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Optimum design of reinforced concrete columns subjected to uniaxial flexural compression

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Abstract. The search for a design that meets both performance and safety, with minimal cost and lesser environmental impact was always the goal of structural engineers. In general, the design of conventional reinforced concrete structures is an iterative process based on rules of thumb established from the personal experience and intuition of the designer. However, such procedure makes the design process exhaustive and only occasionally leads to the best solution. In such context, this work presents the development and implementation of a mathematical formulation for obtaining optimal sections of reinforced concrete columns subjected to uniaxial flexural compression, based on the verification of strength proposed by the Brazilian standard NBR 6118 (ABNT 2007). To minimize the cost of the reinforced concrete columns, the Simulated Annealing optimization method was used, in which the amount and diameters of the reinforcement bars and the dimensions of the columns cross sections were considered as discrete variables. The results obtained were compared to those obtained from the conventional design procedure and other optimization methods, in an attempt to verify the influence of resistance class, variations in the magnitudes of bending moment and axial force, and material costs on the optimal design of reinforced concrete columns subjected to uniaxial flexural compression.

Keywords: optimization; columns; reinforced concrete; simulated annealing.

1. Introduction

The search for an optimal design of an engineering structure or element that meets both performance and safety, with minimal cost and lesser environmental impact, was always the goal pursued by most structural engineers. Currently, with the advances in computer technology, it became possible to investigate a larger number of design variables and constraints, reducing the simplifications and making the mathematical model more representative of the actual state.

Since there are many possible solutions, choosing the most appropriate one can be facilitated by the implementation of mathematical optimization techniques. Nowadays, there are several tools for the analysis and design of structures, but it is not common to incorporate mathematical optimization modules for these tools to dimension optimally a structure.

In the field of structural engineering, the use of mathematical optimization techniques generally aims to minimize the cost or weight of a concrete or steel structure, given the limits prescribed by regulatory standards. Basically, the studies found in the literature are focused on geometry, topology, optimization of the cross sectional dimensions (which represents the majority of studies), or the combination of these cases. Amongst the articles that involve the optimization of reinforced

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concrete columns, stand out the studies by Zielinski *et al.* (1995), who presented a procedure for the optimization of reinforced concrete columns, Argolo (2000), who developed an optimization study of reinforced concrete sections subjected to uniaxial flexion using genetic algorithms, Rodrigues Junior (2005), who proposed a formulation for the optimal design of reinforced concrete columns of tall buildings, and Martínez (2007), who compared several optimization algorithms that allow to obtain the design of reinforced concrete rectangular columns with hollow sections for road and railway viaducts of different heights and spans.

In this context, the present paper proposes to incorporate a heuristic optimization method, the Simulated Annealing method (Kirkpatrick *et al.* 1983), to the process of strength verification of reinforced concrete rectangular columns subjected to uniaxial flexural compression, following the Brazilian standard NBR 6118 - Procedures for the Design of Reinforced Concrete Structures (ABNT 2007). To minimize the cost of the reinforced concrete columns, the amount and diameters of the reinforcement bars and the dimensions of the columns cross sections were considered as discrete variables. The results obtained were compared to those obtained from the conventional design procedure and other optimization methods, in an attempt to verify the influence of resistance class, variations in the magnitudes of bending moment and axial force, and material costs on the optimal design of reinforced concrete columns subjected to uniaxial flexural compression.

2. Background on mathematical optimization

The reduction in the design time and the possibility of simultaneous consideration of a larger number of variables and constraints are some of the major advantages of using mathematical optimization techniques (Vanderplaats 1984).

The mathematical optimization of a given problem can be achieved by finding the best possible solution with the aid of an appropriate algorithm, taking into account the variables within a feasible set of solutions and some imposed constraints.

The basic mathematical formulation of a multidimensional problem of optimization subject to constraints can be described as (Vanderplaats 1984)

$$\text{Minimize: } f(x_i) \qquad i = 1, n \tag{1}$$

$$\text{Subject to: } g_j(x_i) \le 0 \qquad j = 1, m \tag{2}$$

$$h_k(x_i) = 0 \qquad k = 1, l \tag{3}$$

$$x_i^l \le x_i \le x_i^u \tag{4}$$

where f is the objective function; $x = (x_1, x_2, \dots x_n)^T$ is the vector of variables of size n and the other functions are the constraints of the problem (respectively, inequality constraints g, equality constraints h and side constraints of the possible values of x).

Depending on the characteristics of the formulated problem, several techniques can be employed to solve it. Since the 1970s, it has increased the interest in algorithms inspired by the behavior of nature, based on physics and biology, to solve complex optimization problems, especially in situations where methods based on mathematical programming have shown to be inefficient.

