Computers and Concrete, Vol. 8, No. 5 (2011) 525-539
DOI: http://dx.doi.org/10.12989/cac.2011.8.5.525

Fiber reinforced concrete properties - a multiscale approach

Erez Gal* and Roman Kryvoruk

Department of Structural Engineering, Ben-Gurion University, Beer-Sheva, Israel

(Received October 6, 2009, Revised August 22, 2010, Accepted October 20, 2010)

Abstract. This paper describes the development of a fiber reinforced concrete (FRC) unit cell for analyzing concrete structures by executing a multiscale analysis procedure using the theory of homogenization. This was achieved through solving a periodic unit cell problem of the material in order to evaluate its macroscopic properties. Our research describes the creation of an FRC unit cell through the use of concrete paste generic information e.g. the percentage of aggregates, their distribution, and the percentage of fibers in the concrete. The algorithm presented manipulates the percentage and distribution of these aggregates along with fiber weight to create a finite element unit cell model of the FRC which can be used in a multiscale analysis of concrete structures.

Keywords: FRC-fibered reinforced concrete; multiscale analysis; concrete unit cell; elastic properties; mesoscale concrete finite element model.

1. Introduction

Modeling the behavior of concrete structures is a challenging complicated task. Usually the mechanical behavior of concrete and FRC is macroscopically modeled via plastic constitutive relations (e.g. Drucker and Prager 1952, Riedel *et al.* 1999, Thomee *et al.* 2006). These macroscopic models are characterized by the large number of parameters needing calibration in order to analyze the complex behavior of the concrete at different stages of loading and at different damage modes. Using these macroscopic models becomes even more complicated due to the fact that concrete has no single distinctive microstructure, rather a variety of microstructures. This variety includes examples such as: the addition of fibers composed of different materials; variations in aggregate sizes, shapes, and types; and the water-to-cement ratio. The use of multiscale analysis evidently is an appropriate way to model the behavior of concrete structures by coupling between the concrete microstructures and the macroscopic properties needed to analyze concrete structure (e.g. Markovic and Ibrahimbegovic 2004, Ibrahimbegovic and Markovic 2003, Gitman *et al.* 2006, 2007, 2008, Kouznetsova *et al.* 2001, 2002, Feyel 2003, Gutierrez 2004, Ghosh *et al.* 2001, Nadeau 2003, Lee *et al.* 2009, Pichler *et al.* 2007, Mang *et al.* 2003, 2009 de Borst *et al.* 1999, Füssl *et al.* 2008, Asferg *et al.* 2007, Oliver 1996, Jirasek 2000, Wells and Sluys 2001, Moes and Belytschko 2002, de Borst 2002, Meschke and Dumstorff 2007, Simone and Sluys 2004, Wriggers and Moftah 2006, Haffner *et al.* 2006, Wang *et al.* 1999, Cusatis and Cedolin 2007, Shin *et al.* 2008).

The method for obtaining the macroscopic behavior of concrete based on its microstructure is

* Corresponding author, Ph.D., E-mail: erezgal@bgu.ac.il

referred to as *the theory of homogenization*, by which heterogeneous material is replaced by an equivalent homogeneous continuum. The method is performed on a statistically representative sample of material, referred to as a representative volume element or a material unit cell. Numerous theories have been developed to predict the behavior of composite materials starting with the various effective medium properties obtained by the models of Eshelby 1957, Hashin 1962, Mori and Tanaka 1973, self-consistent approaches of Hill 1965, and various mathematical homogenization methods (e.g. Christensen 1979) pioneered by Bensoussan 1978 and Sanchez-Palencia 1980. Unfortunately, most of these analytical models can only give estimates or boundaries for the macroscopic properties, and the simplifying assumptions used bring about the major differences in the results obtained. Computational procedures for executing homogenization have been an active area of research starting with the contribution by Guedes and Kikuchi 1990 for linear elasticity problems. Over the past decade, major contributions have been made to extend the theory of computational homogenization to non-linear regimes (e.g. Terada and Kikuchi 1995, Fish *et al.* 1997, Fish and Shek 1999, Fish and Yu 2001) and to improve the fidelity and computational efficiency of numerical simulations (Terada and Kikuchi 2001, Matsui *et al.* 2004, Aboudi 1991, Aboudi *et al.* 2003, Aboudi 2003, Smit *et al.* 1998, Miehe and Koch 2002, Kouznetsova *et al.* 2001, Feyel and Chaboche 2000, Ghosh *et al.* 1995, 1996, Michel *et al.* 1999, Geers *et al.* 2001, McVeigh *et al.* 2006). This research has established the Finite Element Method (FEM) as one of the most efficient numerical methods, whereby macroscopic responses can be obtained by volumetrically averaging numerical solutions of unit cells (Zohdi and Wriggers 2001,2005).

Aggregate size distribution plays an essential role in concrete mix design and optimization. Proper selection of aggregate size distribution affects the main properties of concrete such as workability of the concrete mix, mechanical strength, permeability, and durability. The size distribution of aggregate particles can be described either by means of grading curves such as the Fuller curve or obtained from sieve analysis (e.g. Wriggers and Moftah 2006, Haffner *et al.* 2006, Wang *et al.* 1999, Cusatis and Cedolin 2007, Gal *et al.* 2008).

FRC is a concrete made of cement, aggregates, and uniformly scattered fibers. Experimental studies estimate the most efficient fiber materials are steel, glass, and organic fibers. In structural engineering, steel fiber reinforced concrete (SFRC) is manufactured by adding steel fibers in the mixing process. SFRC has proved its efficiency with respect to a reduction in shrinkage and cracking, in addition to increases in strength and life cycle (see Romualdi and Batson 1963, Shah and Rangan 1971, Johnston 1974, Swamy 1975, Ramadoss and Nagamani 2008). The analysis and design of FRC structures requires the evaluation of the FRC material properties. Usually empirical approaches based on experimental studies are used to evaluate these properties, whereas purely computational methods are used on a very limited basis. Empirical formulations to evaluate the elastic properties of concrete have been suggested by (Tan *et al.* 1994, Ashour *et al.* 2000, Ezeldin and Balagaru 1992, Mansur *et al.* 1999, Ahmad and Lagoudas 1991, and Teng *et al.* 2004).

