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Identification of reinforced concrete beam-like structures subjected to distributed damage from experimental static measurements

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Abstract. Structural health monitoring of existing infrastructure is currently an important field of research, where elaborate experimental programs and advanced analytical methods are used in identifying the current state of health of critical and important structures. The paper outlines two methods of system identification of beam-like reinforced concrete structures representing bridges, through static measurements, in a distributed damage scenario. The first one is similar to the stiffness method, re-cast and the second one to flexibility method. A least square error (LSE) based solution method is used for the estimation of flexural rigidities and damages of simply supported, cantilever and propped cantilever beam from the measured deformation values. The performance of both methods in the presence of measurement errors is demonstrated. An experiment on an un-symmetrically damaged simply supported reinforced concrete beam is used to validate the developed method. A method for damage prognosis is demonstrated using a generalized, indeterminate, propped cantilever beam.

Keywords: damage identification; beam-like structures; bridges; static measurements.

1. Introduction

A testing program on a civil infrastructural system, for performance evaluation and long-term monitoring involves measurement of global parameters like deflections, rotations and natural frequencies and local parameters like stresses, strains and curvatures. Measurement of global parameters like deflections and frequencies are relatively easy and can be at coarser intervals. The global parameters at limited number of channels can also be remotely monitored and a warning alarm can be sounded, if required, when a noticeable change is seen after pre-processing the collected data and comparing it with the previously collected data. The collected data can be

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classified as a dynamically varying data or a static data, depending on whether the load is applied as a slowly varying one or not. The collected data can be used to refine and update the initial finite element model or can be used as part of a data base system for periodic condition monitoring of the structure. Re-generation of system parameters like stiffness and flexural rigidities (EI) can be done from both the static and dynamic measurements. There are always pros and cons in system identification between static and dynamic measurements. The comparison is out-lined in the following points:

(a) It is easier to measure natural frequencies of a structure even with feeble ambient forces. However, larger paraphernalia is required for static measurements.
(b) Mode shapes are a set of orthogonal as well as linearly independent vectors, whereas static deflection profiles are only a set of linearly independent vectors.
(c) It may be surmised that natural frequency change is proportional to the square root of EI change, whereas, static deflections are inversely proportional to the EI change and hence are more sensitive. In other words, 10% EI reduction, throughout a beam, brings about 5% drop in the flexural frequencies of the beam, whereas, the static central deflection for the same load increases by 11.1%. Resonant displacement, however has the same sensitivity as a static deflection.
(d) Static measurements are un-affected by the data acquisition block size, sampling rate etc, whereas dynamic parameters have to be cautiously measured taking care of the above parameters.
(e) Mean-line shift and the zero-balancing, generally is not a problem in dynamic situations and the DC component can be simply ignored, whereas in static measurements a time dependent DC drift shall pose problems.

The authors are part of the team of an elaborate experimental program, established to monitor the condition of typical Indian Railway bridges so that the load carrying capacity and speeds of the trains could be enhanced.

The available literature on system identification and damage detection using only dynamic measured data is huge. The combined static and dynamic data or static-only data for damage detection are less. The following paragraph summarizes the recent literature but in no means exhaustive. Various structural properties of anisotropic composites are identified using probability of detection (POD) concepts and boundary element numerical simulation, using static data by Rus, *et al.* (2005). Damage-induced variation of static deflection in a beam is used to identify the concentrated damage through an inverse analysis and closed form expression by Caddemi and Morassi (2007) and Caddemi and Greco (2006). As per the procedure developed by them, it is required to measure the displacements at both the sides of a concentrated crack to evaluate the crack position. A Monte-Carlo simulation procedure is also used to evaluate the effect of Gaussian distributed measurement errors on the damage evaluation procedure. Use of wavelets in damage identification using experimentally derived deflection through digital photography by Rucka and Wilde (2006) is an interesting study. Nejad, *et al.* (2005) have used an optimization method that minimizes the difference between the load vector of the damaged and un-damaged structure using static noisy data for damage identification. A static-dynamic combined damage detection technique, without prior information of intact structure is used by Vanlanduit, *et al.* (2003). Sensitivity studies on static deflection curvature, curvature mode shape and strain frequency response function on damages have been used by Yam, *et al.* (2002) in an interesting study. Another combined static-dynamic system identification study using measured damage signature and predicted damage signature is through the work of Wang, *et al.* (2001). Recently, an approach for damage identification using wavelet transform and artificial neural networks, for dynamic parameters is proposed by Lakshmanan, *et al.* (2007a, 2007b, 2008). A Hilbert-Huang transform based system identification and damage detection procedure is proposed by Zhou and Yan (2006) and the validation of the procedure is through a series of experiments conducted using a three storied model frame subjected to two damage patterns. Rajasekaran and Varghese (2005) have proposed a DB-4-Wavelet based damage identification procedure for a beam and plate structural element and the damage is simulated through reduction in elastic modulus for isotropic plate or de-lamination for the composite plate.

2. Damage definitions and the necessity for damage quantification

A structural member can suffer varying degrees of damage due to reasons such as over loading, environmental ageing, corrosion, poor quality of construction, fatigue induced crack growth under cyclic loading, creep etc. On many occasions it is required to take decisions regarding the repair and improvement of the damaged structure. The viability of repair has to be weighed with the cost of new replacement and this is governed by the state of damage suffered by the structure. Estimation of the magnitude of damage, location and its spread thus plays a crucial role in the repair methodology to be adopted. Also, residual strength and remaining life depends on the magnitude and position of damage. Damage is defined, as per International Standards Organization (ISO), as an unfavorable change in the condition of a structure that can affect the structural performance. Structural performance deterioration is also defined as the process that adversely affects the structural performance including reliability over time due to,

- naturally occurring chemical, physical or biological actions
- repeated actions such as those causing fatigue
- normal or severe environmental influences
- Wear due to use
- Improper operation and maintenance of the structure.

An engineer should be provided with tools such that damage could be measured and quantified. As already stated some of the parameters that define a damage are

(a) Increased deflection
(b) Permanent and residual deflection
(c) Increased crack width and length
(d) Decreased stiffness
(e) Decreased Natural frequencies
(f) Increased system damping.
(g) Localized changes in displacement, rotation and curvature mode shapes
(h) Localized changes in the strain distribution and stress flow

3. Inverse problem of evaluation of flexural rigidities from measured deflections and rotations – Method-I

The equilibrium of an externally applied load to the internally developed forces and moments can also be established using application of a unit deformation at a time, restraining all other deforma-

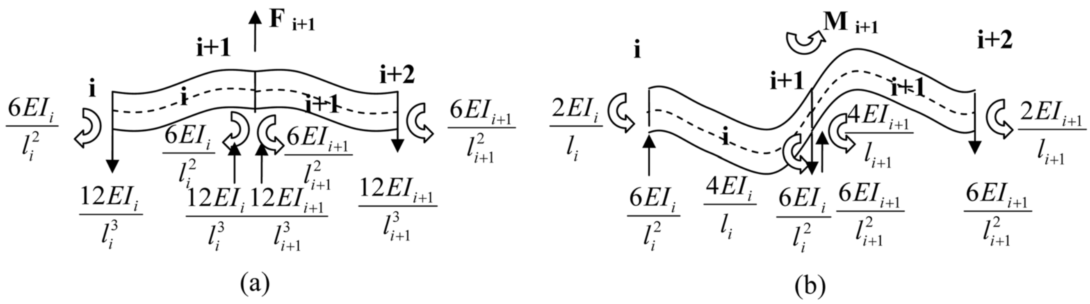

Fig. 1 Forces and moments developed in a structural node due to (a) unit deflection and (b) unit rotation

tions and noting the reactions required to keep such a deformation profile. Fig. 1 shows the internal reactions developed due to application of a unit deflection and unit rotation on a node designated as '$i+1$', inter-connected by two elements 'i' and '$i+1$', whose lengths and EI are l_i , l_{i+1} and El_i, EI_{i+1} respectively. As the formulation is essentially for planar beam-like structures or structural elements, only two elements join at a node. Eq. (1) gives the well known relationship between the applied force at node '$i+1$' to the deformations at the same node and the two adjoining nodes. This equation is re-written, in Eq. (2), separating EI, assuming that deformations are known and EIs are un-knowns. Similarly moment equilibrium relationship is shown in Eq. (3) and the modified equation with un-known EIs is in Eq. (4).

$$-\frac{6EI_i}{l_i^2}\theta_i - \frac{12EI_i}{l_i^3}\delta_i - \frac{6EI_i}{l_i^2}\theta_{i+1} + \frac{12EI_i}{l_i^3}\delta_{i+1}$$

$$+\frac{6EI_{i+1}}{l_{i+1}^2}\theta_{i+1} + \frac{12EI_{i+1}}{l_{i+1}^3}\delta_{i+1} + \frac{6EI_{i+1}}{l_{i+1}^2}\theta_{i+2} - \frac{12EI_{i+1}}{l_{i+1}^3}\delta_{i+2} = F_{i+1} \quad (1)$$

$$\left[-\frac{6}{l_i^2}\theta_i - \frac{12}{l_i^3}\delta_i - \frac{6}{l_i^2}\theta_{i+1} + \frac{12}{l_i^3}\delta_{i+1}\right]EI_i$$

$$+\left[\frac{6}{l_{i+1}^2}\theta_{i+1} + \frac{12}{l_{i+1}^3}\delta_{i+1} + \frac{6}{l_{i+1}^2}\theta_{i+2} - \frac{12}{l_{i+1}^3}\delta_{i+2}\right]EI_{i+1} = F_{i+1} \quad (2)$$

$$+\frac{2EI_i}{l_i}\theta_i + \frac{6EI_i}{l_i^2}\delta_i + \frac{4EI_i}{l_i}\theta_{i+1} - \frac{6EI_i}{l_i^2}\delta_{i+1}$$

$$+\frac{4EI_{i+1}}{l_{i+1}}\theta_{i+1} + \frac{6EI_{i+1}}{l_{i+1}^2}\delta_{i+1} + \frac{2EI_{i+1}}{l_{i+1}}\theta_{i+2} - \frac{6EI_{i+1}}{l_{i+1}^2}\delta_{i+2} = M_{i+1} \quad (3)$$

$$\left[\frac{2}{l_i}\theta_i - \frac{6}{l_i^2}\delta_i + \frac{4}{l_i}\theta_{i+1} - \frac{6}{l_i^2}\delta_{i+1}\right]EI_i$$

$$+\left[\frac{4}{l_{i+1}}\theta_{i+1} + \frac{6}{l_{i+1}^2}\delta_{i+1} + \frac{2}{l_{i+1}}\theta_{i+2} - \frac{6}{l_{i+1}^2}\delta_{i+2}\right]EI_{i+1} = M_{i+1} \quad (4)$$

Altogether, EI of an intermediate element appears in four equations, two for two nodes, that constitute the back-node and fore-nodes and for two conditions of moment and force equilibrium. Practically, if it is not possible to apply a moment, RHS is a zero. An end element may have fewer equations involving its EI. For example a starting element of a simply supported beam shall have only three equations and a similar element in a cantilever shall have only two elements. Also, in a starting element, if there is no applied force, as in a simply supported beam with no end moments applied, the coefficient of EI term will be zero, giving rise to a peculiar condition that (0. EI = 0). Such an equation, though formed, does not provide any useful information. Similar situation arises for a tip element of a cantilever, when no end moment is applied.

