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Flexural ductility and deformability of reinforced and prestressed concrete sections

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Abstract. In designing a flexural member for structural safety, both the flexural strength and ductility have to be considered. For this purpose, the flexural ductility of reinforced concrete sections has been studied quite extensively. As there have been relatively few studies on the flexural ductility of prestressed concrete sections, it is not well understood how various structural parameters affect the flexural ductility. In the present study, the full-range flexural responses of reinforced and prestressed concrete sections are analyzed taking into account the nonlinearity and stress-path dependence of constitutive materials. From the numerical results, the effects of steel content, yield strength and degree of prestressing on the yield curvature and ultimate curvature are evaluated. It is found that whilst the concept of flexural ductility in terms of the ductility factor works well for reinforced sections, it can be misleading when applied to prestressed concrete sections. For prestressed concrete sections, the concept of flexural deformability in terms of ultimate curvature times overall depth of section may be more appropriate.

Keywords: deformability; flexural ductility; prestressed concrete; reinforced concrete.

1. Introduction

In conventional practice, a structure is usually designed for safety to have adequate strength to withstand the design loads at ultimate limit state. However, strength is not the sole parameter governing the safety of a structure. Catastrophic collapse of a structure is often caused not by the design loads at ultimate limit state but by extreme events, such as high energy impact, strong earthquake or terrorist attack. When an extreme event occurs, the loads acting on the structure could far exceed the design loads at ultimate limit state and some members of the structure might have reached the post-peak state at which a member has already exhausted its peak load carrying capacity. At the post-peak state, the ductility or the ability to sustain inelastic deformation without excessive reduction in load carrying capacity would become the most important parameter governing the safety of the structure. Hence, for safety beyond the ultimate limit state, the provision of sufficient ductility is at least as important as the provision of adequate strength.

It is well known that the flexural ductility of a reinforced concrete section depends mainly on whether the section is under-reinforced or over-reinforced. If the section is under-reinforced such that the tension steel yields before the concrete fails in compression, the section would fail in a ductile manner. Conversely, if the section is over-reinforced such that the tension steel does not

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yield even when the concrete fails in compression, the section would fail in a brittle manner. Therefore, the amount and yield strength of the tension steel, which together determine whether the section is under-reinforced or over-reinforced, are the major factors affecting the flexural ductility of a reinforced concrete section.

For a prestressed concrete section, however, the flexural ductility is much more complicated. As for a reinforced concrete section, the flexural ductility of a prestressed concrete section depends on whether the section is under-reinforced or over-reinforced, but it is not so clearly defined. Since prestressing has great effects on the flexural behaviour of the section, it may be envisaged that the prestressing force and steel content should also have some effects on the flexural ductility. Hence, quite obviously, there are more structural parameters affecting the flexural ductility of a prestressed concrete section than a reinforced concrete section.

Extensive numerical and experimental studies on the ductility of reinforced concrete sections and members have been carried out (Desayi *et al*. 1974, Park and Dai 1988, Pam *et al.* 2001, Ashour 2002, Kwan *et al*. 2002, Bernardo and Lopes 2004, Whitehead and Ibell 2004, Rao *et al.* 2008, Bai and Au 2009) but there have been relatively few studies on the ductility of prestressed concrete sections. Thompson and Park (1980) examined the effects of the prestressing steel content and distribution on the ductility of prestressed concrete sections and, based on their theoretical and experimental findings, recommended that a limit on the prestressing steel content should be imposed. They have, however, not considered the possible effect of the prestressing force.

Naaman *et al*. (1986) studied both theoretically and experimentally the effects of the non-prestressing and prestressing steel contents, prestressing force, prestressing steel grade, concrete strength and concrete confinement. From each moment-curvature curve, they derived the yield curvature as the curvature at the intersection point between the initial linear portion and the final linearized portion, and the ultimate curvature as the curvature at maximum moment. They found that decreasing the prestressing force has an unfavourable effect on the ductility, or in other words, increasing the prestressing force would increase the ductility.

Cohn and Riva (1991) studied by numerical analysis the effects of various parameters, including the sectional shape, reinforcement index and prestressing steel to total steel ratio. They defined the yield curvature as the curvature at which the strain increment in the reinforcing or prestressing steel reached a value of 0.2% and the ultimate curvature as the curvature at maximum moment. Based on the numerical results they have come up with, the ductility increases with the prestressing steel to total steel ratio and therefore prestressing has a positive effect on the ductility. However, the degree of prestressing (a dimensionless parameter directly proportional to the prestressing force) was kept constant and not considered in the study.

Zou (2003) conducted a state-of-the-art review on existing ductility indices and addressed their drawbacks in measuring the ductility of beams prestressed by fibre-reinforced polymer (FRP) tendons. A new index was proposed and it was verified by correlating the values of the index against the experimental failure modes of beams prestressed by FRP and steel tendon. However, it was not the focus then to examine the effect of prestressing on ductility of prestressed concrete sections.

Du *et al.* (2008) and Au *et al.* (2009) investigated the ductility of prestressed concrete members with unbonded tendons by numerical analysis. The ductility of a prestressed member with unbonded tendons has been found to be quite different from that of a similar prestressed member with bonded tendons. Hence, the bonding of prestressing tendons has significant effects on the ductility and separate studies are needed for prestressed members with bonded and unbonded tendons. Moreover, extensive parametric study shows that for prestressed members with unbonded tendons, the ductility decreases as the prestressing steel content increases but increases as the prestressing force increases.

Actually the usual practice of measuring the flexural ductility in terms of the ductility factor defined as the ultimate curvature to yield curvature ratio could be misleading, because a reduction in the yield curvature without any increase in the ultimate curvature could produce an apparent increase in ductility. Careful study of the previous research findings leading to the conclusion that increasing the prestressing force would increase the ductility, such as those by Naaman *et al.* (1986), revealed that the apparent increase in ductility was due solely to the reduction in yield curvature rather than any increase in ultimate curvature. Therefore the common belief that prestressing increases ductility has to be critically re-examined. To provide justification for this argument and investigate how the ability of a concrete section to withstand inelastic deformation could be better measured, a parametric study on the effects of various parameters on the full-range moment-curvature behaviour and flexural ductility of reinforced and prestressed concrete sections is carried out, as reported herein.

