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Dynamic behavior of FGM beam using a new first shear deformation theory

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Abstract. A new first-order shear deformation theory is developed for dynamic behavior of functionally graded beams. The equations governing the axial and transverse deformations of functionally graded plates are derived based on the present first-order shear deformation plate theory and the physical neutral surface concept. There is no stretching–bending coupling effect in the neutral surface based formulation, and consequently, the governing equations and boundary conditions of functionally graded beams based on neutral surface have the simple forms as those of isotropic plates. The accuracy of the present solutions is verified by comparing the obtained results with the existing solutions.

Keywords: functionally graded beam; first shear deformation theory; neutral surface position; vibration

1. Introduction

Nowadays functionally graded materials (FGMs) are an alternative materials widely used in aerospace, nuclear, civil, automotive, optical, biomechanical, electronic, chemical, mechanical and shipbuilding industries. In fact, FGMs have been proposed, developed and successfully used in industrial applications since 1980’s (Koizumi 1993). Classical composites structures suffer from discontinuity of material properties at the interface of the layers and constituents of the composite. Therefore the stress fields in these regions create interface problems and thermal stress concentrations under high temperature environments. Furthermore, large plastic deformation of the interface may trigger the initiation and propagation of cracks in the material (Vel *et al*. 2004). These problems can be decreased by gradually changing the volume fraction of constituent materials and tailoring the material for the desired application. In fact, FGMs are materials with spatial variation of the material properties. However, in most of the applications available in the literature, as in the present work, the variation is through the thickness only. Therefore, the early state development of improved production techniques, new applications, introduction to effective micromechanical models and the development of theoretical methodologies for accurate structural predictions, encourage researchers in this field. Many papers, dealing with static and dynamic

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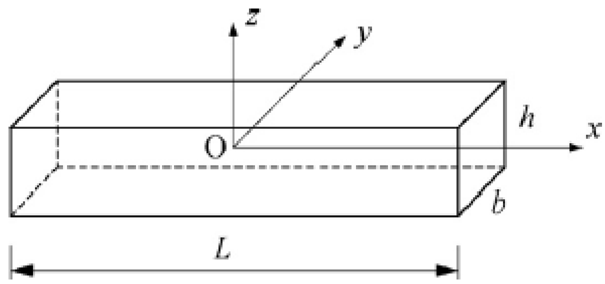

Fig. 1 A beam element

behavior of FGMs, have been published recently. An interesting literature review of above mentioned work may be found in the paper of Birman and Byrd (2007). Taj *et al*. (2013) conducted static analysis of FG plates using higher order shear deformation theory. Recently, Tounsi and his co-workers (Hadji *et al.* 2011, Houari *et al.* 2011, El Meiche *et al.* 2011, Bourada *et al.* 2012, Bachir Bouiadjra *et al.* 2012, Fekrar *et al*. 2012, Tounsi *et al*. 2013, Bouderba *et al.* 2013, Klouche Djedid *et al.* 2014, Nedri *et al.* 2014, Ait Amar Meziane *et al.* 2014, Draiche *et al.* 2014, Sadoune *et al.* 2014, Zidi *et al.* 2014, Ait Yahia *et al.* 2015, Belkorissat *et al.* 2015) developed new shear deformation plates theories involving only four unknown functions. Belabed *et al*. (2014) used an efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates. Tai *et al.* (2014) studied the analysis of functionally graded sandwich plates using a new first-order shear deformation theory. Bousahla *et al*. (2014) investigated a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates. Hebali *et al.* (2014) studied the static and free vibration analysis of functionally graded plates using a new quasi-3D hyperbolic shear deformation theory. Khalfi *et al.* (2014) studied the thermal buckling of solar functionally graded plates on elastic foundation using a refined and simple shear deformation theory. Al-Basyouni *et al.* (2015) investigated size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position. Mahi *et al.* (2015) studied the bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates using a new hyperbolic shear deformation theory. Hamidi *et al.* (2015) used a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Bourada *et al.* (2015) used a new simple shear and normal deformations theory for functionally graded beams.

In the present study, free vibration of simply supported FG beams was investigated by using new first Shear Deformation beam Theory. This theory enforces traction free Boundary conditions at beams surfaces using shear correction factors. Then, the equations governing the axial and transverse deformations of functionally graded plates are derived based on the present first-order shear deformation plate theory and the physical neutral surface concept. Analytical solutions for free vibration are obtained. Numerical examples are presented to verify the accuracy of the present theory.