In this paper, we suggest a multiscale approach based on the homogenization method to evaluate the elastic properties of FRC. The generation of the random geometrical configurations of the aggregate particles and fibers must satisfy the basic statistical characteristics of the real material. In addition, the spatial distribution of the aggregate particles and the fibers must be as macroscopically homogeneous in space and as macroscopically isotropic as possible. In order to produce the geometrical configuration which meets these requirements, the random sampling principle of Monte Carlo's simulation method is used. This principle is applied by taking samples of aggregate particles and fibers from a source whose size distribution follows a certain given grading curve and placing the aggregate particles one by one into the unit cell in such a way that there is no overlapping with particles or fibers already in place. The developed algorithm presented creates a FRC unit cell using concrete paste generic information e.g. the percentage of aggregates, the aggregate distribution, and the percentage of fibers. The algorithm manipulates the percentage of aggregates, fiber weight, and distribution in order to create a finite element unit cell model to be used in a multiscale analysis. Generally, this algorithm adjusts the finite element meshing with respect to the physical unit cell size, creates virtual sieves according to adjusted probability density functions, and transforms the aggregate volumes into a digitized discrete model of spheres. It further transforms the fiber volumes into a digitized discrete model of cylinders, places the spheres and cylinders by using the random sampling principle of Monte Carlo's simulation method in a periodic manner, and creates a finite element model of the FRC unit cell. Finally, the algorithm evaluates the FRC macroscopic material properties using the theory of homogenization.

As this research is focused on the mesoscale of the concrete, it assumes that the cement paste is a homogenized material, since its overreaching goal is toward scaling up to the structural level rather than scaling down to the cement paste level. However, it is important to note that scaling down to the cement paste scale exposes hydration processes which depend on the water-to-cement ratio affecting the mechanical properties of the mortar as the microscopic phases are Clinker, Water, Hydrates, and Air (e.g. see Ulm and Jennings 2008, DeJong and Ulm 2007, Ulm *et al.* 2010a, Ulm *et al.* 2003, Sanahuja *et al.* 2007, Hung *et al.* 2008, Garboczi and Berryman 2000, 2001, Garboczi and Bullard 2000, Bullard and Garboczi 2006, Garboczi *et al.* 2004, 2006, Garboczi and Day 1995, Bentz *et al.* 1998, Constantinides and Ulm 2004, Haecker *et al.* 2005, Sun *et al.* 2007, Smilauer and Bazant 2010, Smilauer and Krejci 2009, Smilauer and Bittnar 2006).

The variation of aggregate shape will not be addressed in this paper as it has been found to have a negligible effect as far as elastic properties are concerned (e.g. see Ulm *et al.* 2010a, Ulm and Jennings 2008, He *et al.* 2009).

The outline of this paper is as follows: Section 2 describes the homogenization method, Section 3 the unit cell generation, Section 4 contains validation and verification of the suggested formulation, and the final section summarizes and concludes our research.

2. Macroscopic material properties of FRC

Past research has tended to focus on the experimental performance of fiber reinforced concrete. However, the calculation of effective elastic modulus of fiber reinforced concrete using an analytical method and an empirical formula has rarely been investigated (Teng *et al.* 2004). As mentioned by He *et al.* 2009, the elastic properties of concrete play a crucial role in structural design and strength analysis. Many studies have shown that mesoscale models are suitable for studying the global elastic properties of concrete; Young's modulus and Poisson's ratio are two crucial ones. Thus, prediction of these elastic modulus has practical significance. To approach this problem, analytical models as well as numerical ones have been developed. Spherically-shaped aggregates are normally considered in modeling approaches because of reduced complications. It is also known that for accurate prediction, all components of concrete should be taken into consideration, i.e. aggregate, matrix, Interfacial Transition Zone (ITZ), and fibers. As this research deals only with the elastic macroscopic properties of FRC, the effect of the ITZ has not been included, since its influence on these properties is minor (see He *et al.* 2009). We do intend to include it in future work.

In our study, macroscopic material properties of the concrete were evaluated using the theory of homogenization. This theory couples the microscopic (unit cell) problem which describes the microscopic material structure and the macroscopic finite element model of the structure by transferring the macroscopic material properties obtained by the solution of the microscale problem to the macroscopic one. In the next section of this paper, a short description of the theory of homogenization is included.

2.1 Asymptotic theory of homogenization

The macroscopic problem was formulated using the following boundary volume problem

$$\begin{aligned} &\text{find } u(x) \text{ on } \Omega \text{ such that} \\ &\bar{L}_{ijlm}\bar{\varepsilon}_{lm,x_j} + \bar{b}_i = 0 \quad \text{on} \quad \Omega \\ &u_i = \bar{u}_i \quad \text{on} \quad \Gamma_u; \ \bar{\sigma}_{ij} n_j = \bar{t}_i \quad \text{on} \quad \Gamma_t \end{aligned} \tag{1}$$

where $\mathbf{x}$ is the macro scale position vector, Ω is the macroscopic domain, and u is the macro scale displacement; $\bar{b}$ is the average unit cell body force, $\bar{L}$ is the macroscopic effective material properties, $\bar{u}_i$ on Γ_u are the essential boundary conditions, $\bar{\sigma}_{ij} n_j = \bar{t}_i$ on Γ_t are the natural boundary conditions with normal n_j, $\bar{\sigma}_{ij}$ represents the macro scale stress components and the macro scale strain components are

$$\bar{\varepsilon}_{mn} = u_{(m,x_m)} = \frac{1}{2}\left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m}\right) \tag{2}$$

Summation convention was employed for repeated indices.

The microscopic problem with periodic boundary conditions was formulated using the following boundary volume problem

$$\begin{aligned} &\text{find } \chi_{imn}(y) \text{ on } \Theta \text{ such that} \\ &[L_{ijlm}\chi_{(k,y_l)mn} + I_{klmn}]_{,y_j} = 0 \ \text{ on } \ \Theta \\ &\chi_{imn}(y) = \chi_{imn}(y+Y) \quad \text{on} \quad \partial\Theta \\ &\chi_{imn}(y) = 0 \quad \text{on} \quad \partial\Theta_{ver} \end{aligned} \tag{3}$$

where $\boldsymbol{y} = \boldsymbol{x}/\zeta$ are the micro scale position vectors as $0 < \zeta << 1$; χ are the micro scale influence functions; Θ is the unit cell domain of size Y, $\partial\Theta$ represents the unit cell boundaries, $\partial\Theta_{ver}$ represents the unit cell vertices, and I_{klmn} is

$$I_{klmn} = (\delta_{mk}\delta_{nl} + \delta_{nk}\delta_{ml})/2 \tag{4}$$

where δ_{mk} is the Kronecker delta.