Totally there are as many linear equations as there are DOFs in the system. The matrix developed for each load-case shall be a rectangular one with number of rows equal to number of DOFs and number of columns equal to number of un-known EIs. If the results of many load cases are required, then the equilibrium equations of other load cases can also be appended and a larger rectangular matrix can be formed.

4. Least square solution of a rectangular set of over-determined linear equations

Once the cluster of linear equations, relating EIs with applied loads are formed, then the solution can be sought using the least square of error (LSE) technique. In the following equation (Eq. 5),

[**A**] is the coefficient matrix involving rotations and deformations which connects EIs to loads.

{**b**} is the un-known EI vector

{**c**} is the applied load vector

{ε} is the error vector and $\{\varepsilon\}^t\{\varepsilon\}$ is the sum of squares of errors at all DOFs

$$\mathbf{Ab} = \mathbf{c}$$
$$\mathbf{Ab} - \mathbf{c} = \varepsilon$$
$$\varepsilon^t \varepsilon = (\mathbf{Ab} - \mathbf{c})^t(\mathbf{Ab} - \mathbf{c}) = (\mathbf{b}^t\mathbf{A}^t - \mathbf{c}^t)(\mathbf{Ab} - \mathbf{c})$$
$$\varepsilon^t\varepsilon = \mathbf{b}^t\mathbf{A}^t\mathbf{Ab} - 2\mathbf{b}^t\mathbf{A}^t\mathbf{c} + \mathbf{c}^t\mathbf{c}$$
$$\frac{\partial(\varepsilon^t\varepsilon)}{\partial \mathbf{b}} = 2\mathbf{A}^t\mathbf{Ab} - 2\mathbf{A}^t\mathbf{c} = 0$$
$$\mathbf{b} = (\mathbf{A}^t\mathbf{A})^{-1}(\mathbf{A}^t\mathbf{c}) \tag{5}$$

Finally, minimization of $\{\varepsilon\}^t\{\varepsilon\}$ leads to a condition such that least square solution vector {**b**} is the inverse of $[\mathbf{A}]^t[\mathbf{A}]$ multiplied with $[\mathbf{A}]^t\{\mathbf{c}\}$. The solution method for the second static system identification method, whenever an over-determined set of equations appear is similar as discussed.

5. Case study for three classes of structural beam elements – Method - I

Methodology mentioned above is verified for three classes of problems, namely, (a) a simply supported beam (b) a cantilever beam and (c) a propped cantilever beam. The beam is divided into 5 sub-elements, such that EI is distributed as (200, 400, 600, 400, 600). There are ten degrees of freedom in the case of simply supported beam (Fig. 2) and cantilever beam and nine DOFs for the propped cantilever beam. Fig. 2 shows the simply supported beam with nodes and elements. It is seen that in the absence of any error in measurements, for all the three cases, the original EI is retrieved back from the given ten or nine nodal information, with an accuracy of 0.001%. Also, the load cases considered are the near-mid-span load case for the simply supported beam and tip-load case for cantilever. It is to be cautioned that in the case of cantilever, there is not much choices and all elements will be strained only when the load is applied at the tip-node. In this analysis, it is possible that other results of various load cases can also be combined and results can be used in

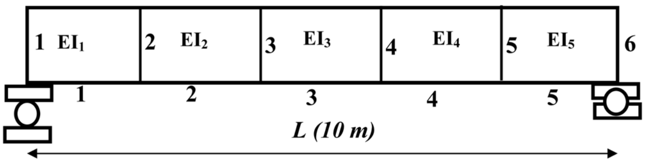

Fig. 2 A simply supported beam taken for static system identification studies

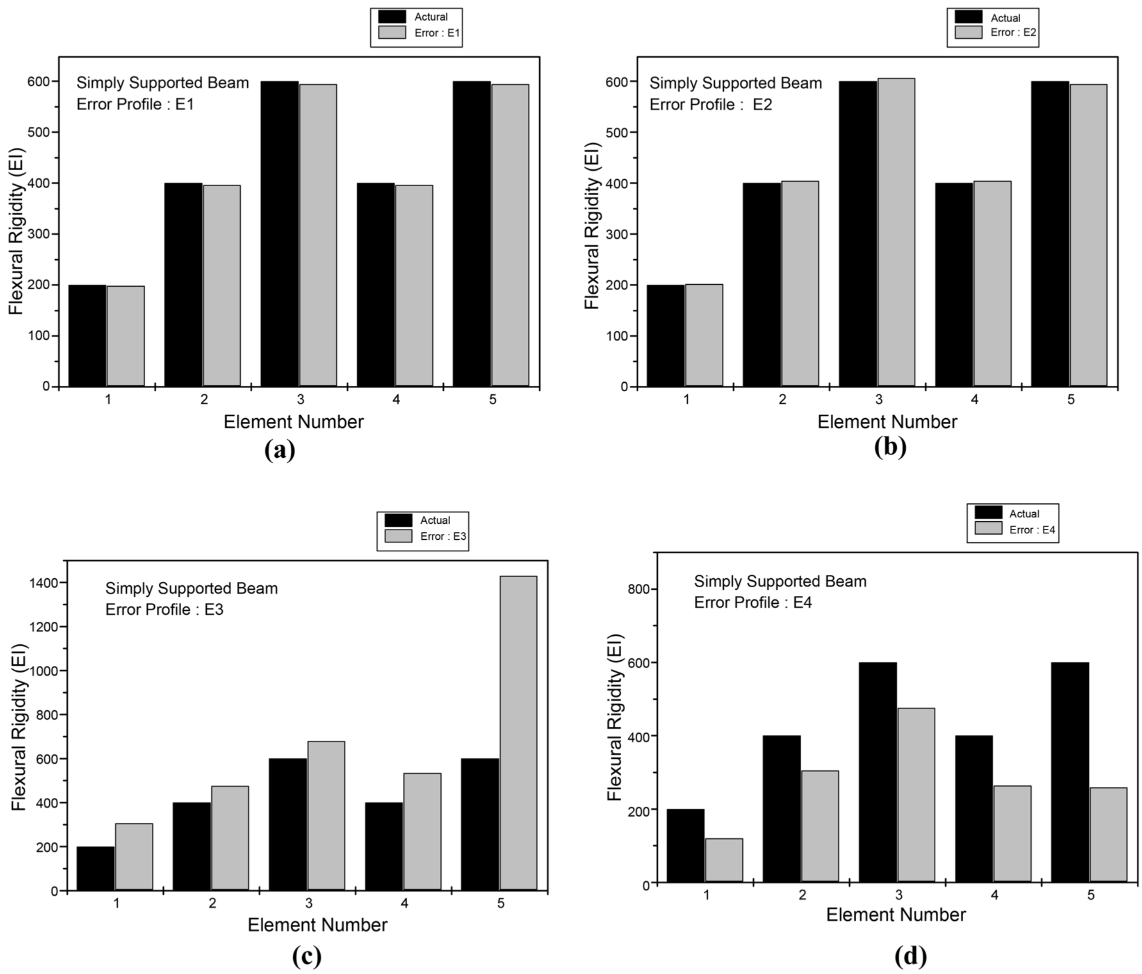

Fig. 3 Flexural rigidities retrieved from an inverse analysis for a simply supported beam for different error patterns ((a) E1, (b) E2, (c) E3, (d) E4)

inverse analysis.

The effect of error in measurement is studied by taking four error patterns in the measured data. There are termed as E1, E2, E3 and E4, which include,

(a) Pattern E1 : Uniform increase of measured rotations and displacements by 1%.

(b) Pattern E2 : Uniform decrease of measured rotations and displacements by 1%.

(c) Pattern E3: Fluctuating Error (+1%, −1%, +1% …..) in measurement.

(d) Pattern E4: Fluctuating Error (−1%, +1%, −1% …..) in measurement.

For the cantilever beam which showed divergence and in-stability in the retrieved EI, additional error patterns are also analyzed, which are given the following designations.

(e) Pattern E41: Fluctuating Error (0.1% in rotations and No error in displacements) in measurement.

(f) Pattern E42: Fluctuating Error (0.1% in rotations and displacements) in measurement.

Fig. 3 shows the retrieved EIs for a simply supported beam for the four error patterns. The fluctuating error does lot more damage than a uniform error and error values are particularly more

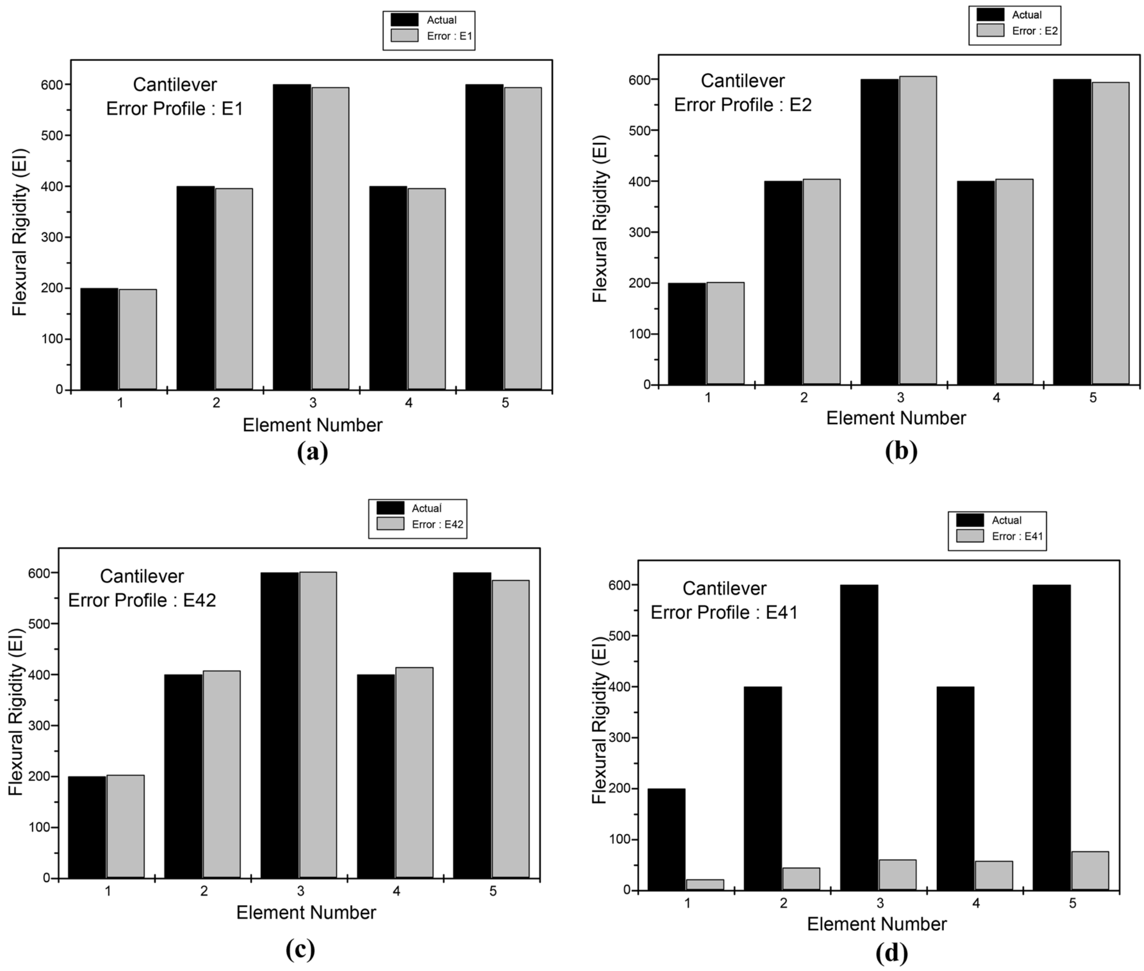

Fig. 4 Flexural rigidities retrieved from an Inverse analysis for a cantilever beam for different error patterns ((a) E1, (b) E2, (c) E42, (d) E41)

