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Neuro-Fuzzy modeling of torsional strength of RC beams

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Abstract. This paper presents Neuro-Fuzzy (NF) based empirical modelling of torsional strength of RC beams for the first time in literature. The proposed model is based on fuzzy rules. The experimental database used for NF modelling is collected from the literature consisting of 76 RC beam tests. The input variables in the developed rule based on NF model are cross-sectional area of beams, dimensions of closed stirrups, spacing of stirrups, cross-sectional area of one-leg of closed stirrup, yield strength of stirrup and longitudinal reinforcement, steel ratio of stirrups, steel ratio of longitudinal reinforcement and concrete compressive strength. According to the selected variables, the formulated NFs were trained by using 60 of the 76 sample beams. Then, the method was tested with the other 16 sample beams. The accuracy rates were found to be about 96% for total set. The performance of accuracy of proposed NF model is furthermore compared with existing design codes by using the same database and found to be by far more accurate. The use of NF provided an alternative way for estimating the torsional strength of RC beams. The outcomes of this study are quite satisfactory which may serve NF approach to be widely used in further applications in the field of reinforced concrete structures.

Keywords: reinforced concrete beam; neuro-fuzzy; torsional strength; building code.

1. Introduction

The monolithic reinforced concrete constructions are subjected to significant torsional moments that affect their strength and deformation. In the literature, numerous analytical and experimental studies have been reported about torsional behavior of reinforced concrete (RC) members subjected to pure tension or combination of torsion with other effects as axial load, shear and bending.

There are many variables affecting the torsional strength of RC beams such as cross-sectional area of beams, dimensions of closed stirrup, spacing of stirrups, cross-sectional area of one-leg of closed stirrup, yield strength of stirrup and longitudinal reinforcement and concrete compressive strength. The effect of these variables on the torsional strength of RC beams has been extensively studied and some empirical approach has been developed related to these variables. For instance, Victor and Muthukrishnan (1973) studied the effect of variations in stirrups on the torsional capacity of RC beams and they proposed an empirical relationship for the contribution by stirrups to torsional capacity. Rasmussen and Baker (1995) examined the behavior of reinforced normal concrete and high strength concrete beams subjected to pure torsion. The test has shown that high strength concrete

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increases the beam torsional capacity and stiffness. McMullen and Rangan (1978) presented the results of torsion test on rectangular RC beams with the aspects ratio and amount reinforcement as main variables. The effect of high strength concrete on the torsional behavior of RC beams under pure tension was investigated by Koutchoukali and Belarbi (2001) and Fang and Shiau (2004). According to these research, the torsional capacity of under - reinforced beams is independent of concrete strength. They also found that the amount of longitudinal reinforcement was more effective in controlling crack width than the amount of transverse reinforcement. The torsional behavior of normal strength concrete beams has also been reported by other researchers (Collins and Mitchell 1980, Hsu 1968, Hsu and Mo 1985).

The test data are often used for validation, calibration or even development of models. Even though the torsional strength of RC beams has been carefully examined experimentally, estimation of torsional strength is still difficult task because of complex behavior of RC beam under torsional action.

The traditional design models and the existing design codes are not capable of addressing efficiently all the structural engineering problems such as torsional capacity of a RC member. As a consequent, the need for meta-modelling in order to address engineering design problems becomes obvious. This meta-modelling has been seriously enlarged by the development and the increasing utilization of artificial intelligence techniques and especially by the artificial intelligence domain known as soft computing, which is comprised by Fuzzy Logic (FL), Artificial Neural Networks (ANN) Neuro-Fuzzy (NF) (combinations of ANN and FL) and Genetic Algorithms (GA) (Evbuowman *et al.* 1996, Regli *et al.* 2000, Simpson *et al.* 2001, Kasabov 1996, Gallant 1993, Rao *et al.* 1999).

Many approaches based on soft computing have been applied in structural design (Murawski *et al.* 2000, Ceylan *et al.* 2010, Mashrei *et al.* 2010, Wang and Elhag 2008, Kim *et al.* 2006), civil engineering (Rajasekaran *et al.* 1996) and the optimization of structures (Ghorbani and Ghasemi 2010) provided sufficient results.

The main aim of this study was to investigate the applicability of Neuro-fuzzy (NF) approach for empirical rule based modeling of torsional strength of RC beams based on experimental results collected from the literature. In this sense, the experimental data of 76 beams subjected to torsion were used from the existing databases of Rasmussen and Baker (1995), Koutchoukali and Belarbi (2001), Fang and Shiau (2004), Hsu (1968). Furthermore, the approaches of some building codes such as ACI-318-2005 (2005), Eurocode-2 (2002), TBC-500-2000 (2000), CSA (1994), BS8110 (1985) and AS3600 (2001) were also examined by comparing their predictions with mentioned experimental studies results. The results obtained by the proposed NF model and building codes were compared with each other.

2. Theories of torsional strength and torsion in the building standards

In the literature, even though several theoretical models were proposed which are based on failure mechanisms, the method of analysis of torsional strength can be approximately divided to two methods as space truss analogy theory and the skew-bending theory. Rausch (1929) developed space truss model in 1929 and it still provides a basis for some design codes. In the space truss model, the torsion is resisted by compression diagonals consisting of the concrete between cracks that spiral around the beam at a constant angle. The theory has been extended later by many scholars in this field (Hsu 1968, Anderson 1935, Elfegren *et al.* 1974). It is assumed in this theory that the concrete beam behaves in torsion similar to a thin-walled box with a constant shear flow in the wall cross-section by producing a constant torsional moment (Nawy 2003). The absence of core does not affect the strength of such members in torsion; hence the acceptability of the space truss analogy approach is based on hollow sections. Therefore, in the process of torsion design of a RC beam, the beam can be considered as the equivalent tubular member.

In 1958, the skew-bending theory that considers the internal deformational behavior of the series of transverse warped surfaces along the beam in detail was proposed by Lessig (1959). The model was further refined by Collins *et al.* (1965) in 1965 as well as Hsu and Zia (1968, 2004). Especially Hsu made a major contribution experimentally to the development of the skew-bending theory as it presently stands. The basic approach of the theory is that the failure of a rectangular section in torsion occurs by bending about an axis that is parallel to the wider face of the section and inclined at about 45° to the longitudinal axis of the beam. In the previous versions of ACI code (1971, 1989), torsional strength of beams was calculated by using this theory. According to the codes, the torsional strength T_n of beams was considered to be made up of two parts: one part is contributed by concrete T_c while the other part is contributed by web reinforcement T_s. Hsu (1968) studied on hollow and solid rectangular beams and observed that the concrete core does not contribute to the ultimate torsional strength. Later he concluded that the concrete contribution T_c was mainly due to the shear resistance of the diagonal concrete struts.

In 1995, ACI code (1995) proposed a radically different design procedure based on the thin-walled tube, space truss analogy which is considerably simpler to understand and apply and is equally accurate. The torsion provisions in the ACI 318 have been revised using the thin walled tube analogy.

According to the current torsion provision of ACI 318-2005 (2005), the meaningful additional torsional strength T_n of RC beams can be achieved only by using both closed stirrups and longitudinal steel bars while the torsion moment T_c resisted by the concrete compression struts is assumed as zero. Thus the concrete contribution is ignored; there is no advantage in using higher concrete strengths in resisting ultimate torsion. The torsional strength T_n is given as follows

$$T_n = \frac{2A_oA_tf_{yv}}{s}cot\theta \tag{1}$$

In Eq.(1), $cot\theta$ can be assumed as

$$cot\theta = \sqrt{\frac{A_lf_{yl}s}{A_tf_{yv}p_h}} \tag{2}$$

In Eqs. (1) and (2), A_o is the gross area enclosed by the shear flow path that can be equal to 0.85 A_{sh}, where A_{sh} is the area enclosed by the centre of stirrups. θ is the angle of compression diagonals, f_{yl} is the yield strength of longitudinal torsional reinforcement, f_{yv} is the yield strength of closed stirrups, A_l is the total area of longitudinal torsional reinforcement, p_h is the perimeter of centerline of outmost closed transverse torsional reinforcement, s is the spacing of stirrups and A_t is the cross sectional area of one-leg of closed stirrup.