2. Method of analysis

2.1 Constitutive model for concrete

The constitutive model of concrete proposed by Attard and Setunge (1996), which is applicable to concrete strength ranging from 20 to 130 MPa, is adopted. In this model, the relationship between concrete stress σ_c and strain ε_c is

$$\frac{\sigma_c}{f_{co}} = \frac{A(\varepsilon_c/\varepsilon_{co})+B(\varepsilon_c/\varepsilon_{co})^2}{1+(A-2)(\varepsilon_c/\varepsilon_{co})+(B+1)(\varepsilon_c/\varepsilon_{co})^2} \tag{1}$$

where f_{co} and ε_{co} are the uniaxial compressive strength and the strain at peak stress, respectively. The formulae for determining the values of A and B are

(a) For the ascending branch of the stress-strain curve

$$A = E_c\varepsilon_{co}/f_{co} \tag{2a}$$

$$B = \frac{(A-1)^2}{0.55} - 1 \text{ and} \tag{2b}$$

(b) For the descending branch of the stress-strain curve

$$A = \frac{f_{ci}(\varepsilon_{ci}-\varepsilon_{co})^2}{\varepsilon_{co}\varepsilon_{ci}(f_{co}-f_{ci})} \tag{2c}$$

$$B = 0 \tag{2d}$$

The parameters E_c, ε_{co}, f_{ci} and ε_{ci} in the above formulae can be determined using the following equations

$$E_c = 4370\,(f_{co})^{0.52} \tag{3a}$$

$$\varepsilon_{co} = 4.11\,(f_{co})^{0.75}\,/\,E_c \tag{3b}$$

$$f_{ci}/f_{co} = 1.41 - 0.17\ \ln(f_{co}) \text{ and} \tag{3c}$$

$$\varepsilon_{ci}/\varepsilon_{co} = 2.50 - 0.30\ \ln(f_{co}) \tag{3d}$$

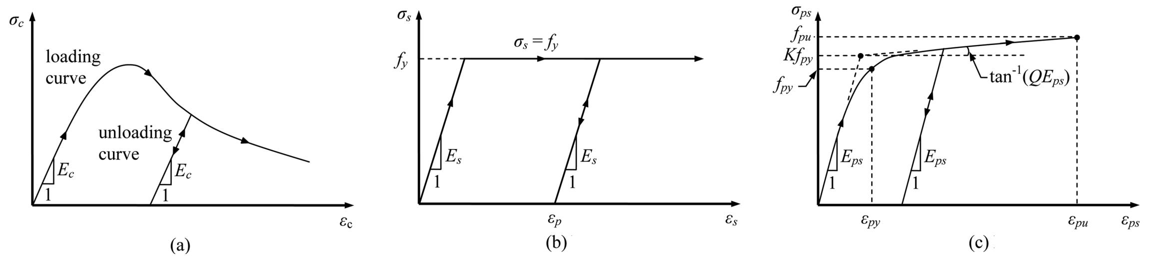

Fig. 1 Stress-strain relation for (a) concrete, (b) non-prestressing steel and (c) prestressing steel tendon

When the concrete strain decreases during strain reversal at post-peak state or during unloading, the stress-strain path is assumed to follow a straight line having a gradient equal to the initial tangent modulus. The constitutive model for concrete accounting for such stress-path dependence is shown in Fig. 1(a).

2.2 Constitutive model for non-prestressing steel

The constitutive behaviour of non-prestressing steel is assumed to be linearly elastic-perfectly plastic. To allow for strain reversal, namely the decrease of strain despite monotonic increase of curvature, stress-path dependence is taken into account by incorporating an unloading path having a gradient equal to the initial tangent modulus. The stress-strain relationship can be described by

(a) Elastic stage: $\sigma_s = E_s\ \varepsilon_s$ (4a)
(b) Beyond yielding: $\sigma_s = f_y$ (4b)
(c) On unloading after yielding: $\sigma_s = E_s\ (\varepsilon_s - \varepsilon_p)$ (4c)

where σ_s, ε_s, E_s and f_y are the stress, strain, elastic modulus and yield strength of the non-prestressing steel, respectively. In Eq. 4(c), ε_p is the residual strain, namely the permanent strain at zero stress, which can be evaluated from the stress and strain values at the previous loading step as

$$\varepsilon_p = \varepsilon_s - \sigma_s/E_s \tag{4d}$$

The constitutive model for the non-prestressing steel is shown in Fig. 1(b).

2.3 Constitutive model for prestressing steel

For the prestressing steel, the constitutive model proposed by Menegotto and Pinto (1973) is adopted. In this model, the stress is related to the strain by the following equations

$$\sigma_{ps} = E_{ps}\varepsilon_{ps}\left\{Q + \frac{1-Q}{[1+(E_{ps}\varepsilon_{ps}/Kf_{py})^N]^{1/N}}\right\} \tag{5a}$$

$$Q = (f_{pu} - Kf_{py})/(E_{ps}\varepsilon_{pu} - Kf_{py}) \tag{5b}$$

where σ_{ps}, ε_{ps}, E_{ps}, f_{py}, f_{pu} and ε_{pu} are the stress, strain, elastic modulus, yield stress, ultimate stress and ultimate strain, respectively, and N, K and Q are empirical coefficients with values depending on the type of tendon used. Naaman (1985) recommended that for 7-wire strands with an ultimate stress f_{pu} of 1863 MPa, the values of N and K may be taken as 7.344 and 1.0618, the value of f_{py}

may be taken as 85% of f_{pu} and the value of ε_{pu} may be taken as 0.069. His recommended values are adopted in the present study. The constitutive model for prestressing steel is shown in Fig. 1(c).

2.4 Moment-curvature analysis

Theoretical moment-curvature analysis of prestressed concrete sections with bonded tendons have been conducted by Thompson and Park (1980) and Cohn and Riva (1991). However, in their analysis, the stress-path dependence of the constitutive behaviour of the materials has been ignored and consequently the stress reductions in the concrete and steel due to strain reversal at the post-peak state have been underestimated. In this regard, Ho *et al.* (2003) have demonstrated that if the stress-path dependence is ignored, the resisting moment at the post-peak state will be overestimated. Hence, in all moment-curvature analysis extended to the post-peak state, stress-path dependence must be properly accounted for.

A computer programme based on the numerical approach of Ho *et al.* (2003) is developed for the present study. Apart from the ordinary non-prestressing steel, prestressing steel has also been incorporated so that the computer programme applies not only to reinforced sections but also to prestressed and partially prestressed sections. Both the non-prestressing steel and prestressing steel are assumed to be perfectly bonded to the concrete. An iterative process with the prescribed curvature applied incrementally is adopted. In each iteration, the strain variation is determined assuming that plane sections remain plane after bending, and the stresses in the concrete and steel are evaluated from their respective constitutive models. Axial equilibrium is used to determine the position of neutral axis after which the resisting moment is calculated. This iterative process is repeated until sufficient length of the full-range moment-curvature curve has been obtained.