2. Functionally graded materials

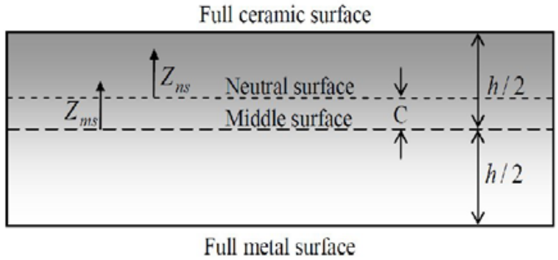

Fig. 1 The position of middle surface and neutral surface for a functionally graded plate.

A straight uniform FG beam of length L, width b, depth h, having rectangular cross-section is shown in Fig. 1. A Cartesian coordinate system $O(x,y,z)$ is defined on the central axis of the beam, where the x axis is taken along the central axis, the y axis in the width direction and the z axis in the depth direction. Due to asymmetry of material properties of FG plates with respect to middle plane, the stretching and bending equations are coupled. But, if the origin of the coordinate system is suitably selected in the thickness direction of the FG plate so as to be the neutral surface, the properties of the FG plate being symmetric with respect to it. To specify the position of neutral surface of FG plates, two different planes are considered for the measurement of z namely, z_{ms} and z_{ns} measured from the middle surface and the neutral surface of the plate, respectively, as depicted in Fig. 2.

The volume-fraction of ceramic V_c is expressed based on z_{ms} and z_{ns} coordinates as

$$V_C = \left(\frac{z_{ms}}{h} + \frac{1}{2}\right)^k = \left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^k \tag{1}$$

Where k is the power law index which takes the value greater or equal to zero and C is the distance of neutral surface from the mid-surface. Material non-homogeneous properties of a functionally graded material plate may be obtained by means of the Voigt rule of mixture. Thus, using Eq. (1), the material non-homogeneous properties of FG plate P, as a function of thickness coordinate, become

$$P(z) = P_M + P_{CM}\left(\frac{z_{ns} + C}{h} + \frac{1}{2}\right)^k, \quad P_{CM} = P_C - P_M \tag{2}$$

where P_M and P_C are the corresponding properties of the metal and ceramic, respectively. In the present work, we assume that the elasticity modules E and the mass density ρ are described by Eq. (2), while Poisson's ratio v, is considered to be constant across the thickness (Benachour *et al.* 2014, Larbi Chaht *et al.* 2014).

The position of neutral surface can be obtained as

$$C = \frac{\int_{-h/2}^{h/2} E(z_{ms}) z_{ms} dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms}) dz_{ms}} \tag{3}$$

2.1 Basic assumptions

The assumptions of the present theory are as follows:
- The origin of the Cartesian coordinate system is taken at the neutral surface of the FG beam.
- The displacements are small in comparison with the height of the beam and, therefore, strains involved are infinitesimal.
- The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x.
- This theory assumes constant transverse shear stress and it needs a shear correction factor to satisfy the plate boundary conditions on the lower and upper surface.

2.2 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained as follows

$$\begin{aligned} u(x,y,z_{ns},t) &= u_0(x,y,t) - z_{ns}\frac{\partial \phi}{\partial x} \\ w(x,y,z_{ns},t) &= w(x,y,t) \end{aligned} \tag{4}$$

where u, w are displacements in the x, z directions, u_0 is the neutral surface displacements. ϕ is function of coordinates x and time t.

The strains associated with the displacements in Eq. (4) are

$$\varepsilon_x = \varepsilon_x^0 + z_{ns}\, k_x \tag{5a}$$

$$\gamma_{xz} = \gamma_{xz}^s \tag{5b}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \qquad k_x = -\frac{\partial^2 \phi}{\partial x^2} \tag{6a}$$

$$\gamma_{xz}^s = \frac{\partial w}{\partial x} - \frac{\partial \phi}{\partial x} \tag{6b}$$

By assuming that the material of FG beam obeys Hooke's law, the stresses in the beam become

$$\sigma_x = Q_{11}(z_{ns})\,\varepsilon_x \quad \text{and} \quad \tau_{xz} = k_S Q_{55}(z_{ns})\,\gamma_{xz} \tag{7a}$$

k_S is a shear correction factor which is analogous to shear correction factor proposed by Mindlin (1951). Using the material properties defined in Eq. (2), stiffness coefficients, Q_{ij} can be expressed as

$$Q_{11}(z_{ns}) = E(z_{ns}) \qquad \text{and} \qquad Q_{55}(z_{ns}) = \frac{E(z_{ns})}{2(1+\nu)} \tag{7b}$$