In Australian Standard AS3600 (2001) and Canadian Standard CSA (1994), the design of RC beams subjected to pure torsion is based on the space truss model and the T_n value is given as the same equation with ACI-318-2005 (2005). Different from ACI 318-2005 (2005), CSA (1994) and AS3600 (2001), in the British Standards BS8110 (1985) for RC structures, the torsional strength shall be calculated from Eq. (3) as

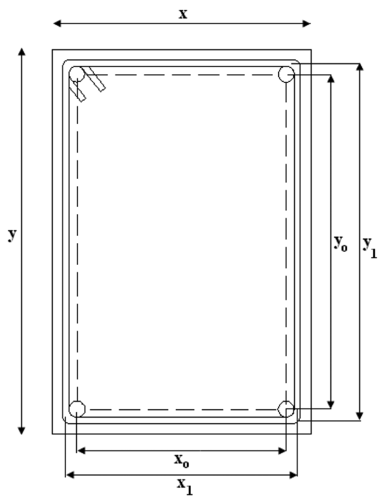

Fig. 1 The cross section of a rectangular reinforced concrete beam

$$T_n = \frac{0.8x_1y_1(0.87f_{ys})A_{sv}}{s} \tag{3}$$

where A_{sv} is the area of the two legs of stirrups at a section and x_1 and y_1 are the center-to-center of the shorter and longer legs of stirrups given in Fig. 1. The torsional strength T_n is described as Eq. (4) in Turkish Building Code TBC-500-2000 (2000).

$$T_n = \frac{2A_lA_ef_{yv}}{2(x_1+y_1)} \tag{4}$$

In Eq. (4), A_e is the area enclosed by lines connecting the centroids of the reinforcing bars at the corner of the section as seen in Fig. 1.

According to the European Standard Eurocode-2 (2002), the torsional strength shall be calculated with three ways and the minimum result is chosen.

$$T_n = f_{ys}(A_{sw}/s)2A_k\, cot\theta \tag{5}$$

$$T_n = f_y(A_s/u_k)2A_k\, tan\theta \tag{6}$$

$$T_n = 1.2(1-f_c/250)f_cA_kt_{ef}\, sin\theta cos\theta \tag{7}$$

Where A_k is the area enclosed by the centre-lines of the effective wall thickness. The effective wall thickness, t_{ef} can be calculated as A/u where A is the total area and u is the perimeter of the cross-section, f_c is the compressive strength of concrete.

3. Selection of database (Description of data)

The experimental database considered here (Table A1) was collected from various studies (Rasmussen and Baker 1995, Koutchoukali and Belarbi 2001, Fang and Shiau 2004, Hsu 1968, Tang 2006, Zang 2002). The test specimens of the database were of solid rectangular beams subjected to pure tension and where none of them was deep beam. The compressive strength of concrete ranged from 25.58 MPa to 109.8 MPa, the stirrup percentage ranged from 0.40% to 2.56%, the yielding stress of longitudinal reinforcement ranged from 314 MPa to 560 MPa and the yielding stress of stirrups ranged from 320 MPa to 672 MPa. The experimental database consists of a total of 76 samples given in details in Table A1. Beams are identified using the notations in the first row, with the first letter of researchers' name. The same series of test was used before by several authors. Tang (2006) developed a radial basis function neural networks to predict the ultimate torsional strength of RC beams, Zhang (2002) and Hossain *et al.* (2006) improved analytical methods for predicting the nonlinear response of RC beams by using the same database.

4. Fuzzy logic

Over the last decade, the fuzzy logic invented by Zadeh (1965) in 1965 has been applied to a wide area covering engineering, process control, image processing, pattern recognition and classification, management, economics and decision making (Rutkowski 2004).

Fuzzy systems can be defined as rule-based systems that are constructed from a collection of linguistic rules which can represent any system with accuracy, i.e., they work as universal approximators. The rule-based system of fuzzy logic theory uses linguistic variables as its antecedents and consequents where antecedents express an inference or the inequality, which should be satisfied and consequents are those, which we can infer and is the output if the antecedent inequality is satisfied. The fuzzy rule-based system is actually an IF–THEN rule-based system, given by, IF antecedent, THEN consequent (Sivanandam 2007).

FL operations are based on fuzzy sets where the input data may be defined as fuzzy sets or a single element with a membership value of unity. The membership values (μ_1 and μ_2) are found from the intersections of the data sets with the fuzzy sets as shown in Fig. 2 that illustrates the graphical method of finding membership values in the case of a single input (Haris 2006).

A fuzzy set contains elements which have varying degrees of membership in the set, unlike the classical or crisp sets where a member either belongs to that set or does not (0 or 1). However a fuzzy set allows a member to have a varying degree of membership and this partial degree membership can be mapped into a function or a universe of membership values (Bai *et al.* 2006).

The implementation of fuzzy logic to real applications considers the following steps (Bai *et al.* 2006).

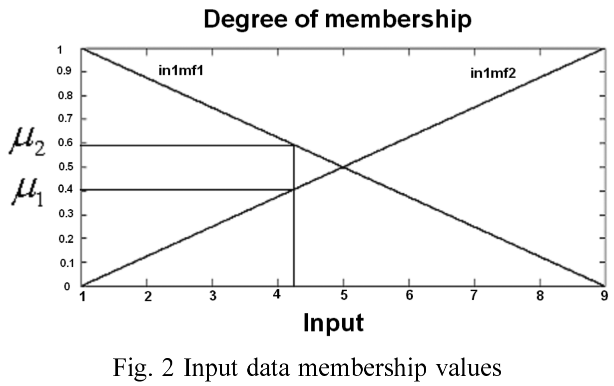

Fig. 2 Input data membership values

1. Fuzzification which requires conversion of classical data or crisp data into fuzzy data or Membership Functions (MFs)
2. Fuzzy inference process which connects membership functions with the Fuzzy rules to derive the fuzzy output
3. Defuzzification which computes each associated output.

4.1 Neuro-fuzzy systems

Fuzzy systems can also be connected with neural networks to form NF systems which exhibit advantages of both approaches. NF systems combine the natural language description of fuzzy systems and the learning properties of neural networks. Various NF systems have been developed that are known in literature under short names. ANFIS developed by Jang *et al.* (1997), (Adaptive Network-based Fuzzy Inference System) is one of these NF systems which allows the fuzzy systems to learn the parameters using adaptive backpropagation learning algorithm (Rutkowski 2004).

Mainly three types of fuzzy inference systems have been widely employed in various applications: Mamdani, Sugeno and Tsukamoto fuzzy models. The differences between these three fuzzy inference systems are due to the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ accordingly Jang *et al.* (1997). In this study, the Sugeno FIS is used where each rule is defined as a linear combination of input variables. The corresponding final output of the fuzzy model is simply the weighted average of each rule's output. For example, in a Sugeno FIS consisting of two input variables as x, y and one output variable f, the rules can be written as follows

Rule 1: If x is A_1, y is B_1 then $f_1 = p_1x + q_1y + r_1$
Rule 2: If x is A_2, y is B_2 then $f_2 = p_2x + q_2y + r_2$

where p_i, q_i and r_i are the consequent parameters of i_{th} rule. A_i, B_i and C_i are the linguistic labels represented by fuzzy sets shown in Fig. 3. The fuzzification and defuzzification processes which were performed by employing the mentioned rules have been demonstrated in Fig. 3. Rule 1 has been employed, firstly. Here, w_1 has been obtained by finding the minimum of the values which was calculated via the intersection of input variable x and membership function A_1 with the intersection of input variable y and membership function B_1. Then Rule 2 has been employed. Here, w_2 has been obtained by finding the minimum of the values which was calculated via the intersection of input variable x and membership function A_2 with the intersection of input variable y and membership function B_2. f_1 and f_2 values obtained from the rules have been subjected to defuzzification process by using weighted average method and thus the output value f has been calculated.