3. Parametric study

In the light of the study of size effect on full-range analyses by Bai (2006), a normalisation approach is adopted with suitable dimensionless parameters so that the findings can be applied to cases of the same material properties and reinforcement arrangement but of different dimensions. The ratios of dimensions in the vertical direction are found to be essential parameters that govern structural behaviour. A parametric study is carried out to examine the effects of various parameters on the flexural strength, yield curvature, ultimate curvature and curvature ductility factor of both

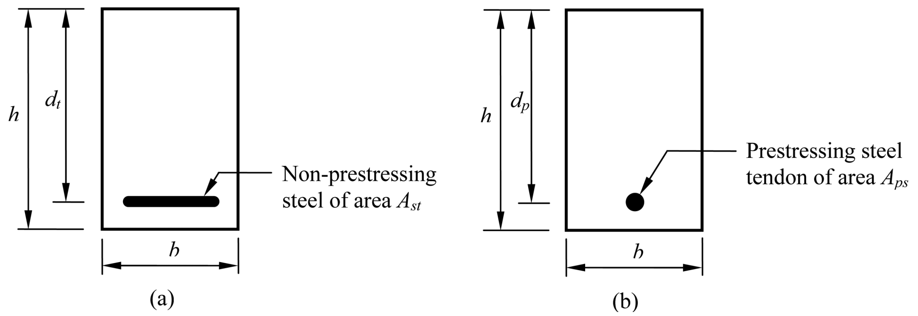

Fig. 2 Sections analyzed (a) reinforced concrete section and (b) prestressed concrete section

reinforced and prestressed concrete sections. Fig. 2 shows the sections analyzed, which are all rectangular with an overall depth h of 1400 mm and a width b of 700 mm. In order to focus on the effects of prestressing, the uniaxial compressive strength of the concrete f_{co} is taken as 60 MPa. Since the elastic moduli of steel E_s and E_{ps} seldom vary, they are just taken as 200 GPa.

For the reinforced concrete sections containing only non-prestressing steel, the effective depth to the tension steel d_t is taken as 0.9 h. The yield strength of tension steel f_{yt} varies from 460 to 620 MPa, whereas the tension steel ratio $\rho_{st} = A_{st}/bd_t$ varies from 1 to 4%, in which A_{st} is the area of tension steel. For the prestressed concrete sections containing only prestressing steel, the effective depth to the prestressing steel d_p is also taken as 0.9 h and the ultimate strength of the prestressing steel f_{pu} is 1860 MPa. The effective prestressing force f_{pe} is so varied that the prestressing force ratio f_{pe}/f_{pu} ranges from 0.3 to 0.7. The prestressing steel ratio $\rho_{ps} = A_{ps}/bd_p$ varies from 0.2 to 1.4%, in which A_{ps} is the area of prestressing steel. These ranges of parameters are so chosen that the maximum compressive stress of the concrete section at transfer would not exceed 0.6 times the concrete strength at transfer and no tensile stress occurs in the concrete section, taking into account the bending moment induced by dead load. These limitations are commonly adopted in the codes of practice, such as Eurocode 2 (European Committee for Standardization, 2004), ACI 318 (ACI Committee 318, 2005) and CSA A23.3 (CSA Technical Committee on Reinforced Concrete Design, 1994).

4. Moment-curvatures curves and ductility factors

From the above analysis, full-range moment-curvature curves of reinforced and prestressed concrete sections, each comprising of a pre-peak branch and a post-peak branch, are generated. Based on these curves, the yield curvature and ultimate curvature were determined for detailed study. Sagging moments and curvatures are taken as positive.

4.1 Moment-curvature curves

Fig. 3(a) shows the moment-curvature curves of reinforced concrete sections with tension steel yield strength $f_{yt} = 580$ MPa and tension steel ratio $\rho_{st} = 1$, 2, 3 or 4%. It can be seen from these

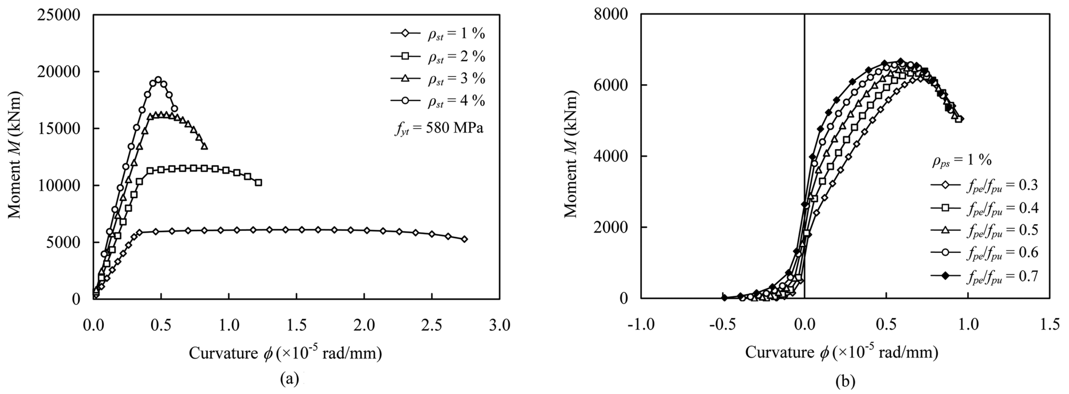

Fig. 3 Moment-curvature curves of (a) reinforced concrete sections and (b) prestressed concrete sections

curves that as ρ_{st} increases, the flexural strength increases but the ductility decreases. For tension steel ratio ρ_{st} not exceeding 3%, the section is under-reinforced and a distinct yield point can be identified on the moment-curvature curve. For tension steel ratio at 4%, the section becomes over-reinforced and there is no distinct yield point on the moment-curvature curve. Hence, the tension steel ratio has great effects on the flexural strength and ductility of reinforced concrete sections.

Fig. 3(b) shows the moment-curvature curves of the prestressed concrete sections with prestressing steel ratio ρ_{ps} = 1% and prestressing force ratio f_{pe}/f_{pu} = 0.3, 0.4, 0.5, 0.6 or 0.7. Unlike those of reinforced sections, the moment-curvature curves of prestressed sections do not start with zero moment at zero curvature, but extend into the negative curvature region. Furthermore, the moment-curvature curves of prestressed sections do not exhibit any distinct yield points. It is also seen that as f_{pe}/f_{pu} increases, the pre-peak branch of the curve gives not only a higher resisting moment but also a more rapid increase in resisting moment with the curvature, whereas the post-peak branch changes very little. Hence, the prestressing force ratio has more effects at the pre-peak state than at the post-peak state.