The homogenized constitutive tensor components which represent the macroscopic material properties is given by

$$\bar{L}_{ijlm} = \frac{1}{|\Theta|}\int_{\Theta} \sigma_{ij}^{mn} d\Theta \tag{5}$$

where σ_{ij}^{mn} are stress influenced functions induced by applying an overall unit strain $\bar{\varepsilon}_{mn}$ defined as

$$\sigma_{ij}^{mn} = \bar{L}_{ijkl}(\chi_{(k,y_j)mn} + I_{klmn}) \tag{6}$$

Solution of the unit cell problem using the finite element method was obtained by resolving the unit cell problem to multiple RHS vectors (six in a 3-D case due to symmetry of indices *mn*). In the matrix implementation, $\bar{L}_{ijkl}$ is a 6×6 matrix where $\underline{ij}$ represents six rows and *mn* six columns. Each column in $\bar{L}_{ijkl}$ can be extracted by multiplying $\bar{L}_{ijkl}$ with a unit overall strain, $\bar{\varepsilon}_{mn} = 1$. For implementation in a commercial package, it is convenient to select $\bar{\varepsilon}_{mn}$ in the form of a unit thermal strain. However, due to the idealization of the homogenized behavior of concrete as being statistically isotropic, (Wrigges and Moftah 2006), one loading state is sufficient for describing its overall linear elastic behavior. Thus, the effective bulk and shear modulus are given by

$$3\kappa = \frac{tr(\boldsymbol{\sigma} \div 3)}{tr(\boldsymbol{\varepsilon} \div 3)} \quad \text{and} \quad 2\mu = \sqrt{\frac{\boldsymbol{\sigma}_D : \boldsymbol{\sigma}_D}{\boldsymbol{\varepsilon}_D : \boldsymbol{\varepsilon}_D}} \tag{7}$$

where κ is the effective bulk modulus, μ is the effective shear modulus and

$$\boldsymbol{\sigma}_D = \boldsymbol{\sigma} - tr(\boldsymbol{\sigma} \div 3)\mathbf{I} \quad \text{and} \quad \boldsymbol{\varepsilon}_D = \boldsymbol{\varepsilon} - tr(\boldsymbol{\varepsilon} \div 3)\mathbf{I} \tag{8}$$

are the deviatory parts of $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$, respectively.

3. Unit cell generation

The suggested framework executing the multiscale analysis of FRC structures by incorporating an original FRC unit cell generator into a commercial software package (e.g. ABAQUS).

3.1 Aggregate distribution

Essential information needed for creating a FRC unit cell is the aggregate distribution. This information is used to monitor the size and amount of the different aggregates within the concrete. In the suggested framework, there are three different ways to supply this information: 1. the Fuller Curve; 2. Sieve analysis; 3. direct access. The following section describes the developed algorithm which manipulates the generic given information in order to create a finite element model of the concrete unit cell.

3.1.1 Fuller curve

In this study, the Fuller curve was used to provide the aggregate distribution such that the distribution was optimal in terms of concrete strength. The curve provided an aggregate distribution function which corresponded to the German standard between the curves A32 to B32 as shown in Fig. 1. This curve enabled the performance of *virtual sieve* analysis using the following eq.

$$P(d) = 100 \cdot \left(\frac{d}{D_{max}}\right)^n \tag{9}$$

Where d is the virtual sieve diameter, D_{max} is the maximum aggregate size, $P(d)$ is the percentage of aggregates which passed through the virtual sieve d, and n is a constant (0.5 in our case). Using this method, we calculated the percentage of aggregates which passed through the virtual sieve of diameter d.

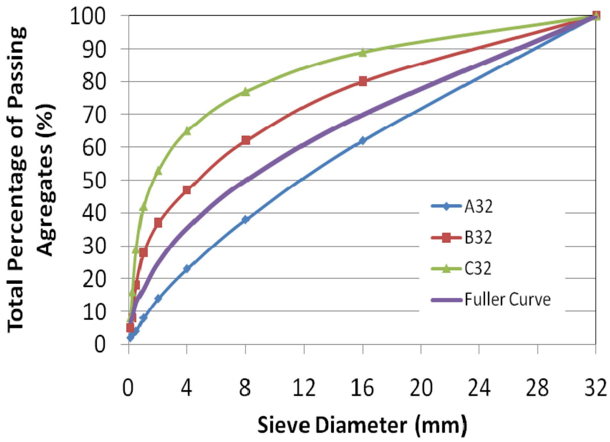

Fig. 1 The fuller curve

3.1.2 Sieve analysis

The aggregate distribution analysis was performed in the laboratory using sieves of different diameters. The bunch of aggregates was passed through a set of sieves which filtered them according to the size of the holes in the sieve. By the end of the process, the aggregates that remained on each sieve were larger or equal in diameter to the size of the holes in that sieve. From the amount of aggregates measured on each sieve, it was possible to evaluate the percentage of aggregates which passed through the sieve of diameter *d*.

Usually the information resulting from this process is not detailed enough to create a well defined unit cell due to the need to transfer these volumes into a set of spheres. Therefore, the differences between the sieves have to be small enough to obtain the required detailed information. For that purpose, we added virtual sieves in order to get the appropriate refinement. The aggregates that remained on sieve *n* had varieties of diameters between *n* to *n*+1. The higher the refinement achieved, the higher the packing factor we obtained. The starting point was the creation of a volumetric density function describing the percentage of aggregates remaining on the virtual sieve within a specific range. The lower limit of the range (of aggregate diameter) was the size of the sieve while the upper limit was the size of the next sieve. Each range had its own distribution function. The assumption was that the distribution function, with the symbolic P_s, is a linear function. Therefore, a probability function was constructed by dividing this function by its integral over the whole range [DA÷DB] as follows

$$\hat{P}_s(d) = \frac{P_s(d)}{\int_{DA}^{DB} P_s \mathrm{d}d} \tag{10}$$

The next step was the conversion of the computed volumes into discrete spheres. For that purpose the number of spheres for a certain diameter *Ns* was computed as follows

$$N_s = \frac{V_s + V_r}{V_B} \tag{11}$$

N_s represents the number of spheres on the specific sieve, V_s the volume of the aggregates on that sieve, V_B the volume of a single aggregate, and V_r is the volume of aggregates left out in the previous sieve (which was insufficient to create a single sphere in the upper level). The volume V_s was calculated as: $V_s = P_s \times P \times V$, where P_s is the percentage of aggregates left on the sieve, P is the percentage of aggregate in the concrete, and V is the actual volume of the unit cell. $V_B = \frac{4\pi \times R^3}{3}$ as R is the radius of the aggregate.