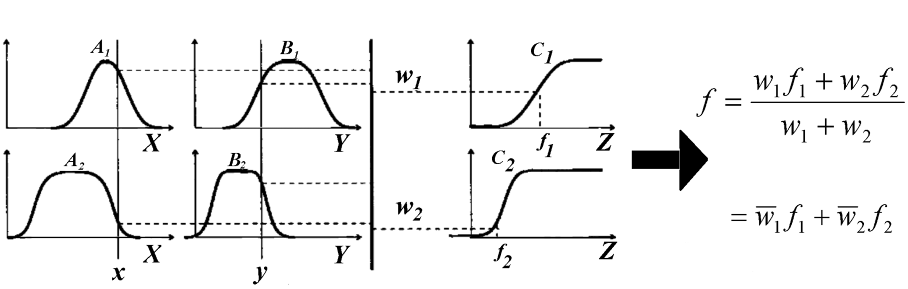

Fig. 3 The Sugeno fuzzy model [36]

4.2 Solving a simple problem with ANFIS

To illustrate how ANFIS works for function approximation, lets suppose one is given a sampling of the numerical values from the simple function below

$$y_i = a^2 + 5b \tag{8}$$

where a and b are independent variables chosen over randomly points in the real interval (1, 9). In this case, a sample of data in the form of 20 pairs (a, b, y_i) is given where a and b are the values of the independent variables in the given interval (1, 9). And y_i is the output of the function given in Eq. (8) and presented in Table 1. The aim is to construct the ANFIS model fitting those values within minimum error for Eq. (1) by using the simplest ANFIS model that is available where the number of rules is 2 for each variable and the type of output membership function is constant.

Inference diagram of the proposed ANFIS model is given in Fig. 4 for the input values of 9 and 5 with the corresponding values of output membership which is chosen as constant. The initial and

Table 1 Ranges of variables of the database

	Minimum	Maximum	Increment
x (mm)	160	350	Variable
y (mm)	275	508	Variable
x_1 (mm)	130	300	Variable
y_1 (mm)	216	469	Variable
f_c (MPa)	26	110	Variable
s (mm)	50	215	Variable
A_t (mm^2)	71	127	Variable
f_{yv} (MPa)	319	672	Variable
A_l (mm^2)	381	3438	Variable
f_{yl} (MPa)	310	638	Variable
ρ_t (%)	0.22	2.56	Variable
ρ_l (%)	0.30	3.51	Variable

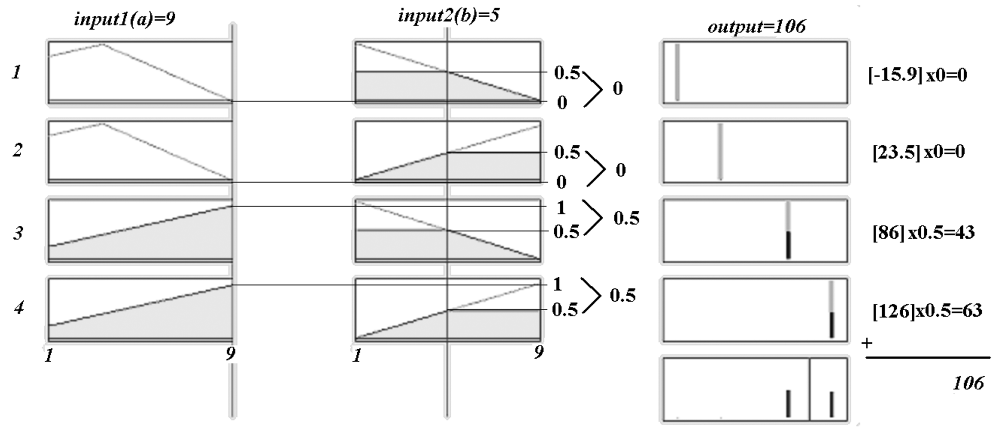

Fig. 4 Fuzzy inference diagram

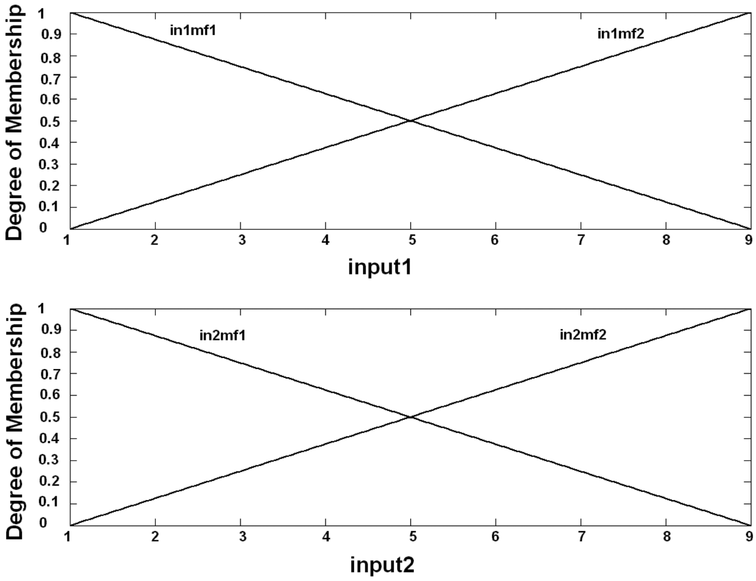

Fig. 5 Initial membership functions

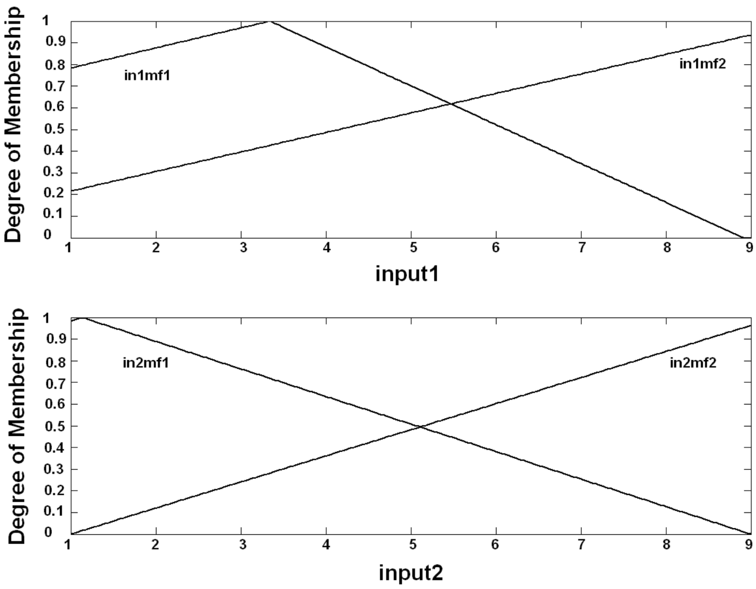

Fig. 6 Final membership functions

final membership values of rules for each input are given in Figs. 5 and 6 respectively. Suppose one will find the output for input values of 9 and 5. As the minimum of the two membership values are taken, the first and second values of output membership will be zero. The third and fourth output membership should be multiplied by 0.5. Thus the final output will be $86 \times 0.5 + 126 \times 0.5 = 106$. The exact result for $a = 9$ and $b = 5$ from Eq. (8) will be $y = 9^2 + 5(5) = 106$.

