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Retrofit strategy issues for structures under earthquake loading using sensitivity-optimization procedures

G.D. Manolis[1]*, C.G. Panagiotopoulos[1], E.A. Paraskevopoulos[2], F.E. Karaoulanis[1], G.N. Vadaloukas[3] and A.G. Papachristidis[3]

[1]Department of Civil Engineering, Aristotle University, Thessaloniki, GR-54124, Greece
[2]Department of Mechanical Engineering, Aristotle University, Thessaloniki, GR-54124, Greece
[3] VK-4M Software Company, 9 Mykinon Str., Athens, GR-15223, Greece

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Abstract. This work aims at introducing structural sensitivity analysis capabilities into existing commercial finite element software codes for the purpose of mapping retrofit strategies for a broad group of structures including heritage-type buildings. More specifically, the first stage sensitivity analysis is implemented for the standard deterministic environment, followed by stochastic structural sensitivity analysis defined for the probabilistic environment in a subsequent, second phase. It is believed that this new generation of software that will be released by the industrial partner will address the needs of a rapidly developing specialty within the engineering design profession, namely commercial retrofit and rehabilitation activities. In congested urban areas, these activities are carried out in reference to a certain percentage of the contemporary building stock that can no longer be demolished to give room for new construction because of economical, historical or cultural reasons. Furthermore, such analysis tools are becoming essential in reference to a new generation of national codes that spell out in detail how retrofit strategies ought to be implemented. More specifically, our work focuses on identifying the minimum-cost intervention on a given structure undergoing retrofit. Finally, an additional factor that arises in earthquake-prone regions across the world is the random nature of seismic activity that further complicates the task of determining the dynamic overstress that is being induced in the building stock and the additional demands placed on the supporting structural system.

Keywords: structural sensitivity; structural uncertainty; finite element method; structural dynamics; earthquake engineering; random vibrations; deficiency indices; minimum-cost optimization.

1. Introduction

Retrofit activities are being carried out with increasing frequency in the last few decades for the existing building stock in European urban areas that can no longer be demolished to make room for new construction because of either economical or historical reasons. Regarding possible intervention strategies facing a structural engineer, three levels of activity can be distinguished (Croci 1998, Brebbia 2003): (a) Selective repair strategy and external restorations to preserve the original facade and function of historical buildings; (b) Strengthening and rehabilitation of the supporting structural skeleton so that conventional buildings are upgraded according to contemporary building code requirements; (c) Standard maintenance and repair as required for the safe operation of the building

* Corresponding Author, Professor, E-mail: gdm@civil.auth.gr

during its useful lifespan.

In this respect, sensitivity analysis yields information that helps identify critical components which influence structural response (Manolis *et al.* 1990). In general, sensitivity analysis is well established from a theoretical viewpoint provided the structural system stays in the linear elastic range. Over the past decade, however, quasi-static non-linear analysis (or 'pushover' analysis) is recognized as a satisfactory approximate method for evaluating the seismic performance of buildings. As a consequence, this method has been incorporated in various seismic codes, and is occasionally combined with the so-called 'capacity spectrum' method in order to determine the performance of a building during earthquakes (Chopra and Goel 1999). In order to account for nonlinearities in a sensitivity analysis, recourse to inelastic material behavior must be made indirectly through the introduction of various types of indices, which also encompass time-dependent response. This seems to be the most rational approach for gradually introducing structural sensitivity analysis capabilities into existing commercial finite element method (FEM) software codes for the purpose of mapping retrofit strategies.

More specifically, the next development stage at the professional FEM-based software level is the introduction of specialized analysis tools. In this respect, we envision FEM programs that encompass the concepts of 'structural sensitivity' in a deterministic environment as well as of 'stochastic structural sensitivity' in a probabilistic environment (Vanmarke *et al.* 1986, Liu *et al.* 1986, Ghanem and Spanos 1991, Kleiber and Hien 1992). This type of software is bound to become a necessary analysis tool in formulating retrofit strategies for all categories of buildings. In essence, the basic concepts behind structural sensitivity are but a continuation of standard structural analysis techniques (Augusti *et al.* 1984, Manolis and Koliopoulos 2001) and help establish the influence of key structural components on the overall kinematic and stress fields that develop in buildings for all categories of loads, including environmentally-induced ones with random characteristics such as earthquakes (Haukaas and Der Kiureghian 2004).

To give some more background-type information, the basic concept behind a sensitivity approach is the ability to formulate material derivatives for stiffness matrices that reveal the degree of influence of a particular structural parameter on the global response of the building under investigation. Sensitivity analysis may also be extended to cover dynamic systems under steady-state vibrations by focusing on the underlying eigenvalue problem. For the general transient problem, it can be directly applied to the discretized equations of motion and material derivatives are taken with respect to the tangent stiffness matrix in what is an incremental, time-stepping approach. The same type of thinking is used in order to define sensitivities for non-linear structural systems (Kleiber *et al.* 1997).

On a more practical level, sensitivity analyses are a rational way in deciding which components of an existing structure must be modified in order to conform to target values of allowable stresses and floor displacements. When nonlinearities develop in the structure because of earthquakes (Kappos 2002), a better measure of these target values is achieved through the use of dimensionless indices such as the recently proposed 'deficiency' indices (KANEPE 2007). These prove especially valuable when the existing building in question is of historical importance and the appropriate retrofit strategy needs to be mapped out in detail, along with an estimation of the cost involved (Mazzolani 2009). The fact is that nowadays in many urban areas in Europe it is no longer possible to demolish older buildings, but instead try to preserve them. This implies new activities in retrofitting existing structures that are starting to be regulated by EU codes (EC8 2003). As expected, new analysis techniques must be developed and used, which in turn require appropriate software tools.

In what follows, we first formulate sensitivity analysis in terms of the FEM to cover both static and dynamic cases, with the latter presented within the context of the eigenvalue problem. Some key concepts underlying sensitivity analyses, such as random vibrations of structural systems with uncertain structural properties modeled by the FEM are presented the Appendix. Next, we discuss software implementation. The challenge here is to program sensitivity analysis modules as independent, self-contained entities and subsequently interface them with existing professional FEM software (STEEL 2005, STRAD 2005) for the benefit of practicing engineers. This is followed by a formal definition of 'deficiency' indices, whose sensitivity behavior is well suited for tracking optimal retrofit venues. Finally, two examples involving a three-bay truss under point loads plus an existing seven-story building under strong seismic loads are presented so as to illustrate the potential of a new generation of FEM codes with sensitivity analysis capabilities.