2.1 Simulated annealing method

The Simulated Annealing is a heuristic method inspired by natural processes and has its origins on the simulation of the mechanical process of annealing metals.

When a metal is heated to high temperatures causing fusion, the atoms move freely. This process is called annealing. The solidification occurs by slow and controlled cooling, in which the atoms are reorganized in an orderly and stable configuration, forming a uniform structure with minimal energy, also resulting in a defects reduction in the material. If the metal is cooled abruptly, the microstructure tends to an unstable state.

Metropolis *et al.* (1953) presented an algorithm to model the annealing process of metals, simulating the energy changes in a system of particles as the temperature decreases to a stable state. The acceptance of this type of solution depends on the probability known as the "Metropolis criterion", calculated by the function

$$p(\Delta E) = \exp\left(\frac{-\Delta E.K}{T}\right) \tag{5}$$

where T is the temperature of the body and K is the Boltzmann constant.

Similarly to the original method of annealing in thermodynamics, the optimization process starts with a high value of T, for which a new solution is generated as T becomes a control parameter. The Boltzmann constant has no analogy in an optimization problem, being eliminated. This new solution will be automatically accepted if it results in a reduction in the function value. On the contrary, if the new function value is greater than its predecessor, the acceptance will be given by a probability criterion, being the acceptance function

$$p = \exp\left(\frac{-\Delta f}{T}\right) \tag{6}$$

A random number r is generated from a uniform probability distribution on the interval (0,1). If this number is less than or equal to p, the solution is accepted, otherwise the solution is rejected.

3. Strength verification of columns subjected to uniaxial flexural compression

The iterative process for the verification of columns strength begins with the knowledge of the external axial force (N_{sd}) and bending moment (M_{sd}), as well as the prior definition of a cross section along with the positions and diameters of the reinforcement bars. The final values of N_{sd} and M_{sd} are obtained by multiplying the respective force and moment by their characteristic partial safety factors provided by the Brazilian standard NBR 8681 (ABNT 2004) for the different actions and combinations involved in the design.

The internal resistant force N_{rd} and bending moment M_{rd} follow the same sign convention adopted for their external counterparts, and are obtained by the following equilibrium equations, according to the description of the deformed configuration

$$N_{rd} = \int_{Ac} \sigma_{cd} \cdot dA_c + \sum_{i=1}^{n} A_{si} \cdot \sigma_{sdi} \tag{7}$$

$$M_{rd} = \int_{Ac} \sigma_{cd} \cdot y_c \cdot dA_c + \sum_{i=1}^{n} A_{si} \cdot \sigma_{sdi} \cdot y_{si} \quad (8)$$

where:

N_{rd} is the resistant axial force;

M_{rd} is the resistant bending moment;

σ_{cd} is the stress acting on the concrete section A_c;

σ_{sdi} is the stress acting on the steel section A_{si};

y_c is the distance from the center of the compressed concrete area to the center of gravity of the section;

y_{si} is the distance from the steel bar i to the center of gravity of the section.

The calculated resistant axial force N_{rd} must be at least equal to the external axial force N_{sd} applied to a known reinforced cross section. The value of N_{sd} is fixed, while N_{rd} varies depending on the depth x_0 of the neutral axis, which is the only unknown parameter in Eq. 9

$$f(x_0) = N_{sd} - A_{cc} \cdot \sigma_{cd} - \sum_{i=1}^{n} A_{si} \cdot \sigma_{sdi} \quad (9)$$

Matching the axial internal and external stresses, Eq. 9 assumes the condition $f(x_0)=0$, whose solution lies in the interval $(0, \infty)$, covering all the flexural compression domain, and in which case it can be solved iteratively. In this work, the solution for the neutral axis position was obtained by using the Golden Section method, a one-dimensional searching method that is characterized by requiring only the calculation of function values at some points, so that the range of values is reduced until a convergence towards a single value occurs at a given tolerance.

Once the depth x_0 of the neutral axis is known, the equilibrium Eqs. (7) and (8) assume the following forms

$$N_{rd} = A_{cc} \cdot \sigma_{cd} + \sum_{i=1}^{n} A_{si} \cdot \sigma_{sdi} \quad (10)$$

$$M_{rd} = S_c \cdot \sigma_{cd} + \sum_{i=1}^{n} A_{si} \cdot \sigma_{sdi} \cdot y_{si} \quad (11)$$

where:

A_{cc} is the area of compressed concrete section;

S_c is the first moment of the compressed part of concrete section.

4. Problem formulation for minimizing the cost of columns

Considering a rectangular cross section, the objective of optimum design is to obtain a configuration that is capable of producing internal forces and moments (N_{rd} and M_{rd}) equal or higher than the applied external loadings (N_{sd} and M_{sd}), with minimal cost.

