 are different, but also their heights are different. For the building structures under consideration (T2-B6, T4-B4 and T6-B2 of both models), the first mode effective masses ($\overline{M}_1^*$) of the corresponding uncoupled systems (Table 1) correspond to effective modal heights equal to 68%, 70% and 75% of the total height respectively, while the second mode effective masses ($\overline{M}_2^*$) correspond to an effective height approximately equal to 19% of the total height. However, as the first mode data are having the dominant effect on the overall response, it was considered appropriate that the approximate $\Theta_{app}$ results should be shown in combination with the accurate values, $\Theta_{com}$, obtained from SAP2000 program at the level (story) which is closer to the first mode effective height of the corresponding

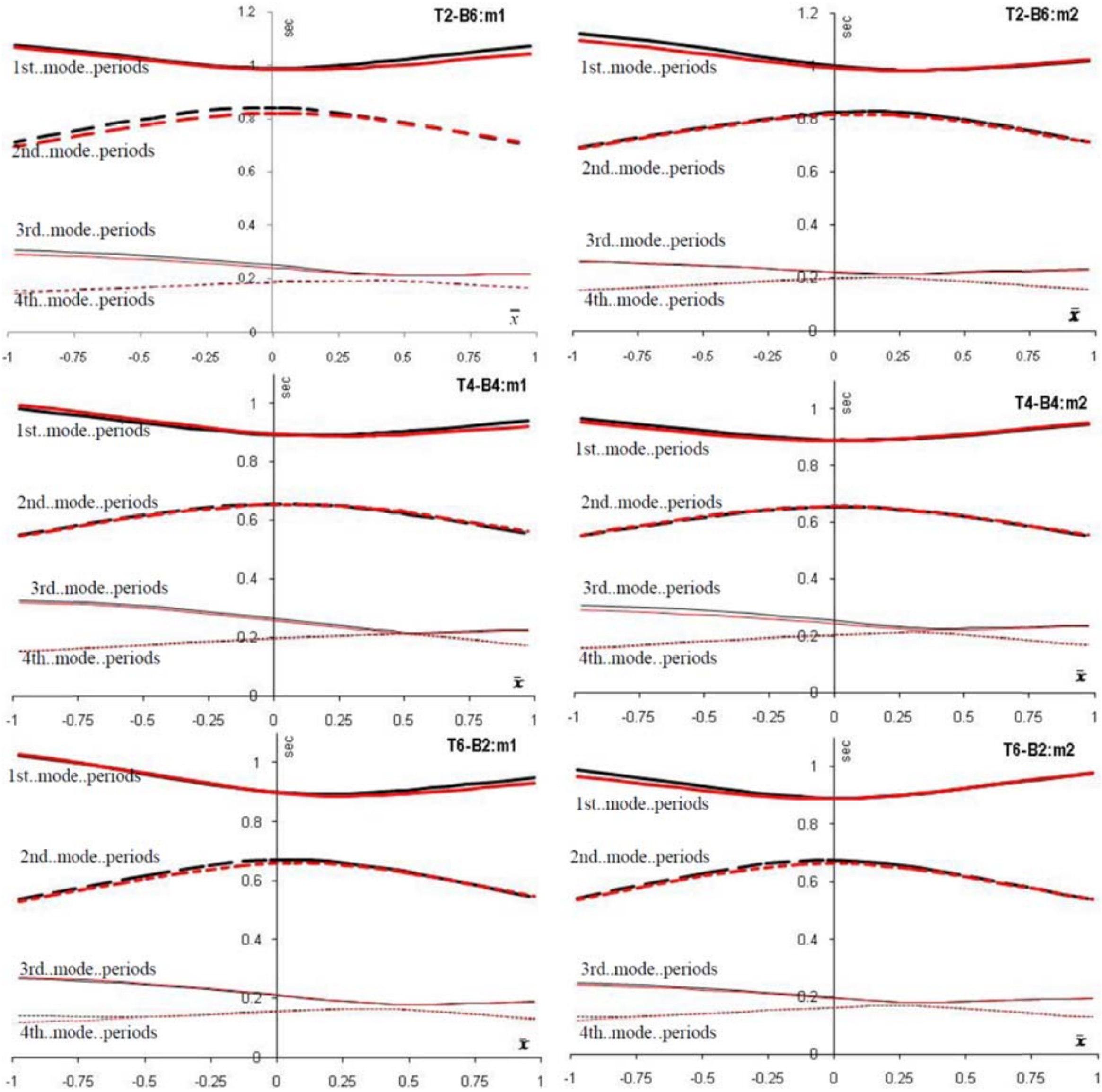


Fig. 4 Vibration periods of the example buildings

model (5th floor for T2-B6 and 6th floor for models T4-B4 and T6-B2). There is no theoretical basis to compare the specific values of $\Theta_{com}$ and $\Theta_{app}$, but their curves provide useful information. As a general observation it can be said that their shape reveals to specific location of Wb which denotes an almost zero rotational behavior. These points are not the same for both $\Theta_{com}$ and $\Theta_{app}$ curves, but with exception of model T4-B4:m1, their location is close to each other. In the models with a rather high base structure (T2-B6:m1 and T2-B6:m2), their location is very close to the location of minimum base torque and also very close to the coordinates of Wb ($\bar{x} = 0.08$ and 0.26 for the aforesaid models respectively) which produces a first mode center of rigidity ($m_1$-CR) coincident to the mass axis. These results are in agreement with the findings in uniform building structures