2. Sensitivity analysis

Sensitivity analysis provides the rate of change for a set of pre-selected structural variables (i.e., kinematic and/or stress fields, known as *state variables*) with respect to a set of structural design parameters (i.e., member properties and their geometric configuration, known as *design variables*). This presupposes that the design variables are inherently uncertain and exhibit a certain amount of variability about their mean design values. Moving to a more general framework of analysis, randomness in the governing equations of equilibrium can be manifested at two levels, namely that of external loads and that of the structure itself (see the Appendix). Either way, randomness filters into the response and the state variables are now described by a mean value and a sequence of statistical moments (Newland 2005). Of course, the description of the sources of the uncertainties and their quantification is in itself a major field of study (Solnes 1997).

The main drawback is that sensitivity analysis is applicable to linear elastic structural systems. Modern design approaches, however, require a structure to behave nonlinearly, especially in the presence of high intensity loads such as earthquake-induces motions. In here, we demonstrate how a sensitivity analysis can be used within the context of a new generation of national building codes such as the Greek codes (EKOS 2000, EAK 2000, KANEPE 2007). As a paradigm, we consider the case of an existing, seven-story reinforced concrete (R/C) building, whose response is carried into the nonlinear range due to strong ground motion. Dimensionless 'deficiency' indices λ_i (KANEPE 2007) are defined at either (1) the element level as the ratio of the maximum bending moment that develops in a beam or column element due to loads that include ground shaking (for a 'q' factor equal to unity) divided by the ultimate moment capacity of that member, or (2) the floor level as the sum of the shear forces computed for the same loads as before at the $i = 1,2,..., n$ floor joints multiplied by the corresponding index λ_i and divided by the total shear force that develops at that floor. These computations are then followed by a sensitivity analysis on the λ_i indices to reveal which structural elements best respond to strengthening efforts. A re-analysis of the retrofitted structure demonstrates the validity of this approach and points to the possibility of optimal retrofit strategies for a fixed amount of financial investment. More specifically, this procedure is done for a set comprising all available design parameters, and sensitivity analysis with respect to the λ_i indices traces the subset of the most influential ones. Since the degree of retrofit is broadly defined as the difference between initial and final structural states, a standard optimization approach is then used to find the most efficient retrofit plan. The degree of retrofit is mathematically described by a cost (or objective) function (Bertsekas 1982), given in terms of the structural design variables of interest that have been identified by the sensitivity analysis. This cost function is minimized, subjected to equality and inequality constraints that derive from either ultimate and serviceability limit states, or from other types of design specifications and constitutes a nonlinear optimization problem (Bertsekas 1995).

3. Finite element formulation

A finite element discretization of a structural system under static loads yields a system of algebraic equations in matrix form. Carrying out the derivatives of this system with respect to a set of design variables, symbolized as 'h', gives the following hierarchy of equations for the change in displacements $\{u\}$ and internal forces $\{f\}_{\mathrm{int}}$

$$[K]\{u\}=\{f\}\ , \qquad [K]\{u\}_{,h}=-[K]_{,h}\{u\}+\{f\}_{,h}\rightarrow\{u\}_{,h}=-[K]^{-1}([K]_{,h}\{u\}+\{f\}_{,h})$$

$$\{f\}_{\mathrm{int},h}=[K]_{,h}\{u\}+[K]\{u\}_{,h} \tag{1}$$

In the above, $[K]$ is the stiffness matrix of the structural system and $\{f\}$ is the vector of external forces. Note that the use of commas indicates partial differentiation with respect to the subscript that follows.

Implementation is carried out (see Manolis *et al.* 2008) using two basic families of finite elements, namely the twelve degree-of-freedom (12-DOF) generalized beam element and the planar twenty four degree-of-freedom (24-DOF) shell element with four nodes, as shown in Fig. 1. This way, it is possible to model multi-story, multi-bay buildings of the mixed type (frames interfacing with shear walls). Also, diaphragm action for flooring systems in a building is a common assumption that can be implemented through imposition of constrained nodal displacements along the horizontal plane of the floor.

The key design parameters, for which all relevant expansions are given in Eq. (1), can be

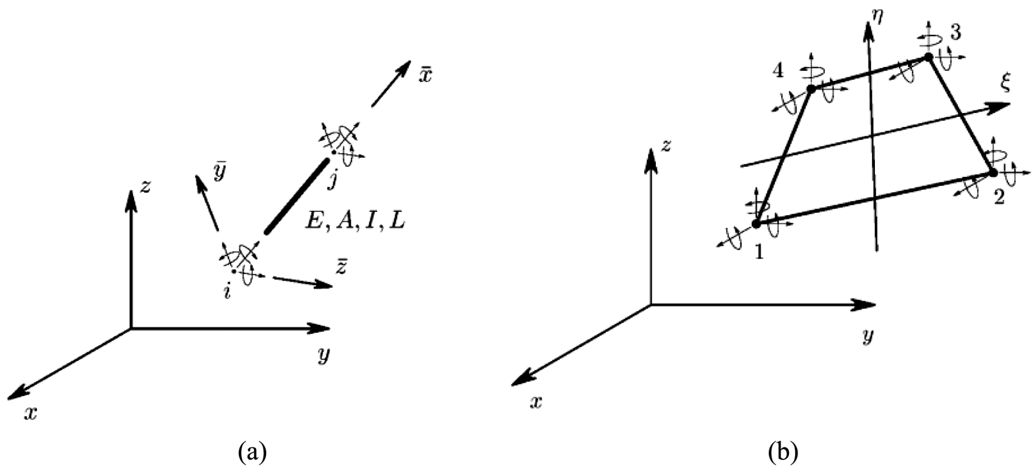


Fig. 1 The generalized (a) 1D beam and (b) 2D shell-type finite elements with added sensitivity analysis capabilities