The formulation of the optimization problem starts out from the knowledge of some input parameters, previously defined and which basically represent the stresses acting on the element and the materials characteristics and costs. These design parameters do not change during the optimization process and are defined as

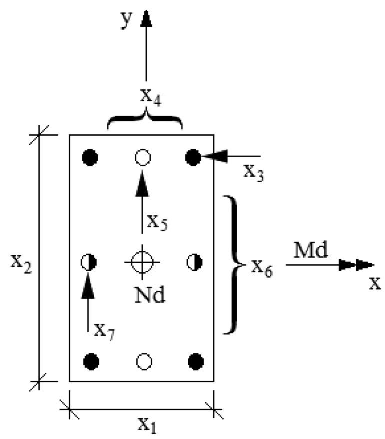

Fig. 1 Design variables

N_{sd} – axial force;
M_{sd} – bending moment in relation to the axis x;
c – cover depht;
f_{yk} – characteristic strength of steel;
E_s – elasticity modulus of steel;
f_{ck} – characteristic strength of concrete;
C_c – unit cost of concrete;
C_s – unit cost of steel;
C_f – unit cost of formwork.

The design variables (x_i) are the values that represent the cross section dimensions and the steel bar diameters as identified in Fig. 1.

Where x_1 and x_2 represent, respectively, the width (b) and the height (h) of the cross section; x_3 is the diameter of the four corner bars; x_4 represents the number of bars in the two layers parallel to x_1; x_5 is the diameter of the bars in the two layers parallel to x_1; x_6 represents the number of layers with two bars parallel to x_2 and x_7 is the diameter of the bars in the layers parallel to x_2.

In this study, all variables were considered as discrete, with the dimensions of the cross section varying in steps of one centimeter and the diameters of the reinforcement bars limited to those available in commercial stores.

The cost function to be minimized in the optimization process considers the total cost of materials (concrete and steel) and formwork, and can be expressed as follows

$$\text{Minimize } F(x) = (x_1 \cdot x_2) \cdot C_c + (4 \cdot x_3 + 2 \cdot x_4 \cdot x_5 + 2 \cdot x_6 \cdot x_7) \cdot (\pi/4) \cdot \gamma_s \cdot C_s + 2 \cdot (x_1 + x_2) \cdot C_f \tag{12}$$

The first term of the function represents the cost of concrete per unit volume (C_c), while the second represents the cost of the longitudinal reinforcement per unit mass (C_s), being γ_s the specific weight of steel. The last term represents the cost of formwork per unit area (C_f). All costs provide a relative value per unit length of the optimized element.

In the process of minimizing the cost function, all constraints imposed to the problem must be respected. Basically, the constraints are related to the strength criteria and construction requirements, as previously mentioned.

All design variables must satisfy the prescriptions of the Brazilian standard NBR 6118 (ABNT 2007) with reference to the limitations of size, spacing and steel ratio. Therefore, x_1 and x_2 are discrete variables represented by the intervals

$x_1 \in (19, 20, ..., 200)$
$x_2 \in (19, 20, ..., 1000)$

being the lower values indicated by standards and the higher values large enough so as to prevent them to interfere in the optimal solution. For the same reason, the variables x_4 and x_6 (number of steel bars in the two layers parallel to x_1 and the number of layers with two bars parallel to x_2, respectively) can take only integer values between 0 and 10.

Also, x_3, x_5 and x_7 are variables that represent the longitudinal steel bars, restricted to the following diameters (mm)

$$x_3, x_5 \text{ and } x_7 \in (10.0, 12.5, 16.0, 20.0, 22.0, 25.0, 32.0, 40.0)$$

Therefore, the constraints can be rewritten in a normalized form as follows

$$g_1 = 1 - N_{rd}/N_{sd} \leq 0 \quad (13)$$

$$g_2 = 1 - M_{rd}/M_{sd} \leq 0 \quad (14)$$

$$g_3 = 1 - b/b_{min} = 1 - x_1/19 \leq 0 \quad (15)$$

$$g_4 = 1 - b_{max}/b = 1 - 200/x_1 \leq 0 \quad (16)$$

$$g_5 = 1 - h/h_{min} = 1 - x_2/19 \leq 0 \quad (17)$$

$$g_6 = 1 - h_{max}/h = 1 - 1000/x_2 \leq 0 \quad (18)$$

$$g_7 = 1 - 5b/h = 1 - 5x_1/x_2 \leq 0 \quad (19)$$

$$g_8 = 1 - e/e_{min} \leq 0 \quad (20)$$

$$g_9 = 1 - e_{max}/e \leq 0 \quad (21)$$

$$g_{10} = 1 - \rho/\rho_{min} \leq 0 \quad (22)$$

$$g_{11} = 1 - \rho_{max}/\rho \leq 0 \quad (23)$$

In previous equations, e represents the spacing between longitudinal bars, and ρ the rate of geometric reinforcement (ratio between the areas of steel and concrete sections). Regarding the constraints, a penalty function technique was adopted, in which constrained problems are transformed into unconstrained ones by adding to the function $f(x)$ a penalty function $P(x)$, which considers a multiplying factor r applied to all the constraints that are not satisfied. Thus, the penalized function $F(x)$ can be written as