5. Implementation

In this study, MATLAB fuzzy logic toolbox was used for ANFIS modeling process. Among the experimental database, 16 samples were used for the test set and the remaining 60 samples were used for the training set in order to perform NF modeling. In the proposed NF model, the input parameters were selected by basing on the previously published studies of Rasmussen and Baker (1995), Koutchoukali and Belarbi (2001), Fang and Shiau (2004), Hsu (1968). These input parameters are dimension of the closed stirrup (x_1, y_1), concrete compressive strength (f_c), steel ratio of stirrups (ρ_t) and steel ratio of longitudinal reinforcement (ρ_l), $cot\theta$ (This parameter is obtained in terms of spacing of stirrups (s), cross-sectional area of one leg of closed stirrup (A_t), yield strength of closed stirrup (f_{yv}), total area of longitudinal torsional reinforcement (A_l), yield strength of longitudinal torsional reinforcement (f_{yl})). The simplest ANFIS model was selected to illustrate the effectiveness of the NF approach. The proposed ANFIS model uses the generalized bell-shaped input membership functions (2) with minimum number of fuzzy rules. The membership functions (fuzzy rules) for inputs are demonstrated in Fig. 7. According to Fig. 7, there are six input parameters each of which contain two bell-shaped membership functions. Besides, the membership of each parameter according to its interval was also indicated in the charts.

The output membership function was chosen as the simplest and available one which is a constant value. The details of the related parameters used for ANFIS model are presented in Table 1. Some features of the proposed ANFIS model are given in Table 2. The obtained statistical parameters of the proposed ANFIS model for Test/Predicted results are indicated in Table 3.

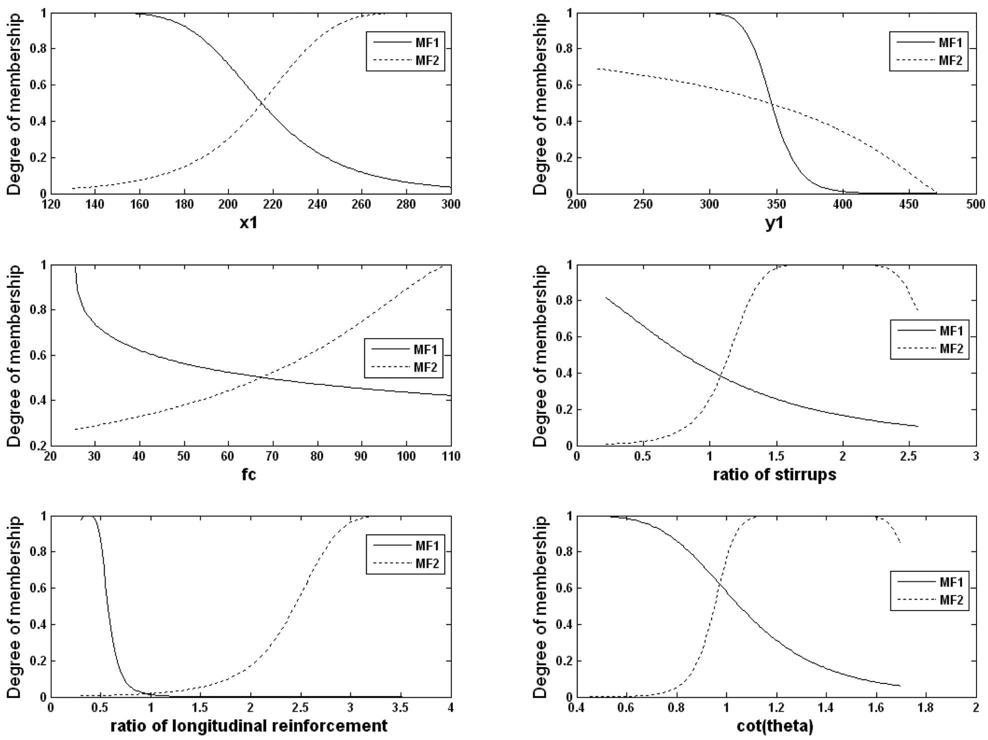

Fig. 7 Membership functions of input parameters

Table 2 Some features of the proposed ANFIS models

Type	SUGENO
Aggregation method	Maximum
Defuzzification method	Weighted average
Input membership function type	Triangular, Gaussian, Generalized bell-shaped
Output membership function type	Constant

Table 3 The obtained statistical parameters of the proposed NF model

	Mean	COV	RMSE	R^2
NF testing set	0.96	0.23	20.1	0.86
NF training set	1.00	0.01	0.34	0.99
NF total set	1.00	0.11	9.21	0.96

6. Discussions

6.1 Code approaches

The prediction accuracy of various standards of building codes related to the torsional strength of the beams for mentioned tested 76 specimens are presented in Table 4. As seen from Table 4, ACI-318-2005 (2005), AS3600 (2001) and CSA (1994) torsional strength expressions have the most powerful estimating capacity. The differences between the codes and test results are based on some reasons such as:

Table 4 Prediction accuracy of existing building codes

Building standards	Expression for torsional strength	COV	RMSE	R^2 (%)
ACI-318-2005	$T_n = \frac{2A_oA_tf_{yv}}{s}cot\theta$	0.29	23.5	85.93
BS8110	$T_n = \frac{0.8x_1y_1(0.87f_{ys})A_{sv}}{s}$	0.29	22.7	81.76
TBC-500-2000	$T_n = \frac{2A_lA_ef_{yv}}{2(x_1+y_1)}$	0.4	38.1	71.07
AS3600	$T_n = \frac{2A_oA_tf_{yv}}{s}cot\theta$	0.29	23.5	85.93
Eurocode-2-01	$T_n = f_{ys}(A_{sw}/s)2A_k\ cot\theta$	0.35	49.1	73.44
Eurocode-2-02	$T_n = f_y(A_s/u_k)2A_k\ tan\theta$	0.30	23.5	85.93
Eurocode-2-03	$T_n = 1.2(1-f_{ck}/250)f_{ck}A_kt_{ef}\ sin\theta cos\theta$	0.56	57.9	61.88
CSA	$T_n = \frac{2A_oA_tf_{yv}}{s}cot\theta$	0.29	23.5	85.93

- In all codes except for Eurocode-2-03 approaches given in Eq. (7), the concrete contribution is ignored after torsional cracking that makes no distinction between the behavior of normal and high strength concretes. Therefore, there is no advantage in using higher concrete strength in resisting ultimate torsion. However, the test series have shown that the ultimate torsional strength of RC beams increases with the increase of concrete quality.
- In the calculation of torsional strength, the main parameter is the shear flow area determined differently in the building codes. Taking the centers of longitudinal bars or centre-to-centre of stirrups for this calculation create a considerable difference in the total result.
- The building codes assume the longitudinal bars and stirrups to be yielded. But in the experiment that represents the real conditions more realist than analytical approaches, neither longitudinal bars nor stirrups yielded or either longitudinal bars or stirrups yielded. Especially high values of yield stresses, larger sizes of reinforcement and weaker concrete gives way the dominance of the neither longitudinal bars nor stirrups yielded.
- In the TBC-500-2000 (2000) and BS8110 (1985), the angle of cracks are neglected (or assumed 45^o). This assumption induces the important differences between the code approaches and test results.
- The theoretical values computed by using code formulations are generally higher than the experimental torsional strength. This can be explained probably by the fact that the thin-walled tube and space truss analogy deviate in the particular and isolated case of over-reinforced beams with low concrete strength.
- The comparison suggests that the most equations overestimate the strength, especially in the case of beams with low concrete strength. This is expected since most of the methods do not taking account the concrete strength in calculating the torsional strength.

6.2 Neuro-fuzzy (NF)

Predicted values achieved through the proposed NF formulations are compared with the experimental results for the torsional strength of the beams in Table A1 and also statistical parameters of the test and training sets of NF formulations are given in Table 3. Based on the findings of the NF the following comparisons can be drawn:

- The results of the proposed NF formulation performed better than building code's results.
- The error between the test and NF model is quite small for mentioned parameters. However, in the comparison of the code and test data, especially for over reinforced concrete, the predicting capability of code has become less.
- The outcomes of NF offer original contributions beside its high estimation capacity.

