# Thermomechanical interactions in a transversely isotropic magneto thermoelastic solids with two temperatures and rotation due to time harmonic sources

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**Abstract.** The present research deals in two dimensional (2D) transversely isotropic magneto generalized thermoelastic solid without energy dissipation and with two temperatures due to time harmonic sources in Lord-Shulman (LS) theory of thermoelasticity. The Fourier transform has been used to find the solution of the problem. The displacement components, stress components and conductive temperature distribution with the horizontal distance are calculated in transformed domain and further calculated in the physical domain numerically. The effect of two temperature are depicted graphically on the resulting quantities.

**Keywords**: transversely isotropic Magneto thermoelastic; nechanical and thermal stresses; inclined load; time harmonic source

#### 1. Introduction

The classical theory of elasticity deals with the systematic study of the stress and strain distribution that develops in an elastic body due to the application of forces or change in temperature. A lot of research and attention has been given to deformation and heat flow in a continuum using thermoelasticity theories during the past few years. It is well known that all the rotating large bodies have an angular velocity, as well as magnetism, therefore, the thermoelastic interactions in a rotating medium under magnetic field is of importance. When sudden heat/external force is applied in a solid body, it transmits time harmonic wave by thermal expansion. The change at some point of the medium is beneficial to detect the deformed field near mining shocks, seismic and volcanic sources, thermal power plants, high-energy particle accelerators, and many emerging technologies. The study of time harmonic source is one of the broad and dynamic areas of continuum dynamics. Therefore, in an unbounded rotating elastic medium with angular velocity, with two temperature, rotation and relaxation time and without energy dissipation in generalized thermoelasticity has been studied in this research.

Marin (1997) had proved the Cesaro means of strain and kinetic energies of dipolar bodies with finite energy. Ailawalia *et al.* (2010) had studied a rotating generalized thermoelastic medium in presence of two temperatures beneath hydrostatic stress and gravity with different kinds of sources

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using integral transforms. Singh and Yadav (2012) solved the transversely isotropic rotating magnetothermoelastic medium equations by cubic velocity equation of three plane waves without anisotropy, rotation, and thermal and magnetic effects. Banik and Kanoria (2012) studied the thermoelastic interaction in an isotropic infinite elastic body with a spherical cavity for the TPL(Three-Phase-Lag) heat equation with two-temperature generalized thermoelasticity theory and has shown variations between two models: the two-temperature GN theory in presence of energy dissipation and two-temperature TPL model and has shown the effects of ramping parameters and two-temperature.

Mahmoud (2012) had considered the impact of rotation, relaxation times, magnetic field, gravity field and initial stress on Rayleigh waves and attenuation coefficient in an elastic half-space of granular medium and obtained the analytical solution of Rayleigh waves velocity by using Lame's potential techniques. Abd-alla and Alshaikh (2015) had discussed the influence of magnetic field and rotation on plane waves in transversely isotropic thermoelastic medium under the GL theory in presence of two relaxation times to show the presence of three quasi plane waves in the medium. Marin *et al.* (2013) has modelled a micro stretch thermoelastic body with two temperatures and eliminated divergences among the classical elasticity and research. Keivani *et al.* (2014) discussed the forced vibration problem of an Euler-Bernoulli beam with a semi-infinite field by considering it a BVP in the frequency domain

Sharma et al. (2015) investigated the 2-D deception in a transversely isotropic homogeneous thermoelastic solids in presence of two temperatures in GN-II theory with an inclined load (linear combination of normal load and tangential load). Delfim et al. (2015) presented a coupled FEM-BEM strategy for elastodynamic problems having infinite-domain models and complex heterogeneous media by using frequency domain analyses and an iterative FEM-BEM coupling technique. Kumar et al. (2016) investigated the impact of Hall current in a transversely isotropic magnetothermoelastic in presence and absence of energy dissipation due to normal force. Kumar et al. (2016) studied the conflicts caused by thermomechanical sources in a transversely isotropic rotating homogeneous thermoelastic medium with magnetic effect as well as two temperature and applied to the thermoelasticity Green-Naghdi theories with and without energy dissipation using thermomechanical sources. Lata et al. (2016) studied two temperature and rotation aspect for GN-II and GN-III theory of thermoelasticity in a homogeneous transversely isotropic magnetothermoelastic medium for the case of the plane wave propagation and reflection. Ezzat et al. (2017) proposed a mathematical model of electro-thermoelasticity for heat conduction with memory-dependent derivative. Kumar et al. (2017) analyzed the Rayleigh waves in a transversely isotropic homogeneous magnetothermoelastic medium in presence of two temperature, with Hall current and rotation. Vinyas et al. (2017) discovered a multiphysics behaviour of magneto-electroelastic (MEE) cantilever beam using thermo-mechanical loading. Akbas (2017) study the nonlinear static deflections of functionally graded (FG) porous under thermal effect using total lagrangian FEM within 2D continuum model in the Newton-Raphson iteration method.

Marin *et.al.* (2017) studied the GN-thermoelastic theory for a dipolar body using mixed initial BVP and proved a result of Hölder's-type stability. Lata (2018) studied the impact of energy dissipation on plane waves in sandwiched layered thermoelastic medium of uniform thickness, with two temperature, rotation and Hall current in the context of GN Type-II and Type-III theory of thermoelasticity. Ezzat and El-Bary (2017) had applied the magneto-thermoelasticity model to a one-dimensional thermal shock problem of functionally graded half-space of based on memory-dependent derivative. Hassan *et al.* (2018) investigated water base nanofluid flow over wavy surface in a porous medium (copper oxides particles) of spherical packing beds. Kumar *et al.* 

(2018) investigated the deformations in a homogeneous transversely isotropic magneto-Visco thermoelastic medium under GN type I and II theories in presence of rotation and two temperature with thermomechanical sources. Despite of this several researchers worked on different theory of thermoelasticity as Marin (1997), Marin (2008), Atwa (2014), Marin (2016), Marin and Baleanu (2016), Bijarnia and Singh (2016), Ezzat *et al.* (2016), Ezzat *et al.* (2012), Ezzat *et al.* (2015), Ezzat and El-Bary (2016), Ezzat and El-Bary (2017), Ezzat *et al.* (2017), Chauthale *et al.* (2017) and Shahani and Torki (2018), Lata and Kaur (2019).