$$F(x) = f(x) + P(x) \quad (24)$$

being

$$P(x) = \sum r|g(x)| \quad (25)$$

Overall, for the simulations performed in this work, several initial solutions were utilized, resulting in the convergence to a single solution. Regarding the optimization method, it was adopted the following parameters, obtained from experiments and previous indication reported in the literature:

– initial temperature (T) = 1000
– temperature reducer (α) = 0.98
– penalty factor (r) = 1000
– stop criterion = Tk/T <0.001

5. Numerical simulations

The proposed approach, as described in previous paragraphs, was implemented from a base program previously developed by Kripka (2003) to optimize general functions with the Simulated Annealing method. The program was associated with a routine for checking the strength capacity of columns subjected to uniaxial flexural compression, using the FORTRAN programming language.

In the next paragraphs, some examples of the numerical simulations carried out in order to test the efficiency of the proposed procedure are presented and briefly discussed. Further details can be found in Bordignon (2010).

5.1 Example 1

Argolo (2000), whose analysis was based on the Brazilian standard NBR 6118 (ABNT 1980), presented an example of uniaxial flexural compression and compared the costs for a 30 × 70 cm rectangular cross section, initially dimensioned with the aid of practical iteration abacuses for three different pairs N_{sd} and M_{sd}, all on the same envelope, resulting in the same area of steel for the three situations. These sections were then optimized for the longitudinal reinforcement, with fixed cross section dimensions, by using the Genetic Algorithm (GA).

The composition of the unit costs of the materials and formwork used in this example refers to March 2000 (in Brazilian Reais, R$), with the same values used by Argolo (2000), as shown in Table 1. Likewise, the pair (N_{sd}; M_{sd}) used in this example corresponds to the second model studied by the author, since it represents a symmetrical distribution of the longitudinal reinforcement over the cross section area, similarly to the condition adopted in the present study.

From this set of conditions, two sections were generated by using the Simulated Annealing method, following the restrictions imposed by the Brazilian standard NBR 6118 (ABNT 2007): the first one with fixed dimensions of 30 ×70 cm, in which only the longitudinal reinforcement was optimized; and the second section, in which both the reinforcement and the cross section dimensions were optimized.

The cross section with fixed dimensions of 30 ×70 cm, whose reinforcement was optimized by the Simulated Annealing method, presented a cost 3.35% lower when compared to the same section

Table 1 Unit costs and acting loads for the example 1

C_c (R$/m³) 25 Mpa	C_s (R$/kg) 500 Mpa	C_f (R$/m²) -	N_{sd} (kN)	M_{sd} (kN · cm)
125.00	1.27	16.49	2142.86	37500

optimized by the Genetic Algorithm method used by Argolo (2000). Also, when the optimal section, freely generated by the Simulated Annealing, was compared to the method used by Argolo (2000), the cost reduction increased to 6.85%.

When the sections generated by the optimal Simulated Annealing method were compared to the results obtained by Argolo (2000) with the aid of practical iteration abacuses, the reduction in the total cost was 29.10% for the section with the optimized reinforcement and fixed dimensions of 30 × 70 cm, and 31.82% for the optimal freely generated section.

It should be noticed, however, that the analysis performed by Argolo (2000), both by the practical method and by the optimal one, did not take into account some construction requirements established by the Brazilian standard NBR 6118 (ABNT 2007) for reinforced concrete columns. Thus, the solutions provided by the author might not be feasible from the standpoint of the formulation used in the present study.

5.2 Example 2

In this example, Zielinski *et al.* (1995) studied a case of uniaxial flexural compression in order to determine the cross-sectional areas of concrete and steel needed to resist the applied loads. The analysis was performed according to the Canadian standard CSA CAN3-A23.3-M84.

The costs of the materials used in the example, along with the acting force and moment are presented in Table 2.

The optimum design, based on mathematical programming (MP) and using the Powell method suggested by the authors, corresponds to a rectangular cross section of 39.57 × 68.36 cm and a steel section of 27.46 cm^2.

This section has been simplified assuming the practical dimensions of 40 × 70 cm, with two reinforcement layers, each one with three steel bars of 25 mm in diameter, resulting in a total steel section of 30.00 cm^2.