2.3 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as Reddy (2002)

$$\delta \int_{t_1}^{t_2} (U - K) dt = 0 \tag{8}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy; and δK is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned}\delta U &= \int_0^L \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \left(\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}\right) dz_{ns}\, dx \\ &= \int_0^L \left(N_x \frac{d\delta u_0}{dx} - M_x \frac{d^2 \delta w_b}{dx^2} + Q_{xz} \frac{d\delta (w - \phi)}{dx} \right) dx\end{aligned} \tag{9}$$

Where N , M and Q are the stress resultants defined as

$$(N_x, M_x) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} (1, z_{ns})\, \sigma_x dz_{ns} \quad \text{and} \quad Q_{xz} = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \tau_{xz} dz_{ns} \tag{10}$$

The variation of the kinetic energy can be expressed as

$$\begin{aligned}\delta K &= \int_0^L \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \rho(z_{ns}) \left[\dot{u} \delta \dot{u} + \dot{w} \delta \dot{w}\right] dz_{ns}\, dx \\ &= \int_0^L \left\{ I_0 \left[\dot{u}_0 \delta \dot{u}_0 + (\dot{w})(\delta \dot{w})\right] - I_1 \left(\dot{u}_0 \frac{d\delta\dot{\phi}}{dx} + \frac{d\dot{\phi}}{dx} \delta \dot{u}_0 \right) \right. \\ &\quad \left. + I_2 \left(\frac{d\dot{\phi}}{dx} \frac{d\delta\dot{\phi}}{dx} \right) \right\} dx\end{aligned} \tag{11}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z_{ns})$ is the mass density; and (I_0, I_1, I_2) are the mass inertias defined as

$$(I_0, I_1, I_2) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \left(1, z_{ns}, z_{ns}^2\right) \rho(z_{ns}) dz_{ns} \tag{12}$$

Substituting the expressions for δU and δK from Eqs. (9) and (11) into Eq.(8) and integrating by parts versus both space and time variables, and collecting the coefficients of δu_0, $\delta\phi$, and δw, the following equations of motion of the functionally graded beam are obtained

$$\delta u_0 : \ \frac{\partial N_x}{\partial x} = I_0 \ddot{u}_0 \tag{13a}$$

$$\delta \phi : \ \frac{d^2 M_x}{dx^2} - \frac{\partial Q_{xz}}{\partial x} = I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_2 \frac{\partial \ddot{\phi}}{\partial x^2} \tag{13b}$$

$$\delta w : \ \frac{\partial Q_{xz}}{\partial x} = I_0 \ddot{w} \tag{13c}$$

Eq. (13) can be expressed in terms of displacements (u_0, ϕ, w) by using Eqs. (4), (5), (6), (7) and (8) as follows

$$A_{11} d_{11} u_0 = I_0 \ddot{u}_0 \tag{14a}$$

$$-D_{11} d_{1111} \phi - A_{55}^S d_{11} (w - \phi) = I_1 \frac{\partial \ddot{u}_0}{\partial x} - I_2 \frac{\partial \ddot{\phi}}{\partial x^2} \tag{14b}$$

$$A_{55}^S d_{11} (w - \phi) = I_0 \ddot{w} \tag{14c}$$

where A_{11}, D_{11}, etc., are the beam stiffness, defined by

$$(A_{11}, D_{11}) = \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} Q_{11} (1, z^2) dz_{ns} \tag{15a}$$

and

$$A_{55}^s = k_S \int_{-\frac{h}{2}-C}^{\frac{h}{2}-C} \frac{E(z_{ns})}{2(1+\nu)} dz_{ns} \tag{15b}$$

3. Analytical solution

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, ϕ, w can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ \phi \\ w \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\lambda x) e^{i\omega t} \\ \psi_m \sin(\lambda x) e^{i\omega t} \\ W_m \sin(\lambda x) e^{i\omega t} \end{Bmatrix} \tag{16}$$

where U_m, ψ_m, and W_m are arbitrary parameters to be determined, ω is the eigenfrequency associated with m th eigenmode, and $\lambda=m\pi/L$.