7. Parametric study

A wide range of parametric studies were performed in this study by using the NF model to investigate the interacting influence of each parameter on the torsional strength of the RC beams. The following results can be drawn based on the parametric study presented:

- It can be clearly understood from Figs. 8 and 9 that the increase in either the short dimension (x_1) or the long dimension (y_1) of the closed stirrup causes more increase on the torsional strength (T_u). Especially, for $x_1 = 200 - 250$ mm and $y_1 = 350 - 400$ mm ranges, the increase in

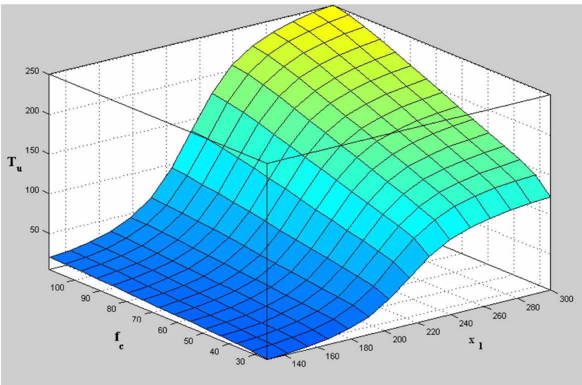

Fig. 8 Relationship between f_c, x_1, T_u

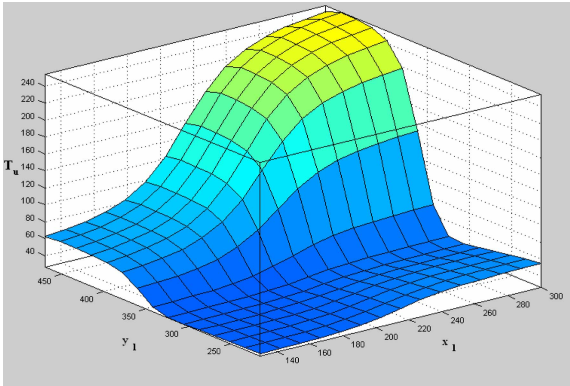

Fig. 9 Relationship between x_1, y_1, T_u

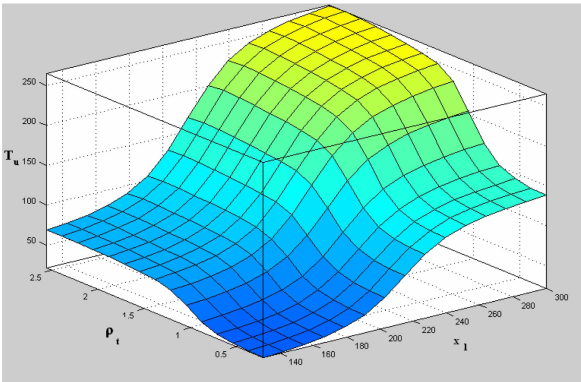

Fig. 10 Relationship between ρ_t, x_1, T_u

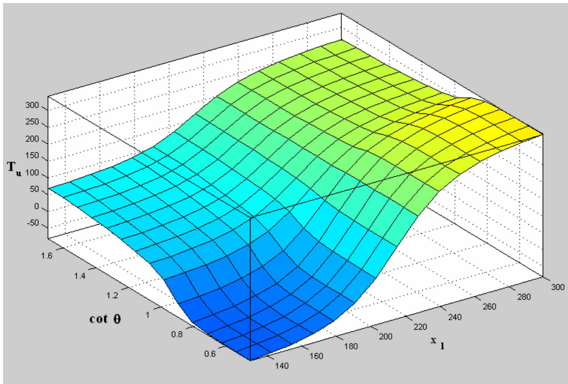

Fig. 11 Relationship between $cot\ \theta$, x_1, T_u

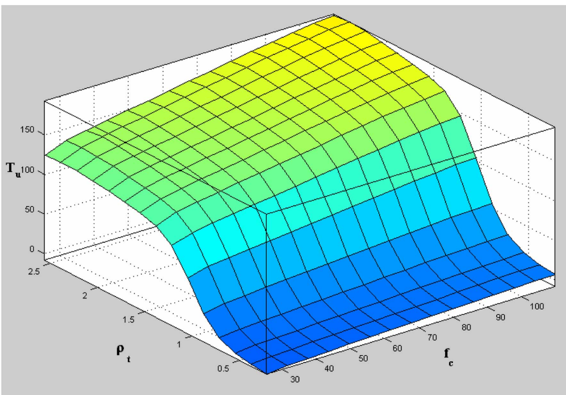

Fig. 12 Relationship between ρ_t, f_c, T_u

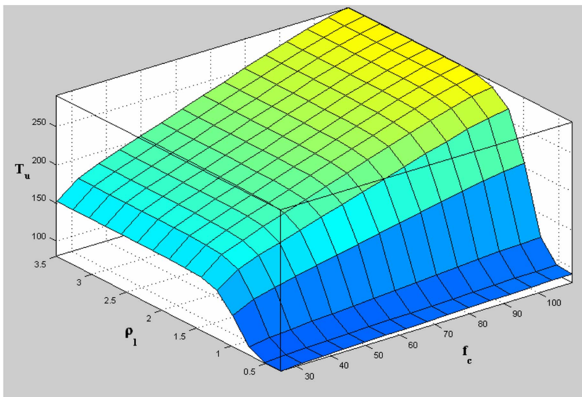

Fig. 13 Relationship between ρ_l, f_c, T_u

T_u becomes more obvious.

- As it can be seen from the relationship given in Fig. 8 between the short dimension of the closed stirrup (x_1) and the compressive strength (f_c), the contribution of f_c on T_u becomes effective with the increasing cross-sectional dimensions.

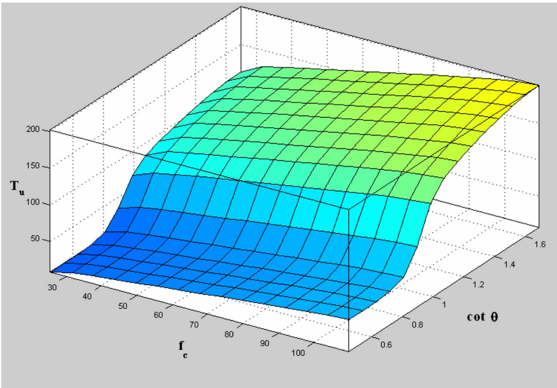

Fig. 14 Relationship between $cot\theta$, f_c, T_u

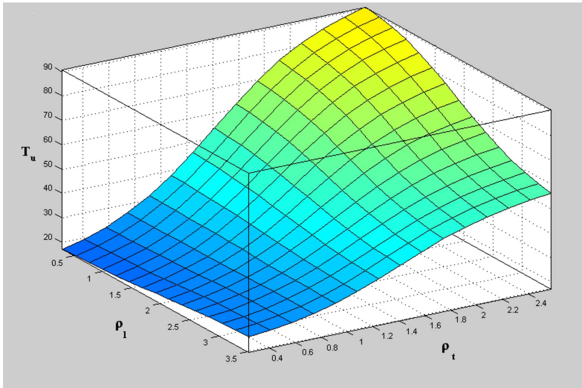

Fig. 15 Relationship between ρ_t, ρ_l, T_u

- The increasing steel ratio of the stirrups (ρ_t) causes to have increase in T_u value which occurs more obvious for the ρ_t values between 1-1.5%. This relationship is given in Fig. 10.
- As it's given in Fig. 11, for a given concrete compressive strength, the angle of crack (θ) increases with a decrease of shorter side of rectangular section.
- In Fig. 12, less affected and more affected torsional strength T_u values are seen because of the change in the compressive strength (f_c) of concrete due to low steel ratio of stirrups (ρ_t) and high steel ratio of stirrups (ρ_t), respectively. Similar to the situation in Fig. 12, Fig. 13 displays the torsional strength T_u affected by the change in the concrete's compressive strength (f_c) due to high steel ratio of longitudinal reinforcement (ρ_l).
- The inclined angle (θ) of crack decreases (i.e., stiffness increases) with the increase of concrete compressive strength. According to Fig. 14, the effect of the concrete's compressive strength (f_c) on the torsional strength (T_u) increases with the increasing *cot* (θ) ($\theta < 45^o$). The increasing concrete compressive strength (f_c) causes to occur decreasing inclined angle and crack slope. Principle compression strength in beam web increases with decreasing inclined angle (θ).
- The increasing steel ratio of the stirrups (ρ_t) causes to have more increment in T_u value than effect of increment steel ratio of longitudinal reinforcement (ρ_l). This relationship is given in Fig. 15.