Table 1 Design variables for a sensitivity analysis

1	2	3	4	5	6	7
Elasticity modulus	Section area	Principal moment of inertia 1	Principal moment of inertia 2	Polar moment of inertia	Mass density	Sub-grade spring constant
E	A	I_{YY}	I_{ZZ}	J	ρ	K_S
(kN/m^2)	(m^2)	(m^4)	(m^4)	(m^4)	(t/m^3)	(kN/m)

formulated as follows: (a) the lateral dimensions of the element cross-section that control the section properties (area A and moment of inertia I); (b) the material parameters (modulus of elasticity E and possibly Poisson's ratio v); (c) the mass density ρ that results from changes in the weight of the structural member in question (as in the use of steel jackets reinforcing part of a concrete column) and finally (d) the connections between the structural elements, such as the degradation of a monolithic joint into a pin-type of joint. It should be pointed out that the first three categories refer to *local* variables whose effect can be grouped as a cumulative uncertainty in the stiffness (e.g., $k = EI/L$) and mass (e.g., $m = \rho AL$) coefficients of the pertinent system matrices. Finally, the last category is characterized as a *global* variable, whose effect is felt as a change in the type of coefficients comprising the system matrix $[K]$, as well as possible changes in their overall size of the numerical model. Problems regarding lack-of-fit of section members (with associated changes in length L) or any other complications that give rise to either non-linear or second-order effects are beyond the scope of this work. In sum, design variables that are potential sources of uncertainty are chosen from the element properties listed in Table 1.

Moving on to a modal analysis of structural systems (see the Appendix for details), we start with the classical formulation in discrete form, where ω_i are the natural frequencies and $\{\phi\}_i$ are the corresponding modal shapes

$$[K]\{\phi\}_i - \omega_i^2[M]\{\phi\}_i = \{0\} \tag{2}$$

with $[M]$ the mass matrix. Carrying out a sensitivity analysis gives the following expression for the dependence of a given eigenvalue ω on design variable h

$$\omega_{i,h} = \left(\{\phi\}_i^T [K]_{,h} \{\phi\}_i - \omega_i^2 \{\phi\}_i^T [M]_{,h} \{\phi\}_i\right) \Big/ \left(2\,\omega_i \{\phi\}_i^T [M]_{,h} \{\phi\}_i\right) \tag{3}$$

The mathematical formulations in Eqs. (1) and (3) requires derivatives that can be evaluated in either of two ways: (a) Direct, closed-form differentiation with respect to the design variables or (b) indirect differentiation using finite difference (FD) schemes.

4. Optimum retrofit interventions

The key objective in any retrofit effort is selection of a particular strategy plus the degree to which retrofit activities will be carried out (Baros and Dritsos 2008). The degree of retrofit is measured as the difference between initial and final structural states. For the purposes of this work, we define the degree of retrofit as an optimization problem in its standard form (Bertsekas 1982)

$$\text{Find } \min_{d_i} \; C(d_i)$$

$$\text{subject to: } E_m\,(d_i) = 0 \text{ and } I_k\,(d_i) \leq 0 \tag{4}$$

where d_i denotes the i^{th} selected design variable, C represents the standard cost function, while E_m and I_k respectively express the m^{th} equality constraint and the k^{th} inequality constraint on the design variables.

To be more specific, in the context of retrofit issues, the standard definition for the design variable d_i is the cross-section area of the i^{th} structural member that is selected for strengthening. This does not necessarily imply that the number of design variables is equal to the totality of structural members. Next, the cost function for the purpose of retrofit may be defined based on the amount of variation in the design variables. Therefore

$$C = \left(\| \{d\}_i - \{d\}_i^0 \|_w \right)^2 = \left(\{d\}_i - \{d\}_i^0 \right)^T [W] \left(\{d\}_i - \{d\}_i^0 \right) \tag{5}$$

where $\{d\}_i^0$ is the vector of the initial values of a design variable (i.e., the cross-sections) and diagonal, positive definite matrix $[W]$ contains weight coefficients associated with each design variable.

Equality and inequality constraints may be defined as either indirect or direct in form, where the former may arise from ultimate limit states (ULS) and serviceability limit states (SLS) requirements imposed on the structure in question. The latter form explicitly refers to the design variables without any reference to structural performance, for example $d_i > d_i^0$ for the i^{th} variable. Note that the number of equality constraints should be less or equal to the number of design variables.

In general, the above formulation corresponds to a nonlinear optimization problem (Bertsekas 1995). Regarding numerical implementation of this problem, standard algorithms (Nocedal and Wright 1999) for minimization of a functional have been implemented using an in-house software code based on the JAVA programming language (Gosling *et al*. 2005).

5. Numerical implementation aspects

The aforementioned structural sensitivity analysis under static loading conditions was programmed in the *C*++ language (2004) and implemented in modular form as a package of independent classes that include data generation, mesh setup, formation of the stiffness matrix and its derivatives, solution procedure, data processing and data output, as shown in Fig. 2 (Manolis *et al*. 2009). All these basic tasks described above were done in a manner consistent with the input / output structure of the two commercial codes marketed by the industrial partner, namely STRAD (2005) and STEEL (2005). This way, it becomes possible during a second phase of the work to interface the sensitivity analysis software with each of the two commercial codes and to set the stage for introducing the eigenvalue sensitivity analysis.

As far as damping is concerned, it is the cumulative effect of a number of mechanisms that are responsible for energy absorption in a structural system. As a result, the conventional linearized damping ratio concept (Chopra 2007) is used, which yields values in the range of 2-4% for steel