Inspite of these, not much work has been carried out in thermomechanical interactions in transversely isotropic magneto thermoelastic solid with two temperature, rotation and relaxation time and without energy dissipation due to time harmonic source in generalized LS theories of thermoelasticity. Keeping these considerations in mind, analytic expressions for the displacements, stresses and temperature distribution in two-dimensional homogeneous, transversely isotropic magneto-thermoelastic solids with two temperatures and without energy dissipation, rotation and various frequencies of time harmonic source.

## 2. Basic equations

For a general anisotropic thermoelastic medium, the constitutive relations in absence of heat source and body forces following Green and Naghdi(1992)are given by

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T. (1)$$

and equation of motion as described by Schoenberg and Censor (1973) for a uniformly rotating medium with an angular velocity and Lorentz force which governs the dynamic displacement *u* is

$$t_{ij,j} + F_i = \rho \{ \ddot{u}_i + (\Omega \times (\Omega \times \mathbf{u})_i + (2\Omega \times \dot{u})_i \},$$
 (2)

where  $\Omega = \Omega n$ , n is a unit vector representing the direction of axis of rotation, The term  $\Omega \times (\Omega \times u)$  is the additional centripetal acceleration due to the time-varying motion only, and the term  $2\Omega \times \dot{u}$  is the Coriolis acceleration. All other terms are as usual  $F_i = \mu_0(\vec{j} \times \vec{H}_0)$ .

The heat conduction equation without energy dissipation using Lord-Shulman (1967) model is

$$K_{ij}\varphi_{,ij} + \rho(Q + \tau_0\dot{Q}) = \beta_{ij}T_0(\dot{e}_{ij} + \tau_0\ddot{e}_{ij}) + \rho C_E(\dot{T} + \tau_0\ddot{T}), \tag{3}$$

where

$$\beta_{ij} = C_{ijkl}\alpha_{ij},\tag{4}$$

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad i, j = 1,2,3.$$

$$T = \varphi - a_{ij} \varphi_{,ij}$$
(5)

 $\beta_{ij} = \beta_i \delta_{ij}$ ,  $K_{ij} = K_i \delta_{ij}$ , i is not summed. Here  $C_{ijkl}(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$  are elastic parameters.

## 3. Formulation and solution of the problem

We consider a homogeneous transversely isotropic magnetothermoelastic medium, permeated

by an initial magnetic field  $\vec{H}_0 = (0, H_0, 0)$  acting along y-axis. The rectangular Cartesian coordinate system (x, y, z) having origin on the surface (z = 0) with z-axis pointing vertically into the medium is introduced. The surface of the half-space is subjected to a thermomechanical force acting at z = 0.

In addition, we consider that

$$\Omega = (0, \Omega, 0).$$

From the generalized Ohm's law

$$J_2 = 0$$

The density components  $J_1$  and  $J_3$  are given as

$$J_1 = -\varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2},\tag{6}$$

$$J_3 = \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2}. \tag{7}$$

In addition, the equations of displacement vector  $(\vec{u}, \vec{v}, \vec{w})$  and conductive temperature  $\varphi$  for transversely isotropic thermoelastic solid in presence of two temperature and without energy dissipation are

$$\vec{u} = u(x, z, t), \vec{v} = 0, \vec{w} = w(x, z, t) \text{ and } \varphi = \varphi(x, z, t). \tag{8}$$

Now using the proper transformation on equations (1)-(3) following Slaughter (2002) are as under:

Eqns. (1) - (3) with the aid of (8), yield

$$C_{11}\frac{\partial^{2} u}{\partial x^{2}} + C_{13}\frac{\partial^{2} w}{\partial x \partial z} + C_{44}\left(\frac{\partial^{2} u}{\partial z^{2}} + \frac{\partial^{2} w}{\partial x \partial z}\right) - \beta_{1}\frac{\partial}{\partial x}\left\{\varphi - \left(a_{1}\frac{\partial^{2} \varphi}{\partial x^{2}} + a_{3}\frac{\partial^{2} \varphi}{\partial z^{2}}\right)\right\} - \mu_{0}J_{3}H_{0} = \rho\left(\frac{\partial^{2} u}{\partial t^{2}} - \Omega^{2}u + 2\Omega\frac{\partial w}{\partial t}\right),$$

$$(9)$$

$$(C_{13} + C_{44}) \frac{\partial^{2} u}{\partial x \partial z} + C_{44} \frac{\partial^{2} w}{\partial x^{2}} + C_{33} \frac{\partial^{2} w}{\partial z^{2}} - \beta_{3} \frac{\partial}{\partial z} \left\{ \varphi - \left( a_{1} \frac{\partial^{2} \varphi}{\partial x^{2}} + a_{3} \frac{\partial^{2} \varphi}{\partial z^{2}} \right) \right\} - \mu_{0} J_{1} H_{0}$$

$$= \rho \left( \frac{\partial^{2} w}{\partial t^{2}} - \Omega^{2} w - 2\Omega \frac{\partial u}{\partial t} \right), \tag{10}$$

$$K_{1} \frac{\partial^{2} \varphi}{\partial x^{2}} + K_{3} \frac{\partial^{2} \varphi}{\partial z^{2}} + \rho \left( Q + \tau_{0} \dot{Q} \right) = \rho C_{E} \left( \dot{T} + \tau_{0} \ddot{T} \right) + T_{0} \frac{\partial}{\partial t} \left\{ \beta_{1} \left( 1 + \tau_{0} \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial x} + \beta_{3} \left( 1 + \tau_{0} \frac{\partial}{\partial t} \right) \frac{\partial w}{\partial z} \right\}, \tag{11}$$

and

$$t_{11} = C_{11}e_{11} + C_{13}e_{13} - \beta_1 T, \tag{12}$$

$$t_{33} = C_{13}e_{11} + C_{33}e_{33} - \beta_3 T, \tag{13}$$

$$t_{13} = 2C_{44}e_{13}, (14)$$

where

$$T = \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2}\right),$$
  
$$\beta_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_3,$$
  
$$\beta_3 = 2C_{13}\alpha_1 + C_{33}\alpha_3.$$

We consider that medium is initially at rest. Therefore, the preliminary and symmetry conditions are given by

$$u(x, z, 0) = 0 = \dot{u}(x, z, 0),$$

$$w(x, z, 0) = 0 = \dot{w}(x, z, 0),$$

$$\varphi(x, z, 0) = 0 = \dot{\varphi}(x, z, 0) \text{ for } z \ge 0, -\infty < x < \infty,$$

$$u(x, z, t) = w(x, z, t) = \varphi(x, z, t) = 0 \text{ for } t > 0 \text{ when } z \to \infty.$$

Assuming the time harmonic behaviour as

$$(u, w, \varphi, Q)(x, z, t) = (u, w, \varphi, Q)(x, z)e^{i\omega t},$$
 (15)

where  $\omega$  is the angular frequency.