After obtaining the detailed information on the number of spheres for each diameter, it was necessary to place them in the unit cell. This step involved the creation of a three-dimensional image of digitized spheres representing the actual aggregate distribution. For this study, a computational volume of 100×100×100, 70×70×70 or 50×50×50 pixels was typically employed. The spherical aggregates obtained from the measured aggregate distribution were placed into the computational volume from largest to smallest in diameter, so that none of the aggregate spheres overlapped. Periodic boundaries were used to eliminate edge effects. If a portion of a particle extended beyond one or more faces of the 3-D box, the remainder of its volume protruded into the opposite face.

3.2 Fiber distribution

The fiber distribution was performed using cylinders of different diameters and lengths. Each cylinder was located according to the following algorithm. First, the cylinder axis was represented as shown in Fig. 2 (Line *AB*) where its direction was derived arbitrarily using random point *B* (i.e. point *A* defined the location of the cylinder while point *B* defined its orientation). The length, *L*, and diameter, 2*R*, of each cylinder was given as an input where point *A* was located (as shown in Fig. 2) at the left hand side of the cylinder. We could check whether each cell within the unit cell was located in the cylinder volume by plotting the cylinder's axis direction, length, and diameter. Our distribution algorithm randomly picked up point *C* and checked if it belonged to this cylinder. For that purpose, we executed the following algorithm:

1. Compute the length of line *AB*:

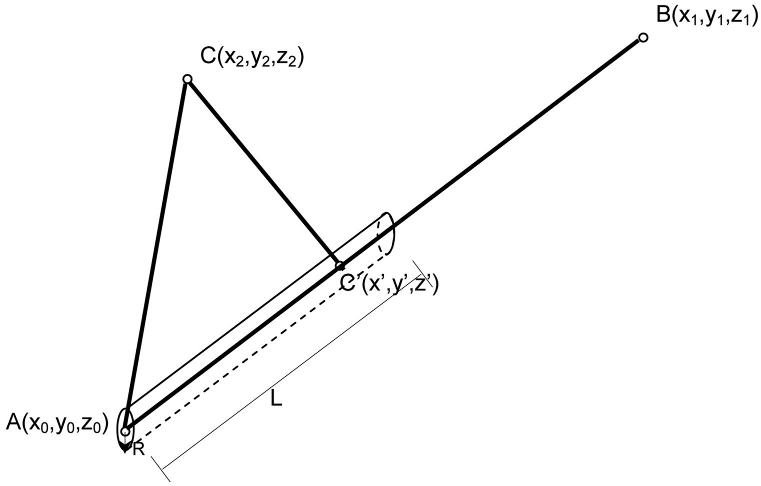

Fig. 2 The fiber orientation

$$|AB| = \sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$$

2. Compute the projection of line AC on line AB using the following scalar product

$$|AC'| = (x_2-x_0)\cdot\frac{x_1-x_0}{|AB|}+(y_2-y_0)\cdot\frac{y_1-y_0}{|AB|}+(z_2-z_0)\cdot\frac{z_1-z_0}{|AB|}$$

3. If $0<|AC'|\le L$

and $\left(x_2-|AC'|\cdot\frac{x_1-x_0}{|AB|}-x_0\right)^2+\left(y_2-|AC'|\cdot\frac{y_1-y_0}{|AB|}-y_0\right)^2+\left(z_2-|AC'|\cdot\frac{z_1-z_0}{|AB|}-z_0\right)^2\le R^2$

then the checked point C is placed within the cylinder representing fiber domain where R and L are the radius and the length of the cylinder.

4. Verification and validation

The FRC-developed macroscopic properties were determined for the unit cells generated according to the presented algorithm. Two types of FRC formulation were tested: in the first the aggregate distribution was obtained using sieve analysis, while in the second one the concrete was formulated as homogenized material. The following uniform strains

$$\varepsilon_0 = \{1\ 0\ 0\ 0\ 0\ 0\}^T$$

were applied to all the unit cell elements together with periodic boundary conditions and according to the theory of homogenization presented in Section 2, the macroscopic properties were calculated. The results obtained from the presented finite element model were compared with experimental results found in the literature. A comparison with experimental studies for the case of plain concrete can be found in (Gal *et al*. 2008).

In the next section, we compare the results obtained by the developed algorithm to the experimental results by (Williamson 1974) and the empirical results by (Teng *et al*. 2004) which correspond well with the same experimental results.

For the first case, we used the material properties given by (Williamson 1974) as presented in Table 1.

Table 2 clearly shows that the proposed numerical simulation is in excellent agreement with the experimental results obtained by (Williamson 1974) for elastic properties of FRC.

For the second case, we used the material properties and experimental data given by (Stock *et al*. 1979) for plain concrete, as presented in Table 3, and added different percentages of steel fibers.

As shown in (Gal *et al.* 2008), the finite element analyses of unit cells based on the Fuller Curve generally give good results in comparison with experimental ones in the case of plain

Table 1 Material properties (Williamson)

Material property	Concrete	Steel fibers
E (GPa)	20.802	200
ν	0.2801	0.30

Table 2 Elastic modulus of FRC

Volume fraction of steel fibers (%)	Current research, E (GPa)	Experimental results E (GPa)	% Discrepancy
1	21.22355	21.5	1.2
1.5	21.47353	21.75	1.27
2.5	22.15326	22.5	1.54

Table 3 Material data of constituents–Fuller Curve

Material property	Mortar material	Aggregate material	fiber material
E (GPa)	11.6	74.5	200
ν	0.20	0.20	0.30

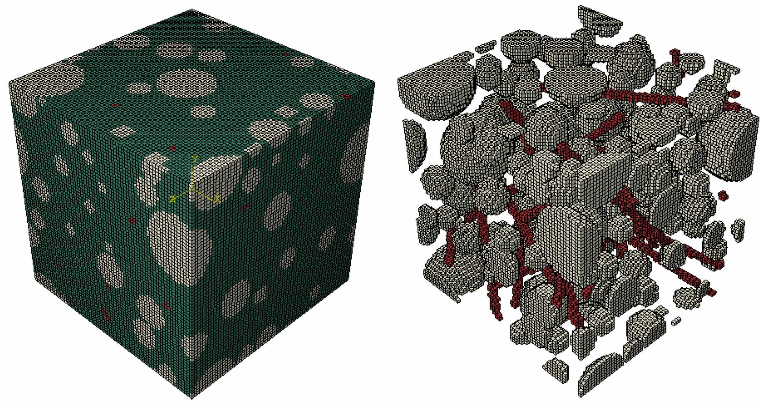

Fig. 3 The unit cell finite element model and the aggregate and fiber distribution

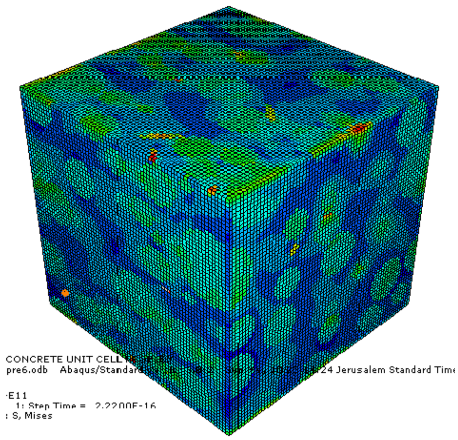

Fig. 4 The stress influence function

concrete.

