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Stoneley wave propagation in transversely isotropic thermoelastic media using new modified couple stress theory and two-temperature theory

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Abstract. This paper is concerned with the study of propagation of Stoneley waves at the interface of two dissimilar transversely isotropic thermoelastic solids using new modified couple stress theory without energy dissipation and with two temperatures. The secular equation of Stoneley waves is derived in the form of the determinant by using appropriate boundary conditions i.e., the stress components, the displacement components, and temperature at the boundary surface between the two media are considered to be continuous at all times and positions. The dispersion curves giving the Stoneley wave velocity and attenuation coefficients with wave number are computed numerically. Numerical simulated results are depicted graphically to show the effect of two temperature on resulting quantities. Copper material has been chosen for the medium M_1 and magnesium for the medium M_2. Some special cases are also deduced from the present investigation.

Keywords: attenuation coefficient; new modified couple stress theory; secular equation; Stoneley wave velocity; Stoneley wave; transversely isotropic; two-temperatures

1. Introduction

Mathematical modeling of surface wave propagation along with the free boundary of an elastic half-space or along the interface between two dissimilar elastic half-spaces has been subject of continued interest for many years. These models are helpful in exploration of valuable materials beneath the earth surface and provide better information about the internal composition of the earth.

Stoneley (1924) firstly studied the existence of these waves propagating at the interface of two solid, solid-liquid medium and derived the Stoneley wave's dispersion equation. The wave propagation in two temperature theory of thermoelasticity was investigated by Warren and Chen (1973). Chadwick and Currie (1974) studied the propagation of Stoneley waves in an infinite body composed of two anisotropic elastic half-spaces in welded contact. Murty (1975) studied the propagation of waves at the interface of unbonded and loosely bonded elastic half spaces. Hawker

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(1978) examined the plane-wave reflection coefficients for a model of the ocean's sub-bottom consisting of a single inhomogeneous fluid layer overlying a semi-infinite solid and suggested the existence of surface wave. Tajuddin (1995) studied the presence of Stoneley waves at the boundary of two micropolar elastic half-spaces. Hornby *et al.* (1989) showed that permeable fractures also give rise to reflected Stoneley waves. Tang *et al.* (1991) have developed a simple model for Stoneley wave propagation in permeable formations. Ting (2004) explored a surface wave propagation in an anisotropic rotating medium. Tomar and Singh (2006) derived the frequency equations for Stoneley waves at unbonded and bonded interfaces between two dissimilar microstretch elastic half-spaces.

Tang and Cheng (2006) investigates the propagation of borehole Stoneley waves across permeable structures. Othman and Song (2006, 2008) presented different hypotheses about magneto-thermo-elastic waves in a homogeneous and isotropic medium. Abo-Dahab (2013, 2015) studied the different forms of surface waves. Abbas and Zenkour (2014) studied the effect of initial stress and rotation for a fiber-reinforcement anisotropic half-space under a thermal shock at its upper surface in the context of Green and Naghdi's theory. Marin and Florea (2014) analysed the temporal behaviour of the solutions in porous micropolar thermoelastic bodies. Abbas (2015) investigated the thermoelastic interactions in a functional graded material due to thermal shock using the fractional order three-phase lag model for functionally graded materials. Abo-Dahab (2015) attempted the propagation of Stoneley waves in magneto-thermoplastic materials with voids. Lata *et al.* (2016) investigated plane waves in an anisotropic thermoelastic medium.

Abd-Alla *et al.* (2017) explored the propagation of surface waves in fiber-reinforced anisotropic media of nth order with rotation and magnetic field. Singh and Tochhawng (2019) studied the propagation of surface waves at the bonded and unbonded interfaces with voids. Marin *et al.* (2019) formulated the mixed backward in time problem in the context of thermoelasticity for dipolar materials. Marin *et al.* (2020) employed the GL model to cover the model of thermoelasticity for dipolar mediums. Abbas *et al.* (2020) studied the Photo-thermal-elastic interactions in an unbounded semiconductor media containing a cylindrical hole under a hyperbolic two-temperature coupled theory of thermo-elasticity and plasma waves. Hobiny and Abbas (2020) analysed the thermal damage of living tissues induced by laser irradiations using a nonlinear dual phase lag model. Saeed and Abbas (2020) investigated the non-linear dual phase lag bioheat transfer model for transient phenomena in a spherical tissue due to the effect of laser heat source. Hobiny and Abbas (2021) established a bio-heat model with fractional derivative and studied the variations of temperature and the thermal damage in spherical tissues during thermal therapy. Lata and Himanshi (2021) studied the Stoneley wave propagation at the interface of two dissimilar homogeneous orthotropic thermoelastic solids with three phase lags using fractional order theory of thermoelasticity. Lata and Singh (2021) have investigated the Stoneley wave propagation at the interface of two dissimilar homogeneous nonlocal magneto-thermoelastic media under the effect of hall current applied to multi-dual-phase lag heat transfer.

Marin *et al.* (2021) studied the effect of porothermoelastic waves under a fractional time derivative and two time delays on temperature increments, stress and the displacement components of the solid and fluid phases. Stoneley wave velocity variation is analyzed by solving the modified Scholte secular equation for velocity of Stoneley waves by Kuznetsov (2022). Himanshi and Lata (2023) studied the plane harmonic waves in a two-dimensional orthotropic magneto-thermoelastic media with fractional order theory of generalized thermoelasticity with two-temperatures and rotation. Debnath *et al.* (2024) investigated the propagation of Stoneley waves through bonded and unbonded interfaces between two dissimilar homogeneous transversely isotropic generalized thermo-elastic diffusion rocks.

Couple stress theory is derived under quasi- static conditions, where the system can be considered to be in equilibrium at each step and basic laws of thermodynamics can be applied. Due to the equilibrium in a quasi-static process, we can precisely define the system's intensive quantities (such as pressure, temperature, specific volume, and specific entropy) at each instant throughout the process; otherwise, different parts of the system would have different values of these quantities. Quasi-static processes make it limit the entropy generated. The parabolic nature of the equations in couple stress theory reflects the fact that the material's behavior is influenced by both the local and nonlocal effects, diffusion-like processes or wave propagation with dispersion due to size effects.

Keeping in view of these applications, dispersion equation for Stoneley waves at the interface of two dissimilar transversely isotropic thermoelastic mediums using new modified couple stress theory with two temperature and without energy dissipation have been derived. Numerical computations are performed for a particular model to study the variation of phase velocity and attenuation coefficient with respect to wave number. The results in this paper should prove useful in the field of material science, designers of new materials as well as for those working on the development of theory of elasticity.