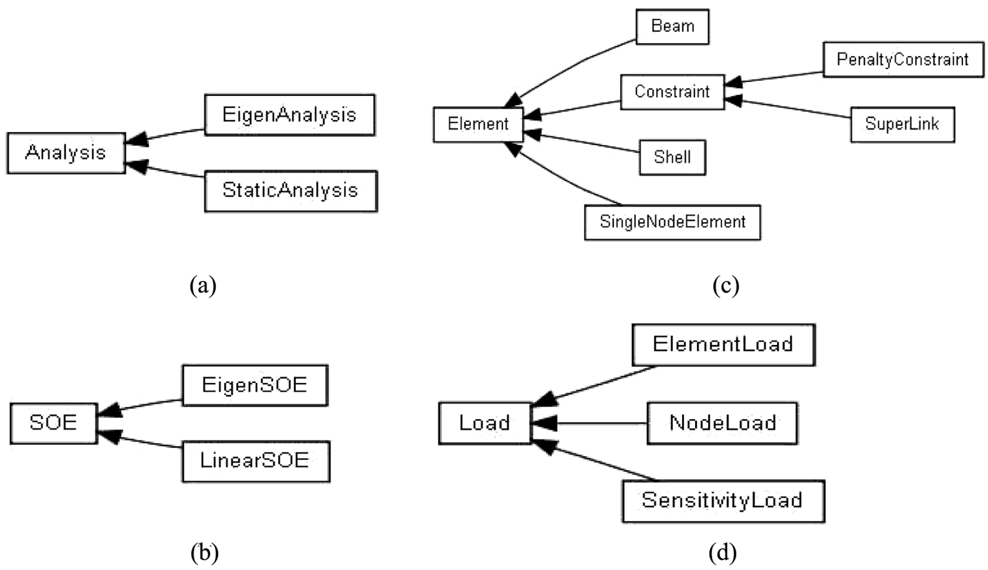


Fig. 2 Software structure listing the hierarchy of classes used for (a) analysis; (b) system equations; (c) finite elements; and (d) loads as implemented in the VK-4M commercial software packages STRAD and STEEL

frames and 3-5% for reinforced concrete frames. Next is the question of the q-factor (or ductility factor) that develops in a structural system as it enters the non-linear range due to intense loads such as strong ground motions. This phenomenon involves joint plastification plus other hysteresis-type of effects. Strictly speaking, it is outside the range of an uncertainty analysis, since no single factor can be isolated as directly responsible for this phenomenon. It is handled in an indirect fashion through use of deficiency indices, as will be discussed in the sequel. We note in passing that a good design for frame-type of buildings with shear walls arranged as a central core and sufficient joint capacity may develop ductility factors in the range of 2-5 (Chopra 2007).

The most complex piece of computer programming has to do with the introduction of supplementary DOF at each nodal point of the discretized structure so as to collect the additional statistical measures (see the Appendix) for the kinematic and stress fields when randomness is considered. It is these measures that in essence show the effect (if any) of stochastic uncertainties in the material parameters on the structural response. This information allows the designer to isolate any elements in the building that are critical to the overall structural behavior and to decide practical matters, such as how to strengthen a specific beam or column.

The next step concerns the eigenvalue analysis, which additionally involves the mass of the structure. In this case, the mass density of a finite element is also allowed to become an uncertain parameter, so that Eq. (A.23) will hold true. In order to simplify the analysis, sensitivity results pertain to the eigevalues only (i.e., the natural periods or frequencies of the structure), while the eigenvectors (i.e., the modal shapes) are considered as unaffected by uncertainty. For the purposes of a response spectrum analysis in earthquake engineering it is considered sufficient, since shifts in the natural periods of a structure is by far the most important piece of information. This is because frequency shifts determine, to a great extent, the change in seismic input to the structure and whether or not a structure will be damaged during a real earthquake.

One necessary step in this development effort is calibration of the sensitivity analysis software against results published in the open literature, as well as compilation of results from a number of trial cases that serve as a user's manual. To give an elementary example, we considered a simple, single-story 3D reinforced concrete (R/C) frame with tie beams at the foundation level that are in turn modeled by Winkler-type springs (Wolf 1994). The top slab was considered rigid, surrounded by beams with monolithic joint connections to the columns below. As was previously mentioned, all development for the sensitivity analysis were programmed in an object-oriented environment (i.e., the *C++* language) and interfaced with FEM program STRAD (2005) through a common input / output format. A flowchart of the sensitivity analysis software is given in Table 2 below. Continuing with the elementary example, only the equivalent soil spring constants were assumed to be uncertain, and their effect on the static response of the frame (nodal displacements, base shears and joint moments) due to gravity loads was studied. Once this first example was fully calibrated, then uncertainty in all material parameters (see Table 1) was allowed to take place, and their separate effects were examined. Then, more floors and bays were added to the original frame so as to produce a realistic high-rise building, and this comprised Example 2 given in the relevant section on numerical examples.

In reference to the cost optimization process that involves minimization of the cost functional given in Eq. (5), we used the Newton-Rapson method for solving this nonlinear problem, while incorporation of the equality constraints was carried out using Lagrange multipliers. Also, the penalty function method was used to impose the inequality constraints of Eq. (4).

Finally, future developments will deal with the implementation of probabilistic analysis capabilities in the FEM software by allowing for the construction of an extra covariance matrix involving all active DOF of the structural model to supplement the vector of nodal mean values. This process will be carried out for standard categories of random loads whose statistical characteristics may be modeled as normal (Gaussian), binomial, geometric, Poisson, as well as other types of distributions (Newland 2005). The last step will then involve an interfacing of both sensitivity and stochasticity so as to produce software that will carry out a sensitivity analysis in the presence of random loads.

Table 2 Basic flowchart for FEM software modules commencing a structural sensitivity analysis

Pseudo-algorithm for a Typical Linear Static Sensitivity Analysis in C++ (code: LHS, RHS → left- and right- hand sides of the equilibrium equation)
for all set of Sensitivity Parameters and for all Load Cases: domain → assembleLHS( ) soe → LUdecompose( ) domain → assembleRHS( ) soe → backsubstitute( ) domain → storeDisp( ) domain → assembleGradLHS( ) soe → LUdecompose( ) domain → assembleGradRHS( ) soe → backsubstitute( ) domain → storeSens( )

6. Deficiency indices

For realistic application examples such as conventional high-rise R/C structures whose skeleton is basically a frame comprising beams and columns and with floors exhibiting rigid-diaphragm behavior, an assessment of the response can be quantified through the use of 'deficiency' indices The first step in the use of such types of indices is to determine the magnitude and distribution of nonlinearities that develop in the main structural elements of the supporting frame because of anticipated strong ground motion. This requires a preliminary elastic analysis of the building so that the local 'deficiency' indices for each structural element (i.e., beams and columns) are computed according to the national Greek retrofit code (KANEPE 2007) as

$$\lambda = S/R_m \tag{6}$$

In the above, S is either (i) the bending moment because of the combined action of all loads including seismically-induced ones, without any reduction to account for member plasticity, implying a value for factor $q = 1$, or (ii) the limit shear force V_S that develops in a structural joint based on the capacities of all individual n members that form the joint. Furthermore, R_m is the corresponding available resistance of a specific structural member computed using mean values for the material properties of R/C. The next step involves computation of sensitivities to pre-selected variables (material properties, cross-section dimensions) for the aforementioned 'deficiency' indices λ_i, a process that requires quasi-linear analysis. We also define an average (or global) 'deficiency' index at a given floor level $<\lambda_k>$ as

$$\langle \lambda_k \rangle = \sum_{i=1}^{n} \lambda_i \cdot V_{si} / \sum_{i=1}^{n} V_{si} \tag{7}$$

Although the standard superposition principle is valid for the kinematic and stress variables at the structural member and floor levels, as well as for their sensitivity indices, this is not true for the 'deficiency' indices because of the underlying nonlinearity in the design process and in the definitions employed by the national Greek retrofit code (KANEPE 2007). Thus, indices λ_i and their sensitivities must be separately computed for each and every loading combination applied to the structure.