Substituting Eq. (16) into Eq. (14a)-(14c), the closed form solutions can be obtained from

$$\left([C]-\omega^2[M]\right)\{\Delta\}=0 \tag{17}$$

where $\{\Delta\}=\{U_m,\psi_m,W_m\}^t$, and $[C]$ and $[M]$ are the symmetric matrixes given by

$$[C]=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix},\ [M]=\begin{bmatrix} m_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \tag{18}$$

where

$$a_{11}=-A_{11}\lambda^2,\ a_{12}=0,\ a_{13}=0,\ a_{22}=-\left(D_{11}\lambda^4+A_{55}^S\lambda^2\right),\ a_{23}=A_{55}^S\lambda^2,\ a_{33}=-A_{55}^S\lambda^2 \tag{19a}$$

$$m_{11}=-I_0,\ m_{12}=I_1\lambda,\ m_{13}=0,\ m_{22}=-I_2\lambda^2,\ m_{23}=0,\ m_{33}=-I_0 \tag{19b}$$

4. Numerical results and discussions

In this section, various numerical examples are presented and discussed to verify the accuracy of present theories in predicting the free vibration response of simply supported FG beams. The FG beam is taken to be made of aluminum and alumina with the following material properties

Ceramic (P_C : Alumina, Al_2O_3): $E_c=380$ GPa; $\rho_c=3960kg/m^3$; $\nu=0.3$;

Metal (P_M : Aluminium, Al): $E_m=70$ GPa; $\rho_m=2702kg/m^3$; $\nu=0.3$;

And their properties change through the thickness of the beam according to power-law. The bottom surfaces of the FG beams are aluminium rich, whereas the top surfaces of the FG beams are alumina rich.

For convenience, the following dimensionless form is used:

$$\bar{\omega}=\frac{\omega L^2}{h}\sqrt{\frac{\rho_m}{E_m}}$$

For the verification purpose, the nondimensional fundamental frequencies $\bar{\omega}$ obtained by the present theory are compared with those given by Simsek (2010) of FG beams for different values of power law index k and span-to-depth ratio L/h and the results are presented in Table 1 and Table 2 (Şimşek 2010).

It can be seen that the present theory and the various theories used by Simsek give almost identical results (Şimşek 2010).

It should be remembered that the frequencies predicted by the shear deformable beam theories are smaller than those predicted by the classical beam theory and the difference between the frequencies of CBT and the shear deformable beam theories decreases as the value of L/h increases (see Tables 2-3). As would be expected, the frequencies are increased when the value of L/h is increased.

Inspection of these tables reveals that an increase in the value of the power-law exponent leads to a decrease in the fundamental frequencies.

The highest frequency values are obtained for full ceramic beam (k=0) while the lowest frequency values are obtained for full metal beam ($k \rightarrow \infty$). This is due to the fact that, an increase in the value of the power-law exponent results in a decrease in the value of elasticity modulus and the value of bending rigidity.

Table 1 Variation of fundamental frequency $\bar{\omega}$ with the power-law exponent FG beam for L/h=5

Theory	k=0	k=0.2	k=0.5	k=1	k=5	k=10	Metal
CBT*	5.3953	5.0206	4.5931	4.1484	3.5949	3.4921	2.8034
FSDBT*	5.1525	4.8066	4.4083	3.9902	3.4312	3.3134	2.6772
PSDBT*	5.1527	4.8092	4.4111	3.9904	3.4012	3.2816	2.6773
Present	**5.1525**	**4.8054**	**4.4079**	**3.9902**	**3.4312**	**3.3134**	**2.6772**

*Results form Ref (Simsek 2010)

Table 2 Variation of fundamental frequency $\bar{\omega}$ with the power-law exponent FG beam for L/h=20

Theory	k=0	k=0.2	k=0.5	k=1	k=5	k=10	Metal
CBT*	5.4777	5.0967	4.6641	4.2163	3.6628	3.5546	2.8462
FSDBT*	5.4603	5.0827	4.6514	4.2051	3.6509	3.5415	2.8371
PSDBT*	5.4603	5.0829	4.6516	4.2050	3.6485	3.5389	2.8372
Present	**5.4603**	**5.0813**	**4.6509**	**4.2051**	**3.6509**	**3.5416**	**2.8371**

*Results form Ref (Simsek 2010)