The figures present that, the relationship tends to be directly proportional with the parabolic type in most cases and is consistent with the findings of the pertinent literature. The wide ranges of these parametric studies also verify the generalization capability and the accuracy of the proposed NF model.

8. Conclusions

This study is a pioneer work that inquires the capability of NF approach for the empirical modeling of torsional strength of RC beams without web reinforcement. The proposed NF (ANFIS) model is based on experimental results (76 samples) collected from separate studies. A computer program was developed to obtain the optimum NF model which evaluates various types and number of membership functions and selects the NF model with the least testing set error. The optimum NF model has been found to have 2 membership functions of generalized bell shaped. The results of the proposed ANFIS model are observed to be more accurate than those of the current design codes and existing torsional strength equations available in literature. Most of the design codes and equations available in literature are based on the regression analysis of predefined functions. However, in the case of NF approach presented in this study, there is no predefined function to be considered. The NF approach generated rules between the input and output values. The outcomes of this study are quite satisfactory which may serve NF approach to be widely used in further applications in the field of reinforced concrete structures.

References

ACI Committee 318-2005 (2005), "Building code requirements for structural concrete (ACI 318-05) and commentary (318R-05)", American Concrete Institute, Farmington Hills, Mich.
ACI Committee 318-71 (1971), "Building code requirements for structural concrete and commentary", American Concrete Institute, Detroit.
ACI Committee 318-89 (1989), "Building code requirements for structural concrete and commentary", American Concrete Institute, Detroit.
ACI Committee 318-95 (1995), "Building code requirements for structural concrete and commentary", American Concrete Institute, Detroit.
Anderson, P. (1935), "Experiments with concrete in torsion", *Trans. ASCE*, **60**, 641-652.
AS3600 (2001), "Concrete structures, Standarts Association of Australia".
Bai, Y.Y., Zhuang, H. and Wang, D. (2006), *Advanced fuzzy logic technologies in industrial applications*, Springer.
BS8110 (1985), "Structural use of concrete-Part 2" British Standards
Canadian Standard Association (1994), *Design of concrete structures: Structure design. CSA Standard*, A23-3-94, Canadian Standard Association, Rexdale, Ontario.
Ceylan, M., Arslan, M.H., Ceylan, R., Kaltakc, M.Y. and Ozbay, Y. (2010), "A new application area of ANN and ANFIS: determination of earthquake load reduction factor of prefabricated industrial buildings", *Civ. Eng. Environ. Syst*., **27**(1), 53-69.
Collins, C.D. Walsh, P.F. Archer, F.E. and Hall, A.S. (1965), "Reinforced concrete beams subjected to conbined torsion and shear", *UNICIV Report*, No.R-14, University of New South Wales.
Collins, C.P. and Mitchell, D. (1980), "Shear and torsion design of prestressed and non-prestressed concrete beams", *PCI J*., **25**(5), 32-100.
Elfegren, L., Karlsson, I. and Losberg, A. (1974), "Torsion bending-shear interaction for concrete beams", *J. Struct. Div. ASCE*, **100**, 1657-1676.
European Standard. Eurocode 2 (2002), Design of concrete structures, prEN, 1992-1-1, Draft for stage 49, Commission of the European Communities, European Committee for Standardization.
Evbuowman, N.F.O., Sivaloganathan, S. and Jebb, A. (1996), "A survey of design philosophies, models, methods and systems", *P. I. Mech. Eng. B-J. Eng.*, **210**(42), 301-320.
Fang, I.K. and Shiau, J.K. (2004), "Torsional behavior of normal and high strength concrete beams", *ACI Struct. J*., **101**(3), 304-313.
Gallant, S.I. (1993), *Neural network learning and expert systems*, The MIT Press.
Ghorbani, A. and Ghasemi, M.R. (2010), "An efficient method for reliability based optimization of structures using adaptive neuro-fuzzy systems", *J. Mod. Simul. Syst.*, **1**(1), 13-21.
Haris, J. (2006), *Fuzzy logic applications in engineering science*, Springer.
Hossain, T. and Mendis, P., Aravinthan, T. and Baker, G. (2006), "Torsional resistance of highstrength concrete beams", *19th Australasian Conference on the Mechanics of Structures and Materials*, Christchurch, NZ.
Hsu, T.T.C. (1968), *Torsion of structural concrete-behavior of reinforced concrete rectangular members*, Torsion of structural concrete SP-18, ACI, 261-306.
Hsu, T.T.C. (1968), "Ultimate torque of reinforced concrete members", *J. Struct. Div. ASCE*, **94**, 485-510.

Hsu, T.T.C. and Mo, Y.L. (1985), "Softening of concrete in torsional members-design recommendation", *ACI J.*, **82**(4), 443-452.

Jang, J.S.R., Sun, C.T. and Mizutani, E. (1997), *Neuro-fuzzy and soft computing*, A computational approach to learning and machine intelligence, Prentice Hall.

Kasabov, K.N. (1996), *Foundation of neural networks, fuzzy systems and knowledge engineering*, MIT Press.

Kim, H.S., Paul, N.R., Lin, P.Y. and Loh, C.H. (2006), "Neuro-fuzzy model of hybrid semi-active base isolation system with FPS bearings and an MR damper", *Eng. Struct.*, **28**(7), 947-958.

Koutchoukali, N.E. and Belarbi, G. (2001), "Torsion of high strength reinforced concrete beams and minimum reinforcement requirement", *ACI Struct. J.*, **98**(4), 462-469.

Lessig, N.N. (1959), "Determination of carriying capacity of reinforced concrete elements with rectangular cross-section subjected to flexure with torsion", *Zhelezonabetona*, **5**, 5-28.

Mashrei, M.A., Abdulrazzaq, N., Turki, Y. Abdalla and M.S. Rahman (2010), "Neural networks model and adaptive neuro-fuzzy inference system for predicting the moment capacity of ferrocement members", *Eng. Struct.*, **32**(6), 1723-1734.

McMullen, A.E. and Rangan, B.V. (1978), "Pure torsion in rectangular section-A reexamination", *ACI J.*, **75**, 511-519.

Murawski, K., Arciszewski, T. and De Jong, K. (2000), "Evolutionary computation in structural design", *Eng. Comput.*, **16**(3-4), 275-286.

Nawy, E.G. (2003), *Reinforced concrete, A fundamental approach*, Pearson Education.

Rajasekaran, S., Suresh, D. and Vijayalakshmi Pai, G.A. (2002), "Application of sequential learning neural networks to civil engineering modeling problems", *Eng. Comput.*, **18**(2), 138-147.

Rao, S.S., Nahm, A., Shi, Z., Deng, X. and Syamil, A. (1999), "Artificial intelligence and expert systems applications in new product development: A survey", *J. Int. Manuf.*, **10**(3-4), 231-244.

Rasmussen, L.J. and Baker, G. (1994), "Assessment of torsional strength in reinforced normal and high-strength concrete beams", *Austral. Civil Eng. Trans., IEAust.*, **36**(2), 165-171.

Rasmussen, L.J. and Baker, G. (1995), "Torsion in reinforced normal and high strength concrete beams Part-I : An experimental test series", *ACI Struct. J.*, **92**(1), 56-62.

Rausch, E. (1929), "Design of reinforced concrete in torsion", Technische Hochschule, Berlin, 53. (in German).

Regli, W.C., Hu, X., Atwood, M. and Sun, W. (2000), "A Survey of design rationale systems: Approaches, representation, capture and retrieval", *Eng. Comput.*, **16**(3-4), 209-235.

Rutkowski, L. (2004), *Flexible neuro-fuzzy systems: Structures, learning and performance evaluation*, Kluwer Academic Publishers.