To simplify the solution, mention below dimensionless quantities are used

$$x' = \frac{x}{L}, \quad u' = \frac{\rho c_1^2}{L\beta_1 T_0} u, \quad t' = \frac{c_1}{L} t,$$

$$w' = \frac{\rho c_1^2}{L\beta_1 T_0} w, \quad T' = \frac{T}{T_0}, t'_{11} = \frac{t_{11}}{\beta_1 T_0}, \quad t'_{33} = \frac{t_{33}}{\beta_1 T_0},$$

$$t'_{31} = \frac{t_{31}}{\beta_1 T_0}, \quad \varphi' = \frac{\varphi}{T_0}, \quad a'_{1} = \frac{a_1}{L^2}, \quad z' = \frac{z}{L},$$

$$a'_{3} = \frac{a_3}{L^2}, \quad h' = \frac{h}{H_0}, \quad \Omega' = \frac{L}{C_1} \Omega.$$
(16)

Making use of (16) in Eqs. (9)–(11), after suppressing the primes, yield

$$\frac{\partial^{2} u}{\partial x^{2}} + \delta_{4} \frac{\partial^{2} w}{\partial x \partial z} + \delta_{2} \left( \frac{\partial^{2} u}{\partial z^{2}} + \frac{\partial^{2} w}{\partial x \partial z} \right) - \frac{\partial}{\partial x} \left\{ \varphi - \left( a_{1} \frac{\partial^{2} \varphi}{\partial x^{2}} + a_{3} \frac{\partial^{2} \varphi}{\partial z^{2}} \right) \right\} = \left( \frac{\varepsilon_{0} \mu_{0}^{2} H_{0}^{2}}{\rho} + 1 \right) (-\omega^{2} u) - \Omega^{2} u + 2\Omega i \omega w, \tag{17}$$

$$\delta_{1} \frac{\partial^{2} u}{\partial x \partial z} + \delta_{2} \frac{\partial^{2} w}{\partial x^{2}} + \delta_{3} \frac{\partial^{2} w}{\partial z^{2}} - \frac{\beta_{3}}{\beta_{1}} \frac{\partial}{\partial z} \left\{ \varphi - \left( a_{1} \frac{\partial^{2} \varphi}{\partial x^{2}} + a_{3} \frac{\partial^{2} \varphi}{\partial z^{2}} \right) \right\} = \left( \frac{\varepsilon_{0} \mu_{0}^{2} H_{0}^{2}}{\rho} + 1 \right) (-\omega^{2} w) - \Omega^{2} w + 2\Omega i \omega u, \quad (18)$$

$$\frac{\partial^{2} \varphi}{\partial x^{2}} + \frac{\kappa_{3}}{\kappa_{1}} \frac{\partial^{2} \varphi}{\partial z^{2}} + \rho \left( 1 + \tau_{0} \frac{c_{1}}{L} i\omega \right) Q = \delta_{5} \frac{\partial}{\partial t} \left( 1 + \tau_{0} \frac{c_{1}}{L} i\omega \right) \left[ \varphi - a_{1} \frac{\partial^{2} \varphi}{\partial x^{2}} - a_{3} \frac{\partial^{2} \varphi}{\partial z^{2}} \right] + \delta_{6} i\omega \left( 1 + \tau_{0} \frac{c_{1}}{L} i\omega \right) \left[ \beta_{1} \frac{\partial u}{\partial x} + \beta_{3} \frac{\partial w}{\partial z} \right], \tag{19}$$

where

$$\delta_{1} = \frac{c_{13} + c_{44}}{c_{11}}, \qquad \delta_{2} = \frac{c_{44}}{c_{11}}, \qquad \delta_{3} = \frac{c_{33}}{c_{11}}, \qquad \delta_{4} = \frac{c_{13}}{c_{11}},$$

$$\delta_{5} = \frac{\rho C_{E} C_{1} L}{K_{1}}, \qquad \delta_{6} = -\frac{T_{0} \beta_{1} L}{\rho C_{1} K_{1}}$$

Apply Fourier transforms defined by

$$\hat{f}(\xi, z, \omega) = \int_{-\infty}^{\infty} f(x, z, \omega) e^{i\xi x} dx$$
 (20)

On Eqs. (17)–(19), we obtain a system of equations

$$[-\xi^{2} + \delta_{2}D^{2} + \delta_{7}\omega^{2} + \Omega^{2}]\hat{u}(\xi, z, \omega) + [\delta_{4}Di\xi + \delta_{2}Di\xi - 2\Omega i\omega]\hat{w}(\xi, z, \omega) + (-i\xi)[1 + a_{1}\xi^{2} - a_{3}D^{2}]\hat{\varphi}(\xi, z, \omega) = 0,$$
(21)

$$[\delta_{1}Di\xi + 2\Omega i\omega]\hat{u}(\xi, z, \omega) + [-\delta_{2}\xi^{2} + \delta_{3}D^{2} + \delta_{7}\omega^{2} + \Omega^{2}]\hat{w}(\xi, z, \omega) - \frac{\beta_{3}}{\beta_{1}}D[1 + a_{1}\xi^{2} - a_{3}D^{2}]\hat{\varphi}(\xi, z, \omega) = 0,$$
(22)

$$[-\delta_6\omega\delta_8\beta_1\xi]\hat{u}(\xi,z,\omega) + [\delta_6i\omega\delta_8\beta_3D]\hat{w}(\xi,z,\omega) + \left[\xi^2 - \frac{\kappa_3}{\kappa_1}D^2 + \delta_5\delta_8i\omega(1 + a_1\xi^2 - a_3D^2)\right]\hat{\varphi}(\xi,z,\omega) = \rho\delta_8\hat{Q}(\xi,z,\omega),$$

$$(23)$$

where

$$\delta_7 = \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1, \qquad \delta_8 = 1 + \tau_0 \frac{c_1}{L} i\omega.$$