4.2 Ductility factors

From the moment-curvature curves, the curvature ductility of the concrete sections analyzed may be evaluated in terms of a ductility factor μ, which is usually defined as the ratio of ultimate curvature ϕ_u to yield curvature ϕ_y, namely

$$\mu = \frac{\phi_u}{\phi_y} \tag{6}$$

However, different researchers have been using different definitions for ϕ_u and ϕ_y.

Regarding the ultimate curvature ϕ_u, some researchers, such as Naaman *et al.* (1986), defined ϕ_u as the curvature at maximum moment with the resisting moment of the section at the post-peak state ignored, while others, such as Du *et al.* (2008), defined ϕ_u as the curvature at which the resisting moment has reached the peak and dropped to 85% of the peak resisting moment. In order to take into account the resisting moment at the post-peak state, the definition used by Du *et al.* (2008) is adopted, as illustrated in Fig. 4.

The major difficulty in the determination of yield curvature ϕ_y is that the moment-curvature curve does not always have a distinct yield point. To overcome this, the yield curvature has been arbitrarily taken as the curvature at the point on the moment-curvature curve marking obvious

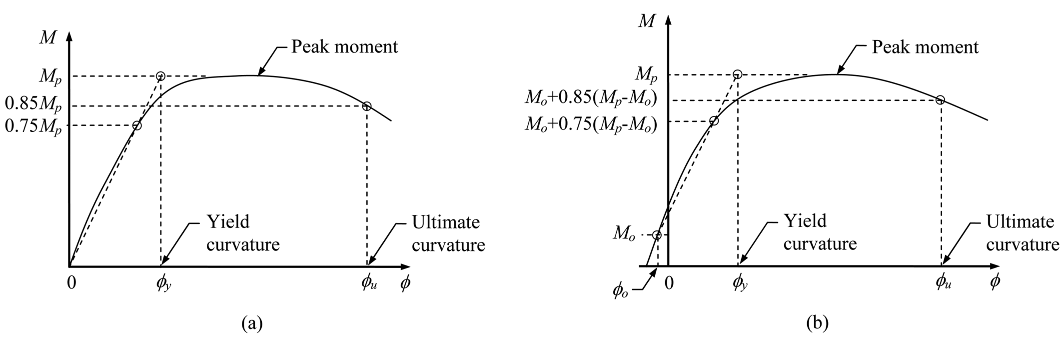

Fig. 4 Definitions of yield curvature and ultimate curvature for (a) reinforced concrete section (not to scale) and (b) prestressed concrete section (not to scale)

transition from elastic to inelastic deformation. For example, Naaman *et al.* (1986) defined the yield curvature as the curvature at the intersection point between the initial linear portion and the final linearized portion of the moment-curvature curve. However, the method they used to find the intersection point for prestressed sections is not the same as that for reinforced sections. The definition used by Naaman *et al.* (1986) is adopted herein, except that a consistent method is employed to find the intersection point for both reinforced and prestressed sections. Prestressed sections behave quite differently from reinforced sections mainly in the existence of positive moment at zero curvature and within the negative curvature region. For a reinforced section, the origin of the moment-curvature curve with zero moment and zero curvature is usually taken as the reference point. However for a prestressed section, there is no such origin that can be taken as the reference point. Hence, the reference point for a prestressed section is arbitrarily chosen as the point at which the stress at the extreme tension fibre is zero at transfer. This is a reasonable assumption for the critical sections of members with eccentric prestressing, which have been properly designed for economy. The moment and curvature at this reference point are denoted by M_o and ϕ_o, respectively. For a reinforced section, this reference point is just the origin with $M_o = 0$ and $\phi_o = 0$. For a prestressed section, this reference point is somewhere in the negative curvature region, as illustrated in Fig. 4.

Having found the reference point (ϕ_o, M_o), the initial linear portion of the moment-curvature curve is constructed by drawing a straight line through the reference point and the point on the pre-peak branch at which the resisting moment M corresponds to 75% of the increment necessary to reach the peak resisting moment M_p, namely when $M = M_o + 0.75(M_p - M_o)$. On the other hand, the final linearized portion is just taken as the horizontal line passing through the peak of moment-curvature curve. The intersection between these two straight lines is taken as the “yield point” from which the yield curvature ϕ_y is determined, as shown in Fig. 4. This method for determination of the yield point is consistent with the methods previously used for both reinforced sections (Kwan *et al*. 2002) and prestressed members (Park and Falconer 1983, Du *et al.* 2008). Similarly the ultimate curvature ϕ_u is taken as the curvature of the point on the post-peak branch with resisting moment M corresponding to 85% of the increment from M_o necessary to reach the peak resisting moment M_p, namely when $M = M_o + 0.85(M_p - M_o)$. Note that the definition of curvatures ϕ_y and ϕ_u for reinforced concrete sections as shown in Fig. 4(a) is a special case of that for prestressed concrete sections as shown in Fig. 4(b) on setting $M_o = 0$ and $\phi_o = 0$.

Taking a simply supported prestressed concrete member as example, prestressing creates negative curvatures and camber. If such a beam is load tested to failure in order to determine the ductility, the initial conditions should have included the effects of dead load and prestressing. If the beam has been properly designed for economy, the bottom fibre should have maximum compressive stress while the top fibre should have roughly zero stress, which satisfies the conditions for the reference point. To encompass prestressed concrete sections as well, the ductility factor μ is rewritten in a more general form as

$$\mu = \frac{\phi_u - \phi_o}{\phi_y - \phi_o} \tag{7}$$

which degenerates to Eq. (6) on noting that $\phi_o = 0$ for reinforced concrete sections. This approach is also consistent with experimental practice. Imagine that a number of prestressed concrete and reinforced concrete beam specimens are tested by displacement control to failure. All displacement measurements are set zero at the beginning of experiment, which implies taking the initial conditions as reference. The above method to define the ductility factor implies that the same procedure applies to both reinforced and prestressed concrete sections.