It should finally be noted here that computations for indices λ_i are carried out for all columns of an R/C frame, assuming that the beam elements are not the crucial components of the structural system for seismic types of loads. Then, looking at the sensitivity data and reconfiguring the section properties of the columns, it becomes possible to identify an optimal retrofit solution such that the recomputed λ_i for the strengthened columns are less or equal to unity.

7. Numerical examples

We consider two basic examples, one involving a truss where sensitivity is examined in terms of the standard state variables (i.e., the displacements) and another involving a building frame where deficiency indices are employed in conjunction with the sensitivity analysis.

7.1 Three-bay steel truss

The task here is to determine the distribution of the cross-section areas of all bars comprising the fictitious three-bay steel truss of Fig. 3 such that the vertical displacement at nodes 2 and 3 are both equal to 0.20 m downwards, starting with initial values for these displacements originally equal to 0.388 m. Thus, there are m = 1, 2 equality constraints, namely U_{Y2} and U_{Y3}, while the design variables are the cross-section areas of all bars, i.e., $i = 1,2,..., 11$. Also, $k = 1,2,... 11$ are inequality constraints indicating that rod element cross-sections can only increase in size. Furthermore, the diagonal matrix of weights $[W]$ is considered as the unit matrix, since all truss bars are of equal importance. For the material properties we prescribe a normalized value of $E = 1000$ kN/m^2, while all member cross-sections originally start at the reference value $A = 1.0$ m^2, and the base length is $L = 2$ m.

The first set of cross-section areas values are computed and presented in Table 3. As a second trial, we consider the same structure but with the set of design variables comprising truss elements 4-9 only. The results of the recomputation are presented in Table 4 below.

We mention here that the cost function C for the first case has a value of $C_1 = 5.61$, while for the second case we have that $C_2 = 16.80$. This could be expected, since in the second case the set of design variables is limited to a subset of the truss elements, thus restricting the range of the solution by roughly 50%.

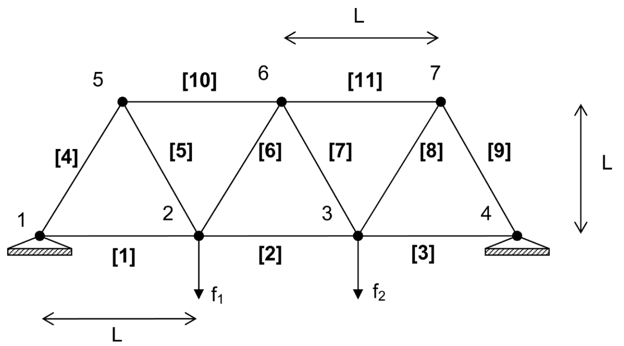


Fig. 3 Steel truss example

Table 3 Member cross-section areas for Case 1

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
1.06	1.24	1.06	2.01	2.01	1.00	1.00	2.01	2.01	1.85	1.85

Table 4 Member cross-section areas for Case 2

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
1.00	1.00	1.00	3.05	3.05	1.00	1.00	3.05	3.05	1.00	1.00

7.2 Seven-story R/C frame

We consider the seven-story asymmetric building of Fig. 4(a) with a basement and rigid diaphragm floor action, where a typical floor plan view is given in Fig. 4(b). Use of static elastic analysis is permitted by the design code EKOS (2003) provided certain conditions are satisfied, among them that $\lambda_i \leq 2.5$. It should be mentioned here that deficiency indices λ_i are also used to define regularity criteria in buildings, in addition to providing a first glimpse of available resistance to seismic actions. For instance, if $\lambda_i > 5.0$ for a large number of structural elements (more than 1/3 of total), it is obvious that the building is deficient and there is no reason for further investigations.

It therefore becomes possible to define a rational intervention strategy aimed at retrofitting the structure by simply devising methods to reduce the numerical values of the deficiency indices of both individual elements and floor assemblies. Starting with the national Greek code for structural interventions (KANEPE 2005), we adopt the following definitions for the seven-story building:

(a) Functionality level of the Structural System (SS): Life Protection

(b) Probability of excedance of the 50-year design earthquake by the SS: 10%

(c) Functionality level of the Non-structural System (NS): Life Protection

(d) Probability of excedance of the 50-year design earthquake by the NS: 10%

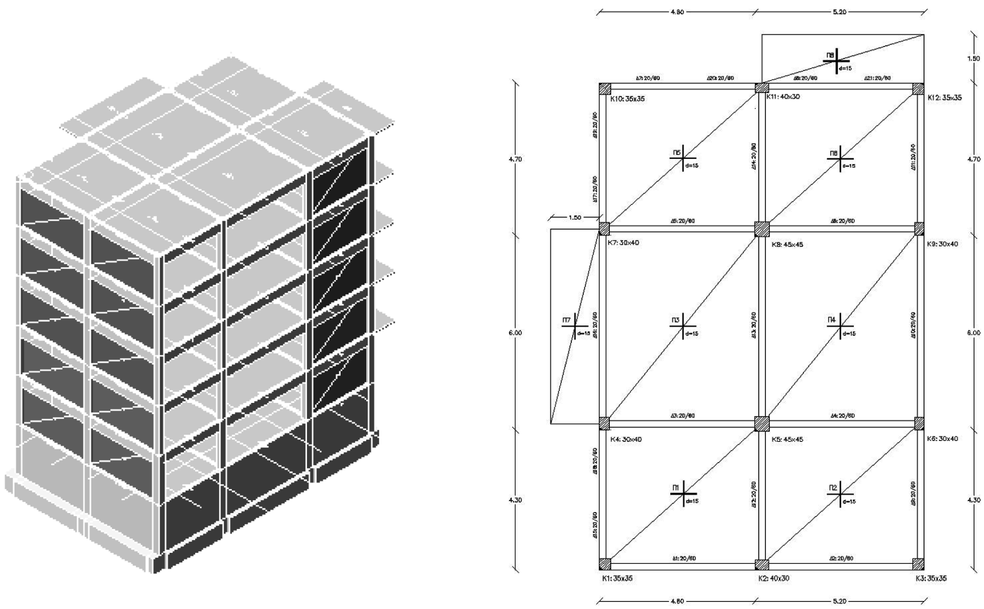
(e) The building is thought to have sustained damages: Section, §.2.1.2 KANEPE Code

(f) Damages have not been taken into account in the structural analysis of the building.