2. Basic equations

Following Chen and Li (2014) and Youssef (2006), the field equations transversely isotropic thermoelastic solid using new modified couple stress theory in the absence of body forces, body couple and without energy dissipation are given by

$$\sigma_{ij} = c_{ijkl}e_{kl} - \beta_{ij}T, \tag{1}$$

$$c_{ijkl}\varepsilon_{kl,j} + \frac{1}{2}e_{ijk}m_{lk,lj} - \beta_{ij}T_{,j} = \rho\ddot{u}_i, \tag{2}$$

$$K_{ij}\varphi_{,ij} - \rho C_E\ddot{T} = \beta_{ij}T_0\ddot{\varepsilon}_{ij}, \tag{3}$$

where

$$\beta_{ij} = c_{ijkl}\alpha_{ij}, \tag{4}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{5}$$

$$m_{ij} = l_i^2G_i\chi_{ij} + l_j^2G_j\chi_{ji}, \tag{6}$$

$$\chi_{ij} = \omega_{i,j}, \tag{7}$$

$$\omega_i = \frac{1}{2}e_{ijk}u_{k,j}, \tag{8}$$

$$T = \varphi - a_{ij}\varphi_{,ij}. \tag{9}$$

Here $\vec{u} = (u_1, u_2, u_3)$ is the displacement vector, $c_{ijkl}(c_{ijkl} = c_{ijlk} = c_{jikl} = c_{jilk})$ are elastic

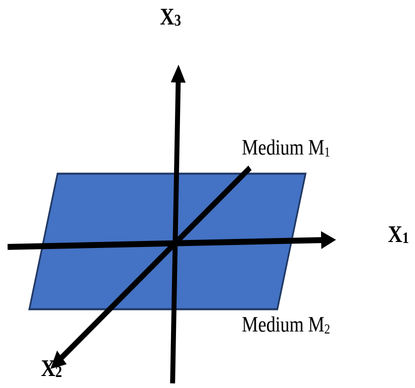

Fig. 1 Geometry of problem

parameters, σ_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, e_{ijk} is alternate tensor, m_{ij} are the components of couple-stress moment, α_{ij} are the coefficients of linear thermal expansion, β_{ij} is thermal tensor, T is the thermodynamic temperature, φ is the conductive temperature, $l_i(\mathrm{i}=1{,}2{,}3)$ are material length scale parameters, χ_{ij} is curvature tensor, ω_i is the rotational vector, ρ is the density, K_{ij} is the materialistic constant, c_E is the specific heat at constant strain, T_0 is the reference temperature assumed to be such that $T/_{T_0} \ll 1$, G_i are the elasticity constants.

For transversely isotropic medium

$$\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3, \beta_3 = 2c_{13}\alpha_1 + c_{33}\alpha_3$$

3. Formulation of the problem

We consider a homogeneous, transversely isotropic new modified couple stress thermoelastic half-space M_1 overlying another homogeneous, transversely isotropic new modified couple stress thermoelastic half-space M_2 connecting at the interface $x_1 = 0$. We take origin of co-ordinate system (x_1, x_2, x_3) on $x_3 = 0$. We choose x_1-axis in the direction of wave propagation in such a way that all the particles on a line parallel to x_2-axis are equally displaced, so that the field component $u_2 = 0$ and u_1, u_3 and φ are independent of x_2. MediumM_2 occupies the region$-\infty < x_1 \leq 0$ and Medium M_1 occupies the region$0 < x_1 \leq \infty$. The plane $x_3 = 0$ represents the interface between the two media M_1 and M_2. We define all the quantities without bar for the medium M_1 and with bar for medium M_2. We have used appropriate transformations following Slaughter (2002) on the set of Eqs. (1)-(3) to derive the equations for transversely isotropic thermoelastic solid with two temperatures and without energy dissipation and we restrict our analysis to the two-dimensional problem with

$$\vec{u} = (u_1, 0, u_3)(x_1, x_3, t). \tag{10}$$

Eqs. (1)-(3) with the aid of (4)-(10) take the form

Equations of motion and heat conduction are

$$c_{11}\frac{\partial^2 u_1}{\partial x_1^2}+\left(c_{44}-\frac{1}{4}l_2^2G_2\nabla^2\right)\frac{\partial^2 u_1}{\partial x_3^2}+\left(c_{13}+c_{44}+\frac{1}{4}l_2^2G_2\nabla^2\right)\frac{\partial^2 u_3}{\partial x_1\partial x_3}-\beta_1\frac{\partial}{\partial x_1}\left(1-a_1\frac{\partial^2}{\partial x_1^2}-a_3\frac{\partial^2}{\partial x_3^2}\right)\varphi=\rho\,\ddot{u}_1, \tag{11}$$

$$c_{33}\frac{\partial^2 u_3}{\partial x_3^2}+\left(c_{44}+c_{13}+\frac{1}{4}l_2^2G_2\nabla^2\right)\frac{\partial^2 u_1}{\partial x_1\partial x_3}+\left(c_{44}+\frac{1}{4}l_2^2G_2\nabla^2\right)\frac{\partial^2 u_3}{\partial x_1^2}-\beta_3\frac{\partial}{\partial x_3}\left(1-a_1\frac{\partial^2}{\partial x_1^2}-a_3\frac{\partial^2}{\partial x_3^2}\right)\varphi=\rho\ddot{u}_3, \tag{12}$$

$$K_1\frac{\partial^2\varphi}{\partial x_1^2}+K_3\frac{\partial^2\varphi}{\partial x_3^2}-\rho c_E\frac{\partial^2}{\partial t^2}\left(1-a_1\frac{\partial^2}{\partial x_1^2}-a_3\frac{\partial^2}{\partial x_3^2}\right)\varphi=T_0\frac{\partial}{\partial t}\left(\beta_1\frac{\partial u_1}{\partial x_1}+\beta_3\frac{\partial u_3}{\partial x_3}\right) \tag{13}$$

The constitutive relations and couple-stress/moment components for transversely isotropic new modified couple stress medium are

$$\sigma_{11}=c_{11}\frac{\partial u_1}{\partial x_1}+c_{13}\frac{\partial u_3}{\partial x_3}-\beta_1\left(1-a_1\frac{\partial^2}{\partial x_1^2}-a_3\frac{\partial^2}{\partial x_3^2}\right)\varphi, \tag{14}$$

$$\sigma_{12}=0, \tag{15}$$

$$\sigma_{13}=c_{44}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right)-\frac{1}{4}l_2^2G_2\left(\frac{\partial^3 u_1}{\partial x_1^2\partial x_3}-\frac{\partial^3 u_3}{\partial x_1^3}+\frac{\partial^3 u_1}{\partial x_3^3}-\frac{\partial^3 u_3}{\partial x_3^2\partial x_1}\right), \tag{16}$$

$$\sigma_{22}=c_{21}\frac{\partial u_1}{\partial x_1}+c_{23}\frac{\partial u_3}{\partial x_3}-\beta_1\left(1-a_1\frac{\partial^2}{\partial x_1^2}-a_3\frac{\partial^2}{\partial x_3^2}\right)\varphi, \tag{17}$$

$$\sigma_{23}=0, \tag{18}$$

$$\sigma_{33}=c_{31}\frac{\partial u_1}{\partial x_1}+c_{33}\frac{\partial u_3}{\partial x_3}-\beta_3\left(1-a_1\frac{\partial^2}{\partial x_1^2}-a_3\frac{\partial^2}{\partial x_3^2}\right)\varphi, \tag{19}$$

$$m_{11}=0, m_{22}=0, m_{33}=0, \tag{20}$$

$$m_{32}=\frac{1}{2}l_2^2G_2\left(\frac{\partial^2 u_1}{\partial x_3^2}-\frac{\partial^2 u_3}{\partial x_1\partial x_3}\right), \tag{21}$$

$$m_{13}=0, \tag{22}$$

$$m_{12}=-\frac{1}{2}l_2^2G_2\left(\frac{\partial^2 u_1}{\partial x_1\partial x_3}-\frac{\partial^2 u_3}{\partial x_1^2}\right), \tag{23}$$