5. Reinforced concrete sections

The variations of the flexural strength and ductility of the reinforced concrete sections, expressed in terms of dimensionless parameters $M_p/f_{co}bh^2$ and μ, with the tension steel ratio ρ_{st} at different steel yield strength f_{yt} are plotted in Figs. 5(a) and 5(b), respectively. From Fig. 5(a), it is observed that the flexural strength parameter $M_p/f_{co}bh^2$ increases with increasing tension steel ratio and/or steel yield strength until the section becomes over-reinforced, and then the flexural strength parameter slowly converges to a constant value of around 0.25 despite further increases in tension steel ratio and steel yield strength. This observation implies that the steel yield strength has significant effect on the flexural strength only when the section is under-reinforced and has basically no effect on the flexural strength when the section is over-reinforced. Fig. 5(b) shows that the flexural ductility μ decreases with increasing tension steel ratio and/or steel yield strength until the section becomes over-reinforced, and then the flexural ductility slowly converges to a constant value of around 1.6 despite further increases in tension steel ratio and steel yield strength. This

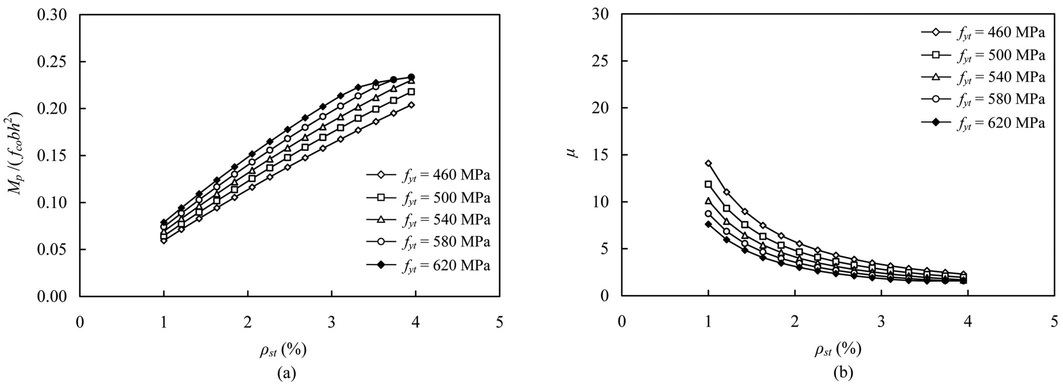

Fig. 5 Reinforced concrete sections: variations of (a) flexural strength and (b) ductility factor

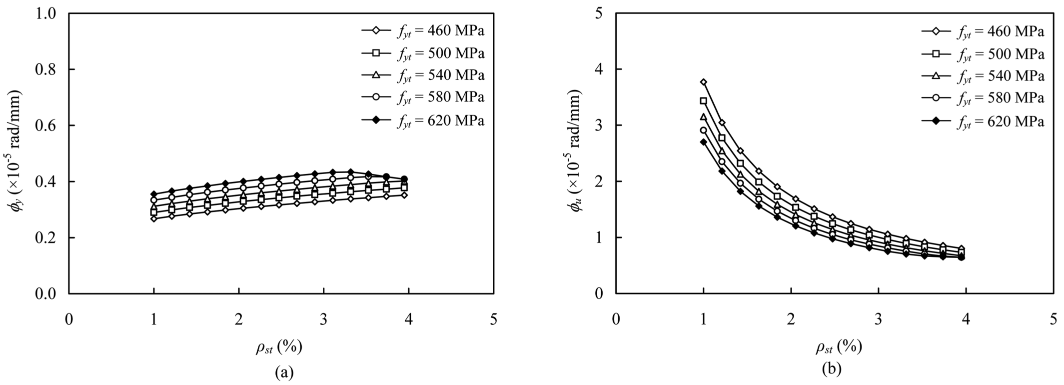

Fig. 6 Reinforced concrete sections: variations of (a) yield curvature and (b) ultimate curvature

observation implies that although the use of more and/or higher strength steel increases the flexural strength while the section is still under-reinforced, it also reduces the flexural ductility.

The variations of the yield curvature ϕ_y and ultimate curvature ϕ_u with the tension steel ratio ρ_{st} at different steel yield strength f_{yt} are plotted in Figs. 6(a) and 6(b), respectively. From Fig. 6(a), it is seen that the yield curvature ϕ_y increases with increasing tension steel ratio and/or steel yield strength until the section becomes over-reinforced. Then the yield curvature decreases as the tension steel ratio further increases and is no longer dependent on the steel yield strength. Hence, the effects of the tension steel ratio and steel yield strength depend on whether the section is under- or over-reinforced. When the section is under-reinforced, the yield curvature increases as the tension steel ratio or steel yield strength increases because a larger curvature is needed to cause yielding of the tension steel at higher tension steel ratio or steel yield strength. When the section is over-reinforced, the yield curvature decreases as the tension steel ratio increases because a higher tension steel ratio leads to a higher initial stiffness and thus an apparent yield point at smaller curvature, and the yield curvature becomes independent of the steel yield strength because the tension steel actually does not yield at all. Fig. 6(b) shows that the ultimate curvature ϕ_u decreases with increasing tension steel ratio and/or steel yield strength until the section becomes over-reinforced, and then the ultimate curvature slowly converges to a constant value of around 0.6×10^{-5} rad/mm despite further increases in tension steel ratio and steel yield strength. Considering Figs. 5 and 6 together, it is evident that the reduction of ductility factor μ as the steel yield strength f_{yt} increases is due to both increase in the yield curvature ϕ_y (the denominator in the definition of μ) and decrease in the ultimate curvature ϕ_u (the numerator in the definition of μ).

6. Prestressed concrete sections

To examine the effects of prestressing, the variations of the flexural strength and ductility of the prestressed concrete sections, expressed in terms of dimensionless parameters $M_p/f_{co}bh^2$ and μ, with the prestressing steel ratio ρ_{ps} at different prestressing force ratio f_{pe}/f_{pu} are plotted in Figs. 7(a) and 7(b), respectively. From Fig. 7(a), it is observed that the flexural strength increases steadily with the prestressing steel ratio ρ_{ps}. For ρ_{ps} below 0.8%, the flexural strength is not sensitive to the prestressing force ratio, but for ρ_{ps} above 0.8%, the flexural strength is slightly higher at a higher

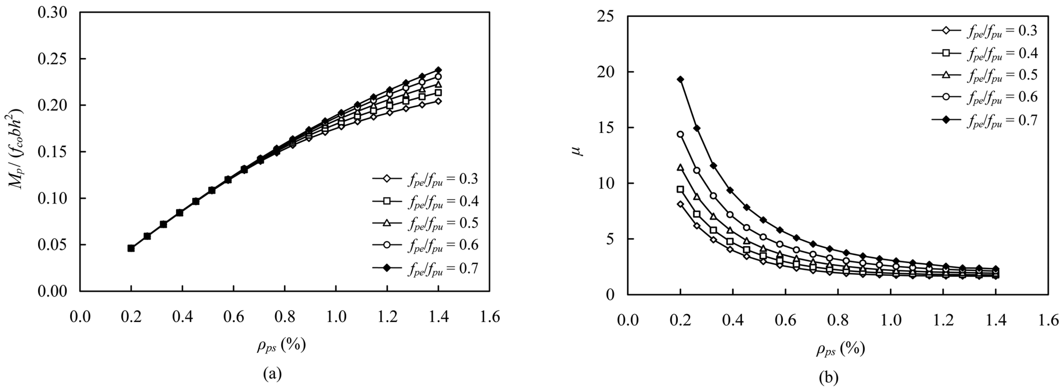

Fig. 7 Prestressed concrete sections: variations of (a) flexural strength and (b) ductility factor