Added were different volumes of steel fibers having the material properties given in Table 3. The finite element generated using the presented algorithm for the case of 20% aggregate volume is shown in Fig. 3, while the von-Misses stress influence function used to obtain the macroscopic properties is given in Fig. 4. In Fig. 4, the red color represents high values of stress which developed in the fibers, the blue color represents low values of stress which developed in the mortar, while the green color represents moderate stress values which developed in the aggregates.

We performed another set of analyses by presenting the FRC using two phases only: plain concrete and fibers. This was done for the plain concrete having a modulus of elasticity of 25, 30, and 35 MPa as shown in Fig. 5 for the case of 2% volume of fibers.

Table 4 clearly shows that the numerical simulation is in excellent agreement with the results obtained by (Teng *et al*. 2004). In addition, the finite element analysis results compare well with the classical boundaries of (Reuss 1929) and (Voigt 1889) which give theoretical ranges for the effective elastic modulus with respect to a certain volume fraction of the constituents.

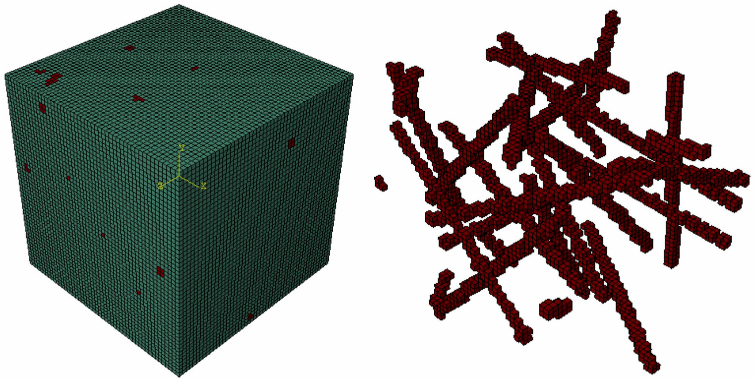

Fig. 5 The unit cell finite element model and the fiber distribution

Table 4 Elastic modulus of FRC

Plain concrete elastic modulus	Fiber volume fraction (%)	Current research E_{fc} (GPa)	Tang *et al.* E_{fc} (GPa)
25	2	26.161729	26.18243
	1.824	26.043681	26.0771
	1	25.578454	25.58725
	0.8912	25.350193	25.52297
	0.3752	25.245406	25.21942
30	2	31.252063	31.31443
	1.824	31.126376	31.19744
	1	30.619859	30.65313
	0.8912	30.3957	30.58167
	0.3752	30.262196	30.24411
35	2	36.32671	36.42241
	1.824	36.19499	36.29592
	0.956	35.853306	35.67582
	0.8912	35.435532	35.62977
	0.3752	35.275739	35.26435

5. Conclusions

This paper has outlined the development of a FRC unit cell used in the multiscale analysis of FRC structures based on the theory of homogenization.

The main advantages of the suggested finite element unit cell with comparison to other homogenization methods are: 1. it is adaptable to all types of microscopic gradient; 2. it is not mandatory to assume that concrete is statistically isotropic in the presented methodology; 3. its extension to more geometrical shape inclusions is straightforward; and 4. its extension to macroscopic anisotropic damage analysis is also straightforward.

The creation of the FRC unit cell finite element model needs only the following input data of the concrete paste: the percentage of aggregates in the concrete, the aggregate distribution, and the percentage of fibers. The suggested algorithm adjusts the finite element meshing with respect to the physical unit cell size, creates virtual sieves according to adjusted probability density functions, and transforms the aggregate and fiber volumes into a digitized discrete model of spheres and cylinders. It then places the spheres and the cylinders using the random sampling principle of Monte Carlo's simulation method in a periodic manner, and evaluates the macroscopic material properties using the theory of homogenization.

The results, obtained using the presented algorithm, are in very good agreement with experimental results. This outcome leads to the conclusion that fully elastic unit cell analysis is appropriate to evaluate the elastic properties of FRC.

It is important to note that scaling down to the cement paste scale explored the hydration processes which affect the mechanical properties of the mortar. The presented research focused *just* on the mesoscale of the concrete (i.e. aggregates, fibers, and mortar) since its overreaching goal was toward scaling up to the structural level rather than scaling down to the cement paste level, and therefore, it assumes here that the cement paste is a homogenized material.

In future works, a non-linear reduced order multiscale analysis will be developed based on the generated unit cell to enable damage analysis of concrete and FRC structures and through microscopy, to explore the post-peak behavior of concrete and FRC.

References

Aboudi, J. (1991), *Mechanics of composite materials - a unified micromechanical approach*, Elsevier, Amsterdam.

Aboudi, J. (2003), "Micromechanical analysis of the finite elastic-viscoplastic response of multiphase composites", *Int. J. Solids Struct.*, **40**, 2793-2817.

Aboudi, J., Pindera, M.J. and Arnold, S.M. (2003), "Higher-order theory for periodic multiphase materials with inelastic phases", *Int. J. Plasticity,* **19**(6), 805-847.

Ahmad, H.A. and Lagoudas, C.L. (1991), "Effective elastic properties of fiber-reinforced concrete with random fibers", *J. Eng. Mech.,* **117**(12), 2931-2938.

Asferg, J.L., Poulsen, P.N. and Nielsen, L.O. (2007), "A consistent partly cracked XFEM element for cohesive crack growth", *Int. J. Numer. Meth. Eng.*, **72**, 464-485.