Argolo (2000) compared these results to those obtained from the implementation of the Genetic Algorithm method, following the same Canadian standard. The section optimized by this method assumed values of 25 × 95 cm in cross section, with three bars of 22 mm in diameter in each one of two reinforcement layers, resulting in a steel area of 22.8 cm², meaning a reduction of 3.56% in the final cost of the section when compared to the optimal result obtained by Zielinski *et al.* (1995), and 7.34% in relation to the practical result suggested by the same authors.

The optimal section generated by the Simulated Annealing method in the present study, following the criteria prescribed by the Brazilian standard NBR 6118 (ABNT 2007), showed a decrease in cost of 20.67% when compared to the optimal section of Zielinski *et al.* (1995), 23.78% over the practical result suggested by the same authors and 17.75% in comparison to the section optimized by Argolo (2000).

Table 2 Costs and acting loads for the example 2

C_c (\$/m³) 25 MPa	C_s (\$/kg) 400 MPa	C_f (\$/m^2) -	N_{sd} (kN)	M_{sd} (kN · cm)
110.00	2.10	27.00	2460	44300

Table 3 Costs and acting loads for the example 3

C_c (\$/m³) 30 MPa	C_s (\$/kg) 400 MPa	C_f (\$/m²) -	N_{sd} (kN)	M_{sd} (kN · cm)
110.00	2.10	27.00	1780	36200

5.3 Example 3

A problem similar to the previous one and presented by the same authors was also analyzed, but with some changes in the acting loads and in the characteristic strength of the concrete, as shown in Table 3.

Just as in the previous example, the design obtained by Zielinski *et al.* (1995) corresponds to a rectangular cross section with dimensions of 31.96 × 59.36 cm and a total steel section of 25.80 cm^2.

This section has been simplified for practical reasons, assuming dimensions of 35 × 60 cm, with two reinforcement layers, each one with three bars of 25 mm in diameter, resulting in a total steel section of 30.00 cm^2, raising the final cost by 9.07%.

The cross section optimized by Argolo (2000) in this example assumed dimensions of 30 × 65 cm, reinforced by five bars of 16 mm of diameter in each one of two layers, resulting in a steel section of 20.11 cm^2 and leading to a cost reduction of 6.01%. This reduction even reaches 13.83% when compared to the practical results suggested by the authors.

For the optimal section generated by the present study, the reduction in cost is 24.12% when compared to the optimal section, and reaches 30.43% when compared to the practical results suggested by the authors. When compared to the section optimized by Argolo (2000) the cost reduction is 19.27%.

5.4 Influence of variation in the strength class of concrete

In this section, the influence of the strength class of concrete on optimized designs is evaluated.

Table 4 shows the unit costs (in Brazilian Reais, R\$) and the acting loads used in this analysis, which were taken from the same source cited in example 1.

To compare the influence of concrete strength in columns subjected to flexural compression, the optimization algorithm generated an optimal section for each class of concrete strength. The different configurations generated are shown in Fig. 2.

Table 4 Unit costs and acting loads

C_c (R\$/m³) -		C_s (R\$/kg) 500 Mpa	C_f (R\$/m²) -	N_{sd} (kN)	M_{sd} (kN · cm)
*C*20	245.00				
*C*25	260.00				
*C*30	275.00	6.56	42.36	2480	32489
*C*35	290.00				
*C*40	305.00				