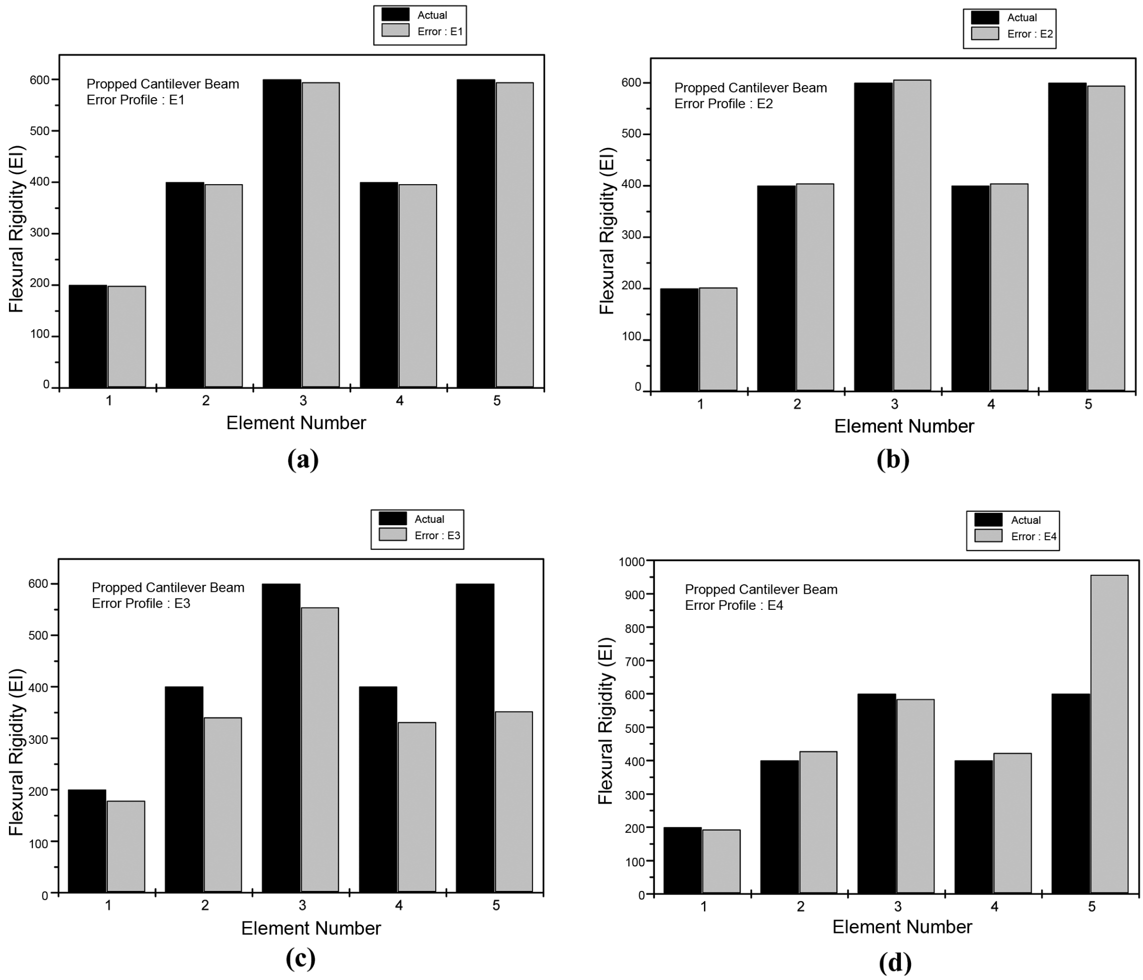

Fig. 5 Flexural rigidities retrieved from an inverse analysis for a propped cantilever for different error patterns ((a) E1, (b) E2, (c) E3, (d) E4)

at end segments. Positive and negative values of error disturb the EI less and more, respectively and equally. Hence the error that comes due to Gaussian noise may get averaged out for large number of data. Fig. 4 shows the trend for a cantilever beam. Since the cantilever beam showed divergence and instability of retrieved EIs even at 1% error, the error analysis is also repeated at 0.1% error. Even at this level of error, a large deviation in results are seen. Fig. 5 shows the results for a propped cantilever beam, whose behaviour is found to be closer to simply supported beam with large errors at supports and the compensating nature of positive and negative errors. It is to be mentioned that in the error analysis, an attempt is made such that all equations are not blindly used and those equation at extreme nodes, where the 0. EI = 0 condition comes are excluded and the analysis repeated.

6. Formulation with only deflection measurements at the load application point for a Simply Supported Beam - Method II

The method described previously and based on re-writing the equilibrium equation in terms of known (measured) deformations and un-known flexural rigidities is good but is not robust. This means that under presence of instrumental noise and resolution errors, it is likely that such a formulation may give erroneous results. Hence a simplified formulation, based on flexibility approach is written, using only deflection measurements and through monitoring of the change in deflection (ratio of changed displacement to the original displacement). Certain other conditions are also imposed such that the value of measured deflection is more and hence the errors are less. These include

(a) Deflection measured only at the loading point is used in the inverse analysis.

(b) Ratio of the change in deflection is used instead of the absolute value of deflection.

Total strain energy due to flexure can be summed up for various regions of the un-damaged beam (Fig. 6) as in the following expression

$$U_{ud} = \int \frac{M^2 dx}{2EI} = \int_A \frac{M^2 dx}{2EI} + \int_B \frac{M^2 dx}{2EI} + \int_C \frac{M^2 dx}{2EI} + \int_D \frac{M^2 dx}{2EI} \tag{6}$$

For the damaged beam, only the region of damage is replaced with an α EI.

$$U_d = \int \frac{M^2 dx}{2EI} = \int_A \frac{M^2 dx}{2EI} + \int_B \frac{M^2 dx}{2EI} + \int_C \frac{M^2 dx}{2\alpha EI} + \int_D \frac{M^2 dx}{2EI} \tag{7}$$

This can be modified and written as,

$$U_d = U_{ud} - \int_C \frac{M^2 dx}{2EI} + \int_C \frac{M^2 dx}{2\alpha EI} = U_{ud} + \frac{1-\alpha}{\alpha} \int_C \frac{M^2 dx}{2EI} = U_{ud} + \frac{\beta}{1-\beta} \int_C \frac{M^2 dx}{2EI} \quad U_d - U_{ud} = \frac{\beta}{1-\beta} \int_C \frac{M^2 dx}{2EI} \tag{8}$$

Here 'α' ($0<\alpha<1$) is the remaining ratio of EI after a damage of 'β EI' has occurred ($1-\alpha=\beta$). The factor $\frac{\beta}{1-\beta}$ is termed as the modified damage factor (β^*) It is to be noted that the strain energy and subsequently deflections are proportional to the modified damage factor and not to the damage factor themselves.

Reckoning 'x' from the right-hand side support and the change of deflection is sought below the applied load itself,

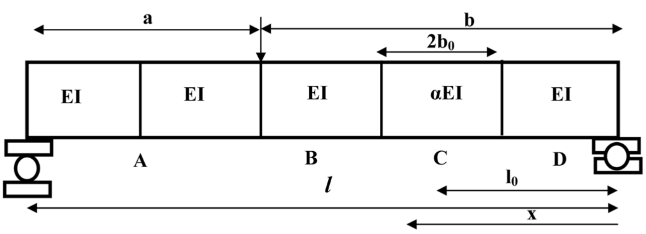

Fig. 6 A simply supported beam taken for static system identification studies

$$\Delta\delta_{ij} = \frac{\partial(U_d - U_{ud})}{\partial P} = \frac{\beta}{1-\beta}\cdot\frac{\partial}{\partial P}\left(\int_C \frac{M^2 dx}{2EI}\right) \tag{9}$$

$$\Delta\delta_{ij} = \frac{\beta_j}{1-\beta_j}\cdot\frac{\partial}{\partial P}\left(\int_{l_0-b_0}^{l_0+b_0} \frac{P^2 a^2}{2EIl^2} x^2 dx\right) \tag{10}$$

where $\Delta\delta_{ij}$ is the incremental increase in deflection at the 'i-th' node due to a damage magnitude β_j at the 'j-th' element.

$$\Delta\delta_{ij} = \frac{\beta_j}{1-\beta_j}\cdot\frac{2Pa_i^2 b_{0,j}}{3EIl^2}(3l_{0,j}^2 + b_{0,j}^2) \tag{11}$$

If there are 'N' damage sites in the beam, the effect of all the damages can be simply summed up and the above equation can be written as,

$$\Delta\delta_i = \sum_{j=1}^{j=N}\frac{\beta_j}{1-\beta_j}\cdot\frac{2Pa_i^2 b_{0,j}}{3EIl^2}(3l_{0,j}^2 + b_{0,j}^2) \tag{12}$$

For a particular case of a uniform and widespread reduction in EI, the equation can be modified and the effects on both the left and right sides of the load are added and the resulting expression is,

$$\Delta\delta_i = \frac{\beta}{1-\beta}\cdot\frac{Pa^2 b^2}{3EIl} \tag{13}$$

The above equation reinforces the well-known fact that for a uniform decrease in EI, throughout the beam, by a factor of α, deflection increases by a facor of $1/\alpha$ and change in deflection is nothing but $((1/\alpha)$-1)

The ratio of increased deflection to the original deflection, for the same loading can similarly be written as,

$$\frac{\Delta\delta_i}{\delta_i} = \sum_{j=1}^{j=N}\frac{\beta_j}{1-\beta_j}\cdot\frac{2b_{0,j}}{b_i^2 l}(3l_{0,j}^2 + b_{0,j}^2) \tag{14}$$

In a system identification procedure, for deriving the system parameters from the known values of ratio of changes in deflections from the original deflection values, above equation can be re-cast into a matrix form with un-known vector composed of $\frac{\beta_i}{1-\beta_i}$ and known vector of $\frac{\Delta\delta_i}{\delta_i}$, both related by a coefficient matrix depending on the geometry of load position and damage. The matrix 'A' as used in this paper is the sensitivity matrix, relating the sensitivity of damage at element 'j' to an increased deflection at node 'i'. The vector 'b' is the un-known $\frac{\beta}{1-\beta}$, modified damage factor (β^*) at various elements and vector 'c' is the known change in deflection, expressed as the ratio of original deflection, occurring at node 'i' due to the effect of damages in all elements and which can be measured.

