Coupled Systems Mechanics, *Vol. 7, No. 5 (2018) 635-647*
DOI: https://doi.org/10.12989/csm.2018.7.5.635

Linear and nonlinear vibrations of inhomogeneous Euler-Bernoulli beam

Ebrahim S. Bakalah[a], F.D. Zaman[b] and Khairul Saleh[*]

Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

(Received March 19, 2018, Revised August 16, 2018, Accepted August 27, 2018)

Abstract. Dynamic problems arising from the Euler-Bernoulli beam model with inhomogeneous elastic properties are considered. The method of Green's function and perturbation theory are employed to find the deflection in the beam correct to the first-order. Eigenvalue problems appearing from transverse vibrations of inhomogeneous beams in linear and nonlinear cases are also discussed.

Keywords: vibration; inhomogeneous; Euler-Bernoulli beam

1. Introduction

Beams and girders are extensively used in civil and mechanical engineering. One of the earliest models was Euler-Bernoulli model that was used to study the bending of beams. The derivation of Euler-Bernoulli beam equation has been given by Duque (2015). The model was based on small deflections of a beam subjected to lateral loads. We may refer to Truesdell (1983) for an account of development of this approach in 1750. The study of beam equation is quite important in a number of engineering situations, see for instance Ebrahimi and Barati (2018), Rizov (2017), Huang *et al.* (2017), Nejad *et al.* (2017), Mohammadimehr and Alimirzaei (2016), and Webb *et al.* (2008). It was of a little consequence in terms of applications till it became a cornerstone of engineering in the late 19th century. Han *et al.* (1999) provided a good description of different models of elastic beams including the Euler-Bernoulli beam. Gupta (1988) proved the existence and uniqueness of solution to the fourth-order equation arising from bending of an elastic beam. Abu-Hilal (2003) studied forced vibration of Euler-Bernoulli beam in the case of different homogenous and elastic boundary conditions for dynamic response due to distributed or concentrated loads.

The use of spectral properties, Green's function and perturbation method has been an important tool in second-order problems arising from vibration, elastic, acoustic and electromagnetic waves. Discussion on these methods may be found in Lindell and Olyslager (2001), Logan (2007), and Stakgold and Holst (2011). Stuwe and Werner (1996) used Green's function to study potential flow

---

∗Corresponding author, Ph.D., E-mail: khairul@kfupm.edu.sa
[a]M.Sc., E-mail: g201309590@kfupm.edu.sa
[b]Professor, E-mail: fzaman@kfupm.edu.sa

http://www.techno-press.org/?journal=csm&subpage=8 ISSN: 2234-2184 (Print), 2234-2192 (Online)

Lindell, I.V. and Olyslager, F. (2001), "Polynomial operators and green functions", *Prog. Electromagn. Res.*, **30**, 59-84.
Logan, J.D. (2007), *Pure and Applied Mathematics in An Introduction to Nonlinear Partial Differential Equations*, 2nd Edition, Hoboken, John Wiley & Sons, New Jersey, U.S.A.
Mohammadimehr, M. and Alimirzaei, S. (2016), "Nonlinear static and vibration analysis of Euler-Bernoulli composite beam model reinforced by FG-SWCNT with initial geometrical imperfection using FEM", *Struct. Eng. Mech.*, **59**(3).
Morrison, S.M. (2007), *Application of the Green's Function for Solutions of Third Order Nonlinear Boundary Value Problems*, Tennessee, Knoxville, U.S.A.
Nejad, M.Z., Hadi, A. and Farajpour, A. (2017), "Consistent couple-stress theory for free vibration analysis of Euler-Bernoulli nano-beams made of arbitrary bi-directional functionally graded materials", *Struct. Eng. Mech.*, **63**(2).
Orucoglu, K. (2005), "A new green function concept for fourth-order differential equations", *Electron. J. Differ. Equat.*, (28), 1-12.
Pietramala, P. (2011), "A note on a beam equation with nonlinear boundary conditions", *Bound. Value Probl.*, 1-14.
Rizov, V.I. (2017), "Fracture analysis of functionally graded beams with considering material non-linearity", *Struct. Eng. Mech.*, **64**(4).
Stakgold, I. and Holst, M.J. (2011), *Green's Functions and Boundary Value Problems*, John Wiley & Sons, New York, U.S.A.
Stuwe, H.C. and Werner, P. (1996), "A Green's function approach to wave propagation and potential flow around obstacles in infinite cylindrical channels", *Math. Meth. Appl. Sci.*, **19**(8), 607-638.
Teterina, O.A. (2013), *The Green's Function Method for Solutions of Fourth Order Nonlinear Boundary Value Problem*, University of Tennessee, Knoxville, U.S.A.
Truesdell, C. (1983), *Rational Mechanics*, Academic Press, New York, U.S.A.
Webb, J.R.L., Infante, G. and Franco, D. (2008), "Positive solutions of nonlinear fourth-order boundary-value problems with local and non-local boundary conditions", *Proceedings of the Royal Society of Edinburgh*.
Zaman, F.D., Asghar, S. and Ahmad, M. (1990), "Love-type waves due to a line source in an inhomogeneous layer trapped between two half spaces", *Boll. Di Geofis. Teor. Ed Appl.*, **XXXII**(N), 127-128.
Zaman, F.D. and Al-Zayer, R.M. (2004), "Dispersion of Love waves in a stochastic layer", *Nuovo Cimento-Socista Ital. Di Fis. Sez.*, **C**(27), 143-154.
Zaman, F.D., Masood, K. and Muhiameed, Z. (2006), "Inverse scattering in multilayer inverse problem in the presence of damping", *Appl. Math. Comput.*, **176**(2), 455-461.

AI