Strain components and rotation components are

$$e_{11}=\frac{\partial u_1}{\partial x_1}, e_{22}=0, e_{33}=\frac{\partial u_3}{\partial x_3}, e_{12}=0, e_{23}=0, e_{31}=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right), \quad \omega_1=0, \omega_2=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_3}-\frac{\partial u_3}{\partial x_1}\right), \omega_3=0. \tag{24}$$

where $\nabla^2=\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}\right)$. In the above equations we use contracting subscript notation $(1\to 11, 2\to 22, 3\to 33, 4\to 23, 5\to 31, 6\to 12)$ to relate c_{ijkl} to c_{mn}.

To facilitate the solution following dimensionless quantities are used

$$x_i' = \frac{x_i}{L}, u_i' = \frac{\rho c_1^2}{L\beta_1 T_0} u_i, \varphi' = \frac{\varphi}{T_0}, t' = \frac{c_1}{L} t, \sigma_{ij}' = \frac{ij}{\beta_1 T_0}, m_{ij}' = \frac{m_{ij}}{L\beta_1 T_0}, i,j = 1,2,3. \quad (25)$$

where $c_1^2 = \frac{c_{11}}{\rho}$ and L is constant of dimension of length.

Using the dimensionless quantities defined by (25) in the Eqs. (11)-(13), and after suppre-ssing the primes we obtain

$$\frac{\partial^2 u_1}{\partial x_1^2} + \delta_1 \frac{\partial^2 u_1}{\partial x_3^2} + \delta_2 \frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \frac{l_2^2 G_2}{4L^2 c_{11}} \nabla^2 \left(\frac{\partial^2 u_1}{\partial x_3^2} - \frac{\partial^2 u_3}{\partial x_1 \partial x_3}\right) - \frac{\partial}{\partial x_1}\left(1 - a_1 \frac{\partial^2}{\partial x_1^2} - a_3 \frac{\partial^2}{\partial x_3^2}\right)\varphi = \frac{\partial^2 u_1}{\partial t^2}, \quad (26)$$

$$\delta_4 \frac{\partial^2 u_3}{\partial x_3^2} + \delta_2 \frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \delta_1 \frac{\partial^2 u_3}{\partial x_1^2} + \frac{l_2^2 G_2}{4L^2 c_{11}} \nabla^2 \left(\frac{\partial^2 u_1}{\partial x_1 \partial x_3} - \frac{\partial^2 u_3}{\partial x_1^2}\right) - p_5 \frac{\partial}{\partial x_3}\left(1 - a_1 \frac{\partial^2}{\partial x_1^2} - a_3 \frac{\partial^2}{\partial x_3^2}\right)\varphi = \frac{\partial^2 u_3}{\partial t^2}, \quad (27)$$

$$\frac{\partial^2 \varphi}{\partial x_1^2} + p_3 \frac{\partial^2 \varphi}{\partial x_3^2} = \zeta_1 \frac{\partial^2 u_1}{\partial t^2 \partial x_1} + \zeta_2 \frac{\partial^2 u_3}{\partial t^2 \partial x_3} + \zeta_3 \frac{\partial^2}{\partial t^2}\left(1 - a_1 \frac{\partial^2}{\partial x_1^2} - a_3 \frac{\partial^2}{\partial x_3^2}\right)\varphi. \quad (28)$$

where

$$p_3 = \frac{K_3}{K_1}, p_5 = \frac{\beta_3}{\beta_1}, \zeta_1 = \frac{T_0 \beta_1^2}{K_1 \rho}, \zeta_2 = \frac{T_0 \beta_1 \beta_3}{K_1 \rho}, \quad \zeta_3 = \frac{c_E c_{11}}{K_1}, \ \rho c_1^2 = c_{11}.$$

For the medium M_1

We assume the solution of the form

$$(u_1, u_3, \varphi) = (u_1^*, u_3^*, \varphi^*) e^{\iota\xi(x_1 - ct)}. \quad (29)$$

where ξ is wave number and $\omega = \xi c$ is the angular frequency and c is the phase velocity of the wave. Using (29) in Eqs. (26)-(28) and satisfying the radiation condition $\widehat{u_1}, \widehat{u_3}, \ \hat{\varphi} \to 0$ as $x_3 \to \infty$, we obtain

$$\left(\xi^2(c^2 - 1) + \delta_1 \frac{d^2}{dx_3^2}\right) u_1^* + +\iota\xi\delta_2 \frac{du_3^*}{dz} - \frac{l_2^2 G_2}{4L^2 c_{11}}\left(-\xi^2 + \frac{d^2}{dz^2}\right)\left(\iota\xi \frac{du_3^*}{dx_3} + \frac{d^2 u_1^*}{dx_3^2}\right) - \left(1 + a_1\xi^2 - a_3 \frac{d^2}{dx_3^2}\right)\varphi^* = 0, \quad (30)$$

$$\iota\xi\delta_2 \frac{du_1^*}{dz} + \left(\xi^2(c^2 - \delta_1) + \delta_4 \frac{d^2 u_3^*}{dx_3^2}\right) u_3^* + \frac{l_2^2 G_2}{4L^2 c_{11}}\left(-\xi^2 + \frac{d^2}{dz^2}\right)\left(\iota\xi \frac{du_1^*}{dx_3} + \xi^2 u_3^*\right) - p_5\left(1 + a_1\xi^2 - a_3 \frac{d^2}{dx_3^2}\right)\varphi^* = 0, \quad (31)$$

$$\zeta_1 \xi^2 c^2 \iota\xi u_1^* + \zeta_2 \xi^2 c^2 \frac{du_3^*}{dx_3} + (\zeta_3 \xi^2 c^2 - \xi^2 + \zeta_3 \xi^4 a_1 c^2 + (p_3 - \zeta_3 \xi^2 c^2 a_3)\frac{d^2}{dx_3^2})\varphi^* = 0. \quad (32)$$