Saridakisa, K.M., Dentsorasa, A.J., Radelb, P.E., Saridakisb, V.G. and Exintarib, N.V. (2006), "Neuro-fuzzy casebased design: An application in structural design", *Intell. Prod. Mach. Syst.*, 2nd IProms Virtual Conferences, 407-412.

Simpson, T.W., Peplinski, J.D., Koch, P.N. and Allen, J.K. (2001), "Metamodels for computer-based engineering design: survey and recommendations", *Eng. Comput.*, **17**(2), 129-150.

Sivanandam, S.N., Sumathi, S. and Deepa, S.N. (2007), *Introduction to fuzzy logic using MATLAB*, Springer.

Tang, C.W. (2006), "Using radial basis function neural networks to model torsional strength of reinforced concrete beams", *Comput. Concrete*, **3**(5), 335-355.

TBC-500-2000 (2000), *Requirements for design and construction of reinforced concrete structures*, Turkish Standards TS-500, Ankara.

Victor, D.J. and Muthukrishnan, R. (1973), "Effect of stirrups on ultimate torque of reinforced concrete beams", *ACI J.*, **70**(4), 300-306.

Wang, Y.M. and Elhag, T.H.M. (2008), "An adaptive neuro-fuzzy inference system for bridge risk assessment", *Expert Syst. Appl.,* **34**(4), 3099-3106.

Zadeh, L.A. (1965), "Fuzzy sets", *Inform. Control.*, **8**(3), 338-353.

Zang, Y. (2002), "Torsion in high strength concrete rectangular beams", Master thesis, University of Nevada.

Zia, P. and Hsu, T.T.C. (2004), "Design for torsion and shear in prestressed concrete", *PCI J. Prescast/Prest. Concrete Inst.,* **49**(3), 34-42.

CC

Notation

A	: total area
A_o	: gross area enclosed by the shear flow path
A_e	: area enclosed by lines connecting the centroids of the reinforcing bars at the corner of the section
A_k	: area enclosed by the centre-lines of the effective wall thickness
A_l	: total area of longitudinal torsional reinforcement
A_{sh}	: area enclosed by the centre of stirrups
A_{sv}	: area of the two legs of stirrups at a section (=$2A_t$)
A_t	: cross sectional area of one-leg of closed stirrup
a_i	: outputs of neural network
f_c	: compressive strength of concrete
f_{yl}	: yield strength of longitudinal torsional reinforcement
f_{yv}	: yield strength of closed stirrups
k	: number of samples in training or test data
m	: number of segments in training or test data
n	: number of outputs of neural network for training and test procedures
ρ_l	: steel ratio of longitudinal reinforcement
ρ_t	: steel ratio of stirrups
p_h	: perimeter of centerline of outmost closed transverse torsional reinforcement
R^2	: correlation coefficient
s	: spacing of stirrups
s_x	: normalized value of variable
T_c	: torsion moment resisted by the concrete compression struts
T_n	: nominal torsional strength
$T_{u(\text{estimated})}$	: predicted ultimate torsional strength
$T_{u\ (\text{experimental})}$	: measured ultimate torsional strength
t_{ef}	: the effective wall thickness
t_i	: desired outputs
u	: perimeter of the cross-section
x	: short dimension of the cross section
y	: long dimension of the cross section
x_1	: center-to-center of the shorter and longer legs of stirrups
y_1	: center-to-center of the longer legs of stirrups
z	: variable values
$z_{\min}$	: variable minimum values
$z_{\max}$	: variable maximum values
θ	: angle of compression diagonals

Appendix

Table A1 Experimental database [2,5,6,8,28,29]

Ref.	No	x	y	x_1	y_1	f_c	S	A_t	f_y	A_l	f_{yl}	ρ_t	ρ_l	T_u (Test)	T_u (NF)	T_u(Test)/ T_u(NF)
2	FS-1	350	500	300	450	78.5	100	71.33	440	1196.6	440	0.61	0.68	92.00	92.00	1.00
	F2-2	**350**	**500**	**300**	**450**	**78.5**	**100**	**71.33**	**440**	**2027.2**	**410**	**0.61**	**1.16**	**115.10**	**112.74**	**1.02**
	FS-3	**350**	**500**	**300**	**450**	**78.5**	**50**	**71.33**	**440**	**2027.2**	**410**	**1.22**	**1.16**	**155.30**	**155.94**	**1.00**
	FS-4	350	500	300	450	78.5	50	71.33	440	2865	520	1.22	1.64	196.00	195.99	1.00
	FS-5	350	500	300	450	78.5	55	126.7	440	3438	560	1.97	1.96	239.00	239.00	1.00
	FS-6	350	500	300	450	68.4	90	71.33	420	1719	500	0.68	0.98	126.70	126.70	1.00
	FS-7	350	500	300	450	68.4	80	126.7	360	1719	500	1.36	0.98	135.20	135.21	1.00
	FS-8	**350**	**500**	**300**	**450**	**68.4**	**90**	**71.33**	**440**	**2865**	**500**	**0.68**	**1.64**	**144.50**	**134.76**	**1.07**
	FS-9	350	500	300	450	35.5	100	71.33	440	1191.6	440	0.61	0.68	79.70	79.71	1.00
	FS-10	350	500	300	450	35.5	100	71.33	440	2027.2	410	0.61	1.16	95.20	95.22	1.00
	FS-11	350	500	300	450	35.5	50	71.33	440	2027.2	410	1.22	1.16	116.80	116.79	1.00
	FS-12	350	500	300	450	35.5	50	71.33	440	2865	520	1.22	1.64	138.00	138.01	1.00
	FS-13	**350**	**500**	**300**	**450**	**35.5**	**55**	**126.7**	**440**	**3438**	**560**	**1.97**	**1.96**	**158.00**	**165.00**	**0.96**
	FS-14	350	500	300	450	35.5	90	71.33	420	1719	500	0.68	0.98	111.70	111.68	1.00
	FS-15	350	500	300	450	35.5	80	126.7	360	1719	500	1.36	0.98	125.00	125.00	1.00
	FS-16	350	500	300	450	35.5	90	71.33	420	2865	500	0.68	1.64	117.30	117.29	1.00
5	KB-1	203	305	165	267	39.6	108	71.33	373	506.8	386	0.92	0.82	19.40	19.40	1.00
	KB-2	**203**	**305**	**165**	**267**	**64.6**	**108**	**71.33**	**399**	**506.8**	**386**	**0.92**	**0.82**	**18.90**	**18.56**	**1.02**
	KB-3	203	305	165	267	75	108	71.33	373	506.8	386	0.92	0.82	21.10	21.11	1.00
	KB-4	203	305	165	267	80.6	108	71.33	399	506.8	386	0.92	0.82	19.40	19.38	1.00
	KB-5	203	305	165	267	93.9	108	71.33	386	506.8	386	0.92	0.82	21.00	21.00	1.00
	KB-6	203	305	165	267	76.2	102	71.33	386	506.8	386	0.98	0.82	18.40	18.41	1.00
	KB-7	**203**	**305**	**165**	**267**	**72.9**	**95**	**71.33**	**386**	**649.46**	**373**	**1.05**	**1.05**	**22.50**	**19.56**	**1.15**
	KB-8	203	305	165	267	75.9	90	71.33	386	760.2	373	1.11	1.23	23.70	23.69	1.00
	KB-9	203	305	165	267	76.7	70	71.33	386	794.4	380	1.42	1.28	24.00	24.02	1.00
6	RB-1	160	275	130	245	41.7	90	78.54	665	1543.9	620	1.49	3.51	16.60	16.44	1.01
	RB-2	160	275	130	245	38.2	90	78.54	669	1543.9	638	1.49	3.51	15.30	15.81	0.97
	RB-3	**160**	**275**	**130**	**245**	**36.3**	**90**	**78.54**	**672**	**1543.9**	**605**	**1.49**	**3.51**	**15.30**	**16.00**	**0.96**
	RB-4	160	275	130	245	61.8	90	78.54	665	1543.9	612	1.49	3.51	20.00	19.05	1.05
	RB-5	160	275	130	245	57.1	90	78.54	665	1543.9	614	1.49	3.51	18.50	18.45	1.00
	RB-6	160	275	130	245	61.7	90	78.54	665	1543.9	612	1.49	3.51	19.10	19.04	1.00
	RB-7	160	275	130	245	77.3	90	78.54	658	1543.9	617	1.49	3.51	20.10	21.01	0.96
	RB-8	**160**	**275**	**130**	**245**	**76.9**	**90**	**78.54**	**656**	**1543.9**	**614**	**1.49**	**3.51**	**20.70**	**20.96**	**0.99**
	RB-9	160	275	130	245	76.2	90	78.54	663	1543.9	617	1.49	3.51	21.00	20.86	1.01
	RB-10	160	275	130	245	109.8	90	78.54	655	1526.8	618	1.49	3.51	24.70	24.43	1.01
	RB-11	160	275	130	245	105	90	78.54	660	1526.8	634	1.49	3.51	23.60	24.26	0.97
	RB-12	160	275	130	245	105.1	90	78.54	655	1543.9	629	1.49	3.51	24.80	24.33	1.02