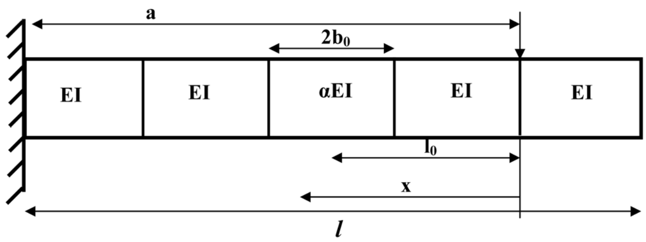

Fig. 7 A Cantilever beam taken for static system identification studies

7. Formulation with only deflection measurements at the load application point for a cantilever beam - Method II

It is seen from Fig. 7, that the co-ordinate system for the cantilever beam is slightly different from the simply supported one, wherein the position of damage is always reckoned from the load point. This is mainly due to the reason that a damage on the right-side of load point does not affect the deflection and the strain energy distribution of the beam. Similar to the former case, if $\Delta\delta_{ij}$ is defined as the incremental increase in deflection at the 'i-th' node due to a damage magnitude β_j at the 'j-th' element.

$$\Delta\delta_{ij} = \frac{\beta_j}{1-\beta_j} \cdot \frac{2Pb_{0,j}}{3EI}(3l_{0,j}^2 + b_{0,j}^2) \tag{15}$$

If there are 'N' damage sites in the beam, the effect of all the damages can be summed up and the above equation can be written as,

$$\Delta\delta_i = \sum_{j=1}^{j=N} \frac{\beta_j}{1-\beta_j} \cdot \frac{2Pb_{0,j}}{3EI}(3l_{0,j}^2 + b_{0,j}^2) \tag{16}$$

For a particular case of uniform and widespread reduction in EI, the equation can be modified as,

$$\Delta\delta_i = \frac{\beta}{1-\beta} \cdot \frac{Pa^3}{3EI} \tag{17}$$

Expressing in a non-dimensionalised form, ratio of increased deflection to the original deflection, for the same loading can be written as,

$$\frac{\Delta\delta_i}{\delta_i} = \sum_{j=1}^{j=N} \frac{\beta_j}{1-\beta_j} \cdot \frac{2b_{0,j}}{a_i^3}(3l_{0,j}^2 + b_{0,j}^2) \tag{18}$$

8. Case study for two classes of structural beam elements – Method - II

It is worthwhile to see the sensitivity matrix 'A', for the five-segment, simply supported (A_{ss}) and cantilever beams (A_c). They are :

$$A_{ss} = \begin{bmatrix} 0.2 & 0.4625 & 0.2375 & 0.0875 & 0.0125 \\ 0.05 & 0.35 & 0.4222 & 0.1555 & 0.0222 \\ 0.0222 & 0.1555 & 0.4222 & 0.35 & 0.05 \\ 0.0125 & 0.0875 & 0.2375 & 0.4625 & 0.2 \end{bmatrix} \quad (19)$$

$$A_C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.875 & 0.125 & 0.0 & 0.0 & 0.0 \\ 0.7037 & 0.2593 & 0.03707 & 0.0 & 0.0 \\ 0.5781 & 0.2969 & 0.1094 & 0.0156 & 0.0 \\ 0.4880 & 0.2960 & 0.1520 & 0.056 & 0.008 \end{bmatrix} \quad (20)$$

Each row of the matrix (*i*-th row) corresponds to a particular node and A_{ij} gives the ratio of change in displacement in the '*i*-th' node, when a unit damage (β^*) is induced at the '*j*-th' element Fig. 8 shows the scenario for eight-segment simply supported and cantilever beams. Fig. 8 shows the contribution of damage at various elements towards change in deflection at any node for these cases (Load and deflection measurement points are assumed same). The following observa- tions can be made from this figure and the equations.

(a) An element '*N*' has a back-node '*N*' and a fore-node '$N+1$'. In a simply supported beam, the deflection change at any node is governed more by the neighborhood elements. For example, node-3 deflection is affected maximum by element-3 followed by 2,4,5,6,1,7,8 in that order.

(b) For the cantilever, deflection at any node is affected maximum by the first element (close to fixed support). For a node '*N*', (load and deflection also at '*N*') element equal to and beyond '*N*' has no influence.

(c) For any node, the contribution of all the elements adds to 1.0, which means that the sum of all elements in a row of '*A*' matrix adds to 1.0. This leads to the conclusion, that for the same β^*, for all the elements, the ratio of changed displacement is also β^*.

(d) The condition number for the five segment cantilever beam, A_c (ratio between the largest and smallest Eigen value), which gives the stability of the matrix is 2437, indicating that the above matrix may be sensitive to small changes. (For the eight segment cantilever beam cond (A_c)= 1.47E5)

Simply supported beam is subjected to the damage patterns (Fig. 9), P1 = {0.0, 0.05, 0.07, 0.10, 0.10, 0.07, 0.05, 0.0}, P2 = {0.0, 0.20, 0.25, 0.30, 0.30, 0.25, 0.20, 0.0}, P3 = {0.0, 0.15, 0.25, 0.10, 0.15, 0.14, 0.25, 0.0} and P4={0.0, 0.05, 0.10, 0.05, 0.10, 0.05, 0.10, 0.0}. These patterns are both small and medium damage cases and also symmetric and un-symmetric. Also, the end-elements are assumed as not damaged. It is seen in Fig. 9, that in the absence of error ingress, the method is able to retrieve back the damages very well. The four error patterns E1, E2, E3 and E4 are similar as described earlier and are used in contaminating the measurements. Figs. 10-13 show the performance of the method in the midst of error ingress. The observations are:

(a) Uniform error is less harmful in damage retrieval

(b) Amongst fluctuating errors, a larger damage shows less variation, whereas a damage of 5% or less is affected the most.

The condition number of the A_c matrix, for cantilever is large and some constraints need to be imposed. It is assumed that in the eight segment cantilever beam, the last two elements are assumed as

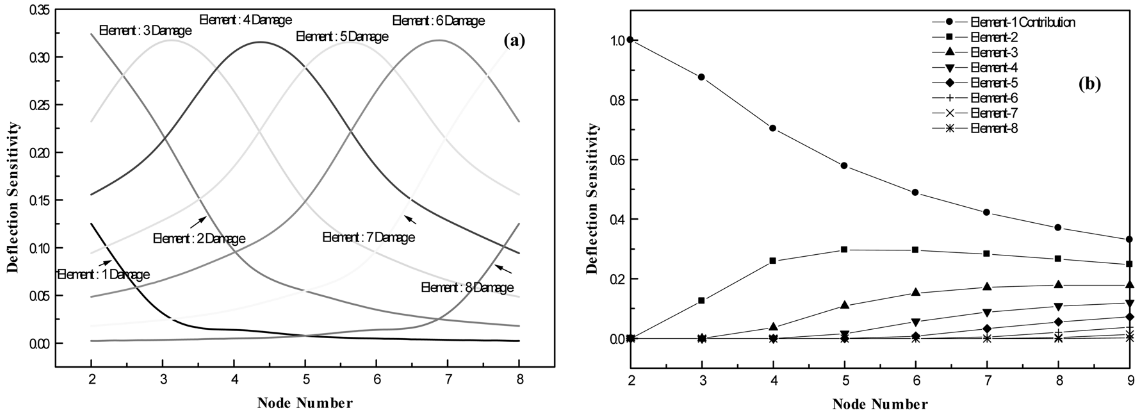

Fig. 8 Sensitivity of deflection-change ratio at each node due to damage at different elements for (a) simply supported and (b) cantilever beams

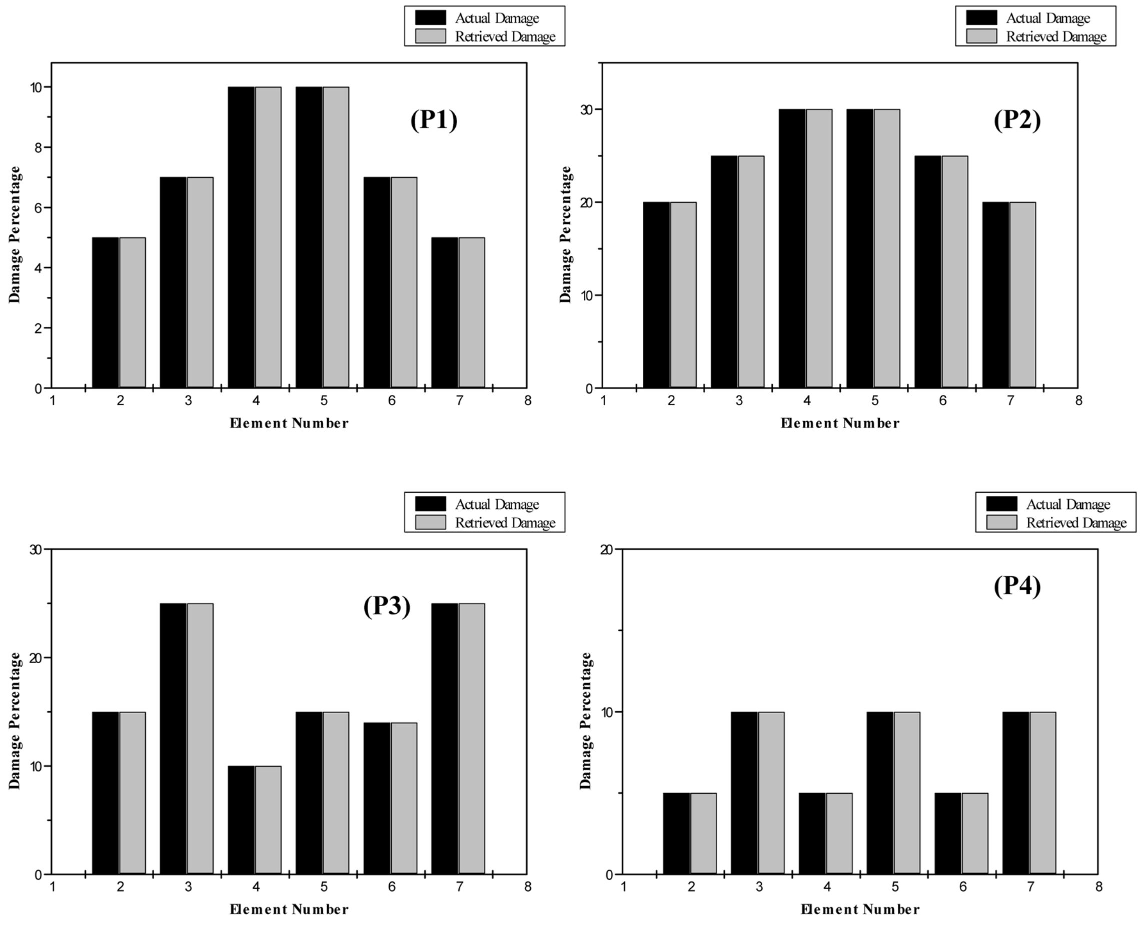

Fig. 9 Various Damage patterns for damage identification for a simply supported beam