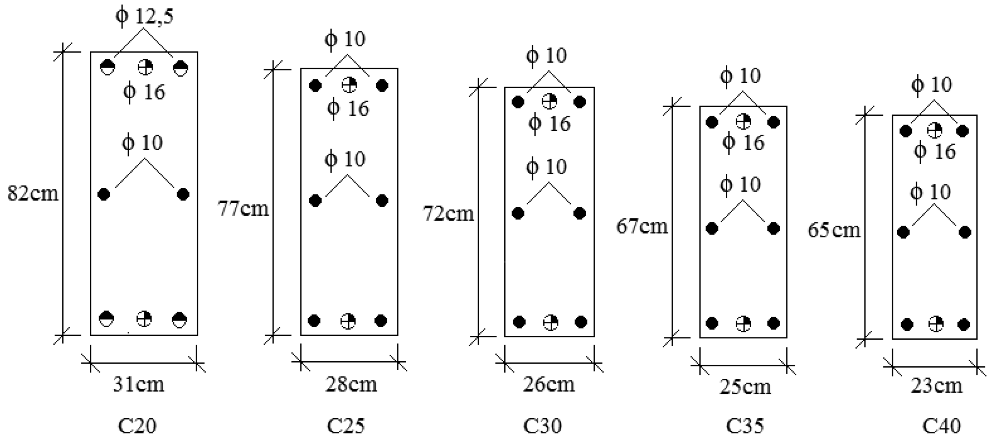

Fig. 2 Optimal sections for different classes of concrete resistance

Table 5 Results for different classes of concrete resistance

	*C*20		*C*25		*C*30		*C*35		*C*40	
	Section	(R$/m)	Section	(R$/m)	Section	(R$/m)	Section	(R$/m)	Section	(R$/m)
Concrete (cm²)	2542	62.28	2156	56.06	1872	51.48	1675	48.58	1495	45.60
Steel (cm²)	10.50	54.07	8.73	4.97	8.73	44.97	8.73	44.97	8.73	44.98
Formwork (cm)	226	95.73	210	88.96	196	83.03	184	77.94	176	74.55
ρ	0.0041		0.0041		0.0047		0.0052		0.0058	
Total cost (R$/m)	212.09		189.99		179.48		171.49		165.13	
Difference	0%		-10%		-15%		-19%		-22%	

Analyzing this figure, it becomes evident that the optimization algorithm seeks for a reduction in the consumption of concrete as their resistance and cost increases.

The results obtained in this simulation are summarized in Table 5, in which it is possible to notice that the section corresponding to the concrete class *C*40 resulted in a reduction in final cost of about 22%, when compared to the section generated for the concrete class *C*20, both subjected to the same acting loads.

5.5 Influence of variation in the magnitude of acting loads

In this example, one of the acting loads had a fixed value, while its pair varied. By adopting this procedure, the section was analyzed under uniaxial flexural compression with small and large eccentricities.

In the first analysis, the value of axial force was kept fixed at 2480 kN, while the bending moment varied from 5000 to 35000 kN · cm.

Fig. 3 shows the differences amongst the costs obtained for different pairs of acting loads and strength classes, demonstrating the efficiency of the upper classes of concrete strength when columns are subjected to flexural compression. This was due not only to the greater resistance, but also because the algorithm optimized their use. Although the unit costs were higher for the upper

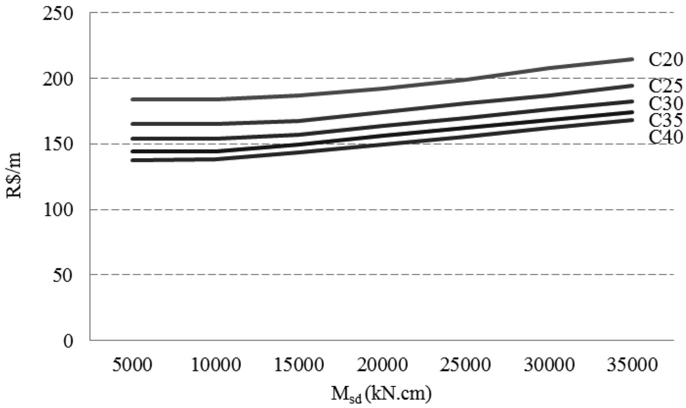

Fig. 3 Total cost of section per linear meter (constant axial force)

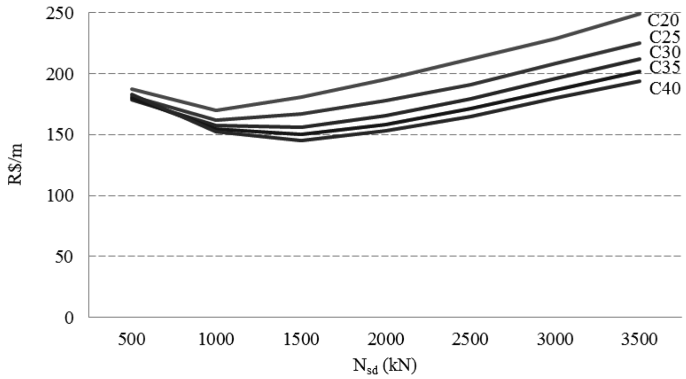

Fig. 4 Total cost of section per linear meter (constant bending moment)

classes, the optimization algorithm leads to a lower consumption of formwork, which represents a significant portion of the cost composition.

In a second analysis, the value of the bending moment was kept fixed at 32489 kN · cm, while the axial force varied from 500 kN to 3500 kN.

The results generated by the optimization algorithm lead to an increased cross-sectional area of concrete as the value of axial force increased. This behavior was characteristic of sections using concrete classes *C*20 and *C*25.

For the other classes of concrete strength, the cross-sectional area tended to decrease, reaching a minimum when the pair was formed by a bending moment of 32489 kN · cm and an axial force of 1500 kN, since at this point there was also a reduction in steel consumption and formwork. The consumption of materials increased again as the bending moment increased.