The Eqs. (30)-(32), have nontrivial solutions if the determinant of the coefficient $(u_1^*, u_3^*, \varphi^*)$ vanishes, which yield the following characteristic equation

$$\left(P \frac{d^8}{d_{x_3}^8} + Q \frac{d^6}{d_{x_3}^6} + R \frac{d^4}{d_{x_3}^4} + S \frac{d^2}{d_{x_3}^2} + T\right)(u_1^*, u_3^*, \varphi^*) = 0 \quad (33)$$

where

$$P = -p_2p_{10}p_{12} + \zeta_2\xi^2c^2p_5a_3 - \iota\xi p_2^2\xi^2p_{12},$$
$$Q = -p_7p_{10}p_{12} - \zeta_2\xi^2c^2p_5(1 + a_1\xi^2)p_2 + \zeta_2\xi^2c^2p_5a_3p_7 + p_2(-p_9p_{12} + p_{10}p_{11}) - p_2\iota\xi(-p_8p_{12} + p_2\iota\xi p_{11} + \iota\zeta_1\xi^2c^2p_5a_3) - p_2p_8\iota\xi p_{12} - \iota a_3\zeta_2\xi^3c^2,$$
$$R = -p_6p_{10}p_{12} + p_7(-p_9p_{12} + p_{10}p_{11}) + p_2p_9p_{11} - \zeta_2\xi^2c^2p_5(1 + a_1\xi^2)p_7 + \zeta_2\xi^2c^2p_5a_3p_6 - p_8(-p_8p_{12} + p_2p_{11}\iota\xi - \iota\zeta_1\xi^2c^2p_5a_3) - \iota\xi p_{12}\big(p_8p_{11} + \iota\zeta_1\xi^2c^2p_5(1 + a_1\xi^2)\big) - \xi(1 + a_1\xi^2)\zeta_2\xi^4c^2 - a_3(\zeta_2\xi^2c^2p_9 - \iota\zeta_1\xi^2c^2p_{10}),$$
$$S = p_6(-p_9p_{12} + p_{10}p_{11}) + p_7p_9p_{11} - p_8(p_8p_{11} + \iota\zeta_1\xi^2c^2p_5(1 + a_1\xi^2) - \iota\xi(1 + a_1\xi^2)\zeta_2\xi^2c^2p_9 - \iota\zeta_1\xi^2c^2p_{10}) + a_3\zeta_1\xi^3c^2p_9,$$
$$T = p_9p_6p_{11} + \zeta_1\xi^4c^2p_9(1 + a_1\xi^2),$$
$$p_2 = \frac{l_2^2G_2}{4L^2c_{11}}, p_6 = \xi^2(c^2 - 1), p_7 = \delta_1 - p_2\xi^2, p_8 = \iota\xi\delta_2 - p_2\iota\xi^3, p_9 = \xi^2(c^2 - \delta_1) - p_2\xi^4, p_{10} = p_2\xi^2 + \delta_4, p_{11} = (\zeta_3\xi^2c^2 - \xi^2 + \zeta_3\xi^4a_1c^2, p_{12} = \zeta_3\xi^2c^2a_3 + p_3.$$

Following Kuznetsov (2002), Under the assumption $0 < c < c_4^{lim}$ representation (29) is valid for non- multiple roots of Eq. (33) as well as multiple roots.
where

$$c_i^{lim} \equiv \inf_{\psi\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]}\left(\cos^{-1}(\psi)\sqrt{\rho^{-1}\lambda_i(w.C.w)}\right),$$

$w = \sin(\psi)\nu + \cos(\psi)n', \nu$ is the outward normal to boundary of half-space, n' is the unit vector in the direction of propagation of wave, $C = c_{ijkl}$ is the fourth order elasticity tensor, and $\lambda_i(w.C.w), k = 1,2,3,4$ are the eigen values of the matrix $(w.C.w)$ arranged in descending order. Assume that

$$(u_1^*, u_3^*, \varphi^*) = \sum_{i=1}^4(1, R_i, S_i)A_ie^{-\lambda_ix_3}, \tag{34}$$

$$R_i = \frac{P^*\lambda_i^6 + Q^*\lambda_i^4 + R^*\lambda_i^2 + S^*}{A^*\lambda_i^4 + B^*\lambda_i^2 + C^*}, \quad S_i = \frac{P^{**}\lambda_i^6 + Q^{**}\lambda_i^4 + R^{**}\lambda_i^2 + S^{**}}{A^*\lambda_i^4 + B^*\lambda_i^2 + C^*}, \tag{35}$$

where

$$P^* = -p_2p_{12}, Q^* = p_2p_{11} - p_7p_{12},$$
$$R^* = -p_6p_{12} + p_7p_{11} - \zeta_1\xi^4c^2a_3, S^* = p_6p_{11} + \zeta_1\xi^4c^2(1 + a_1\xi^2)$$
$$P^{**} = p_2p_{10} + p_2^2\xi^2, Q^{**} = p_7p_{10} + p_2p_9 - 2p_2\iota\xi p_8,$$
$$R^{**} = p_6p_{10} + p_7p_9 - p_8^2, S^{**} = p_6p_9,$$
$$A^* = p_{10}p_{12} + \zeta_2\xi^2c^2p_5a_3, B^* = -p_9p_{12} - p_{10}p_{11} - \zeta_2\xi^2c^2p_5(1 + a_1\xi^2), C^* = p_9p_{11}.$$

For the medium M_2

We attach bars for the medium M_2 and write the appropriate values $\bar{u}_1, \bar{u}_3, \bar{\varphi}$ for the medium $M_2(x_3 < 0)$ satisfying the radiation condition $\widehat{\bar{u}_1}, \widehat{\bar{u}_3}, \hat{\bar{\varphi}} \to 0$ as $x_3 \to \infty$ given by

$$(\bar{u}_1, \bar{u}_3, \bar{\varphi}) = \sum_{i=1}^4(1, \bar{R}_i, \bar{S}_i)\bar{A}_ie^{-\bar{\lambda}_ix_3}e^{\iota\xi(x_1-ct)}. \tag{36}$$

where $\overline{R_\iota}, \overline{S_\iota}, \overline{P^*}, \overline{Q^*}, \overline{R^*}, \overline{S^*}, \overline{P^{**}}, \overline{Q^{**}}, \overline{R^{**}}, \overline{S^{**}}, \overline{A^*}, \overline{B^*}, \overline{C^*}$ are obtained from Eqs. (34) and (35) by attaching bar to all the quantities.

5. Boundary conditions

We assume that the half spaces are in perfect contact. Thus, there is continuity of components of displacement vector, normal stress, tangential stress, temperature, rotation vector and couple stress components at the interface $x_3 = 0$

$$\sigma_{33} = \bar{\sigma}_{33} \tag{37}$$

$$\sigma_{31} = \bar{\sigma}_{31} \tag{38}$$

$$m_{32} = \bar{m}_{32} \tag{39}$$

$$u_1 = \bar{u}_1 \tag{40}$$

$$u_3 = \bar{u}_3 \tag{41}$$

$$k_3 \frac{\partial \varphi}{\partial x_3} = \overline{k_3} \frac{\partial \bar{\varphi}}{\partial x_3} \tag{42}$$

$$\varphi = \bar{\varphi} \tag{43}$$

$$\omega_2 = \bar{\omega}_2 \tag{44}$$

6. Derivation of secular equations

Making use of Eqs. (29) and (30) for medium M_1 and corresponding equations with bar for medium M_2 in Eqs. (37)-(44) along with (29)-(34) we obtain a system of simultaneous homogeneous secular equations