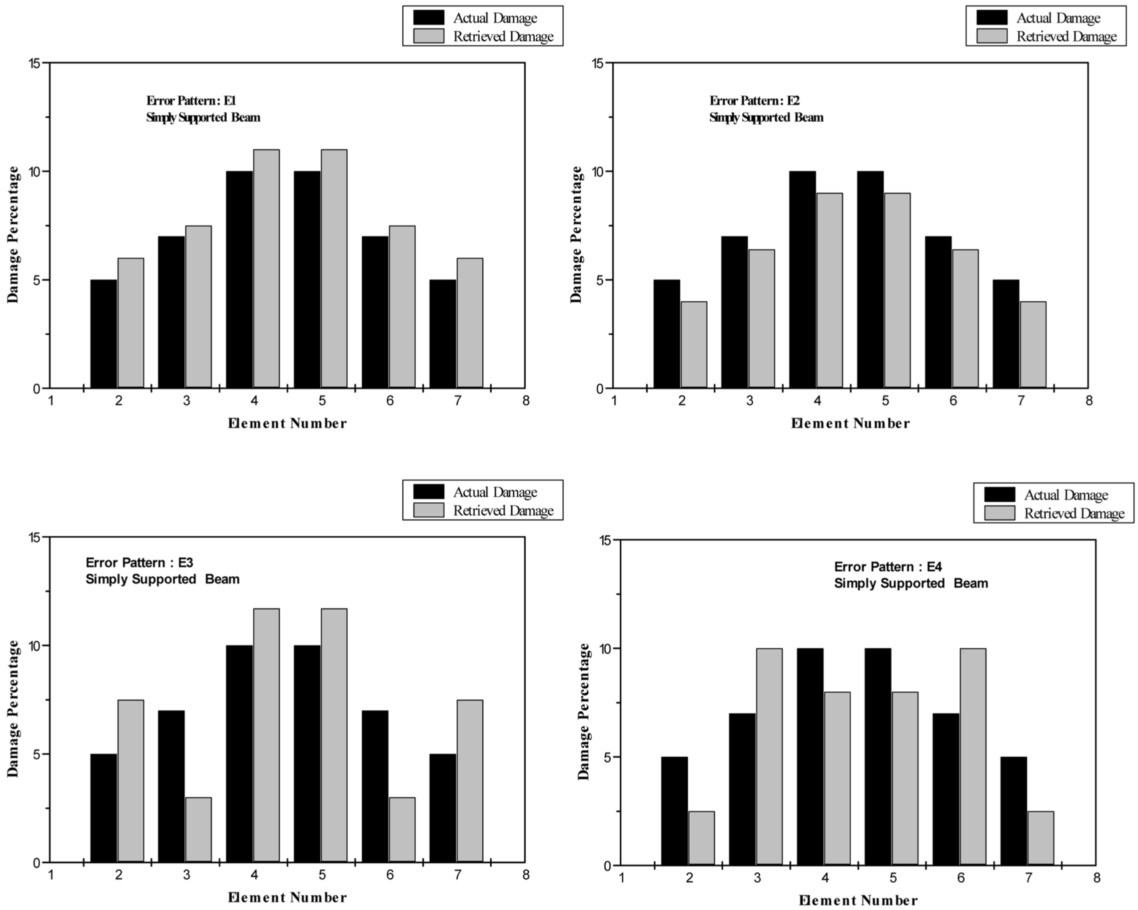

Fig. 10 Performance of damage identification method for the simply supported beam for damage pattern – P1 and for various error patterns (E1 to E4)

not damaged, fourth to sixth elements undergo the same damage; second and third elements also undergo the same damage. These constraints reduce the ratio between the largest and smallest diagonal element and bring stability to the matrix. Cantilever beam is subjected to the damage patterns (Fig. 14), P1 = {0.25, 0.20, 0.20, 0.15, 0.15, 0.15, 0.0, 0.0}, P2 = {0.10, 0.07, 0.07, 0.05, 0.05, 0.05, 0.0, 0.0} and P3 = {0.20, 0.05, 0.05, 0.15, 0.15, 0.15, 0.0, 0.0}. These patterns are both small and medium damage cases and also uniformly decreasing and undulating. It is seen in Fig. 14, that in the absence of error ingress, the method is able to retrieve back the damages very well. The four error patterns E1, E2, E3 and E4 are similar as described earlier and are used in contaminating the measurements. Figs. 15-17 show the performance of the method in the midst of error ingress. The observations are:

(a) Uniform error is less harmful in damage retrieval

(b) Amongst fluctuating errors, a larger damage shows less variation.

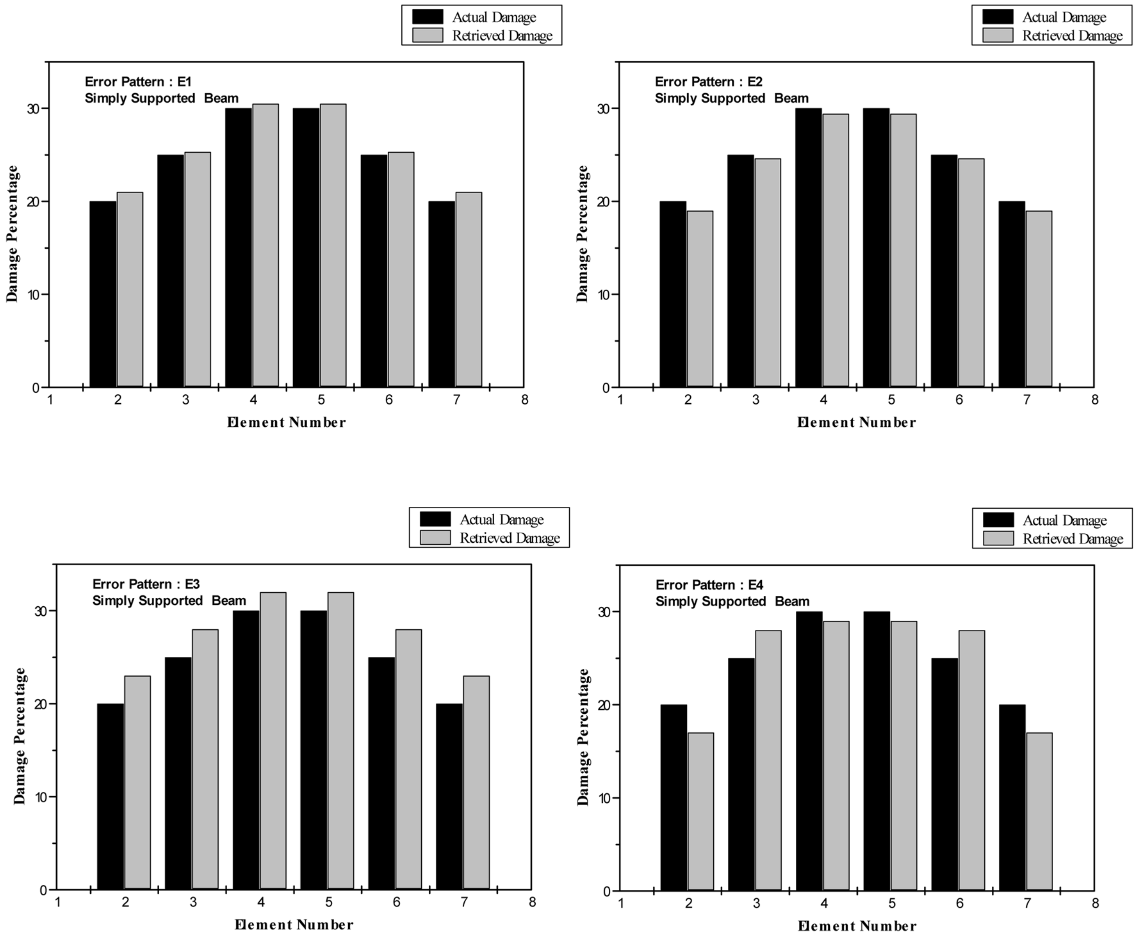

Fig. 11 Performance of damage identification method for the simply supported beam for damage pattern – P2 and for various error patterns (E1 to E4)

9. Static load application in practical cases

The above mentioned methods (Method-I and Method-II) for damage identification through static measurements require a suitable loading scheme for practical applications. In the case of bridges, a test vehicle of known load and precise accurate knowledge of wheel loads and their positions can be thought of as a static loading scheme. Lin and Yang (2005) suggest a passing vehicle with an attached trailer for both load application and dynamic mesurements. However, Lin and Yang (2005) confine the study to only dynamic measurements and the suggested method requires an extended variation that can be adopted for static load applications. In cases, where it is impossible to measure the static deflections about a non-moving reference, a long distance theodolite or a total station can be used to measure the suffered deflection of the bridge when subjected to the known test load.