By taking  $\hat{Q}(\xi, z, s) = 0$ , the non trivial solution of homogeneous equations (21)-(23) exists if determinant of coefficient matrix  $(\hat{u}, \hat{w}, \hat{\varphi})$  of (21)-(23) is equal to zero i.e.,

$$AD^6 + BD^4 + CD^2 + E = 0, (24)$$

where

$$D = \frac{d}{dz},$$

$$A = \delta_2 \delta_3 \vartheta_7 - \vartheta_5 \delta_2 \frac{\beta_3}{\beta_1} a_3,$$

$$B = \delta_3 \vartheta_1 \vartheta_7 - a_3 \vartheta_1 \vartheta_5 \frac{\beta_3}{\beta_1} + \delta_2 \delta_3 \vartheta_6 + \delta_2 \vartheta_7 \vartheta_3 - \vartheta_5 \vartheta_9 \delta_2 - \vartheta_8 \delta_1 i \xi \vartheta_7 + \vartheta_8 \vartheta_4 \frac{\beta_3}{\beta_1} a_3 - a_3 \xi^2 \vartheta_5 \delta_1 - a_3 \delta_2 \vartheta_4 i \xi.$$

$$\mathsf{C} = \delta_3 \vartheta_1 \vartheta_6 + \vartheta_1 \vartheta_3 \vartheta_7 - \vartheta_1 \vartheta_5 \vartheta_9 + \delta_2 \vartheta_6 \vartheta_3 + \vartheta_4 \vartheta_8 \vartheta_9 - \vartheta_8 \delta_1 i \xi \vartheta_6 - 4 \Omega^2 \omega^2 \vartheta_7 + \vartheta_2 \delta_1 i \xi \vartheta_5 - \vartheta_2 \vartheta_4 \delta_3 - a_3 \vartheta_4 i \xi \vartheta_3,$$

$$E = \vartheta_3 \vartheta_1 \vartheta_6 - 4\Omega^2 \omega^2 \vartheta_6 - \vartheta_2 \vartheta_4 \vartheta_3,$$

$$\begin{split} \vartheta_1 &= \, \xi^2 + \delta_7 \omega^2 + \Omega^2, \\ \vartheta_2 &= -i \xi (1 + a_1 \xi^2), \\ \vartheta_3 &= -\delta_2 \xi^2 + \delta_7 \omega^2 + \Omega^2, \\ \vartheta_4 &= -\delta_6 \delta_8 \omega \beta_1 \xi, \\ \vartheta_5 &= \delta_6 \delta_8 i \omega \beta_3, \\ \vartheta_6 &= \, \xi^2 + \delta_5 \delta_8 i \omega (1 + a_1 \xi^2), \\ \vartheta_7 &= -\frac{K_3}{K_1} - a_3 \delta_5 \delta_8 i \omega, \\ \vartheta_8 &= \delta_1 i \xi, \\ \vartheta_9 &= -(1 + a_1 \xi^2) \frac{\beta_3}{R}. \end{split}$$

The roots of the Eq. (24) are  $\pm \lambda j$ , (j = 1, 2, 3), the solution of the Eq. (24) is calculated by using the radiation conditions that  $\tilde{u}, \tilde{w}, \tilde{\varphi} \to 0$  as  $z \to \infty$  yields

$$\hat{u}(\xi, z, \omega) = \sum_{i=1}^{3} A_i e^{-\lambda_j z}, \tag{25}$$

$$\widehat{w}(\xi, z, \omega) = \sum_{j=1}^{3} d_j A_j e^{-\lambda_j z}, \tag{26}$$

$$\hat{\varphi}(\xi, z, \omega) = \sum_{i=1}^{3} l_j A_j e^{-\lambda_j z},$$
(27)

where  $A_i(\xi, \omega)$ , j = 1, 2, 3 being undetermined constants and  $d_i$  and  $l_j$  are given by

$$\begin{split} d_j &= \frac{\delta_2 \zeta_7 \lambda_j^4 + (\vartheta_7 \vartheta_1 - a_3 \vartheta_4 i \xi + \delta_2 \vartheta_6) \lambda_j^2 + \vartheta_1 \vartheta_6 - \vartheta_4 \vartheta_2}{\left(\delta_3 \vartheta_7 - \frac{\beta_3}{\beta_1} a_3 \vartheta_5\right) \lambda_j^4 + (\delta_3 \vartheta_6 + \vartheta_3 \vartheta_7 - \vartheta_5 \vartheta_9) \lambda_j^2 + \vartheta_3 \vartheta_6} \\ l_j &= \frac{\delta_2 \delta_3 \lambda_j^4 + (\delta_2 \zeta_3 + \vartheta_1 \delta_3 - \delta_1 \vartheta_8 i \xi) \lambda_j^2 - 4 \Omega^2 \omega^2 + \vartheta_3 \vartheta_1}{\left(\delta_3 \vartheta_7 - \frac{\beta_3}{\beta_1} a_3 \vartheta_5\right) \lambda_j^4 + (\delta_3 \vartheta_6 + \vartheta_3 \vartheta_7 - \vartheta_5 \vartheta_9) \lambda_j^2 + \vartheta_3 \vartheta_6} \end{split}$$

## 4. Boundary conditions

Thermal source and normal force are applied on the half-space (z = 0) surface.

$$t_{33}(x,z,t) = -F_1 \psi_1(x) e^{i\omega t}, \tag{28}$$

$$t_{31} = 0, (29)$$

$$\frac{\partial \varphi}{\partial z}(x,z,t) = F_2 \psi_2(x) e^{i\omega t},\tag{30}$$

where  $F_1$  is the magnitude of the force applied,  $F_2$  is the constant temperature applied on the boundary,  $\psi_1(x)$  specifies the source distribution function along x-axis,  $\psi_2(x)$  specifies the source distribution function along z-axis.