Ashour, S.A., Wafa, F.F. and Kamal, M.I. (2000), "Effect of concrete compressive strength and tensile reinforcement ratio on the flexural behavior of fibruous concrete beams", *Eng. Struct.*, **22**(9), 1145-1158.

Benssousan, A., Lions, J.L. and Papanicoulau, G. (1978), *Asymptotic analysis for periodic structures*, North-Holland, Amsterdam.

Bentz, D.P., Garboczi, E.J. and Lagergren, E.S. (1998), "Multi-scale microstructural modelling of concrete diffusivity: identification of significant variables", *Cement Concrete Aggr.*, **20**(1), 129-139.

Bullard, J.W. and Garboczi, E.J. (2006), "A model investigation of the influence of particle shape on portland cement hydration", *Cement Concrete Res.*, **36**(6),1007-1015.

Christensen, R.M. and Lo, K.H. (1979), "Solution for effective shear properties in three phase sphere and cylinder models", *J. Mech. Phys. Solids*, **27**, 315-330.

Constantinides, G. and Ulm, F.J. (2004), "The effect of two types of C-S-H on the elasticity of cement-based materials: results from nanoindentation and micromechanical modeling", *Cement Concrete Res.*, **34**(1), 67-80.

Cusatis, G. and Cedolin, L. (2007), "Two-scale study of concrete fracturing behavior", *Eng. Fracture Mech.,* **74**, 3-17.

de Borst, R., Pamin, J. and Geers, M.G.D. (1999), "On coupled gradient-dependent plasticity and damage theories with a view to localization analysis", *Eur. J. Mech. A - Solids*, **18**(6), 939-962.

de Borst, R. (2002), "Some recent issues in computational failure mechanics", *Int. J. Numer. Meth. Eng.*, **52**, 63-95.

DeJong, M.J. and Ulm, F.J. (2007), "The nanogranular behavior of C-S-H at elevated temperatures (up to 700°C)", *Cement Concrete Res.,* **37**, 1-12.

Drucker, D.C. and Prager, W. (1952), "Soil mechanics and plastic analysis or limit design", *Q. Appl. Math.*, **10**, 157-165.

Eshelby, J.D. (1957), "The determination of the field of an ellipsoidal inclusion and related problems", *Proc. R. Soc. Lond. A.*, **241**, 376-396.

Ezeldin, A.S. and Balagaru, P.N. (1992), "Normal and high strength fiber reinforced concrete under compression", *J. Mater. Civil Eng.*, **4**(4), 415-429.

Feyel, F. (2003), "A multilevel finite element method (FE2) to describe the response of highly non-linear structures using generalized continua", *Comput. Method. Appl. M.*, **192**, 3233-3244.

Feyel, F. and Chaboche, J.L. (2000), "FE2 Multiscale approach for modeling the elastoviscoplastic behavior of long fiber Sic/Ti composite materials", *Comput. Method. Appl. M.,* **183**, 309-330.

Fish, J. and Shek, K.L. (1999), "Finite deformation plasticity of composite structures: computational models and adaptive strategies", *Comput. Method. Appl. M.*, **172**, 145-174.

Fish, J., Shek, K., Pandheeradi, M. and Shephard, M.S. (1997), "Computational plasticity for composite structures based on mathematical homogenization: theory and practice", *Comput. Method. Appl. M.*, **148**, 53-73.

Fish, J. and Yu, Q. (2001), "Multiscale damage modeling for composite materials: theory and computational framework", *Int. J. Numer. Meth. Eng.*, **52**, 161-192.

Füssl, J., Lackner, R., Eberhardsteiner, J. and Mang, H.A. (2008), "Failure modes and effective strength of two-phase materials determined by means of numerical limit analysis", *Acta Mech.*, **195**, 185-202.

Gal, E., Ganz, A., Chadad, L. and Krivoruk, R. (2008), "Development of a concrete unit cell", *Int. J. Multiscale Comp. Eng.*, **6**(5), 499-510.

Garboczi, E.J. and Berryman, J.G. (2000), "New effective medium theory for the diffusivity or conductivity of a multi-scale concrete microstructure model", *Concrete Sci. Eng.,* **2**(6), 88-96.

Garboczi, E.J. and Berryman, J.G. (2001), "Elastic moduli of a material containing composite inclusions: effective medium theory and finite element computations", *Mech. Mater.,* **33**, 455-470.

Garboczi, E.J. and Bullard, J.W. (2000), "Shape analysis of a reference cement", *Cement Concrete Res.*, **34**(10), 1933-1937.

Garboczi, E.J., Bullard, J.W. and Bentz, D.P. (2004), "Status of virtual testing of cement and concrete in the US – 2004", *Concrete Int.,* **26**(12), 33-37.

Garboczi, E.J., Douglas, J.F. and Bohn, R.B. (2006), "A hybrid finite elementanalytical method for determining the intrinsic elastic moduli of particles having moderately extended shapes and a wide range of elastic properties", *Mech. Mater.*, **38**(8-10), 786-800.

Garboczi, E.J. and Day, A.R. (1995), "An algorithm for computing the effective linear elastic properties of heterogeneous materials: three-dimensional results for composites with equal phase Poisson ratios", *J. Mech. Phys. Solids.,* **43**(9), 1349-1362.

Geers, M.G.D., Kouznetsova, V. and Brekelmans, W.A.M. (2001), "Gradient-enhanced computational homogenization for the micro-macro scale transition", *J. Phys. IV,* **11**, 145-152.

Ghosh, S., Lee, K. and Moorthy, S. (1995), "Multiple scale analysis of heterogeneous elastic structures using homogenization theory and voronoi cell finite element method", *Int. J. Solids Struct.,* **32**, 27-62.

Ghosh, S., Lee, K. and Moorthy, S. (1996), "Two scale analysis of heterogeneous elasticplastic materials with asymptotic homogenization and voronoi cell finite element model", *Comput. Method. Appl. M.*, **132**, 63-116.

Ghosh, S., Lee, K. and Raghavan, P. (2001), "A multi-level computational model for multi-scale damage analysis in composite and porous materials", *Int. J. Solids. Struct.*, **38**, 2335-2385.

Gitman, I.M., Askes, H. and Sluys, L.J. (2007), "Representative volume: existence and size determination engineering", *Fracture Mech.*, **74**, 2518-2534.

Gitman, I.M., Gitman, M.B. and Askes, H. (2006), "Quantification of stochastically stable representative volumes for random heterogeneous materials", *Arch. Appl. Mech.,* **75**, 79-92.