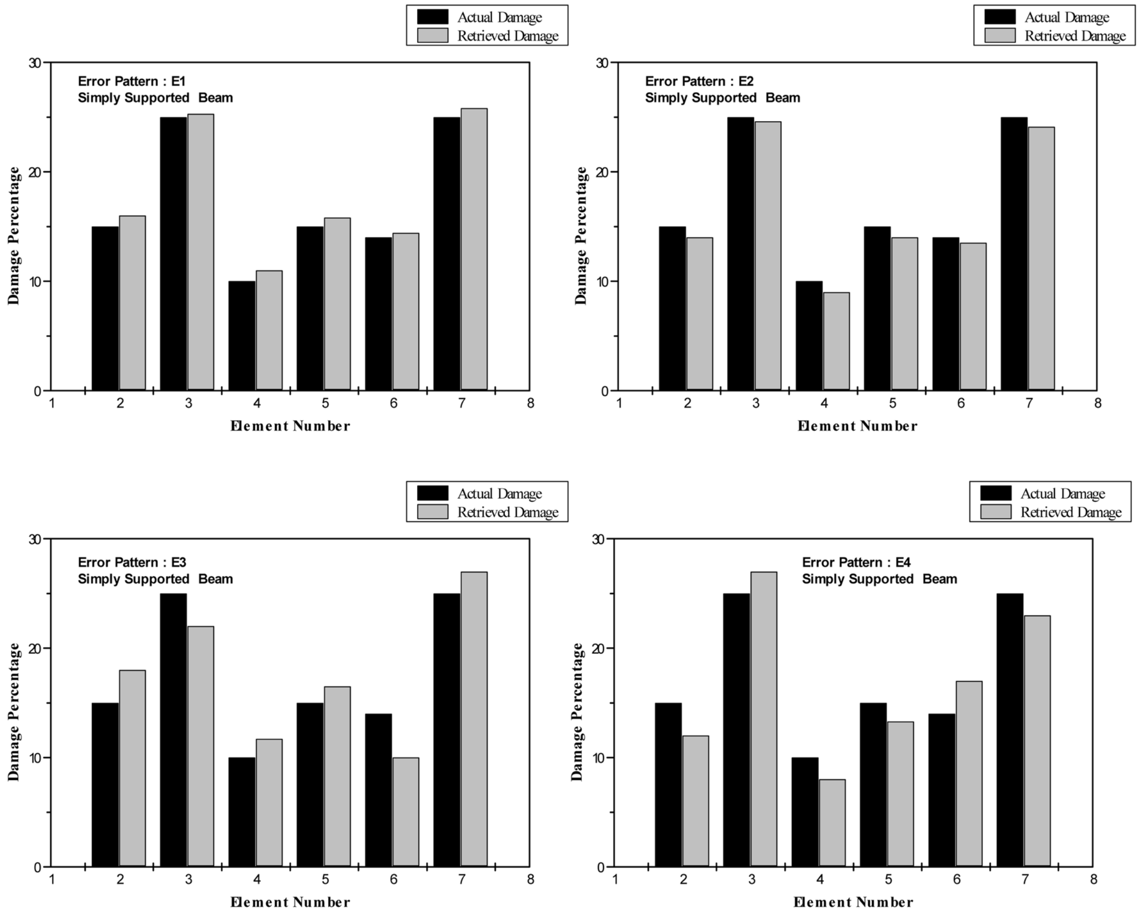

Fig. 12 Performance of damage identification method for the simply supported beam for damage pattern – P3 and for various error patterns (E1 to E4)

10. Comparative performance of Method-I and Method-II

Though both the methods lead to similar results, there are some essential differences and Table 1 summarises the differences between the methods.

As seen above, method-I does not require comparison of deformations from the healthy state of the structure, but regenerates flexural rigidities absolutely. However method-II requires changes in deformations with the healthy structure taken as datum. A little bit of robustness is gained owing to this feature of method-II and a physial in-sight of what is happening could be visualised.

The reason for the poor performance of the catilever beam, both under methods-I and II could be explained from the matrix given in Eq. (20) (Five-segment cantilever) and from Fig. 8(b) (Eight-segment Cantilever). In Eq. (20), the last row of the matrix gives the contribution of increase in tip-displacement due to damage in element-1 (first column, fifth-row), element-2 (second column, fifth-row) and so on. It is observed that a damage in the fifth element (near the tip) influences the change in tip displacement sixty times less than a damage in the first element (near the support). It is still

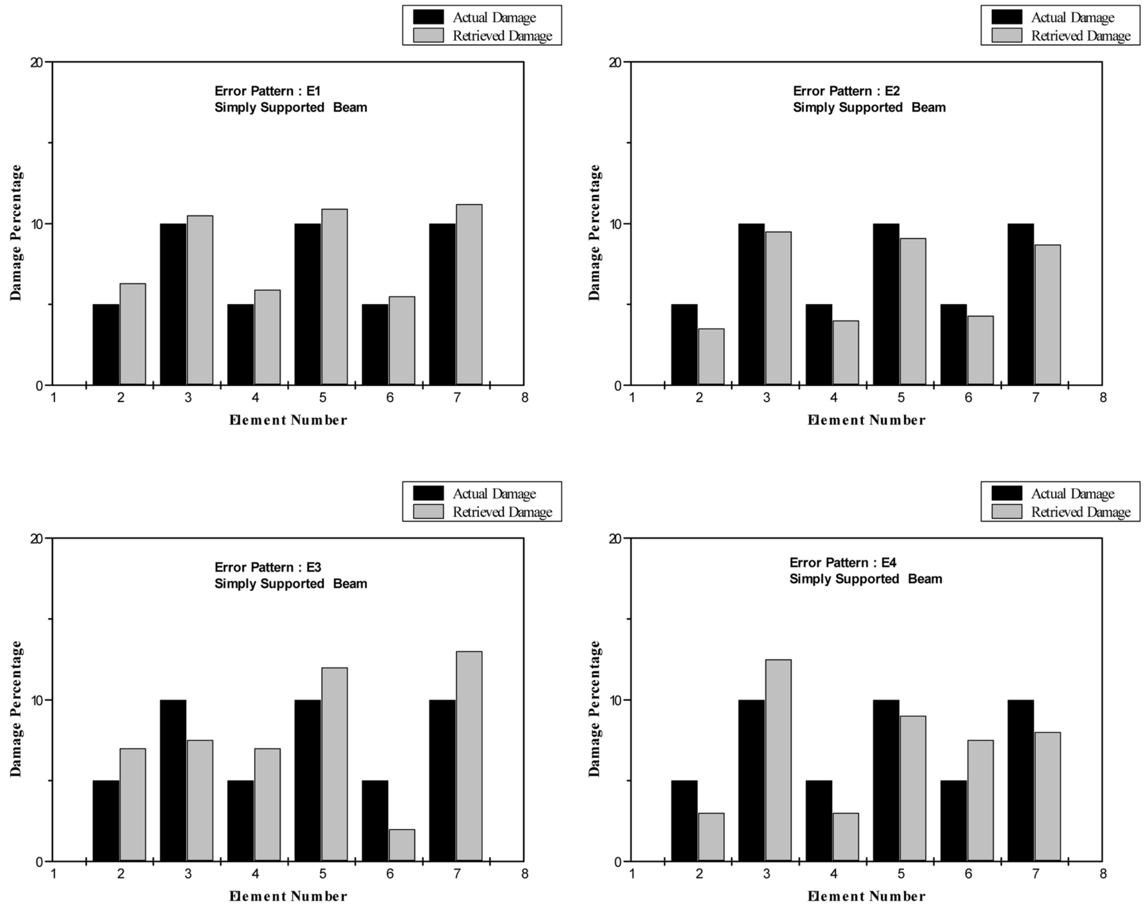

Fig. 13 Performance of damage identification method for the simply supported beam for damage pattern – P4 and for various error patterns (E1 to E4)

Table 1 Comparion of Method-I Versus Method-II

Features	Method-I	Method-II
Basis of formulation	Direct Stiffness-based	Flexibility-based
Required Input	Loads and resulting displacements and rotations, not necessarily at the point of load application	Loads and ratio of changes in resulting displacements, only at the point of load application. (Rotations need not be measured)
Obtained Output	Direct Flexural Rigidities (EI)	Changes in flexural rigidities, synonymous with damages
Comments on the method	A sort of brute-force scheme	A physical in-sight of whatever happening is possible

worse for an eight segment cantilever beam, wherein the ratio is 1000 or more. This renders the sensitivity matrix (A) ill-conditioned and affects the result. Such a scenario does not happen in a simply supported beam to this alarming effect. Although it can be said that those regions of high

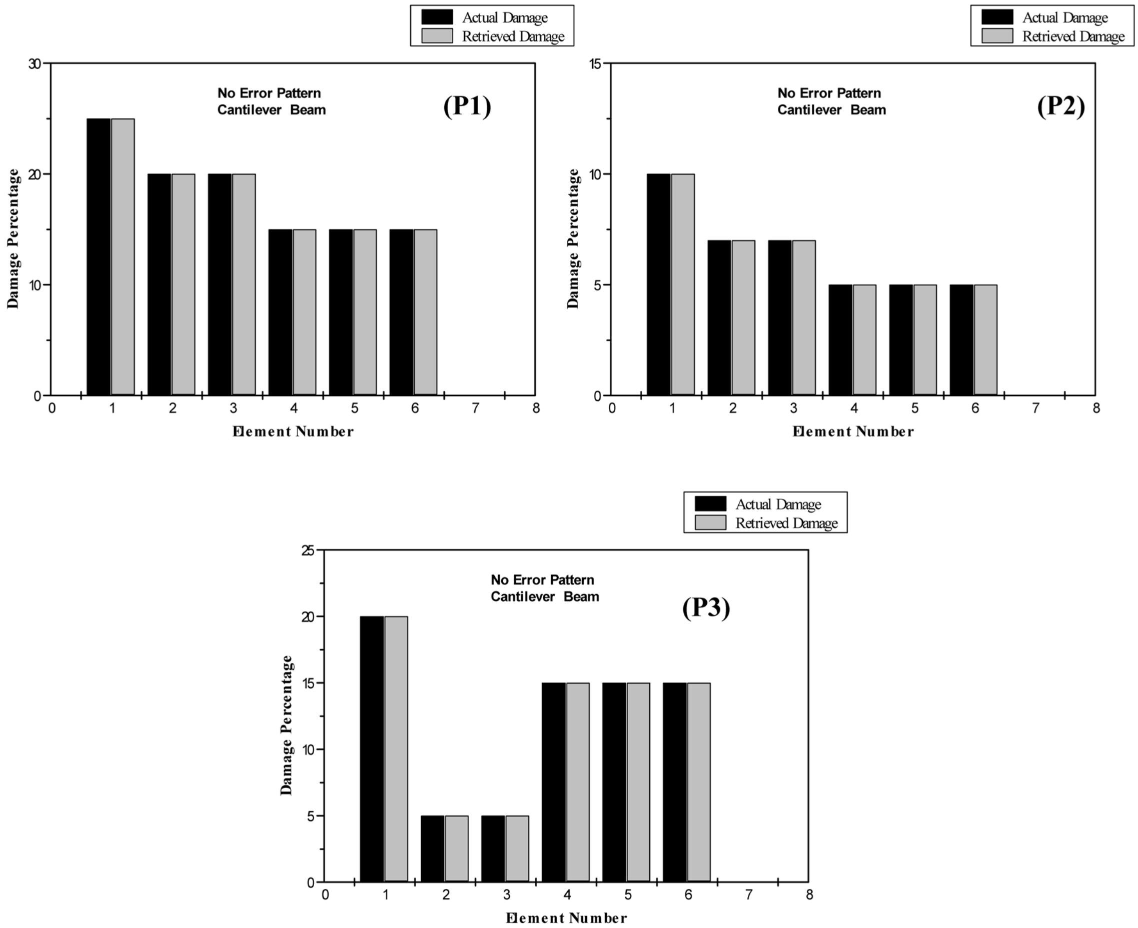

Fig. 14 Various damage patterns for damage identification for a cantilever beam

strain energy density (near support for cantilever and mid-span for simply supported beam) contribute larger to the sensitivity matrix and a damage occuring in these regions is also easily detectable.