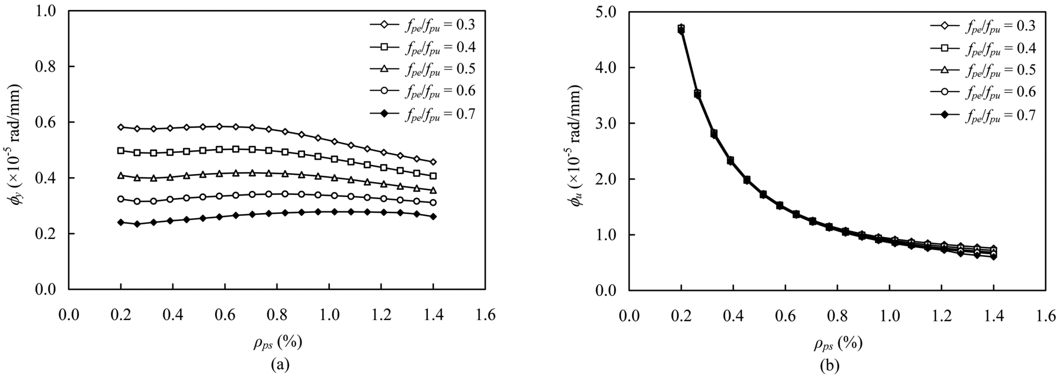

Fig. 8 Prestressed concrete sections: variations of (a) yield curvature and (b) ultimate curvature

prestressing force ratio. Fig. 7(b) shows that the flexural ductility decreases as the prestressing steel ratio ρ_{ps} increases, but increases as the prestressing force ratio f_{pe}/f_{pu} increases. Hence, in both reinforced and prestressed sections, an increase in the tension steel ratio or prestressing steel ratio reduces the flexural ductility. However, it appears that an increase in prestressing force ratio f_{pe}/f_{pu} improves the flexural ductility, which is the same phenomenon observed by Naaman *et al.* (1986).

The above observation of higher flexural ductility at higher prestressing force should be treated with caution. Flexural ductility is often measured in terms of the ductility factor μ, which can be increased by an increase in ultimate curvature ϕ_u and/or a decrease in yield curvature ϕ_y. If the ductility factor μ is increased due to an increase in ultimate curvature ϕ_u, one may consider that the flexural ductility has improved in view of the larger amount of energy absorption before failure. However if the ductility factor μ is increased solely by a reduced yield curvature ϕ_y, then it is questionable if the flexural ductility has really improved. To illustrate this point, the variations of the yield curvature ϕ_y and ultimate curvature ϕ_u with the prestressing steel ratio ρ_{ps} at different prestressing force ratio f_{pe}/f_{pu} are plotted in Figs. 8(a) and 8(b), respectively. Fig. 8(a) shows that the yield curvature changes only slightly as the prestressing steel ratio increases but decreases substantially as the prestressing force ratio increases. This is because, as the prestressing force increases, the pre-peak branch of the moment-curvature curve is shifted further to the left, leading to substantial decrease in the yield curvature. On the other hand, Fig. 8(b) shows that the ultimate curvature decreases as the prestressing steel ratio increases but is virtually insensitive to the prestressing force ratio. Considering Figs. 7 and 8 together, it is evident that the increase in the ductility factor μ as the prestressing force ratio f_{pe}/f_{pu} increases is due solely to decrease in yield curvature ϕ_y. Such apparent increase in the ductility factor without increase in the ultimate curvature should not be construed as any improvement in the flexural ductility at all. For this reason, the common measure of the flexural ductility in terms of the ductility factor should be reviewed.

7. Effects of *x/d*

Although the combined effects of the various steel-related parameters are fairly complicated, it has been found in previous studies (Desayi *et al.* 1974, Park and Dai 1988, Pam *et al.* 2001, Kwan *et al.* 2002, Bai and Au 2009) that the combined effects of the tension steel ratio and steel yield

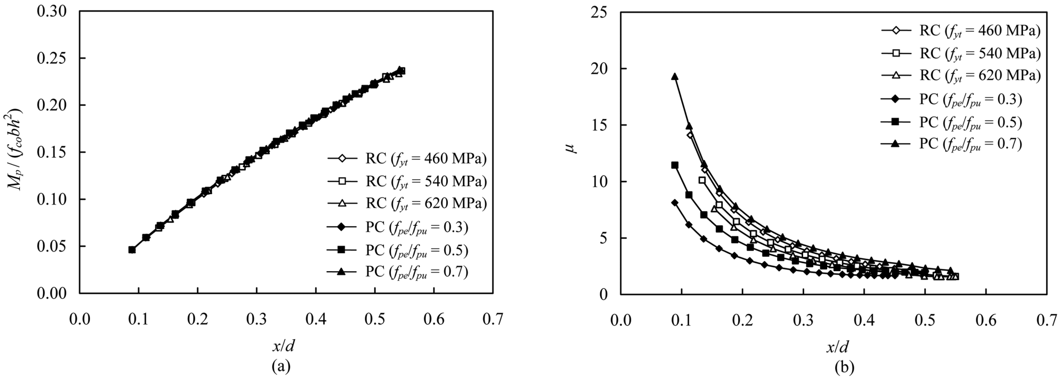

Fig. 9 Effects of x/d ratio of reinforced concrete (RC) sections and prestressed concrete (PC) sections on (a) flexural strength and (b) ductility factor

strength on the flexural behaviour of reinforced sections may be evaluated in terms of the ratio of neutral axis depth x at peak moment to effective depth d to tension steel (for reinforced or prestressed section). In fact, the maximum allowable x/d ratio is often used in codes of practice to stipulate the minimum ductility required for reinforced concrete sections. It is therefore desirable to find out whether the combined effects of the prestressing steel ratio and prestressing force ratio on the flexural behaviour of prestressed sections may also be evaluated in terms of the x/d ratio.