(g) Any intervention strategy will focus on repairing sustained damage and then in some selective strengthening of key structural members.

(h) The SS fully participates in any seismically-induced event.

(i) All masonry infill walls provide added stiffness to the SS.



(a) (b)

Fig. 4 (a) 3D view of the seven-story building and (b) typical floor plan

Table 5 Deficiency index λ_i for ground floor columns for load combination $G + 0.3Q + E_Y$

K1	*K2*	*K3*	*K4*	*K5*	*K6*	*K7*	*K8*	*K9*	*K10*	*K11*	*K12*
1.67	1.414	1.043	0.766	1.286	2.257	1.797	1.149	2.015	1.734	1.345	1.453

Table 6 Deficiency index λ_i for ground floor columns

K1	K2	K3	K4	K5	K6	K7	K8	K9	K10	K11	K12
0%	0%	0%	2%	43%	43%	0%	43%	43%	0%	0%	0%
0.895	0.895	0.888	0.9	0.923	0.913	0.897	0.926	0.917	0.898	0.891	0.892

(j) The level of accuracy in the description of all geometric and material properties of the SS is considered satisfactory.

(k) The 'deficiency' indices of the individual vertical (i.e., column) elements are computed at the local coordinate system as the inverse of the ratio of design resistance to required resistance.

The deficiency indices at the floor levels are computed in the two principal directions along which the earthquake motion has been prescribed, by using the individual deficiency indices of all separate vertical members framing into that floor. The ensuing values are normalized by the percentage of total floor shear force that develops in each column and in both perpendicular directions, as shown in Table 5. We observe that most of the twelve columns (K_i) at the ground floor level are deficient when the load combination involving dead, live and seismic loading $G + 0.3Q + E_Y$ is examined.

Continuing through with the example, we now set about the following repair/strengthening target: redesign all first floor columns by choosing new cross-sections so that their revised deficiency indices assume new values in the range $8.5 \leq \lambda_i \leq 9.5$. Therefore, the design variables are the cross-sections of the twelve columns ($i = 1,...,12$), whereas the conditions on deficiency indices constitute the set of indirect inequality constraints for this problem. Also, a set of direct constraints is enforced by assuming that the cross-sections can only increase. Furthermore, the diagonal matrix of weights $[W]$ is considered as the identity matrix since all members are of equal importance. Results are recovered in Table 6. It can be observed that the deficiency indices have all been brought down to optimal values, and the amount of extra reinforcement (as percentage) of certain key columns has also been computed.

8. Conclusions

This work presented a methodology based on combination of sensitivity analysis (for the response variables and their associated indices) with optimization procedures in order to estimate the minimum amount of structural intervention necessary in a building that satisfies pre-set constraints on its retrofit program. This way, it becomes possible to map out an optimum retrofit schedule for an existing structural system. In sum, the sensitivity analysis presented herein has a number of advantages with respect to planned structural interventions, which are as follows: (i) It is a fast and effective way of realizing sets of multi-variable analyses for use in repair scenarios; (ii) it is an algorithmic extension of the intuitive feeling that a practicing engineer develops regarding the way a structure behaves; (iii) it can quickly and accurately yield the rate of change of state variables in reference to changes in the design variables; (iv) the change in the state variables can be filtered into local or global 'deficiency' indices and their variability for the structure in question; and (v) it is a valuable tool that can be used in optimizing structural behavior and in performing inverse-type of analyses. Sensitivity analysis has important applications in earthquake-resistant design, since it is possible to extend these concepts to fully inelastic, time-dependent structural response.

As discussed before, the development of a new generation of commercial FEM software based on the concepts of structural sensitivity and stochastic analysis will have practical implications in the civil engineering design profession. Specifically, software of this type can be used as an aid in addressing the following problems: (i) Repair and retrofit of historical structures so as to restore their original appearance and function; (ii) strengthening of existing buildings in order to upgrade their structural behavior in a manner that is consistent with new versions of national and European building codes; and (iii) repairs, both emergency-type and long-term, following structural damage due to environmentally-induced loads such as earthquakes. In all cases, a carefully conducted sensitivity analysis reveals the influence of specific changes in the various structural components of a building system on the kinematic and traction fields that develop for preset categories of loads, which can then be filtered into various dimensionless indices representative of the inelastic behavior of the structural system. This can be coupled with optimization procedures so as to minimize the amount of intervention to the absolute necessary. Finally, although the emphasis is on deterministic loads, loads of random nature can also be handled by the present development.

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Appendix Analysis of MDOF systems with uncertain properties and random input

The response of a multi-degree-of-freedom (MDOF) system to non-stationary random input parallels the classical random vibration analysis (Nigam 1983) of single DOF systems. The equations of motion and initial conditions are

$$\left.\begin{array}{ll}[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\} = \{f(t)\} & t>t_0 \\ \{x(t_0)\} = \{x_0\},\ \ \{\dot{x}(t_0)\} = \{\dot{x}_0\} & \end{array}\right\} \tag{A.1}$$

where, $[M]$, $[C]$, $[K]$ are symmetric, positive-definite $N \times N$ matrices representing mass, damping and stiffness, respectively, while $\{x\}$ and $\{f\}$ are $N \times 1$ column vectors denoting input and output processes. Also, $\{f(t)\}$ is a vector of Gaussian random variables with mean and covariance

$$\left.\begin{array}{l}\{m_f(t)\} = E[\{f(t)\}] \\ [K_{ff}(t_1,t_2)] = E\left[\{f(t_1)-m_f(t_1)\}\{f(t_2)-m_f(t_2)\}^T\right]\end{array}\right\} \tag{A.2}$$

In the above, superscript T denotes transposition and E is the expectation operator.