11. Element discretisation and its effect on the results of the formulation

In this study, the discretisation of beam-like structures is kept as five or eight, coinciding the experimentally measured points with the anlytically computed nodes. This is an approximation and an inherent weakness of the formulation. It is assumed that all segments are of equal lengths and whenever, only a portion of segment is damaged, the predicted damage will give only a smeared average effect. Then the question arises as to why not increase the number of segments. There are both pros and cons to this. The measurement, obviously has to be increased to cater towards more un-knowns. The measurement cannot be also very close as the instrument resolution may limit as to how close the points can be measured.

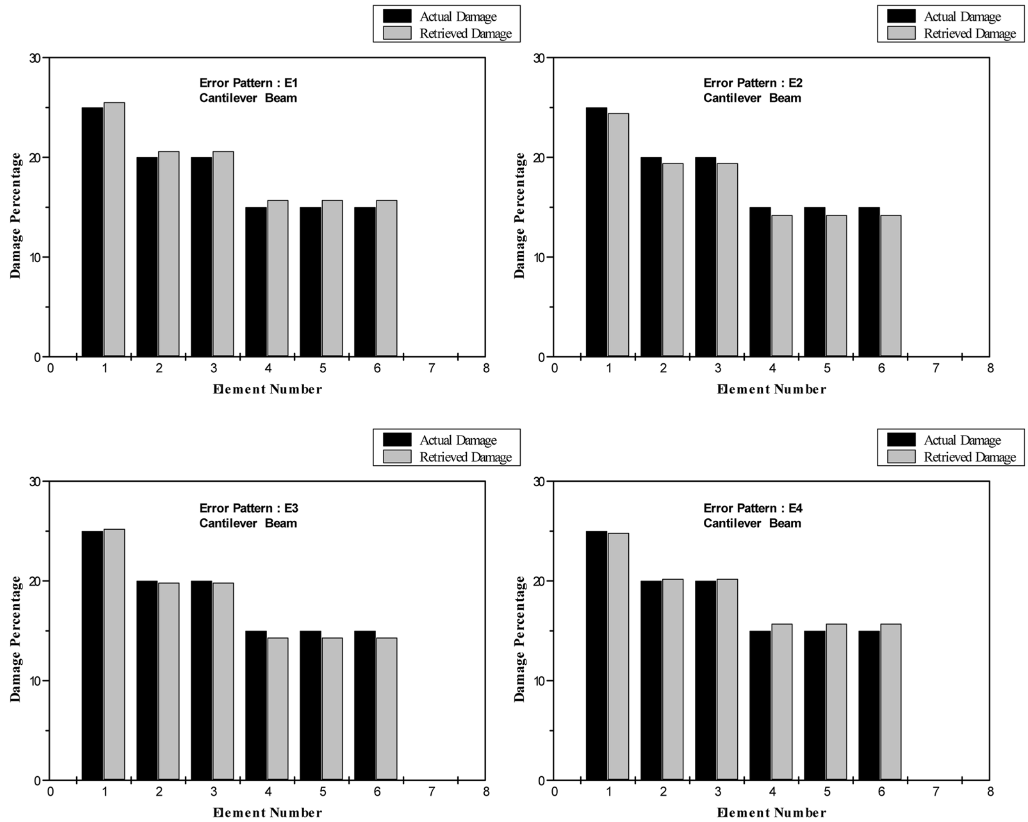

Fig. 15 Performance of damage identification method for the cantilever beam for damage pattern – P1 and for various error patterns (E1 to E4)

12. Experimental validation with a simply supported RC beam

The procedure outlined and the expression developed are validated by conducting a simple experiment on a reinforced concrete beam, simply supported and through measurement of static deflection before and after damage (Fig. 18). Rectangular beam is $100\times200\times3000$ mm, with centre to centre distance between the supports equalling 2800 mm. The beam is eccentrically loaded in a displacement controlled Instron actuator, well beyond the yield point of steel, such that cracks do not close and an eccentric damage pattern results. Static deflections are measured at seven nodes using a HBM LVDT of 10 mm stroke and a HBM carrier amplifier, MVD-555, such that resolution may go well below 1 micron. Table 2 shows the measurement in mm before and after damage as well as the ratio of change in displacements. Using this ratio (vector 'b') and having already computed the rectangular sensitivity matrix ('A'), modified damage factor is obtained and shown. The modified damage factor and the actual damage factor are also given and shown. Both the deflction pattern as well as predicted 'β' are eccentric. As a cross check to this re-generated EI, natural frequencies are computed using the this obtained values of EI and compared with the measured values before and after damage. It is seen that the ratio of change in Eigen values (squared natural frequancies) between the damaged and un-damaged states, for the first three bending modes, computed analytically, using the measured EI values are {0.68, 0.76, 0.71}. Measured ratio of differences in Eigen values are {0.695, 0.73, 0.71}, thus validating the developed method.

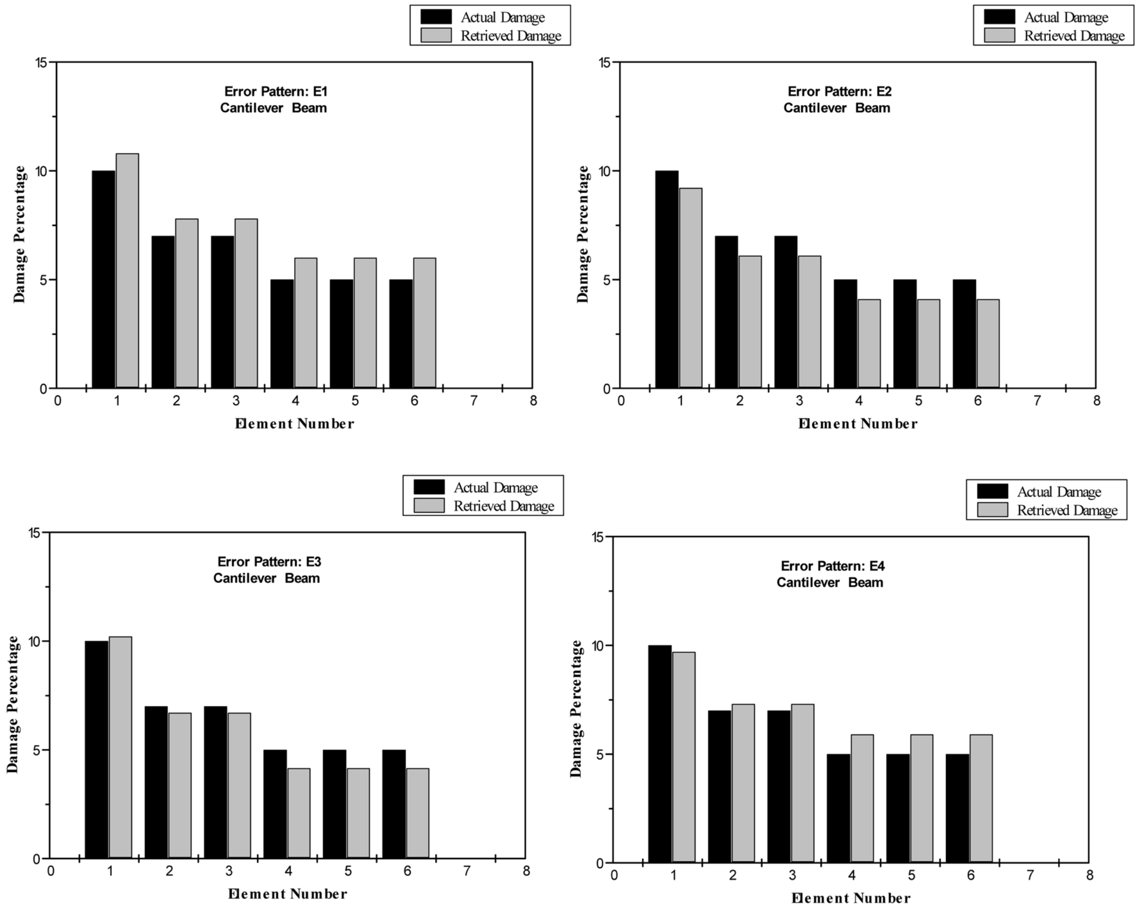

Fig. 16 Performance of damage identification method for the cantilever beam for damage pattern – P2 and for various error patterns (E1 to E4)

Table 2 Results of experiment on damage identification from static deflection measurement

Node	Deflection (mm) Un-damaged	Deflection (mm) -damaged	Ratio of Deflection Change	Element	Modified damage Factor β^*	Actual Damage Factor β
1	0.00	0.00	0.00	1	0.000	0.000
2	0.050	0.076	0.53	2	0.010	0.010
3	0.146	0.252	0.72	3	0.175	0.148
4	0.229	0.458	1.00	4	1.450	0.591
5	0.260	0.583	1.24	5	2.320	0.698
6	0.229	0.502	1.19	6	0.680	0.405
7	0.146	0.283	0.94	7	0.099	0.090
8	0.050	0.085	0.70	8	0.000	0.000
9	0.000	0.00	0.00			

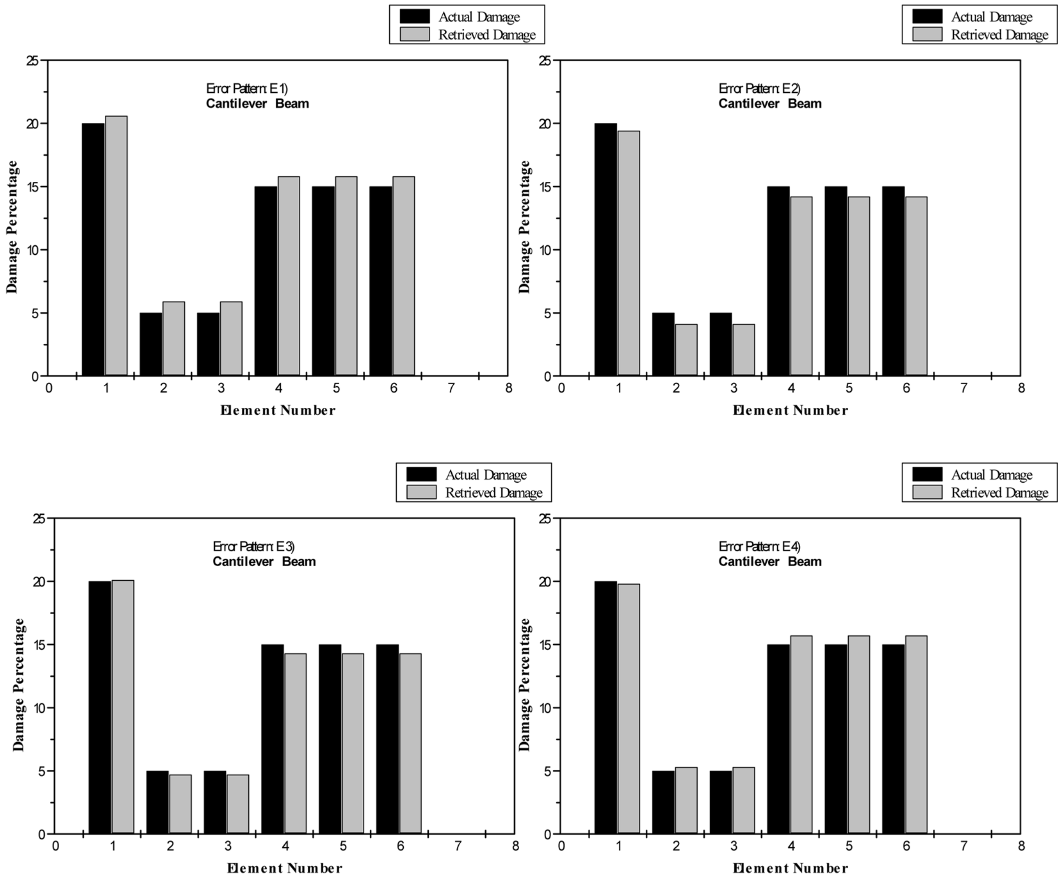

Fig. 17 Performance of damage identification method for the cantilever beam for damage pattern-P3 and for various error patterns (E1 to E4)