Applying the Laplace and Fourier transform defined by (19) and (20) on the boundary conditions (28)-(30), (12)-(14) and with the help of Eqs. (25)-(27), we find the components of displacement, stress and conductive temperature as

$$\hat{u} = \frac{F_1 \hat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{i=1}^3 \Gamma_{1i} e^{-\lambda_i z} \right] e^{i\omega t} + \frac{F_2 \hat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{i=1}^3 \Gamma_{2i} e^{-\lambda_i z} \right] e^{i\omega t}, \tag{31}$$

$$\widehat{w} = \frac{F_1 \widehat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{i=1}^3 d_i \Gamma_{1i} e^{-\lambda_i z} \right] e^{i\omega t} + \frac{F_2 \widehat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{i=1}^3 d_i \Gamma_{2i} e^{-\lambda_i z} \right] e^{i\omega t}, \tag{32}$$

$$\hat{\varphi} = \frac{F_1 \hat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{i=1}^3 l_i \Gamma_{1i} e^{-\lambda_i z} \right] e^{i\omega t} + \frac{F_2 \hat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{i=1}^3 l_i \Gamma_{2i} e^{-\lambda_i z} \right] e^{i\omega t}, \tag{33}$$

$$\widehat{t_{11}} = \frac{F_1 \widehat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{i=1}^3 S_i \Gamma_{1i} e^{-\lambda_i z} \right] e^{i\omega t} + \frac{F_2 \widehat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{i=1}^3 S_i \Gamma_{2i} e^{-\lambda_i z} \right] e^{i\omega t}, \tag{34}$$

$$\widehat{t_{13}} = \frac{F_1 \widehat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{i=1}^3 N_i \Gamma_{1i} e^{-\lambda_i z} \right] e^{i\omega t} + \frac{F_2 \widehat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{i=1}^3 N_i \Gamma_{2i} e^{-\lambda_i z} \right] e^{i\omega t}, \tag{35}$$

$$\widehat{t_{33}} = \frac{F_1 \widehat{\psi}_1(\xi)}{\Gamma} \left[ \sum_{i=1}^3 M_i \Gamma_{1i} e^{-\lambda_i z} \right] e^{i\omega t} + \frac{F_2 \widehat{\psi}_2(\xi)}{\Gamma} \left[ \sum_{i=1}^3 M_i \Gamma_{2i} e^{-\lambda_i z} \right] e^{i\omega t}, \tag{36}$$

where

$$\begin{split} \Gamma_{11} &= -N_2 R_3 \, + R_2 N_3, \\ \Gamma_{12} &= N_1 R_3 \, - R_1 N_3, \\ \Gamma_{13} &= -N_1 R_2 \, + R_1 N_2, \\ \Gamma_{21} &= M_2 N_3 \, - N_2 M_3, \\ \Gamma_{22} &= -M_1 N_3 \, + N_1 M_3, \\ \Gamma_{23} &= M_1 N_2 \, - N_1 M_2, \\ \Gamma &= -M_1 \Gamma_{11} - M_2 \Gamma_{12} - M_3 \Gamma_{13} \end{split}$$

$$\begin{split} N_{j} &= -\delta_{2}\lambda_{j} + i\xi d_{j}, \\ M_{j} &= i\xi - \delta_{3}d_{j}\lambda_{j} - \frac{\beta_{3}}{\beta_{1}}l_{j}\big[(1 + a_{1}\xi^{2}) - a_{3}\lambda_{j}^{2}\big], \\ R_{j} &= -\lambda_{j}l_{j}\big[(1 + a_{1}\xi^{2}) - a_{3}\lambda_{j}^{2}\big], \\ S_{j} &= -i\xi - \delta_{4}d_{j}\lambda_{j} - l_{j}\big[(1 + a_{1}\xi^{2}) - a_{3}\lambda_{j}^{2}\big]. \end{split}$$

### 5. Special Cases

a. Mechanical force on half-space surface

By taking F2 = 0 in Eqs. (31)-(36), we obtain the components of displacement, normal stress, tangential stress and conductive temperature due to mechanical force.

b. Thermal source on the half-space surface

By considering F1 = 0 in Eqs. (31)-(36), we obtain the components of displacement, normal stress, tangential stress and conductive temperature due to thermal source.

## 5.1 Concentrated force

We obtained the solution with concentrated normal force on the half space by taking

$$\psi_1(\mathbf{x}) = \delta(\mathbf{x}), \psi_2(\mathbf{x}) = \delta(\mathbf{x}) \tag{37}$$

Applying Fourier transform defined by (19)-(20) and (37), we obtain

$$\widehat{\Psi}_1(\xi) = 1, \widehat{\Psi}_2(\xi) = 1.$$
 (38)

Using (38) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

## 5.2 Uniformly distributed force

We obtained the solution with uniformly distributed force applied on the half space by taking

$$\psi_1(x), \psi_2(x) = \begin{cases} 1 \text{ if } |x| \le m \\ 0 \text{ if } |x| > m \end{cases}$$
 (39)

The Fourier transforms of  $\psi_1(x)$  and  $\psi_2(x)$  with respect to the pair  $(x, \xi)$  for the case of a uniform strip load of non-dimensional width 2m applied at origin of co-ordinate system x=z=0 in the dimensionless form after suppressing the primes becomes

$$\widehat{\Psi}_1(\xi) = \widehat{\Psi}_2(\xi) = \left\{ \frac{2\sin(\xi m)}{\xi} \right\}, \ \xi \neq 0$$
(40)

Using (40) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

## 5.3 Linearly distributed force

We obtained the solution with linearly distributed force applied on the half space having 2 m as

the width of the strip load by taking

$$\{\psi_1(x), \psi_2(x)\} = \begin{cases} 1 - \frac{|x|}{m} & \text{if } |x| \le m \\ 0 & \text{if } |x| > m \end{cases}$$
 (41)

By using (15) and applying the transform defined by (20) on (41), we get

$$\widehat{\psi}_1(\xi) = \widehat{\psi}_2(\xi) = \left\{ \frac{2\{1 - \cos(\xi m))}{\xi^2 m} \right\}, \ \xi \neq 0$$
 (42)

Using (42) in (31)-(36), the components of displacement, stress and conductive temperature are obtained.