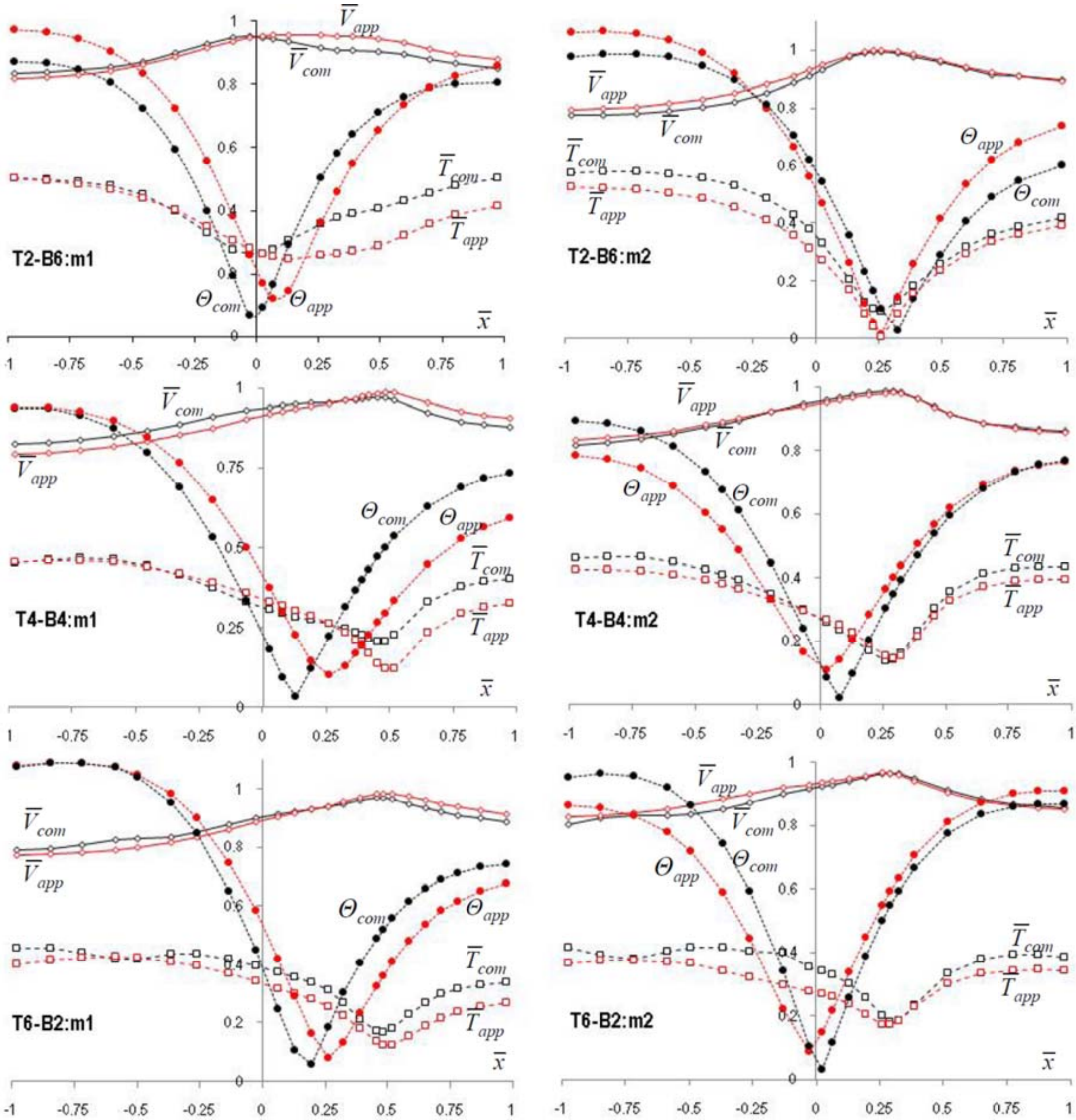

Fig. 5 Normalized elastic base shears, torques and rotations (×10$^{-2}$ rads) of example buildings

(Georgoussis *et al*. 2013a, Georgoussis 2014), where, when the structural layout produces an $m_1$-CR point very close to the mass axis, the torsional response is practically negligible. In the models with a taller tower structure (T4-B4:m1, T4-B4:m2, T6-B2:m1 and T6-B2:m2) the location of Wb of almost zero rotation deviates from that of minimum base torque. However, in these models the approximate method ($\Theta_{app}$ curve) presents a minimum at a location which is almost identical to the coordinate of Wb ( $\bar{x} = 0.26$ , 0.01, 0.25 and -0.03 for the aforesaid models respectively) which again produces a first mode center of rigidity ($m_1$-CR) coincident to the mass axis. In three of the aforementioned models (T4-B4:m2, T6-B2:m1 and T6-B2:m2) this point is also close to that of minimum rotation found by the accurate results ($\Theta_{com}$ curve). Only in model T4-B4:m1the curves of $\Theta_{com}$ and $\Theta_{app}$ indicate points of minimum rotation which are a bit further apart.

## 5. Observed non-linear response

The inelastic response of the assumed model structures was investigated under the ground motion of Kobe 1995 (component KJM000), selected from the strong ground motion database of the Pacific Earthquake Engineering Research (PEER) Center (hppt://peer.berkely.edu) and scaled to a PGA=0.5 g (unidirectional excitation along the $y$-axis). All the nonlinear response history analyses were performed by means of the program SAP2000-V11, using inelastic link elements at the assumed locations of plastic hinges. The moment-rotation relationships of these elements were assumed bilinear with a post-yielding stiffness ratio of the generalized load-deformation curve, equal to 4%. The reason for assuming this bi-linear form of these relationships was to accelerate the convergence of the step by step time history analyses. The aforesaid analyses were performed using