Table A1 Continued

Ref.	No	x	y	x_1	y_1	f_c	S	A_t	f_y	A_l	f_{yl}	ρ_t	ρ_l	T_u (Test)	T_u (NF)	T_u(Test)/ T_u(NF)
	HS-1	**254**	**381**	**215.9**	**342.9**	**27.58**	**152.4**	**71.33**	**341.29**	**508**	**313.71**	**0.54**	**0.52**	**22.30**	**25.85**	**0.86**
	HS-2	254	381	215.9	342.9	28.61	181.1	126.7	319.92	635	316.47	0.81	0.66	29.30	29.51	0.99
	HS-3	254	381	215.9	342.9	28.06	127	126.7	319.92	762	327.5	1.15	0.79	37.50	38.82	0.97
	HS-4	254	381	215.9	342.9	30.54	92.2	126.7	323.56	889	319.92	1.59	0.92	47.30	47.59	0.99
	HS-5	254	381	215.9	342.9	29.03	69.9	126.7	321.3	1016	332.33	2.09	1.05	56.20	56.10	1.00
	HS-6	**254**	**381**	**215.9**	**342.9**	**28.82**	**57.2**	**126.7**	**322.67**	**1143**	**331.64**	**2.56**	**1.18**	**61.70**	**60.94**	**1.01**
	HS-7	254	381	215.9	342.9	25.99	127	126.7	318.54	508	319.92	1.15	0.52	26.90	27.50	0.98
	HS-8	254	381	215.9	342.9	26.75	57.2	126.7	319.92	508	321.99	2.56	0.52	32.50	32.09	1.01
	HS-9	254	381	215.9	342.9	28.82	152.4	126.7	342.67	762	319.23	0.96	0.79	29.80	29.91	1.00
	HS-10	254	381	215.9	342.9	26.48	152.4	126.7	341.98	1143	334.4	0.96	1.18	34.40	34.41	1.00
	HS-11	**254**	**381**	**215.9**	**342.9**	**26.61**	**152.4**	**71.33**	**337.84**	**508**	**333.02**	**0.54**	**0.52**	**22.40**	**26.78**	**0.84**
	HS-12	254	381	215.9	342.9	25.58	181.1	126.7	330.95	635	322.67	0.81	0.66	27.70	27.65	1.00
	HS-13	254	381	215.9	342.9	28.41	127	126.7	333.02	762	341.67	1.15	0.79	40.20	39.18	1.03
	HS-14	254	381	215.9	342.9	30.61	92.2	126.7	333.02	889	330.26	1.59	0.92	47.90	47.70	1.00
	HS-15	254	381	215.9	342.9	29.85	149.4	71.33	353.01	635	326.12	0.55	0.66	30.40	30.31	1.00
	HS-16	**254**	**381**	**215.9**	**342.9**	**30.54**	**104.9**	**71.33**	**357.15**	**762**	**328.88**	**0.79**	**0.79**	**40.60**	**31.33**	**1.30**
	HS-17	254	381	215.9	342.9	26.75	139.7	126.7	326.12	889	321.99	1.05	0.92	43.80	43.79	1.00
	HS-18	254	381	215.9	342.9	26.54	104.9	126.7	326.81	1016	318.54	1.39	1.05	49.60	49.04	1.01
	HS-19	254	381	215.9	342.9	27.99	82.6	126.7	330.95	1143	335.09	1.77	1.18	55.70	56.13	0.99
7	HS-20	254	381	215.9	342.9	29.37	69.9	126.7	340.6	2288	317.85	2.09	2.36	60.10	60.12	1.00
	HS-21	**254**	**381**	**215.9**	**342.9**	**45.23**	**98.6**	**71.33**	**348.87**	**635**	**325.43**	**0.84**	**0.66**	**36.00**	**28.31**	**1.27**
	HS-22	254	381	215.9	342.9	44.75	127	126.7	333.71	762	343.36	1.15	0.79	45.60	45.56	1.00
	HS-23	254	381	215.9	342.9	44.95	92.2	126.7	326.12	889	315.09	1.59	0.92	58.10	58.26	1.00
	HS-24	254	381	215.9	342.9	45.02	69.9	126.7	325.43	1016	310.26	2.09	1.05	70.70	70.50	1.00
	HS-25	254	381	215.9	342.9	45.78	57.2	126.7	328.88	1143	325.43	2.56	1.18	76.70	76.72	1.00
	HS-26	**254**	**508**	**215.9**	**469.9**	**29.79**	**187.5**	**71.33**	**339.22**	**508**	**321.99**	**0.4**	**0.39**	**26.80**	**104.27**	**0.26**
	HS-27	254	508	215.9	469.9	30.89	120.7	71.33	333.71	635	322.67	0.63	0.49	40.30	40.34	1.00
	HS-28	254	508	215.9	469.9	26.82	155.7	126.7	327.5	762	338.53	0.87	0.59	49.60	49.52	1.00
	HS-29	254	508	215.9	469.9	28.27	114.3	126.7	341.98	889	325.43	1.18	0.69	64.90	64.87	1.00
	HS-30	254	508	215.9	469.9	26.89	85.9	126.7	327.5	1016	330.95	1.57	0.79	72.00	72.03	1.00
	HS-31	**254**	**508**	**215.9**	**469.9**	**29.92**	**127**	**126.7**	**349.56**	**1144**	**334.4**	**1.06**	**0.89**	**39.10**	**49.47**	**0.79**
	HS-32	254	508	215.9	469.9	30.96	146.1	126.7	322.67	1430	319.23	0.92	1.11	52.70	52.70	1.00
	HS-33	254	508	215.9	469.9	28.34	104.9	126.7	328.88	1716	321.99	1.28	1.33	63.30	63.30	1.00
	HS-34	254	508	215.9	215.9	27.03	215.9	71.33	341.29	381	341.29	0.22	0.3	11.30	11.39	0.99
	HS-35	254	508	215.9	215.9	26.54	117.6	71.33	344.74	508	334.4	0.41	0.39	15.30	15.25	1.00
	HS-36	**254**	**508**	**215.9**	**215.9**	**26.89**	**139.7**	**126.7**	**329.57**	**635**	**330.95**	**0.61**	**0.49**	**20.00**	**20.25**	**0.99**
	HS-37	254	508	215.9	215.9	27.17	98.6	126.7	327.5	762	336.46	0.86	0.59	25.30	24.89	1.02
	HS-38	254	508	215.9	215.9	27.23	73.2	126.7	328.88	889	328.19	1.16	0.69	29.70	29.47	1.01
	HS-39	254	508	215.9	215.9	27.58	54.1	126.7	327.5	1016	315.78	1.57	0.79	34.20	34.34	1.00

*Bold samples are used testing set