Gitman, I.M., Askes, H. and Sluys, L.J. (2008), "Coupled-volume multi-scale modeling of quasi-brittle material", *Eur. J. Mech. A - Solids,* **27**, 302-327.

Guedes, J.M. and Kikuchi, N. (1990), "Preprocessing and postprocessing for materials based on the homogenization method with adaptive finite element methods", *Comput. Method. Appl. M.*, **83**, 143-198.

Gutierrez, M.A. (2004), "Energy release control for numerical simulations of failure in quasi-brittle solids", *Commun. Numer. Meth. Eng.*, **20**, 19-29.

Haecker, C.J., Garboczi, E.J., Bohn, R.B., Sun, Z., Voigt, T. and Shah, S.P (2005), "Modeling the linear elastic properties of cement paste", *Cement Concrete Res.,* **35**(10), 1948-1960.

Haffner, S., Eckardt, S., Luther, T. and Koknke, C. (2006), "Mesoscale modeling of concrete: geometry and numerics", *Comp. Struct.,* **84**, 450-461.

Halpin, J.C. and Kardos, J.L. (1976), "Halpin-Tsai equations: a review", *Polym. Eng. Sci.*, 1976, **16(**5), 344-352.

Hashin, Z. (1962), "The elastic moduli of heterogeneous materials", *J. Appl. Mech*., **29**, 143-150.

He, H., Guo, Z., Stroeven, P., Stroeven, M. and Sluys, L.H. (2009), "Influence of particle packing on elastic properties of concrete", *Proceeding of The First International Conference on Computational Technologies in Concrete Structures (CTCS'09)*, Jeju, Korea, May, 1177-1198.

Hill, R. (1965), "A self-consistent mechanics of composite materials", *J. Mech. Phys. Solid*s, **13**, 357-372.

Hung, L.T., Dormieux, L., Jeannin, L., Burlion, N. and Barthélémy, J.F. (2008), "Nonlinear behavior of matrix-inclusion composites under high confining pressure: application to concrete and mortar", *C.R. Mecanique*, **336**, 670-676.

Ibrahimbegovic, A. and Markovic, D. (2003), "Strong coupling methods in multiphase and multiscale modeling of inelastic behavior of heterogeneous structures", *Comput. Method. Appl. M.,* **192**, 3089-3107.

Jirasek, M. (2000), "Comparative study on finite elements with embedded discontinuities", *Comput. Method. Appl. M.,* **188**, 307-330.

Johnston, C.D. (1974), "A review of mechanical properties, fiber reinforced concrete", *Steel Fiber Reinforced Concrete*, ACI Publication*,* **SP-44**, 127-142.

Kouznetsova, V., Geers, M.G.D. and Brekelmans, W.A.M. (2002), "Multi-scale constitutive modeling of heterogeneous materials with a gradient tenhanced computational homogenization scheme", *Int. J. Numer. Meth. Eng.*, **54**, 1235-1260.

Kouznetsova, V., Brekelmans, W.A. and Baaijens, F.P.T. (2001), "An approach to micro-macro modeling of heterogeneous materials", *Comp. Mech.,* **27**, 37-48.

Lee, J., Xi, Y., Willam, K. and Jung, Y. (2009), "A multiscale model for modulus of elasticity of concrete at high temperatures", *Cement Concrete Res.,* **39**, 754-762.

Mang, H.A., Aigner, E., Eberhardsteiner, J., Hackspiel, C., Hellmich, C., Hofstetter, K., Lackner, R., Pichler, B., Scheiner, S. and Stürzenbecher, R. (2009), "Computational multiscale analysis in civil engineering", *Int. Multiscale Mech.,* **2**(2), 109-128.

Mang, H.A., Lackner, R., Meschke, G. and Mosler, J. (2003), "Computational modeling of concrete structures", Karihaloo BL, Ritchie RO, Milne I, de Borst R, Mang HA, (editors), *Comprehensive structural integrity, Numerical and computational methods*, **3**. Oxford: Elsevier Science, 541-606.

Mansur, M.A., Chin, M.S. and Wee, T.H. (1999), "Stress-strain relationship of high strength fiber concrete in compression", *J. Mater. Civil Eng.*, **11**(1), 21-29.

Markovic, D. and Ibrahimbegovic, A. (2004), "On micro-macro interface conditions for micro scale based FEM for inelastic behavior of heterogeneous materials", *Comput. Method. Appl. M.,* **193**, 5503-5523.

Matsui, K., Terada, K. and Yuge, K. (2004), "Two-scale finite element analysis of heterogeneous solids with periodic microstructures", *Comp. Struct.,* **82**, 593-606.

Meschke, G. and Dumstorff, P. (2007), "Energy-based modeling of cohesive and cohesionless cracks via X-FEM", *Comput. Method. Appl. M.,* **196**, 2338-2357.

Moes, N. and Belytschko, T. (2002), "Extended finite element method for cohesive crack growth", *Eng. Fracture Mech.*, **69**, 813-833.

Miehe, C. and Koch, A. (2002), "Computational micro-to-macro transition of discretized microstructures undergoing small strain", *Arch. Appl. Mech.,* **72**, 300-317.

Mori, T. and Tanaka, K. (1973), "Average stress in the matrix and average elastic energy of materials with misfitting inclusions", *Acta Metall*., **21**, 571-574.

Nadeau, J.C. (2003), "A multiscale model for effective moduli of concrete incorporating ITZ water–cement ratio gradients, aggregate size distributions, and entrapped voids", *Cement Concrete Res.*, **33**, 103-113.

Oliver, J. (1996), "Modeling strong discontinuities in solid mechanics via strain softening constitutive equations. Part 1: Fundamentals, Part 2: Numerical simulation", *Int. J. Numer. Meth. Eng.*, **39**, 3575-3623.

Pichler, C., Lackner, R. and Mang, H.A. (2007), "A multiscale micromechanics model for the autogenous-shrinkage deformation of early-age cement-based materials", *Eng, Fracture Mech,,* **74**, 34-58.

Pichler, B., Hellmichz, C. and Mang, H.A. (2007), "A combined fracture-micromechanics model for tensile strain-softening in brittle materials, based on propagation of interacting microcracks", *Int. J. Numer. Anal. Meth. Geomech.*, **31**, 111-132.

Ramadoss, P. and Nagamani, K. (2008), "A new strength model for the high-performance fiber reinforced concrete", *Comput. Concrete*, **5**(1), 21-36.

Reuss, A. (1929), "Berechnung der fliessgrenz von mischkristallen auf grund der plastizitatsbedingung fur einkristalle", *Z. Angew Math. Mech*., **9**, 49-58.