If the system of Eq. (A.1) possesses classical normal modes, the matrix of normalized eigenvectors $[A]$ defines a set of modal coordinates as

$$\{y\} = [A]^{-1}\{x\} = [A]^T[M]\{x\} \tag{A.3}$$

Use of modal coordinates in conjunction with Eq. (A.1) results in an uncoupled system of equations of motion given by

$$[I]\{\ddot{y}\}+\left[\bar{C}\right]\{\dot{y}\}+\left[\bar{K}\right]\{y\} = [A]^T\{f(t)\} = \{q(t)\} \tag{A.4}$$

where $[I]$ is the identity matrix and $[\bar{C}]$ and $[\bar{K}]$ are now diagonal matrices. Taking the i^{th} row of the above system gives a single DOF equation as

$$\ddot{y}_i + 2\omega_i\zeta_i\dot{y}_i + \omega_i^2 y_i = q_i(t) = \sum_{j=1}^{N} A_{ij} f_j(t) \tag{A.5}$$

where ω is the natural frequency and ζ is the coefficient of damping for that particular mode.

The MDOF system response in modal coordinates is

$$\{y\} = [U(t-t_0)]\{y(t_0)\}+[H(t-t_0)]\{\dot{y}(t_0)\}+\int_{t_0}^{t}[H(t-\tau)]\{q(\tau)\}\,d\tau \tag{A.6}$$

where $[U]$ and $[H]$ are diagonal matrices with elements

$$\left.\begin{aligned} U_{ii}(t) &= \exp\left(-\zeta_i \omega_i t\right)\left(\cos \bar{\omega}_i t + \zeta_i \omega_i \sin \bar{\omega}_i t / \bar{\omega}_i\right) \\ H_{ii}(t) &= \exp\left(-\zeta_i \omega_i t\right) \sin \bar{\omega}_i t / \bar{\omega}_i \\ \bar{\omega}_i &= \omega_i \sqrt{1-\zeta_i^2} \end{aligned}\right\} \tag{A.7}$$

The former matrix accounts for the initial conditions, while the latter one contains the unit impulse response functions. Reverting to physical coordinates via the transformation of Eq. (A.3) gives

$$\begin{aligned} \{x\} = [A]\left[U\left(t-t_0\right)\right][A]^T [M]\{x_0\} + [A]\left[H\left(t-t_0\right)\right][A]^T [M]\{\dot{x}_0\} \\ + \int_{t_0}^{t} [A]\left[H\left(t-\tau\right)\right][A]^T \left\{f\left(\tau\right)\right\} d\tau \end{aligned} \tag{A.8}$$

Given the above solution for the mean-square response, the stochastic means are formulated as

$$\begin{aligned} \left\{m_x(t)\right\} = [A]\left[U\left(t-t_0\right)\right][A]^T [M]\{x_0\} + [A]\left[H\left(t-t_0\right)\right][A]^T [M]\{\dot{x}_0\} \\ + \int_{t_0}^{t} [A]\left[H\left(t-\tau\right)\right][A]^T \left\{m_f\left(\tau\right)\right\} d\tau \end{aligned} \tag{A.9}$$

Next, the covariance matrix is given by the stochastic average of the outer product of the aforementioned zero-mean response vector evaluated at two different times, i.e.

$$\begin{aligned} \left[K_{xx}\left(t_1,t_2\right)\right] &= E\left[\left\{x\left(t_1\right)-m_x\left(t_1\right)\right\}\left\{x\left(t_2\right)-m_x\left(t_2\right)\right\}^T\right] \\ &= \int_0^{t_1-t_0}\int_0^{t_2-t_0} [A][H\left(\xi_1\right)][A]^T\left[K_{ff}\left(t_1-\xi_1,t_2-\xi_2\right)\right][A][H\left(\xi_2\right)][A]^T \, d\xi_1 \, d\xi_2 \end{aligned} \tag{A.10}$$

For a linear system, Gaussian input results in Gaussian output and the probability density function (PDF) for the ith component of the response $\{x(t)\}$ is

$$p(x_i) = \exp\left(-\left(x_i(t) - m_{xi}(t)\right)^2\right) / \left(\sqrt{2\pi}\ \sigma_{xi}(t)\right) \tag{A.11}$$

where the variance is defined as a diagonal component of the covariance matrix evaluated at $t_1 = t_2 = t$, i.e., $\sigma_{xi}^2(t) = K_{ii}(t,t)$. Finally, if the components of the input vector $\{f\}$ are jointly normally distributed, so are the components of the output vector $\{x\}$ with a joint *PDF* given by

$$\begin{gathered} p(x_1, x_2, \ldots, x_N) = \\ \exp\left(-0.5\left\{x(t)-m_x(t)\right\}^T\left[K_{xx}(t)\right]^{-1}\left\{x(t)-m_x(t)\right\}\right) / \left(\left(2\pi\right)^{N/2}\left(\det\left[K_{xx}(t)\right]\right)^{1/2}\right) \end{gathered} \tag{A.12}$$

For stationary random processes, all the above statistical averages are time-independent.

Consider now structures with material randomness expressed in the general form $z = z^0(1+\gamma)$, where z^0 is the deterministic value of a specific material property (e.g., elastic modulus) or of a structural component (e.g., moment of inertia) and γ is a random, zero-mean small fluctuation about z^0. Following Vanmarke *et al.* (1986) and Liu *et al.* (1986), we start with the static case and the stiffness matrix is expanded about the uncertainty using Taylor series as

$$[K] = \left[K^0\right] + \sum_{i=1}^{n}[K]_{,i}\,\gamma_i + 0.5\sum_{i=1}^{n}\sum_{j=1}^{n}[K]_{,ij}\,\gamma_i\gamma_j \tag{A.13}$$

In the above, n denotes the total number of random parameters γ_i. Using standard notation, superscript 0 in the first term denotes a deterministic quantity and subsequent terms in the expansion denote higher order rates of change, which are evaluated by differentiating $[K]$ with respect to the random parameters γ_i. The important point here is that upon solution, uncertainty is reflected in the nodal displacements and subsequently filters into both the kinematic and stress fields that develop in the structure. The same type of expansion as above can also be used for the nodal displacements, i.e.

$$\{x\} = \left\{x^0\right\} + \sum_i \{x\}_{,i}\,\gamma_i + 0.5\sum_i\sum_j \{x\}_{,ij}\,\gamma_i\gamma_j \tag{A.14}$$

Since random input was examined previously, $\{f\}$ is assumed to be deterministic here.