#### 6. Inversion of the transformation

For obtaining the result in physical domain, invert the transforms in Eqs. (31)-(36) using

$$\tilde{f}(x,z,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \, \hat{f}(\xi,z,\omega) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\xi x) f_e - i\sin(\xi x) f_o| \, d\xi,$$

where  $f_0$  is odd and  $f_e$  is the even parts of  $\hat{f}(\xi, z, s)$  respectively.

#### 7. Numerical results and discussion

To demonstrate the theoretical results and effect of rotation, relaxation time and two temperature, the physical data for cobalt material, which is transversely isotropic, is taken from Dhaliwal & Singh (1980) is given as

$$\begin{split} c_{11} &= 3.07 \times 10^{11} Nm^{-2}, \ c_{33} = 3.581 \times 10^{11} Nm^{-2}, \ c_{13} = 1.027 \times 10^{10} Nm^{-2}, \\ c_{44} &= 1.510 \times 10^{11} Nm^{-2}, \ \beta_1 = 7.04 \times 10^6 Nm^{-2} deg^{-1}, \\ \beta_3 &= 6.90 \times 10^6 Nm^{-2} deg^{-1}, \ \rho = 8.836 \times 10^3 Kgm^{-3}, \\ C_E &= 4.27 \times 10^2 j Kg^{-1} deg^{-1}, \ K_1 = 0.690 \times 10^2 Wm^{-1} K deg^{-1}, \\ K_3 &= 0.690 \times 10^2 Wm^{-1} K^{-1}, \quad T_0 = 298 \ \mathrm{K, H_0} = 1 \mathrm{Jm^{-1} nb^{-1}}, \\ \epsilon_0 &= 8.838 \times 10^{-12} \mathrm{Fm^{-1}}, \quad L = 1. \end{split}$$

Using the above values, the graphical representations of displacement component u, normal displacement w, conductive temperature  $\varphi$ , stress components  $t_{11}$ ,  $t_{13}$  and  $t_{33}$  for transversely isotropic magneto-thermoelastic medium have been studied and the effect of inclination and rotation has been depicted.

Case 1: Mechanical force with rotation and with two temperature Sub case i: Concentrated force

Figs. 1-6 shows the variations of the displacement components (u and w), Conductive temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magneto-thermoelastic medium with mechanical force and concentrated force and with combined effects of two temperature, relaxation time, rotation, time harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (u and w) illustrate the

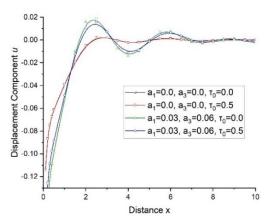


Fig. 1 Variations of displacement component u with distance x

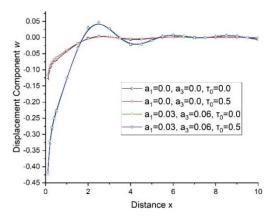


Fig. 2 Variations of displacement component w with distance x

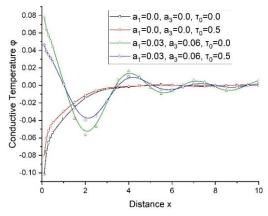


Fig. 3 Variations of conductive temperature  $\phi$  with distance x

same pattern but having different magnitudes with and without temperature. Conductive temperature  $\varphi$ shows the different behaviour for two temperature and without two temperatures.

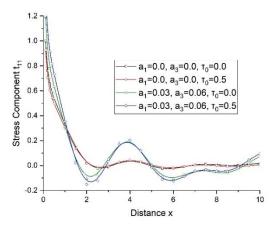


Fig. 4 Variations of stress component  $t_{11}$  with distance x

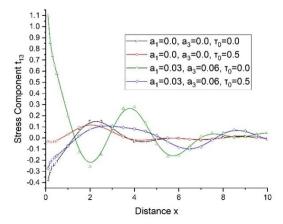


Fig. 5 Variations of stress component  $t_{13}$  with distance x

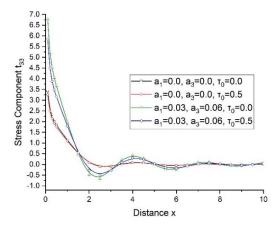


Fig. 6 Variations of stress component  $t3_3$  with distance x

Stress components (  $t_{11}$ ,  $t_{13}$  and  $t_{33}$ )in Figs. 4-6 vary (increases or decreases ) during the initial

range of distance near the loading surface of the time harmonic source and follow small oscillatory pattern for rest of the range of distance. Zero value of  $\tau_0$  with two temperatureshows more stress near loading surface.

#### Sub case ii: Linearly distributed force

Figs. 7-12 shows the variations of the displacement components (u and w), Conductive temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magnetothermoelastic medium with mechanical force (linearly distributed force) and with combined effects of two temperature, relaxation time, rotation, time harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (u and w) and Conductive temperature  $\varphi$  illustrate the same pattern but having different magnitudes with and without temperature. Stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) in Figs. 10-12 varies (increases or decreases) during the initial range of distance near the loading surface of the time harmonic source and follow.

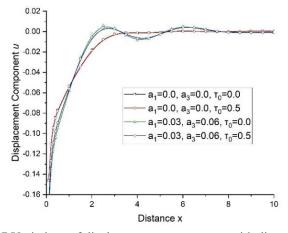


Fig. 7 Variations of displacement component u with distance x

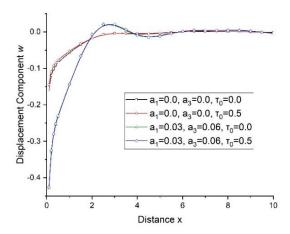


Fig. 8 Variations of displacement component w with distance x

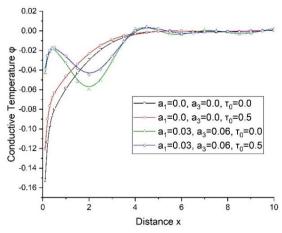


Fig. 9 Variations of conductive temperature  $\phi$  with distance x

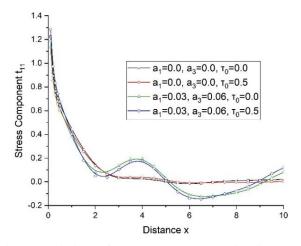


Fig. 10 Variations of stress component  $t_{11}$  with distance x

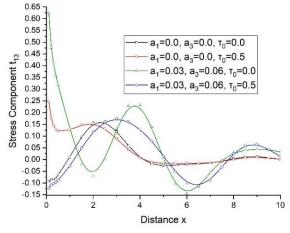


Fig. 11 Variations of stress component  $t_{11}$  with distance x

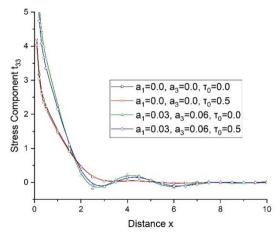


Fig. 12 Variations of stress component  $t_{33}$  with distance x

small oscillatory pattern for rest of the range of distance. Zero value of  $\tau_0$  with two temperatureshows more stress near loading surface.