To study the effects of the x/d ratio on the flexural strength and ductility of both reinforced and prestressed sections, the values of $M_p/f_{co}bh^2$ and μ obtained for the representative reinforced and prestressed sections analyzed and reported here are plotted against the x/d ratio in Figs. 9(a) and 9(b), respectively. Fig. 9(a) shows that all the data points plotted, including those of reinforced and prestressed sections, lie on virtually the same curve. In fact, it can be shown that the data points of the other reinforced and prestressed sections analyzed in the study also lie on this curve, although they have been omitted for clarity. Hence, it may be concluded that for both reinforced and prestressed sections, the flexural strength is governed solely by the x/d ratio, irrespective of whether the section is reinforced or prestressed. However, from Fig. 9(b), it can be seen that the relationship between the ductility factor μ and the x/d ratio depends on whether the section is reinforced or prestressed, and it varies significantly with the steel yield strength or the prestressing force ratio.

To study the effects of the x/d ratio on the yield and ultimate curvatures of both reinforced and prestressed sections, the values of ϕ_y and ϕ_u obtained for the reinforced and prestressed sections analyzed and reported here are plotted against the x/d ratio in Figs. 10(a) and 10(b), respectively. From Fig. 10(a), it can be observed that the relationship between the yield curvature ϕ_y and the x/d ratio depends on whether the section is reinforced or prestressed, and the yield curvature varies significantly with the steel yield strength or the prestressing force ratio. Particularly, for reinforced sections, the yield curvature at a fixed x/d ratio is larger at higher steel yield strength; whereas for prestressed sections, the yield curvature at a fixed x/d ratio is smaller at higher prestressing force ratio. Hence, for both reinforced and prestressed sections, the yield curvature cannot be evaluated as a simple function of the x/d ratio. Nevertheless, Fig. 10(b) shows that all the data points of ultimate curvature for the reinforced sections lie on one curve, whereas all those of the prestressed sections lie on another curve. These two curves, with one for reinforced sections and the other for prestressed sections, are so close together that for practical applications they may be merged into one single

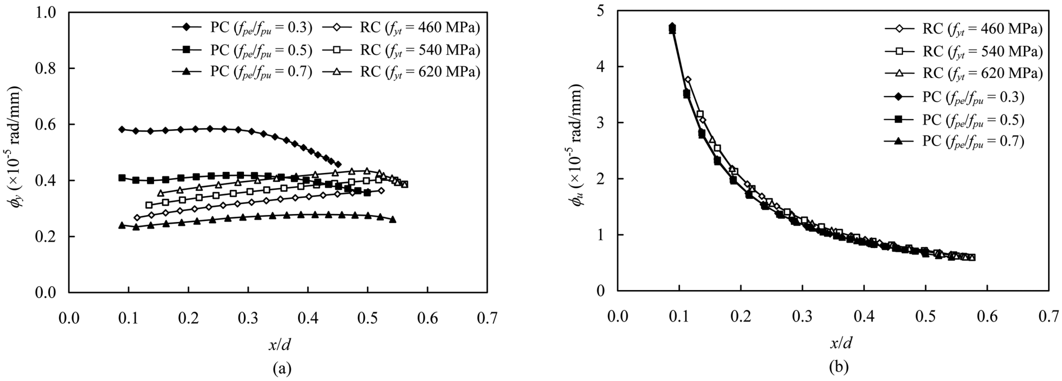

Fig. 10 Effects of x/d ratio of reinforced concrete (RC) sections and prestressed concrete (PC) sections on (a) yield curvature and (b) ultimate curvature

curve. Hence one may conclude that, regardless of whether the section being considered is reinforced or prestressed, the ultimate curvature may be evaluated as a simple function of the x/d ratio.

In summary, one may regard that in general the flexural strength and ultimate curvature of a prestressed concrete section are essentially the same as those of a reinforced concrete section having the same x/d ratio. However, the flexural ductility and yield curvature of a prestressed concrete section are highly dependent on the prestressing force ratio and therefore cannot be directly compared to those of a reinforced concrete section having the same x/d ratio. In fact, the occasional higher flexural ductility of a prestressed concrete section (i.e. curve in Fig. 9(b) for $f_{pe}/f_{pu} = 0.7$) than a reinforced concrete section having the same x/d ratio is rather misleading; the flexural ductility of the prestressed concrete section appears to be higher only because of the reduction in yield curvature caused by prestressing (i.e. curve in Fig. 10(a) for $f_{pe}/f_{pu} = 0.7$). To overcome this anomaly, one should avoid any reliance on the yield curvature for ductility evaluation. The ability of a concrete section to sustain inelastic deformation without excessive reduction in load carrying capacity should preferably be evaluated in terms of the ultimate curvature.

8. Strength-ductility-deformability performance

Since the steel-related parameters affect the flexural strength and ductility at the same time, the ductility performance of reinforced and prestressed sections should be compared on the equal strength basis. For this purpose, the concurrent flexural strength (in terms of $M_p/f_{co}bh^2$) and flexural ductility (in terms of μ) that can be achieved by the reinforced and prestressed sections analyzed are plotted in Figs. 11(a) and 11(b), respectively. Each curve in Fig. 11(a) shows the concurrent flexural strength and ductility that can be achieved at different steel yield strength, while each curve in Fig. 11(b) shows the concurrent flexural strength and ductility that can be achieved at different prestressing force ratio. From these curves, it is evident that as the flexural strength increases, the flexural ductility decreases, and vice versa. Particularly, in the case of reinforced sections, a higher steel yield strength at the same flexural strength leads to a lower flexural ductility; and in the case of prestressed sections, a higher prestressing force ratio at the same flexural strength leads to a higher flexural ductility apparently. However, such observed effects of the steel yield strength and prestressing force ratio should be interpreted with extreme care as explained below.

To overcome the above confusion associated with the equivalent yield point, the concept of deformability is advocated as an alternative measure of the ability of a reinforced or prestressed section to sustain inelastic deformation. To measure the maximum deformation that a section can sustain without excessive reduction in load carrying capacity, one may adopt the ultimate curvature ϕ_u, which is the curvature at which the resisting moment has dropped to a point corresponding to 85% of the maximum imposed moment after reaching the peak resisting moment. However, the ultimate curvature is dependent on the depth of section. It is proposed herein to multiply the ultimate curvature ϕ_u by the overall depth h and take the product $\phi_u h$ as a dimensionless measure of curvature deformability. In this regard, it is noteworthy that Cohn and Riva (1991) always multiplied the curvature by the depth of section to convert the curvature into a dimensionless value.