Substitution of the above two expansions in the static equilibrium equation and subsequent ordering of the resulting terms (Hinch 1991) gives the following system of equations

$$\left.\begin{aligned}
&\left[K^0\right]\left\{x^0\right\} = \{f\}\\
&\left[K^0\right]\{x\}_{,i} = -[K]_{,i}\left\{x^0\right\}\\
&\left[K^0\right]\{x\}_{,ij} = -[K]_{,i}\{x\}_{,j} - [K]_{,j}\{x\}_{,i} - [K]_{,ij}\left\{x^0\right\}
\end{aligned}\right\} \tag{A.15}$$

The structure of the above system of equations is quite standard and all unknown displacement terms can be obtained sequentially, starting from the deterministic solution $\{x^0\}$ and substituting the newly found terms in the right-hand side of the following equation. As a result, the deterministic stiffness matrix needs to be inverted only once, resulting in an efficient solution scheme. Also, the non-zero terms in $[K]_{,i}$ and $[K]_{,ij}$ are relatively few, so that the right-hand sides can be quickly formed. Following the displacement field solution, the unknown stresses within a finite element can be found after stress terms $\{\sigma^0\}$, $\{\sigma\}_{,i}$ and $\{\sigma\}_{,ij}$ have been evaluated in the usual way from their corresponding nodal displacements in the form

$$\{\sigma\} = \left\{\sigma^0\right\} + \sum_i \{\sigma\}_{,i} + 0.5\sum_i\sum_j \{\sigma\}_{,ij} \tag{A.16}$$

Based on the above equation, the expectation and variance for stresses respectively are

$$E\left[\{\sigma\}\right] = \left\{\sigma^0\right\} + 0.5\sum_i\sum_j \{\sigma\}_{,ij}\,E\left[\gamma_i\gamma_j\right] \tag{A.17}$$

and

$$E\left[\{\sigma\}\{\sigma\}^T\right] = \sum_i \sum_j \{\sigma\},_i \{\sigma\},_j E\left[\gamma_i\gamma_j\right] + \sum_i \sum_j \sum_k \{\sigma\},_i \{\sigma\},_{jk}^T E\left[\gamma_i\gamma_j\gamma_k\right]$$

$$+0.25 \sum_i \sum_j \sum_k \sum_\ell \{\sigma\},_{ij} \{\sigma\}^T,_{kl} \left(E\left[\gamma_i\gamma_j\gamma_k\gamma_\ell\right] - E\left[\gamma_i\gamma_j\right] E\left[\gamma_k\gamma_\ell\right]\right) \tag{A.18}$$

Since local changes in a structural parameter cause nonlinear changes in the structural response, a second order Taylor series expansion such as the one used here is necessary to cover such behavior. Third order expansions are preferable (Augusti *et al.* 1984), but computation becomes prohibitively expensive since the sixth moments of the random variables γ_i are necessary.

Dynamic effects can be viewed within the context of the classical eigenvalue problem for linear systems

$$\left([K] - \lambda[M]\right)\{\varphi\} = \{0\} \tag{A.19}$$

where $[M]$ is the mass matrix, λ are the eigenvalues and $\{\varphi\}$ are their corresponding eigenvectors. As before, uncertainty in the stiffness and mass matrices filters, upon solution, to the eigenproperties of the structure. By expanding both eigenvalues and eigenvectors in a Taylor series about the randomness γ we recover the following results

$$\lambda = \lambda^0 + \sum_i \lambda,_i \gamma_i + 0.5 \sum_i \sum_j \lambda,_{ij} \gamma_i\gamma_j \tag{A.20}$$

and

$$\{\varphi\} = \left\{\varphi^0\right\} + \sum_i \{\varphi\},_i \gamma_i + 0.5 \sum_i \sum_j \{\varphi\},_{ij} \gamma_i\gamma_j \tag{A.21}$$

Substitution of the above two expressions in Eq. (A.19) along with the expansion of Eq. (A.13) for the stiffness and a similar expansion for the mass, gives the following system of equations

$$\left.\begin{aligned}
&\left(\left[K^0\right] - \lambda^0\left[M^0\right]\right)\left\{\varphi^0\right\} = \left[H^0\right]\left\{\varphi^0\right\} = \{0\} \\
&\left[H^0\right]\{\varphi\},_i = -\left([K],_i - \lambda^0[M],_i - \lambda,_i\left[M^0\right]\right)\left\{\varphi^0\right\} \\
&\left[H^0\right]\{\varphi\},_{ij} = -\left([K],_i - \lambda^0[M],_i - \lambda,_i\left[M^0\right]\right)\{\varphi\},_j \\
&-\left([K],_j - \lambda^0[M],_j - \lambda,_j\left[M^0\right]\right)\{\varphi\},_i \\
&-\left([K],_{ij} - \lambda,_i[M],_j - \lambda,_j[M],_i - \lambda,_{ij}\left[M^0\right] - \lambda^0[M],_{ij}\right)\left\{\varphi^0\right\}
\end{aligned}\right\} \tag{A.22}$$

By taking advantage of symmetry in system matrix $[H^0]$, the derivatives of the eigenvalues $\lambda,_i$ can be computed from the second of Eq. (A.22) as

$$\lambda,_i = \left\{\varphi^0\right\}^T \left([K],_i - \lambda^0[M],_i\right)\left\{\varphi^0\right\} / \left(\left\{\varphi^0\right\}^T [M]\left\{\varphi^0\right\}\right) \tag{A.23}$$

The derivatives of the eigenvectors $\{\varphi\}_{,i}$ from the second of Eq. (A.22) is not, however, feasible because of the singularity in $[H^0]$. To overcome this drawback, a reduction in the rank of $[H^0]$ is necessary. The same situation holds for the evaluation of $\{\varphi\}_{,ij}$ since only the right-hand side of Eq. (A.22) changes. As with the static case, each new eigenvalue solution depends on the previously obtained eigenproperties.