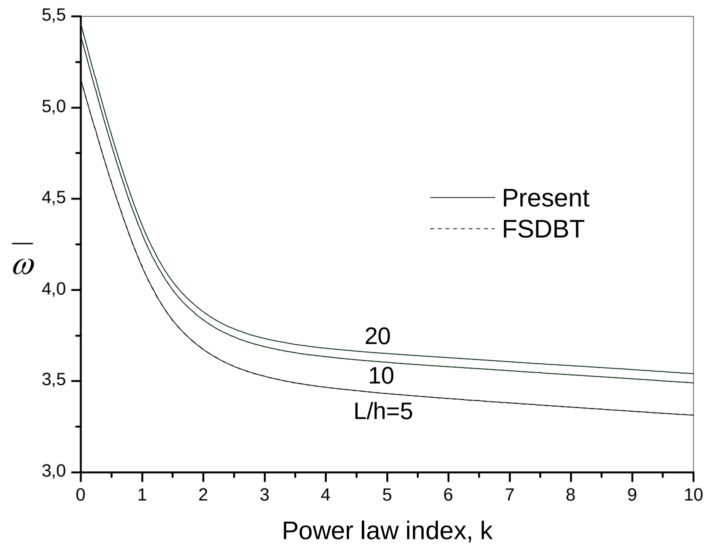

Fig. 3 Variation of the nondimensional fundamental frequency $\bar{\omega}$ of FG beam with power law index k and span-to-depth ratio L/h

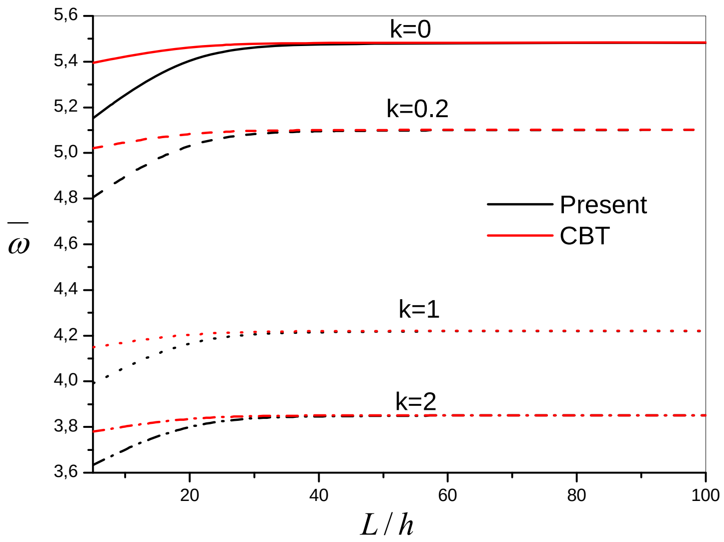

Fig. 4 Variation of the fundamental frequency of FG beam with *L*/*h* ratio for various values of the power-law exponent *k*

Fig. 3 shows the non-dimensional fundamental natural frequency $\bar{\omega}$ versus the power law index *k* for different values of span-to-depth ratio *L*/*h* using both the present theory and FSDBT. An excellent agreement between the present theory and FSDBT is showed from Fig. 3. It can be observed that the frequency decreases with increasing the power law index. The full ceramic beams (*k*=0) lead to a highest frequency. However, the lowest frequency values are obtained for full metal beams ($k\rightarrow\infty$). This is due to the fact that an increase in the value of the power law index results in a decrease in the value of elasticity modulus.

Fig. 4 show the variation of the fundamental frequency of the FG beam with *L*/*h* ratio and the power-law exponent, respectively by using CBT and the present theory. As seen from these figure, there is a remarkable difference between the frequencies of CBT and those of shear deformable beam theories when the slenderness ratio of the FG beam is less than *L*/*h*=20. This means that for short beams (i.e., $L/h\leq 10$), in particular, shear deformable beam theories should be used in the analysis. Also, the fundamental frequency of the FG beam is saturated after the value of *L*/*h*=20, and all the beam theories give almost the same frequencies when the FG beam has the slenderness ratio which is greater than *L*/*h*=20. Also, note that the power-law exponent plays an important role on the fundamental frequency of the FG beam.

5 Conclusions

An New first shear deformation beam theory (FSDT) is developed for dynamic behavior of FG beames. Based on the present theory and the neutral surface concept, the equations of motion are derived from Hamilton's principle. The accuracy of neutral surface-based model is verified by comparing the obtained results with those reported in the literature. Finally, it can be concluded that the NFSDT is not only accurate but also simple in predicting the dynamic behavior of FG beames.

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