$$\sum_{i=1}^{4} \eta_{qi} A_i + \eta_{qi+3} \bar{A}_i = 0, q = 1,2,3,4,5,6,7,8. \tag{45}$$

$$\eta_{1i} = \frac{c_{13}}{\rho c_1^2} \iota\xi - \frac{c_{33}}{\rho c_1^2} R_i \lambda_i - p_5 \left(1 + a_1 \xi^2 - a_3 \lambda_i^2\right) S_i,$$

$$\eta_{1(i+3)} = -\frac{\overline{c_{13}}}{\bar{\rho} \bar{c}_1^2} \iota\xi + \frac{\overline{c_{33}}}{\bar{\rho} \bar{c}_1^2} R_i \lambda_i + \overline{p_5} \left(1 + a_1 \xi^2 - a_3 \bar{\lambda}_i^2\right) S_i,$$

$$\eta_{2i} = \frac{c_{44}}{\rho c_1^2} (\iota\xi R_i + \lambda_i) + \frac{l_2^2 G_2}{4 \rho c_1^2} (\lambda_i \iota\xi + \xi^2 R_i),$$

$$\eta_{2(i+3)} = -\frac{\overline{c_{44}}}{\bar{\rho} \bar{c}_1^2} \left(\iota\xi \bar{R}_i + \bar{\lambda}_i\right),$$

$$\eta_{3i} = \frac{l_2^2 G_2}{\rho c_1^2 L^2} \left(\lambda_i^2 + \iota\xi R_i \lambda_i\right),$$

$$\eta_{3(i+3)} = \frac{l_2^2 G_2}{\bar{\rho} \bar{c}_1^2 L^2} \left(\lambda_i^2 + \iota\xi R_i \lambda_i\right),$$

$$\eta_{4i} = 1,$$

$$\eta_{4(i+3)} = -1,$$

$$\eta_{5i} = R_i$$

$$\eta_{5(i+3)} = -\bar{R}_i$$

$$\eta_{6i} = -k_3 S_i \lambda_i$$

$$\eta_{6(i+3)} = \bar{k}_3 \bar{S}_i \bar{\lambda}_i$$

$$\eta_{7i} = S_i$$

$$\eta_{7(i+3)} = -\bar{S_\iota}$$
$$\eta_{8i} = \frac{1}{2}(-\lambda_i - \iota\xi R_i)$$
$$\eta_{8(i+3)} = -\frac{1}{2}\left(-\bar{\lambda}_i - \iota\xi\bar{R_\iota}\right), i = 1,2,3,4.$$

The system of Eq. (45) has a non-trivial solution if the determinant of unknowns $A_i, \bar{A}_i, i = 1,2,3,4$ vanishes i.e., $|\eta ij|_{8\times8}=0$. The whole information regarding the wave number, phase velocity and attenuation coefficient of Stoneley waves are described by secular equations.

7. Particular cases

i. If $a_1 = a_3 = 0$ in the Eq. (45), we obtain the corresponding expressions for phase velocity, attenuation coefficient, components of displacement, conductive temperature, components of stress and couple stress components for transversely isotropic new modified couple stress thermoelastic solid without two temperatures and without energy dissipation.

ii. If $a_1 \neq 0\ or\ a_3 \neq 0$ in the Eq. (45), we obtain the corresponding expressions for phase velocity, attenuation coefficient, components of displacement, conductive temperature, component of stress and couple stress components for transversely isotropic new modified couple stress thermoelastic solid with two-temperature and without energy dissipation

8. Numerical results and discussion

Following Youssef (2006), transversely isotropic Copper material is chosen for the purpose of numerical calculation for the medium M_1 with length dimensional parameter L=1.

Quantity	Value	Unit
$\boldsymbol{c_{11}}$	18.78×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{12}	8.76×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{13}	8.0×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{33}	17.2×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
c_{44}	5.06×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
C_E	0.6331×10^{3}	$\mathrm{JKg^{-1}K^{-1}}$
α_1	2.98×10^{-5}	$\mathrm{K^{-1}}$
α_3	2.4×10^{-5}	$\mathrm{K^{-1}}$
β_1	7.543×10^{6}	$\mathrm{Nm^{-1}K^{-1}}$
β_3	9.0208×10^{6}	$\mathrm{Nm^{-1}K^{-1}}$
ρ	8.954×10^{3}	$\mathrm{Kgm^{-3}}$
K_1	0.433×10^{3}	$\mathrm{Wm^{-1}K^{-1}}$
K_3	0.450×10^{3}	$\mathrm{Wm^{-1}K^{-1}}$
T_0	293	K
G_1	0.1	$\mathrm{Kgm^{-1}s^{-2}}$
G_2	0.2	$\mathrm{Kgm^{-1}s^{-2}}$

Following Dhaliwal and Singh (1980), magnesium material has been taken for the medium M_2, as

Quantity	Value	Unit
$\overline{c_{11}}$	5.974×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
$\overline{c_{12}}$	2.624×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
$\overline{c_{13}}$	2.17×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
$\overline{c_{33}}$	6.17×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
$\overline{c_{44}}$	3.278×10^{10}	$\mathrm{Kgm^{-1}s^{-2}}$
$\bar{C}_E$	1.04×10^{3}	$\mathrm{JKg^{-1}K^{-1}}$
$\bar{\alpha}_1$	2.98×10^{-5}	$\mathrm{K^{-1}}$
$\bar{\alpha}_3$	2.4×10^{-5}	$\mathrm{K^{-1}}$
$\overline{\beta_1} = \overline{\beta_3} = \beta$	2.68×10^{6}	$\mathrm{Nm^{-1}K^{-1}}$
ρ	1.74×10^{3}	$\mathrm{Kgm^{-3}}$
$\overline{K_1} = \overline{K_3} = K$	0.433×10^{3}	$\mathrm{Wm^{-1}K^{-1}}$
T_0	298	K
$\overline{G_1}$	0.2	$\mathrm{Kgm^{-1}s^{-2}}$
$\overline{G_2}$	0.3	$\mathrm{Kgm^{-1}s^{-2}}$
$\overline{G_3}$	0.4	$\mathrm{Kgm^{-1}s^{-2}}$
$\overline{l_1} = \overline{l_2} = \overline{l_3}$	0.3	nm