the numerical implicit Wilson-θ time integration method, with the parameter θ taken equal to 1.4. By means of these analyses, base shears, torques and rotations were calculated for all the possible locations of wall Wb of the assumed models, as shown in Fig. 6. The lateral load resisting bents were assumed to have only in-plane stiffness and their strength assignment was based on a planar static

Table 3 Yield moments at critical sections of individual bents

| Model | Dir | Bent | Col. yield moment (basement) | Beam yield moment/floor | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| T2-B6 | $y$ | Wa,Wb | 33771 | | | | | | | | |
| | | FR | 436 | 352 | 558 | 695 | 769 | 793 | 788 | 808 | 640 |
| | | Wcy | 8260 | | | | | | | | |
| | | FRcy | 438 | 366 | 586 | 739 | 818 | 896 | 727 | | |
| | $x$ | CW | 10071 | 256 | 422 | 514 | 550 | 545 | 510 | 465 | 433 |
| | | Wcx | 10086 | | | | | | | | |
| T4-B4 | $y$ | Wa,Wb | 28630 | | | | | | | | |
| | | FR | 369 | 296 | 470 | 583 | 659 | 711 | 719 | 739 | 586 |
| | | Wcy | 6995 | | | | | | | | |
| | | FRcy | 372 | 326 | 522 | 702 | 614 | | | | |
| | $x$ | CW | 8253 | 208 | 340 | 412 | 445 | 450 | 430 | 400 | 375 |
| | | Wcx | 8265 | | | | | | | | |
| T6-B2 | $y$ | Wa,Wb | 25362 | | | | | | | | |
| | | FR | 325 | 251 | 424 | 568 | 652 | 697 | 706 | 725 | 575 |
| | | Wcy | 6124 | | | | | | | | |
| | | FRcy | 306 | 322 | 359 | | | | | | |
| | $x$ | CW | 6680 | 164 | 293 | 389 | 433 | 440 | 421 | 392 | 368 |
| | | Wcx | 6688 | | | | | | | | |

analysis under a set of floor forces determined from Eq. (4.11) of EC8-2004 and summing to a base (design) shear, $V_d$, equal to 20% of the total weight of the assumed model structure. More specifically, allowing for plastic hinges at the bases of walls and detailing CW and frames FR, FRcy according to the strong column-weak beam philosophy (that is, allowing plastic hinges at the ends of the clear spans of the beams and at the foot of the ground floor columns), this static analysis leads to the yield moments shown in Table 3.