Riedel, W., Thoma, K., Hiermaier, S. and Schmolinske, E. (1999), "Penetration of reinforced concrete by BETA-B-500 numerical analysis using a new mecroscopic concrete model for hydrocode", *Proceedings of 9 International Symposium, Interaction of the Effects of Munitions with Structures,* Berlin, May.

Romualdi, J.P. and Batson, G.B. (1963), "Mechanics of crack arrest in concrete, Journal of Engineering Mechanics", *Division of Proceedings of the American Soci. Civ. Eng.*, **89**(EM3), 147-168.

Sanahuja, J., Dormieux, L. and Chanvillard, G. (2007). "Modeling elasticity of a hydrating cement paste", *Cement Concrete Res.*, **37**, 1427-1439.

Sanchez-Palencia, E. (1980), *Non-homogeneous media and vibration theory: lecture notes in physics*, Springer, NJ.

Shah, S.P. and Rangan, B.V. (1971), "Fiber reinforced concrete properties", *ACI J.,* **68**, 126-135.

Shin, K.J., Lee, K.M. and Chang, S.P. (2008), "Numerical modeling for cyclic crack bridging behavior of fiber reinforced cementitious composites", *Struct. Eng. Mech.,* **30**(2), 147-164.

Simone, A. and Sluys, L.J. (2004), "The use of displacement discontinuities in a rate dependent medium", *Comput. Method. Appl. M.,* **193**, 3015-3033.

Smit, R.J.M., Brekelmans, W.A.M. and Meijer, H.E.H. (1998), "Prediction of the mechanical behavior of nonlinear heterogeneous systems by multilevel finite element modeling", *Comput. Method. Appl. M.,* **155**, 181-192.

Smilauer, V. and Bazant, Z. (2010), "Identification of viscoelastic C-S-H behavior in mature cement paste by FFT-based homogenization method", *Cement Concrete Res.*, **40**, 197-207.

Smilauer, V. and Krejci, T. (2009), "Multiscale model for temperature distribution in hydrating concrete", *Int. J. Multiscale Comp. Eng.,* **7**, 135-151.

Smilauer, V. and Bittnar, Z. (2006), "Microstructure-based micromechanical prediction of elastic properties in hydrating cement paste", *Cement Concrete Res.,* **36**, 1708-1718.

Stock, A.F., Hannant, D.J. and Williams, R.I.T. (1979), "Effect of aggregate concentration upon the strength and modulus of elasticity of concrete", *Mag. Concrete Res.*, **31**, 225-234.

Sun, Z., Garboczi, E.J. and Shah, S.P. (2007), "Modeling the elastic properties of concrete composites: experiment, differential effective medium theory, and numerical simulation", *Cement Concrete Compos.*, **29**, 22-38.

Swamy, R.N. (1975), "Fiber reinforced concrete of cement and concrete", *Materaux et Constructions Mater.*

Struct., **8**(45), 235-254.

Tan, K.H., Paramasivam, P. and Tan, K.C. (1994), "Instantaneous and long-term deflections of steel fiber reinforced concrete beams", *ACI Struct. J.*, **91**(4), 384-393.

Teng, T.L., Chu, Y.A., Chang, F.A. and Chin, H.S. (2004), "Calculating the elastic moduli of steel-fiber reinforced concrete using a dedicated empirical formula", *Comp. Mater. Sci.*, **31**, 337-346.

Terada, K. and Kikuchi, N. (1995), "Nonlinear homogenization method for practical applications", S. Ghosh and M. ostoja-Starzewski, EDS, *Computational Methods in Micromechanics*, *ASME*, New York, AMD-212/MD-62, 1-16.

Terada, K. and Kikuchi, N. (2001), "A class of general algorithms for multi-scale analysis of heterogeneous media", *Comput. Method. Appl. M.,* **190**, 5427-5464.

Thomee, B., Schikora, K. and Bletzinger, K.U. (2006), "Material modeling of steel fiber reinforced concrete", *Comput. Concrete,* **3**(4), 197-212.

Ulm, F.J., Eric Lemarchand, E. and Heukamp, F.H. (2003), "Elements of chemomechanics of calcium leaching of cement-based materials at different scales", *Eng. Fracture Mech.*, **70** , 871-889.

Ulm, F.J. and Jennings, H.M. (2008), "Does C-S-H particle shape matter? A discussion of the paper 'Modelling elasticity of a hydrating cement paste', by Julien Sanahuja, Luc Dormieux and Gilles Chanvillard. CCR 37 (2007) 1427-1439", *Cement Concrete Res.*, **38**, 1126-1129.

Ulm, F.J., Pellenq, R.J.M. and Vandamme, M. (2010), "C-concrete: from atoms to concrete structures", Nenad Bicanic, René Borst, Herbert Mang, Gunther Meschke (Edt.), *Computational Modelling of Concrete Structures*, CRC Press, Austria, 65-75.

Ulm, F.J., Vandamme, M., Hamlin Jennings, M., Vanzo, J., Bentivegna, M., Krakowiak, K.J., Constantinides, G., Bobko, C.P. and Van Vliet, K.J. (2010), "Does microstructure matter for statistical nanoindentation techniques?", *Cement Concrete Compos.,* **32**, 92-99.

Voigt, W. (1889), "Uber die beziehung zwischen den beiden elastizitatskonstanten isotroper korper", *Wied. Ann.*, **38**, 573-587.

Wang, Z.M., Kwan, A.K.H. and Chan, H.C. (1999), "Mesoscopic study of concrete I: generation of random aggregate structure and finite element mesh", *Comp. Struct.*, **70**, 533-544.

Wells, G.N. and Sluys, L.J. (2001), "A new method for modeling of cohesive cracks using finite elements", *Int. J. Numer. Meth. Eng.,* **50**(12), 2667-2682.

Williamson, G.R. (1974), "The effect of steel fibers on the compressive strength of concrete", *Int. Symp. on Fiber Reinforced Concrete*, SP44-11, American Concrete Institute, 195-207.

Wriggers, P. and Moftah, S.O. (2006), "Mesoscale models for concrete: homogenisation and damage behavior", *Finite Elem. Anal. Des*., **42**, 623-636.

Zohdi, T.I. and Wriggers, P. (2001), "Computational micro-macro material testing", *Arch. Comput. Method. E.*, **8**, 131-228.

Zohdi, T.I. and Wriggers, P. (2005), *Introduction to computational micromechanics*, Springer, Berlin.

CC