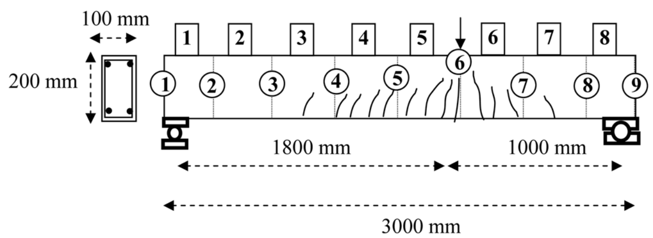

Fig. 18 A Simply supported reinforced concrete beam used in damage identification experiment

13. Damage prognosis for a generalized case – a propped cantilever

Assuming that some measurements are carried out for a healthy structure, the parameters that define the sensitivity of the measured values to a futuristic damage pattern are established. Later on, when those measurable parameters are periodically available from the structure, working on the sensitivity matrix already established, the damage pattern in terms of position and magnitude can be established. This concept on damage prognosis is an ideal wishful thinking and it is studied here

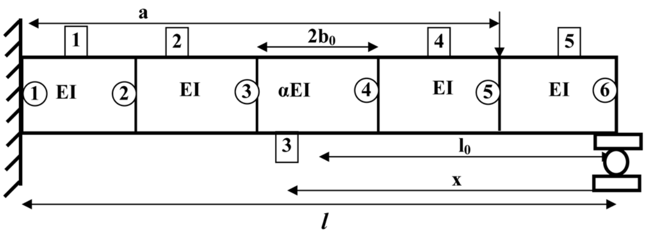

Fig. 19 A propped cantilever beam taken for static system identification studies

(through analytical simulation), whether it is possible to be realized. A generalized and in-determinate propped cantilever is chosen for the purpose.

In a determinate structure, the bending moment variation in the region of damage is in-variant with reference to the damage and hence the change in deflection due to damage becomes a linear function of damage (modified damage factor). However, in an indeterminate structure, bending moment variation in the region of damage is affected by the damage, although with smaller damage values, the deflection change might be a weak non-linear function of damage. This statement is verified using a five-segment propped cantilever beam (Fig. 19). The damaged element is varied from element 1 to element 5 (corresponding to Fig. 20 (a) to 20 (e)) and the ratio of change in deflection at each node (Node-2 to Node-5) is observed. The negative damage in the Fig. 20 is actually an increase of EI. The following points can be observed from the figures:

(a) Deflection at a node is affected more by the damage in the neighborhood elements.

(b) For a small amount of damage and around origin, the curve is linear.

(c) Reduction in section or increase in section for smaller damage (quantified by the same (β^*) essentially brings rise to the same change in displacements.

Emboldened by the last two points, the sensitivity matrix is formulated, using a small value of β^* (in the negative direction, for an increase in section) and the ratio of change in displacement ratio is normalized with β^*. In a real life bridge, due to rotation restraint of bearings and where the bridge may not be exactly simply supported, additional plates can be added, temporarily to increase the EI and the sensitivity matrix can be established.

Totally three damage patterns are investigated with and without the presence of errors. The error patterns are E1, E2, E3 and E4 as already discussed. It is also assumed that the last element (5) and second element have not suffered damage.

The three damage patterns are P1 = {0.25, 0.0, 0.15, 0.14 , 0.0}, P2 = {0.10, 0.0, 0.05, 0.07, 0.0} and P3 = {0.30, 0.0, 0.05, 0.15, 0.0]. In the absence of errors, above values are exactly retrieved from the calculated sensitivity matrix. In the presence of error

P1_E1 = {0.25, 0.0, 0.16, 0.15, 0.0}, P1_E2 = {0.24, 0.0, 0.15, 0.13, 0.0},

P1_E3 = {0.25, 0.0, 0.14, 0.14, 0.0}, P1_E4 = {0.24, 0.0, 0.16, 0.15, 0.0}

For pattern P2

P2_E1 = {0.11, 0.0, 0.06, 0.08, 0.0}, P2_E2 = {0.09, 0.0, 0.045, 0.058, 0.0}

P2_E3 = {0.11, 0.0, 0.04, 0.064, 0.0}, P2_E4 = {0.09, 0.0, 0.062, 0.076, 0.0}

For pattern P3

P3_E1 = {0.29, 0.0, 0.04, 0.16, 0.0}, P3_E2 = {0.28, 0.0, 0.03, 0.14, 0.0}

P3_E3 = {0.29, 0.0, 0.02, 0.15, 0.0}, P3_E4 = {0.28, 0.0, 0.045, 0.158, 0.0}

For example P1_E1 means damage pattern P1 subjected to error pattern E1.

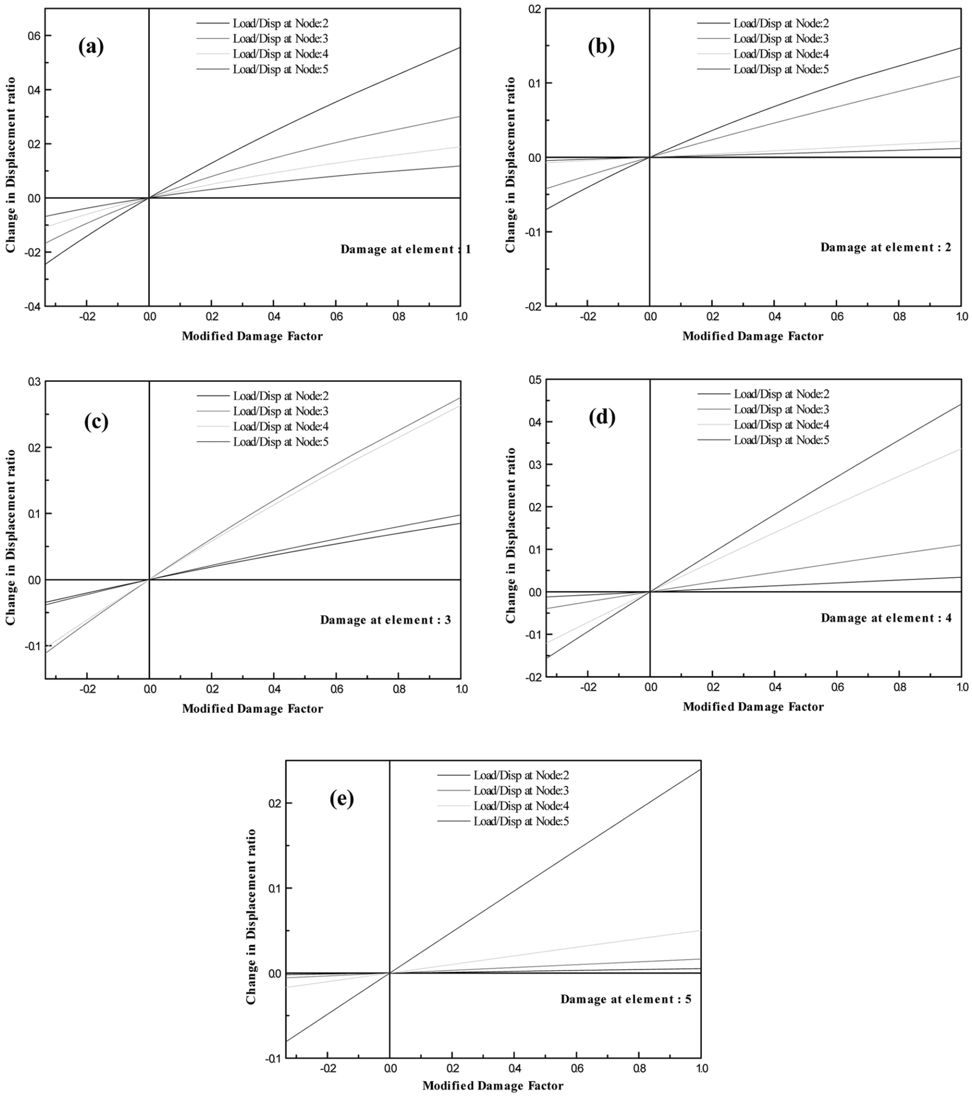

Fig. 20 Sensitivity of static deflection to damages of varying magnitudes for a propped cantilever beam (Negative damage is increase of section)

14. Conclusions

Two different methods are proposed, based on stiffness and flexibility formulations such that the inverse problem of extracting the changes in flexural rigidities (EI) can be solved, using known values of deformations. First method, directly retrieves EIs and the second method, based on deflection at the applied load point retrieves the loss ratio of EI. The divergence of results in the presence of errors due to noise and instrument resolution could be more in the first method. The matrix problem is typically over-determined, with more numbers of known values and less number of un-known EI values. A least squared error (LSE) procedure, also called as pseudo-inverse is made use of to solve the equations. The error ingress into these equations in the form of artificially added errors is studied and the distortion in results, in the presence of errors is more at low damage values, typically less than 5%.

Even though a Monte-Carlo simulation is required to study the effects of Gaussian distributed errors, it is observed that the average of a flip-flop error pattern tends towards actual values of EI and hence it is concluded that a large averaging for an un-systemetic error pattern may actually clean up the measurement from any error.

Towards establishing the robustness of the developed method an experiment is conducted on an un-symmetrically damaged reinforced concrete beam, with an applied static force less than that required for yielding of steel at critical sections. The EIs predicted by the second method is also verified by the changes in the frequencies of the system.

In a generalized case of an indeterminate structural element, where the bending moment variation in the region of damage is theoretically a function of damage, closed form expressions are difficult to formulate. However at small values of damage, typically less than 30%, the modified damage factor β^*, (β/(1-β)) is found to be a near-linear function of deflection (both in positive and negative regions near origin). This can be advantageously made use of in a damage prognosis scenario and damage-deflection sensitivity matrix can be developed, with measured deflections using artificially enhanced EI. This is demonstrated with a propped cantilever beam.

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