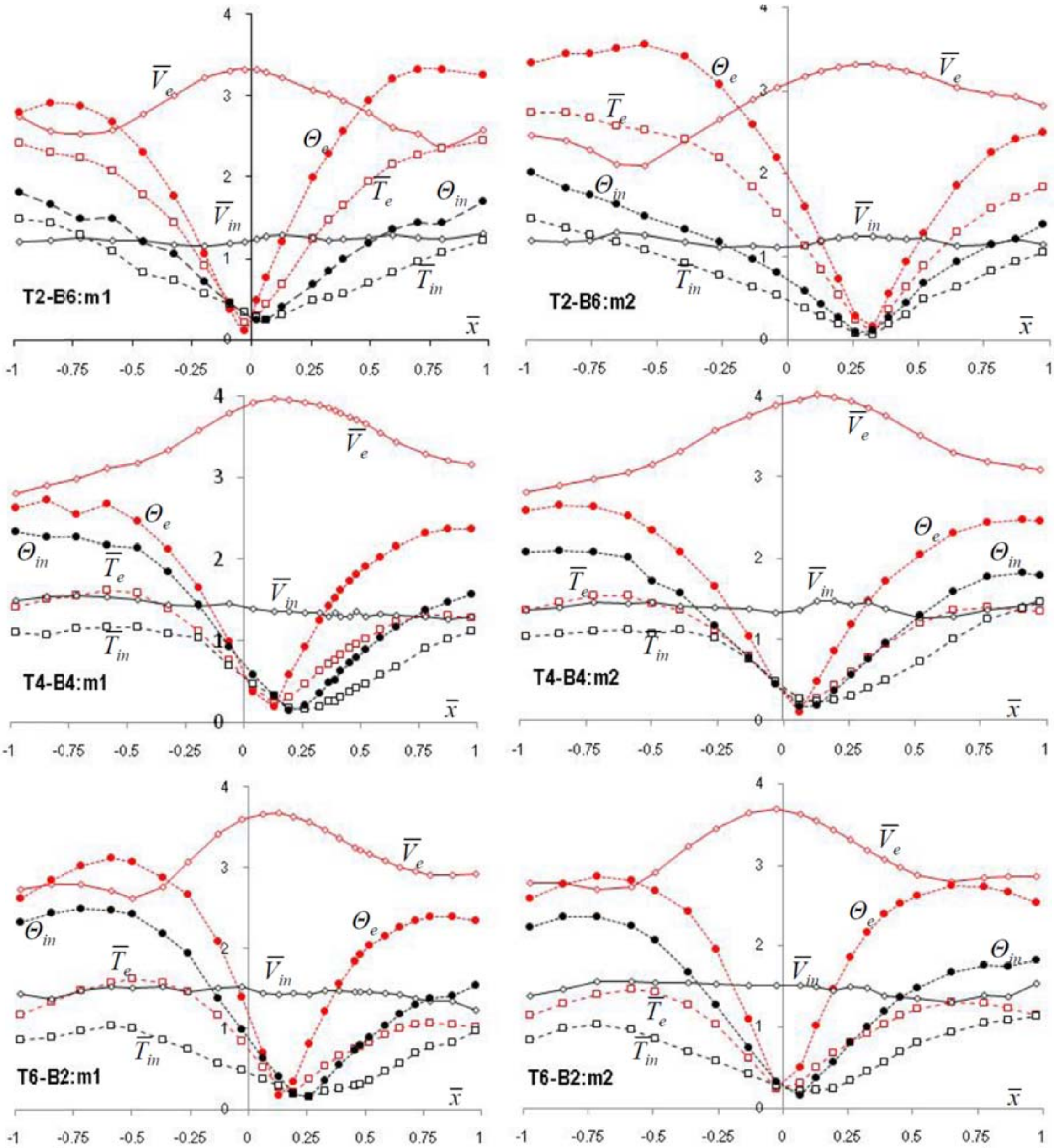


Fig. 6 Elastic and inelastic base shears, torques and top rotations ($\times 10^{-2}$ rads) under the Kobe excitation