#### Sub case iii: Uniformly distributed force

Figs. 13-18 shows the variations of the displacement components (u and w), Conductive temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magnetothermoelastic medium with mechanical force (uniformly distributed force) and with combined effects of two temperature, relaxation time, rotation, time harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (u and w) and Conductive temperature  $\varphi$  illustrate the same pattern but having different magnitudes with and without temperature. Stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) in figures 16 to figure 18 varies (increases or decreases) during the initial range of distance near the loading surface of the time harmonic source and follow small oscillatory pattern for rest of the range of distance. Zero value of  $\tau_0$  with two temperatureshows less stress near loading surface.

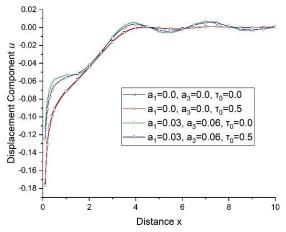


Fig. 13 Variations of displacement component u with distance x

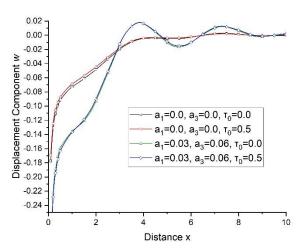


Fig. 14 Variations of displacement component w with distance x

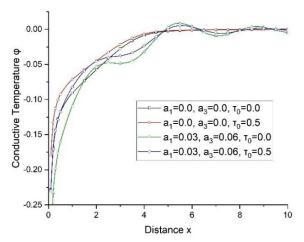


Fig. 15 Variations of conductive temperature  $\phi$  with distance x

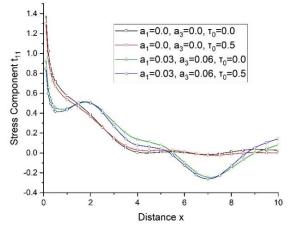


Fig. 16 Variations of stress component  $t_{11}$  with distance x

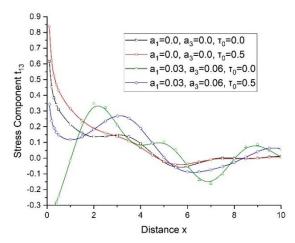


Fig. 17 Variations of stress component  $t_{13}$  with distance x

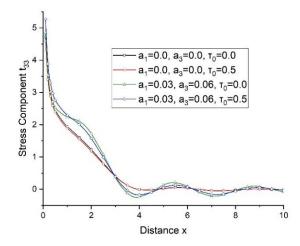


Fig. 18 Variations of stress component  $t_{33}$  with distance x

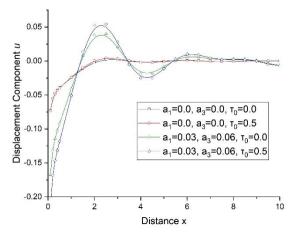


Fig. 19 Variations of displacement component u with distance x

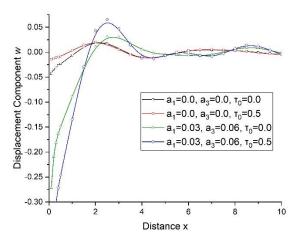


Fig. 20 Variations of displacement component w with distance x

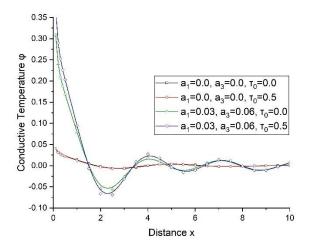


Fig. 21 Variations of stress component  $t_{11}$  with distance x

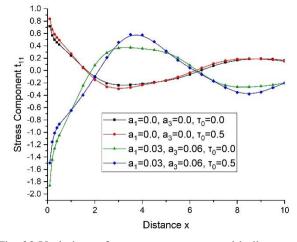


Fig. 22 Variations of stress component  $t_{11}$  with distance x

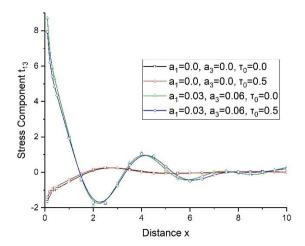


Fig. 23 Variations of stress component  $t_{13}$  with distance x

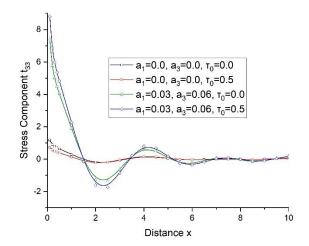


Fig. 24 Variations of stress component  $t_{33}$  with distance x

Case II: Thermal source with rotation and with two temperature Sub case i: Concentrated Force

Figs. 19-24 shows the variations of the displacement components (u and w), Conductive temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magnetothermoelastic medium with thermal source (concentrated force) and with combined effects of two temperature, relaxation time, rotation, time harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (u and w) and Conductive temperature  $\varphi$  illustrate the same pattern but having different magnitudes with and without temperature. Stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) in Figs. 22-24 show the different behaviour for two temperature and without two temperatures.