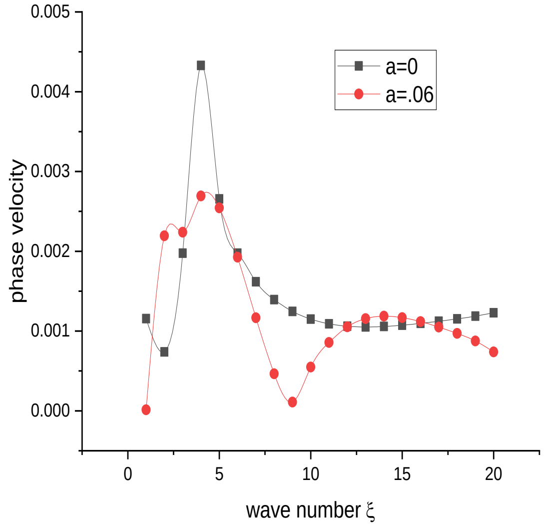

Fig. 2 Variation of phase velocity with the wave number ξ

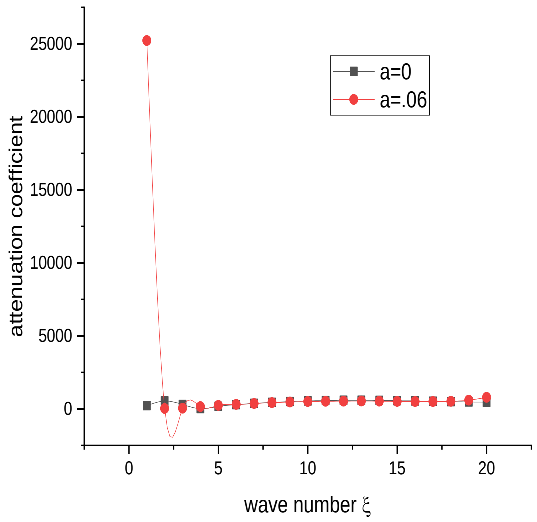

Fig. 3 Variation of attenuation coefficient with the wave number ξ

Software GNU octave has been used to determine the component of displacement u_3, stress component σ_{33}, couple stress m_{32}, conduction temperature, rotation components, phase velocity and attenuation coefficient depth of stoneley wave. Variation of resulting quantities with wave number has been made for transversely isotropic thermoelastic body by using two- temperature parameters as $a_1 = a_3 = 0$ and $a_1 = a_3 = .06$.

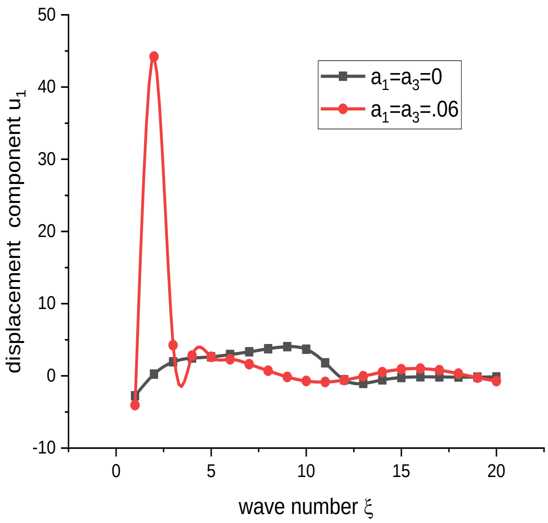

Fig. 4 Variation of displacement component u_1 with the wave number ξ

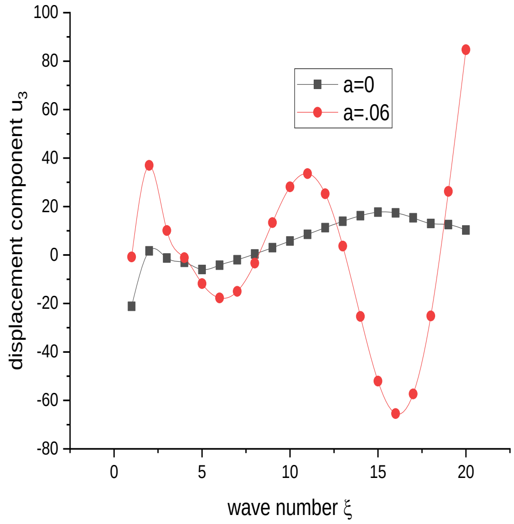

Fig. 5 Variation of displacement component u_3 with the wave number ξ

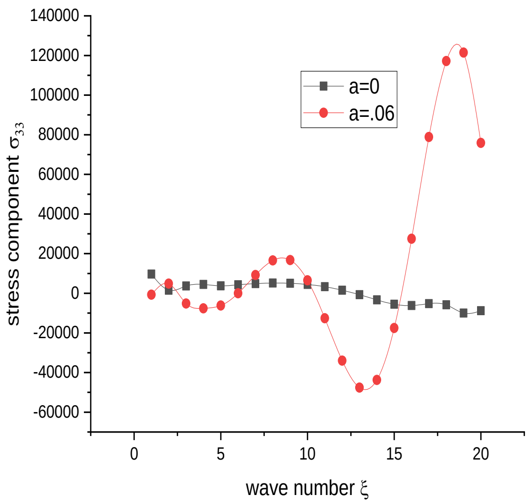

Fig. 6 Variation of stress component σ_{33} with the wave number ξ

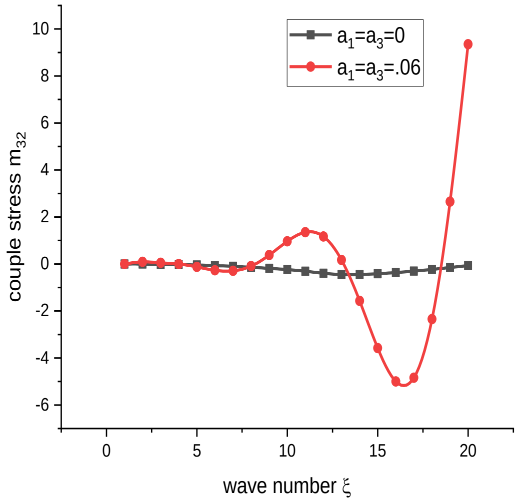

Fig. 7 Variation of couple stress m_{32} with the wave number ξ

The solid line in black with center symbol square (- -) ■ corresponds to $a_1 = a_3 = 0$ and red line with center symbol circle (- -)● corresponds to the $a_1 = a_3 = .06$.

Fig. 2 demonstrates the phase velocity w.r.t. ξ. The value of phase velocity for $a_1 = a_3 = 0$ decreases in $0 \le \xi \le 2.5$, increases sharply in the range $2.5 \le \xi \le 5$, decreases smoothly in the $5 \le \xi \le 15$, then starts increasing. For $a_1 = a_3 = .06$ trend is oscillatory in the range $0 \le \xi \le 10$ and inverse behavior to $a_1 = a_3 = 0$ is noticed in the remaining range.