In Fig. 4, it is clearly shown that there is an optimal combination of loads acting in the section subjected to flexural compression, which does not involve the lower values of axial force and bending moment. This behavior can be explained by the fact that at this optimal point all materials are being used at their full capacity.

However, the achievement of an actual optimal combination of acting loads, or close to that, depends on several factors that influence the structural analysis, the principal being the freedom in structure conception.

5.6 Analysis of variation in the unit costs of materials

In this section, the influence of the variation in the unit costs of materials on the final cost of the optimal section is evaluated. As a reference, two optimal sections were generated, both capable of resisting the acting loads and using the materials and respective unit costs presented in Table 1. In one section, the concrete used is of strength class *C*20 and in the other, the concrete is class *C*40.

Based on the reference sections, the cost of each individual material was either increased or decreased by 20% and 50%, generating four new optimal sections, with different final costs.

5.6.1 Analysis of variation in the unit cost of concrete

The simulation using concrete class *C*20, which had its unit cost either increased or decreased by 20%, resulted in an increase in the final cost of optimal section of 5.9% and a decrease of 5.7%, respectively, while the 50% variation caused a 14.7% change in the cost of optimal section.

When the same analysis was performed for optimal sections generated using concrete class *C*40, the results demonstrated an increase in final cost of 5.2% and 12.9%, as the unit cost is increased by 20% and 50%, respectively. When the unit cost is decreased in the same proportions, the final cost of optimal section is reduced by 5.5% and 13.9%, respectively.

In relation to the consumption of materials in optimal sections generated for both classes of concrete, it was remarkable the capacity of the optimization algorithm in reducing the concrete area as the unit cost of concrete increased.

The variation in the unit cost of concrete also caused a decrease in the steel section area for the concrete class *C*20 and a 50% unit cost increase, since the amount of steel is forced to the minimum rate stipulated by the standard. In the sections using concrete class *C*40, the area of steel remained constant, leading to higher reinforcement rates as the concrete area decreased.

In both cases, the perimeter of the section tended to increase as the algorithm searched for reducing the cross-sectional area. As a consequence, the relative cost of concrete increased. This behavior was more evident when the unit cost of concrete was increased by 50%, as the algorithm tried to maintain the resistant capacity of the section with a minimum area of concrete, leading to sections of more rectangular shapes.

5.6.2 Analysis of variation in the unit cost of reinforcement

Just as in the previous analysis, the unit cost of reinforcement were either increased or decreased by 20% and 50%, and compared to reference generated sections using concrete classes *C*20 and *C*40.

In optimal sections using concrete class *C*20, the final cost increased 5.1% due to a 20% increase in the unit cost of reinforcement, and 12.7% for a unit cost increase of 50%. This last value is very close to the 12.37% increase observed by Argolo (2000) in similar comparisons using the Genetic Algorithm.

When the analysis was performed using concrete class *C*40, the results demonstrated an increase in final costs of 5.4% and 13.6%, as the unit cost of the reinforcement was increased by 20% and 50%, respectively. When the unit cost was reduced in the same proportions, final costs reduction was found to be numerically the same as for the unit cost increase, i.e., 5.4% and 13.6%, respectively.

Regarding the consumption of materials, it should be noticed that an increase of 6.47% in the steel section, as well as a reduction of 2.05% in the concrete area occurred for the reference section using concrete class *C*20 and for a 50% reduction in the unit cost of reinforcement. For the other reference sections, material consumption remained constant.

In sections with concrete class *C*40, the consumption of this material increased as the unit cost of reinforcement took higher values, reducing the formwork perimeter and keeping the area of steel unchanged for all optimal sections. This occurred because there were no other possible combinations of longitudinal reinforcement bars capable of reducing the final cost of the optimal section, compensating the higher cost of reinforcement with a reduction in formwork perimeter.

5.6.3 Analysis of the variation in the unit cost of formwork

Sections that use concrete class *C*20 with either an increase or a decrease in formwork unit cost of 20%, presented final costs with a corresponding absolute variation of 9.0%. When the unit cost variation was reduced by 50%, the reduction in final cost reached 24.2%, while an increase of 50% caused an increase of 22.2% in the final cost of the optimal section.

In sections with concrete class *C*40, the variation in unit cost of formwork caused similar changes in the final cost of the optimal section when compared to the concrete class *C*20. The results showed increases of 9.0% and 22.7%, as the unit cost of the formwork is increased by 20% and 50%, respectively. When the unit cost is reduced in the same proportions, the final costs of the optimal sections decreased 9.4% and 23.6%, respectively.