It should be noted here that the proposed approximate method for evaluating basic dynamic data and possibly an optimum structural configuration is useful at the preliminary stage of a practical application, where the structural model has to be formed. At this stage an insight of the overall (elastic and post-elastic) response is required. Therefore, particular design limitations dependent on the ductility properties of the structural members (reinforcement -longitudinal and lateral- detailing, axial load ratio, concrete strength etc.) and other code drift limitations are beyond of the scope of this paper and they are not investigated. As a consequence, the magnitude of the plastic deformations (rotations) at the assumed plastic hinges and furthermore the maximum story drifts are not examined with respect to the code limits and the ductility capacity of the member sections.

In order to compare elastic and inelastic behaviors, the elastic responses of the assumed models under the same excitation are also presented in Fig. 6. Three response parameters, obtained by time history analyses assuming a 5% damping ratio, are shown: top rotations, Θ, normalized base shears and normalized base torques. The red lines represent the peak elastic response (top rotations: $\Theta_e$, are shown by dotted lines, normalized base shears: $\overline{V}_e = V_e/V_d$ by solid lines and normalized base torques: $\overline{T}_e = T_e/r_b V_d$ by dashed lines) and the corresponding black lines represent the peak inelastic behavior ($\Theta_{in}, \overline{V}_{in} = V_{in}/V_d, \overline{T}_{in} = T_{in}/r_b V_d$).

It can be seen that the response of the inelastic systems is smoother and the overall rotational behavior is smaller than that obtained by the elastic behavior. This finding confirms observations on single story systems that after yielding asymmetric systems have the tendency to deform further in a translational mode (e.g., Kan and Chopra 1981, Ghersi and Rossi 2001). In the models with a rather tall base structure (T2-B6:m1 and T2-B6:m2), minimum elastic and inelastic responses (in terms of base torsion and top rotation) are observed at the same location of wall Wb, which is very close to that predicted by the proposed approximate method (as shown in top diagrams of Fig. 5). In other words, an almost translational response at the end of the linear phase is preserved into the inelastic phase, when the bent strength assignment is based on a planar static analysis as described above. This is because of the almost concurrent yielding of these bents.

In the models with equal number of floors in the base and tower structures, the variation of inelastic base torques becomes further smoother and least values appear in a wider range of positions of Wb (when $\bar{x}$ varies in the range 0.19 to 0.39 for model T4-B4:m1 and in the range of 0.065 to 0.325 for model T4-B4:m2), but minimum rotational response, in both the elastic and inelastic phase, appears when the location of Wb is close to that predicted by the proposed method (see mid diagrams of Fig. 5). In the models with a short base structure, the diagram of inelastic base torques is apparently flatter and least values appear in an even wider range of positions of Wb (when $\bar{x}$ varies in the range 0.13 to 0.45 for model T6-B2:m1 and in the range of -0.026 to 0.325 for model T6-B2:m2). The minimum value of inelastic top rotation is again close to the location of Wb predicted by the proposed methodology.

## 6. Conclusions

An approximate method is presented for the analysis of multi-story asymmetric setback buildings. Basic dynamic data (periods and base shears) can be estimated with reasonable accuracy and, to some extent, base torques. Furthermore, the method provides an overview on the rotational response of such buildings and it may be found useful at the preliminary stage of a practical design in predicting the structural configuration of minimum rotational behavior. It is based on the analysis of two equivalent, single-story asymmetric modal systems, the masses of which are determined from the first two vibration modes of the uncoupled multi-story structure and the radius of gyration is computed as a Rayleigh quotient as described in an earlier paper. The stiffness of the supporting elements, at the locations of the real bents, when they represent full-height resisting bents, are determined from the corresponding individual bents when they are assumed to carry, as planar frames, the mass of the complete structure, but an indirect procedure is used for the curtailed bents.

It is demonstrated, that the predicted structural configuration of minimum rotation implies that not only its elastic response during a ground motion is more or less translational, but, also, that this response is preserved in the inelastic phase, when the strength assignment of the lateral load resisting bents is stiffness proportional (that is, it is derived from a planar static analysis under a set of lateral forces simulating an equivalent seismic loading). This is a consequence of the almost concurrent yielding of these bents. This is demonstrated in common 8-story setback buildings under a characteristic ground motion.

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