Fig. 3 demonstrates the attenuation coefficient w.r.t. ξ. For $a_1 = a_3 = 0$ attenuation coefficient decreases sharply in $0 \le \xi \le 2.5$, increases in $2.5 \le \xi \le 5$ and linear in remaining range. For

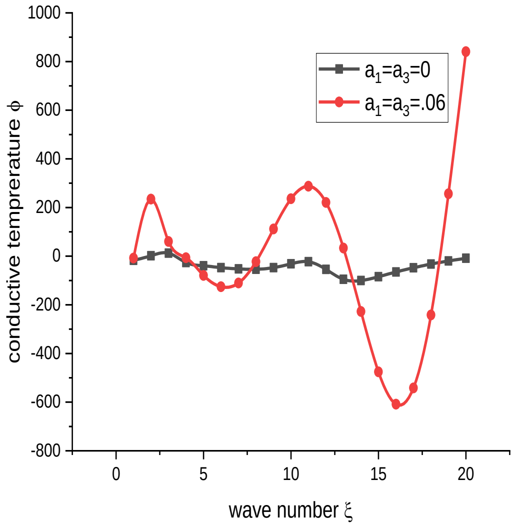

Fig. 8 Variation of conductive temperature φ with the wave number ξ

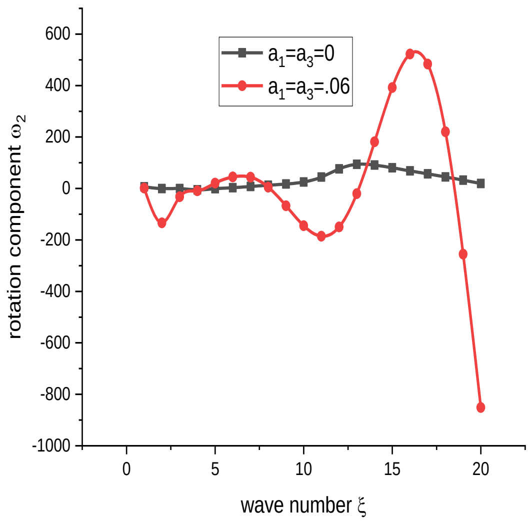

Fig. 9 Variation of rotation component ω_2 with the wave number ξ

$a_1 = a_3 = .06$ variation is linear.

Fig. 4 demonstrates the variations in displacement component u_1 with ξ. For displacement component u_1 variation is oscillatory. More variations in amplitude are observed for $a_1 = a_3 = .06$.

Fig. 5 demonstrates the variations in displacement component u_3 with ξ. For $a_1 = a_3 = 0$ u_3 increase in $0 \leq \xi \leq 2.5$, decrease in $2.5 \leq \xi \leq 5$, increase in $2.5 \leq \xi \leq 15$ and decreases in remaining range. For $a_1 = a_3 = .06$ variation is oscillatory.

Fig. 6 demonstrates the variations in stress component σ_{33} with ξ. For $a_1 = a_3 = 0$ behavior is steady state. For $a_1 = a_3 = .06$ behavior is oscillatory.

Fig. 7 demonstrates the variations in stress component m_{32} with ξ. For $a_1 = a_3 = 0$ variation is nearby linear for $0 \leq \xi \leq 20$. For $a_1 = a_3 = .06$ behavior is oscillatory with amplitude going on increasing as ξ increases.

Fig. 8 demonstrates the variations in conductive temperature φ with ξ. Variations are ascending oscillatory in nature.

Fig. 9 demonstrates the variation in rotation component ω_2 with ξ. For conductive temperature φ variations are ascending oscillatory. Presence of two temperature increases the amplitude of variation.

9. Conclusions

In the present work, the propagation of Stoneley waves in a transversely isotropic thermoelastic solid by using new modified couple stress theory with two temperatures has been studied. The effect of two-temperature parameters on the Stoneley wave phase velocity, attenuation coefficient, displacement components, stress components and couple stress components has been investigated. From the observations it is deduced that:

• For small value of non-dimensional wave number, the effect of two-temperature parameter has a significant impact on the variations of resulting quantities.
• The phase velocity and attenuation coefficient of Stoneley waves is also influenced by the two-temperature parameter. For high value of wave number attenuation coefficient is small.
• Behavior of resulting quantities is more oscillatory in the presence of two-temperature parameters.
• More variations in amplitude are observed with the non-zero value of two-temperature parameters.
• amplitude of variations is found to be raised in the presence of two-temperature parameters.

Stoneley wave's analysis provides information about the positions of fractures and permeability of the formation. These waves are also helpful in the assessment of valuable materials under the earth's surface. The results of this research may provide useful information for experimental scientists, researchers and seismologists which are working in this field.

References

Abbas, I.A. (2015), "Generalized thermoelastic interaction in functional graded material with fractional order three-phase lag heat transfer", *J. Cent. South Univ.*, **22**, 1606-1613. https://doi.org/10.1007/s11771-015-2677-5.

Abbas, I.A. and Zenkour, A.M. (2014), "The effect of rotation and initial stress on thermal shock problem for a fiber-reinforced anisotropic half-space using Green-Naghdi theory", *J. Comput. Theor. Nanosci.*, **11**(2), 331-338. https://doi.org/10.1166/jctn.2014.3356.

Abbas, I.A., Saeed, T. and Alhothuali, M. (2020), "Hyperbolic two-temperature photo-thermal interaction in a semiconductor medium with a cylindrical cavity", *Silicon*, **13**, 1871-1878. https://doi.org/10.1007/s12633-020-00570-7.

Abd-Alla, A.M., Abo-Dahab, S.M. and Khan, A. (2017), "Rotational effects on magneto-thermoelastic stoneley, love and Rayleigh waves in fibre-reinforced anisotropic general viscoelastic media of higher order", *Comput. Mater. Contin.*, **53**(1), 49-72. https://doi.org/10.3970/cmc.2017.053.052.

Abo-Dahab, S.M. (2013), "Surface waves in coupled and generalized thermoelasticity", *Encyclopedia Therm. Stress.*, **2**(1), 4764-4774. https://doi.org/10.1007/978-94-007-2739-7_371.

Abo-Dahab, S.M. (2015), "Propagation of Stoneley waves in magneto-thermoplastic materials with voids and two relaxation time", *J. Vib. Control*, **21**(6), 1144-1153. https://doi.org/10.1177/1077546313493651.

Boley, B.A. and Tolins, I.S. (1962), "Transient coupled thermoelastic boundary value problem in the half space", *J. Appl. Mech.*, **29**, 637-646. https://doi.org/10.1115/1.3640647.

Chadwick, P. and Currie, P.K. (1974), "Stoneley waves at an inter-face between elastic crystals", *Quart. J. Mech. Appl. Math.*, **27**(4), 497-503. https://doi.org/10.1093/qjmam/27.4.497.

Chen, P.J. and Gurtin, M.E. (1968), "On a theory of heat conduction involving two parameters", *Zeitschrift für Angewandte Mathematik und Physik (ZAMP)*, **19**, 614-627. http://doi.org/10.1007/BF01594969.

Chen, P.J. Gurtin, M.E. and Williams, W.O. (1968), "A note on non simple heat conduction", *Z. fur Angew. Math. Phys. (ZAMP)*, **19**, 969-970. http://doi.org/10.1007/BF01602278.

Chen, P.J., Gurtin, M.E. and Williams, W.O. (1969), "On the thermodynamics of non simple elastic materials with two temperatures", *Z. fur Angew. Math. Phys.*, **20**, 107-112. https://doi.org/10.1007/BF01591120.

Debnath, S., Singh, S.S. and Othman, M.I.A. (2024), "Stoneley waves in isotropic crustal rocks under the theory of generalized thermoelastic diffusion", *Int. J. Comput. Mater. Sci. Eng.*, 2350044. https://doi.org/10.1142/S2047684123500446.

Hawker, K.E. (1978), "Influence of Stoneley waves on plane-wave reflection coefficients: Characteristics of bottom reflection loss", *J. Acoust. Soc. Am.*, **64**, 548-555. https://doi.org/10.1121/1.382006.