As the unit cost of formwork increases, the optimization algorithm seeks to an optimal cross-sectional shape that results in the smallest possible perimeter, even if this cross section presents a greater area of concrete and, in some cases, forces a greater consumption of steel to satisfy the minimum rate of reinforcement. This behavior was also observed for the sections using concrete classes *C*20 and *C*40 and could be easily justified as the portion represented by formwork is the biggest contributor to the final cost of the section.

In general, it was noticed that 20% of variation in the unit costs of materials changed just a little the consumption of materials and the dimensions of the optimal cross section. This behavior is modified when the variations in unit costs are of 50%, which is more evident for the sections using concrete class *C*20.

It should be emphasized that the simulation results discussed herein are valid for the materials and acting loads analyzed in the present work, being altered in the case of new combinations of acting loads and materials characteristics.

6. Conclusions

This work dealt with the problem of optimization of rectangular reinforced concrete columns subjected to uniaxial flexural compression, following the requirements of the Brazilian standard NBR 6118 (ABNT 2007), and using the Simulated Annealing optimization method.

Based on the results obtained from the implemented formulation and comparisons performed, it was possible to conclude that:

- the optimization process implemented showed the ability of assisting in decision making, also eliminating some of the uncertainties in determining the parameters that lead to more efficient and economic designs;
- the Simulated Annealing method was efficient, especially in the treatment of the constraints imposed to the problem;
- when compared to the practical dimensioning performed with the aid of iteration abacuses and other optimization techniques, the Simulated Annealing method showed superior efficiency in the search for lower cost cross sections;

- the possibility of using several diameters of reinforcement bars in the column cross section contributed to the reduction of final costs;
- the optimum cross sections proposed by the Simulated Annealing method are feasible for possible practical implementation;
- optimal sections subjected to uniaxial flexural compression were found to be those using the higher concrete strength classes; sections using lower strength classes presented high costs due to the increased demand for materials;
- the formwork appeared as the major contributor to the final cost of the optimized sections, approaching 45.6% of the final cost, followed by concrete, with 28.7%, and steel, with 25.7%;
- due to the high cost of the reinforcement, the optimal sections tended to present low rates of reinforcement; in most sections, these rates approached the minimum stipulated by the Brazilian standard NBR 6118 (ABNT 2007);
- the topology of the optimal cross section was only slightly affected by the variation of 20% in the unit costs of materials, but major changes were found for costs variation of 50%;
- there is an optimal combination of acting loads, which along with the optimal dimensions of the cross section, leads to optimal global structures, although these optimal values depend on several factors, including structural conception.

References

Araújo, J.A. (2003), "Reinforced concrete", v.3, Dunas, Rio Grande. (in Portuguese).

Argolo, W.P. (2000), "Optimal design of reinforced concrete section subjected to axial load and moment using Genetic Algorithms", Dissertation (Master in Engineering) – University Federal of Rio de Janeiro, Rio de Janeiro. (in Portuguese).

Brazilian Association of Technical Standards (2007), "Procedures for the design of reinforced concrete structures", NBR 6118, Rio de Janeiro. (in Portuguese).

Bordignon, R. (2010), "Optimization of reinforced concrete rectangular columns subjected to uniaxial flexural compression", Dissertation (Master in Engineering) – University of Passo Fundo, Passo Fundo. (in Portuguese).

Kirkpatrick, S., Gelatt, C.D. and Vecchi, M.P. (1983), "Optimization by simulated annealing", *Sci.*, **220**(4598), 671-680.

Kripka, M. (2003), "Cost optimization of reinforced concrete structures by the grid method", *V Symposium on Concrete Struct.* University of São Paulo, São Paulo. (in Portuguese).

Martínez Martin, F. J. (2007) "Heuristic optimization of rectangular reinforced concrete columns", Thesis (Ph.D. in Engineering) – Polytechnic University of Valencia, Valencia. (in Spanish)

Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H. and Teller, E. (1953), "Equation of state calculations by fast computing machines", *J. Chem. Phys.*, **21**(6), 1087-1092.

Montoya, P.J., Meseguer, A.G. and Cabré, F.M. (1994), "Reinforced concrete", **2**(13). Ed. Gustavo Gili, Barcelona. (in Spanish).

Rodrigues Júnior, S.J. (2005), "Optimization of reinforced concrete columns in tall buildings", Thesis (Doctor in Engineering) – Pontifical Catholic University of Rio de Janeiro, Rio de Janeiro. (in Portuguese).

Vanderplaats, G.N. (1984), "Numerical optimization techniques for engineering design", McGraw-Hill Book Company.

Zieliski, Z.A., Long, W. and Troitsky, M.S. (1995), "Designing reinforced concrete short-tied columns using the optimization technique", *ACI Struct. J.*, **92**(5), 619-625.

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