Sub case ii: Linearly distributed force

Figs. 25-30 shows the variations of the displacement components (u and w), Conductive

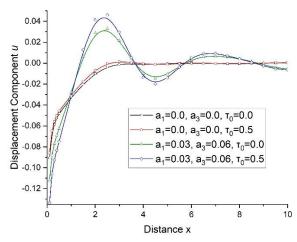


Fig. 25 Variations of displacement component u with distance x

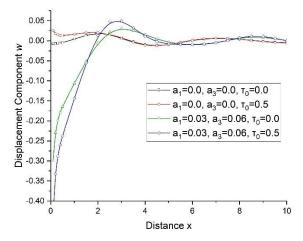


Fig. 26 Variations of displacement component w with distance x

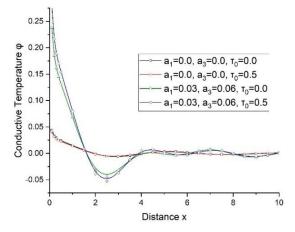


Fig. 27 Variations of conductive temperature  $\phi$  with distance x

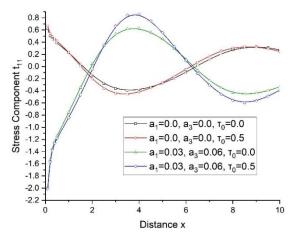


Fig. 28 Variations of stress component  $t_{11}$  with distance x

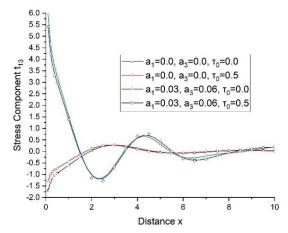


Fig. 29 Variations of stress component  $t_{13}$  with distance x

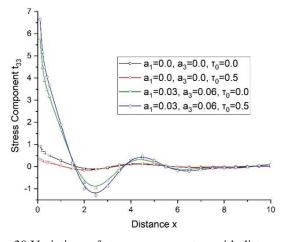


Fig. 30 Variations of stress component  $t_{33}$  with distance x

temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magneto-thermoelastic medium with thermal source (linearly distributed force) and combined effects of two temperature, relaxation time, rotation, time harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (u and w) illustrate the same pattern but having different magnitudes with and without temperature. Conductive temperature  $\varphi$  decreaseduring the initial range of distance near the loading surface of the time harmonic source and follow small oscillatory pattern for rest of the range of distance. Stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) in Figs. 28-30 show the different behaviour for two temperature and without two temperatures.

#### Sub case iii: Uniformly Distributed Force

Figs. 31-36 shows the variations of the displacement components (u and w), Conductive temperature  $\varphi$  and stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) for transversely isotropic magnetothermoelastic medium with thermal source and uniformly distributed force and with combined

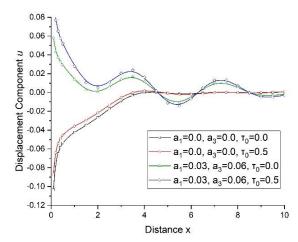


Fig. 31 Variations of displacement component u with distance x

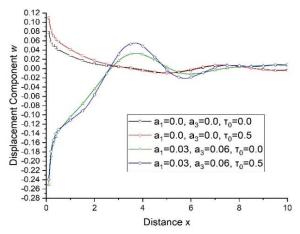


Fig. 32 Variations of displacement component w with distance x

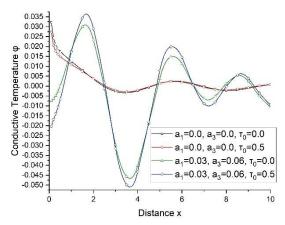


Fig. 33 Variations of conductive temperature  $\phi$  with distance x

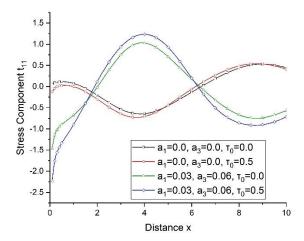


Fig. 34 Variations of stress component  $t_{11}$  with distance x

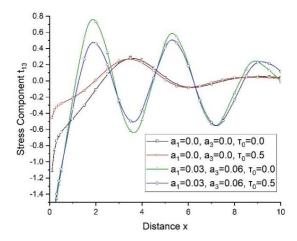


Fig. 35 Variations of stress component  $t_{13}$  with distance x

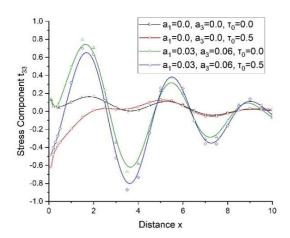


Fig. 36 Variations of stress component  $t_{33}$  with distance x

effects of two temperature, relaxation time, rotation, time harmonic source in generalized thermoelasticity without energy dissipation respectively. The displacement components (u and w)and Conductive temperature $\varphi$ illustrate the different pattern with and without temperature. Stress components ( $t_{11}$ ,  $t_{13}$  and  $t_{33}$ ) in Figs. 34-36 show the different pattern with and without temperature. Stress component shows small oscillatory pattern without two temperature and large oscillatory pattern with two temperature.

## 8. Conclusions

From above research, it is observed that two temperatures and rotation plays a key role for the oscillation of physical quantities both close to the point of use of source as well as just as far from the source. The physical quantities amplitude differ with change in two temperatures. In presence of two temperature and time harmonic source, the displacement components and stress components show different nature with respect to x. The result gives an inspiration to study magneto-thermoelastic materials as an innovative domain of applicable thermoelastic solids. The shape of curves shows the impact of two temperatures, relaxation time and rotation with time harmonic source on the body and fulfils the purpose of the study. When sudden heat/external force is applied in a solid body, it transmits time harmonic wave by thermal expansion. The outcomes of this research are extremely helpful in the 2-D problem with dynamic response of time harmonic sources in transversely isotropic magneto-thermoelastic medium with rotation and two temperature which beneficial to detect the deformed field near mining shocks, seismic and volcanic sources, thermal power plants, high-energy particle accelerators, and many emerging technologies.

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# **Nomenclature**

 $\vec{u}$ 

$\delta_{ij}$	Kronecker delta
C <sub>ijkl</sub>	Elastic parameters
$\beta_{ij}$	Thermal elastic coupling tensor
Γ	Absolute temperature
$T_0$	Reference temperature
ρ	conductive temperature
tij	Stress tensors
Pij	Strain tensors
$u_i$	Components of displacement
)	Medium density
$C_E$	Specific heat
$a_{ij}$	Two temperature parameters
$a_{ij}$	Linear thermal expansion coefficient
$K_{ij}$	Materialistic constant
$K^*_{ij}$	Thermal conductivity
$\omega$	Frequency
<b>7</b> 0	Relaxation Time
Ω	Angular Velocity of the Solid
$F_i$	Components of Lorentz force
$\overrightarrow{H_0}$	Magnetic field intensity vector
$\vec{J}$	Current Density Vector

Displacement Vector