Hobiny, A. and Abbas, I.A. (2021), “Analytical solutions of fractional bioheat model in a spherical tissue”, *Mech. Bas. Des. Struct. Mach.*, **49**(3), 430-439. https://doi.org/10.1080/15397734.2019.1702055.

Hobiny, A.D. and Abbas, I.A. (2020) “Nonlinear analysis of dual-phase lag bio-heat model in living tissues induced by laser irradiation”, *J. Therm. Stress.*, **43**, 503-511. https://doi.org/10.1080/01495739.2020.1722050.

Hornby, B.E., Johnson, D.L., Winkler, K.H. and Plumb, R.A. (1989), “Fracture evaluation using reflected Stoneley-wave arrivals”, *Geophys.*, **54**, 1274-1288. https://doi.org/10.1190/1.1442587.

Kaur, I. and Lata, P. (2020), “Stoneley wave propagation in transversely isotropic thermoelastic medium with two temperature and rotation”, *GEM-Int. J. Geomath.*, **11**(4), 1-17. https://doi.org/10.1007/s13137-020-0140-8.

Kuznetsov, S.V. (2002), “Forbidden planes for Rayleigh waves”, *Quart. Appl. Math*., **60**(1), 87-97. http://doi.org/10.1090/qam/1878260.

Kuznetsov, S.V. (2020), “Stoneley wave velocity variation”, *J. Theor. Comput. Acoust.*, **30**(01), 2050030. http://doi.org/10.1142/S2591728520500309.

Lata, P. and Himanshi (2021), “Stoneley wave propagation in an orthotropic thermoelastic media with fractional order theory”, *Compos. Mater. Eng.*, **3**(1), 57-70. https://doi.org/10.12989/cme.2021.3.1.057.

Lata, P. and Singh, S. (2021), “Stoneley wave propagation in nonlocal isoropic magneto-thermoelastic solid with multi-dual-phase lag heat transfer”, *Steel Compos. Struct.*, **38**(2), 141-150. https://doi.org/10.12989/scs.2021.38.2.141.

Lata, P., Kumar, R. and Sharma, N, (2016), “Plane waves in an anisotropic thermoelastic”, *Steel Compos. Struct.*, **22**(3), 567-587. https://doi.org/10.12989/scs.2016.22.3.567.

Marin, M. and Florea, O. (2014), “On temporal behaviour of solutions in thermoelasticity of porous micropolar bodies”, *Analele Stiintifice ale Univ. Ovidius Constanta, Ser. Mat.*, **22**(1), 169-188. https://doi.org/10.2478/auom-2014-0014.

Marin, M., Craciun, E.M. and Pop, N. (2020), “Some results in Green-Lindsay thermoelasticity of bodies with dipolar structure”, *Math.*, **8**, 497-509. http://doi.org/10.3390/math8040497.

Marin, M., Ellahi, R. and Chirila, A. (2017), “On solutions of saint-venant’s problem for elastic dipolar bodies with voids”, *Carpathian J. Math.*, **33**(2), 219-232. https://doi.org/10.37193/CJM.2017.02.09.

Marin, M., Hobiny, A. and Abbas, I.A. (2021), “The effects of fractional time derivatives in porothermoelastic materials using Finite Element Method”, *Math.*, **9**, 1606. https://dx.doi.org/10.3390/math9141606.

Marin, M., Vlase, S., Ellahi, R. and Bhatti, M.M. (2019), “On the partition of energies for the backward in time problem of thermoelastic materials with a dipolar structure”, *Symmetry*, **11**(7), 863. https://doi.org/10.3390/sym11070863.

Murty, G.S. (1975), “Wave propagation at unbonded interface between two elastic half-spaces”, *J. Acoust. Soc. Am.*, **58**(5), 1094-1095. https://doi.org/10.1016/0031-9201(75)90076-X.

Ning, L., Kewen, W., Peng, L., Hongliang, W., Zhou, F., Huajun, F. and David, S. (2021), “Experimental study on attenuation of Stoneley wave under different fracture factors”, *Pet. Explor. Dev.*, **48**(2), 299-307. https://doi.org/10.1016/S1876-3804(21)60024-1.

Othman, M.I. and Song, Y.Q, (2006), “The effect of rotation on the reflection of magneto-thermoelastic waves under thermoelasticity without energy dissipation”, *Acta Mechanica*, **184**, 89-204. http://doi.org/10.1007/s00707-006-0337-4.

Othman, M.I. and Song, Y.Q. (2008), “Reflection of magneto-thermoelastic waves from a rotating elastic half-space”, *Int. J. Eng. Sci.*, **46**, 459-474. https://doi.org/10.1016/j.ijengsci.2007.12.004.

Saeed, T. and Abbas, I.A. (2020), “Finite element analyses of nonlinear DPL bioheat model in spherical tissues using experimental data”, *Mech. Bas. Des. Struct. Mach.*, **50**, 1287-1297. https://doi.org/10.1080/15397734.2020.1749068.

Sharma, N., Kumar, R. and Lata, P. (2015), “Effect of two temperature and anisotropy in an axisymmetric problem in transversely isotropic thermoelastic solid without energy dissipation and with two-temperature”, *Am. J. Eng. Res.*, **4**(7), 176-187.

Singh, S. and Tochhawng, L. (2019), “Stoneley and Rayleigh waves in thermoelastic materials with voids”,

J. Vib. Control, **25**(14), 2053-2062. https://doi.org/10.1177/1077546319847850.
Stoneley, R. (1924), “Elastic waves at the surface of separation of two solids”, *Proc. Roy. Soc. London*, **106**(738), 416-428. https://doi.org/10.1098/rspa.1924.0079.
Tajuddin, M. (1995), “Existence of stoneley waves at an unbounded interface between two micropolar elastic half spaces”, *J. Appl. Mech.*, **62**(1), 255-257.
Tang, X.M. and Cheng, C.H. (2006), “Borehole Stoneley wave propagation across permeable structures”, *Geophys. Prospect*., **41**(2), 165-187. https://doi.org/10.1111/j.1365-2478.1993.tb00864.x.
Tang, X.M., Cheng, C.H. and Toksoz, M.N. (1991), “Dynamic permeability and borehole Stoneley waves: A simplified Biot-Rosenbaum model”, *J. Acoust. Soc. Am.*, **90**, 1632-1646. https://doi.org/10.1190/1.1889947.
Ting, T.C. (2004), “Surface waves in a rotating anisotropic elastic half-space”, *Wave Motion*, **40**, 329-346. https://doi.org/10.1016/j.wavemoti.2003.10.005.
Tomar S.K. and Singh, D. (2006), “Propagation of Stoneley waves at an interface between two microstretch elastic half-spaces”, *J. Vib. Control*, **12**(9), 995-1009. https://doi.org/10.1177/1077546306068689.
Warren, W.E. and Chen, P. (1973), “Wave propagation in the two-temperature theory of thermoelasticity”, *Acta Mechanica*, **16**, 21-33. https://doi.org/10.1007/BF01177123.

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