As before, since the steel-related parameters affect the flexural strength and deformability at the same time, the deformability performance of reinforced and prestressed sections should be compared on equal strength basis. For this purpose, the concurrent flexural strength (in terms of $M_p/f_{co}bh^2$) and flexural deformability (in terms of $\phi_u h$) that can be achieved by the reinforced and prestressed sections analyzed are plotted in Figs. 12(a) and 12(b), respectively. This time, all the data points in Fig. 12(a) fall on the same curve, indicating that the steel yield strength has no effect on the concurrent flexural strength and deformability that can be achieved. Likewise, all the data points in Fig. 12(b) fall on the same curve, indicating that the prestressing force ratio has no effect on the concurrent flexural strength and deformability that can be achieved. Hence, it may be concluded that on the equal strength basis, the steel yield strength and prestressing force ratio have no effect on the deformability. Finally, it can be shown that the two curves in Fig. 12, though plotted separately for clarity, are almost identical to each other, revealing that on the equal strength basis, reinforced and prestressed sections actually have very similar deformability. The belief that prestressing can improve the ability of a section to withstand inelastic deformation is a misconception. As for reinforced sections, due care should be exercised in the provision of sufficient ductility or deformability in the design of prestressed sections so as to avoid brittle failure.

9. Conclusions

Full-range moment-curvature analysis is carried out on reinforced concrete sections with only non-prestressing steel reinforcement and prestressed concrete sections with only bonded prestressing tendons, which takes into account material nonlinearity and stress-path dependence. The effects of non-prestressing steel content, prestressing steel content, steel yield strength and prestressing force are examined. From each moment-curvature curve, the peak resisting moment, yield curvature and ultimate curvature are determined, from which the dimensionless flexural strength parameter $M_p/f_{co}bh^2$ and the flexural ductility factor μ can be evaluated.

As the tension steel ratio and/or steel yield strength of a reinforced section increase, the flexural strength increases but the ductility factor decreases until the section becomes over-reinforced. It is also found that the reduction in ductility factor as the steel yield strength increases is due to both increase in yield curvature and decrease in ultimate curvature. For prestressed sections, it is observed that the flexural strength increases with the prestressing steel ratio but is insensitive to the prestressing force ratio except at high prestressing steel ratio. The ductility factor decreases as the prestressing steel ratio increases, but apparently increases as the prestressing force ratio increases.

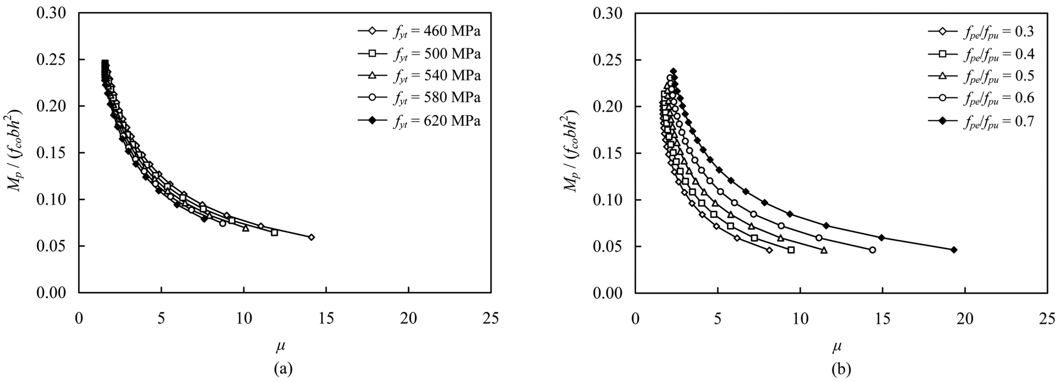

Fig. 11 Flexural strength-ductility performance of (a) reinforced concrete sections and (b) prestressed concrete sections

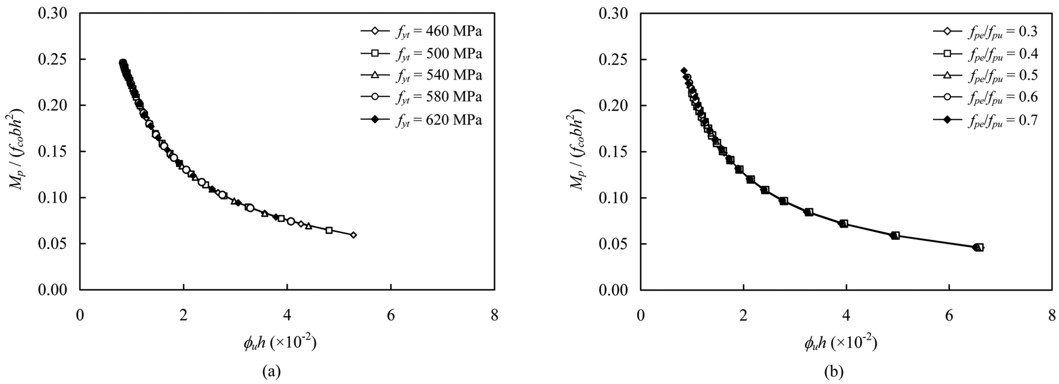

Fig. 12 Flexural strength-deformability performance of (a) reinforced concrete sections and (b) prestressed concrete sections

However, the increase in ductility factor as the prestressing force ratio increases is due solely to decrease in yield curvature and should not be construed as improvement in flexural ductility.

Correlation of the flexural strength, ductility factor, yield curvature and ultimate curvature with the neutral axis depth ratio x/d at maximum moment reveals that for both reinforced and prestressed sections, the flexural strength and ultimate curvature are uniquely related to the x/d ratio, but the ductility factor and yield curvature are not. More importantly, the flexural strength and ultimate curvature of a prestressed section are virtually the same as those of a reinforced section having the same x/d ratio. Furthermore, comparison of the ductility factors of reinforced and prestressed sections on the equal strength basis reveals that the ductility factor of a prestressed section can be higher than that of a reinforced section, thus giving rise to the impression that prestressing can increase ductility. Since there is actually no increase in ultimate curvature by prestressing, this is just an illusion. To avoid this, it is proposed to measure the ability of a section to withstand inelastic flexural deformation in terms of a dimensionless deformability factor, defined as the ultimate curvature multiplied by the overall depth. Comparing the deformability factors of reinforced and prestressed sections on the equal strength basis, it becomes clear that the deformability of a prestressed section is virtually the same as that of a reinforced section.

As for reinforced sections, due care should be exercised to provide sufficient deformability in the design of prestressed sections so as to avoid brittle failure. Since the deformability of prestressed sections are related to the x/d ratio in the same way as reinforced sections, the simplest way of providing minimum deformability for prestressed sections is to follow the current practice for reinforced sections of limiting the x/d ratio in the design codes.

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