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Nonlinear finite element analysis of four-pile caps supporting columns subjected to generic loading

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Abstract. The paper presents the development of an adaptable strut-and-tie model that can be applied to the design or analysis of four-pile caps that support axial compression and biaxial flexure from a supported rectangular column. Due to an absence of relevant test data, the model is validated using non-linear finite element analyses (NLFEA). The results indicate that the use of the proposed model would lead to safe and economical designs. The proposed model can be easily extended to any number of piles, providing a rational procedure for the design of wide range of pile caps.

Keywords: pile caps; strut-and-tie models; flexure strength; shear strength; concrete design.

1. Introduction

In traditional design practice, pile caps are assumed to act as beams spanning between piles. The depth of a cap is then selected to provide adequate shear capacity and the required amount of longitudinal reinforcement is calculated using engineering beam theory. Quite recently, methods for the design of pile caps have been developed that are based on the strut-and-tie approach. These include the methods in the Canadian CSA Code (1984), by Schlaich, *et al.* (1987), in the AASHTO LRFD code (1994), in the Spanish concrete code EHE (1999), by Reineck (2002), and in the

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American ACI318 building code (2002). These methods assume that an internal load resisting truss, so-called strut-and-tie model, carries the forces through the pile cap in which concrete compressive struts act between the column and piles and steel ties (reinforcement) act between piles.

Results of elastic analyses, as example that one obtained by Iyer & Sam (1992), illustrate that there is a complex state of straining in these three-dimensional pile caps and that the Strut-and-Tie Theory provides a rational basis for design. Adebar, *et al*. (1990), Adebar & Zhou (1996), Bloodworth, *et al.* (2003) and Caves & Fenton (2004) have provided experimental evidence demonstrating that the use of sectional approaches based on engineering beam theory are not appropriate for the design of pile caps. As further illustrated in the research conducted by Blévot & Fremy (1967), Clarke (1972), Suzuki, *et al* (1998, 1999, 2000) and Suzuki & Otsuki (2002), many pile caps designed to fail in flexure by engineering beam theory have been reported to fail in shear. This is highly undesirable behavior as there is neither warning cracks nor pronounced deformations before these types of brittle shear failures occur.

These unexpected shear failures can be explained in two ways. Firstly, engineering beam theory was originally developed for structural elements with significant deformation capacity. As a consequence, if this theory is applied to elements with limited deformation capacity such as pile caps, the calculated effective depth will tend to overestimate the concrete contribution from shear. Secondly, engineering beam theory usually leads to more longitudinal reinforcement than would be calculated by using a strut-and-tie approach, and for the specific situation of four-pile caps, Clarke (1972) concluded that this difference can be higher than 20%. Consequently, pile caps designed using engineering beam theory have a tendency to be over reinforced and as consequence, shear failures may occur as a result of longitudinal splitting of compression struts before yielding of the longitudinal reinforcement.

Although the strut-and-tie approach provides a more rational basis for the design of pile caps, it is only commonly applied for the design of simple pile caps such as pile caps supporting square columns subjected to axial load. This is believed to be due to the complexity and uncertainties as to the appropriate strut-and-tie model to use for more complex loading conditions. Thus, designers have chosen to rely on the use of engineering beam theory for the design of even slightly more complex pile caps, including four-pile caps that support axial compression and biaxial flexure from a single rectangular column.

To address the situation of pile caps supporting columns under general situation (axial compression and biaxial flexure), an adaptable strut-and-tie model for four-pile caps is proposed in this paper. Unfortunately there is no experimental test data on the performance of this type of four-pile caps. Thus, non-linear finite element analysis (NLFEA) has been applied to make the best possible prediction of the behavior of these pile caps. A NLFEA program was selected for use that was specifically written for predicting the behavior of a three-dimensional continuum of structural concrete subjected to a complex state of stress. This program will be validated herein by available test data. The result of the analyses of four-pile caps supporting axial compression and biaxial flexure from a single column will illustrate the appropriateness of the proposed model. This model can be further extended for the design of more complex pile caps.

2. An adaptable strut-and-tie model to the design of four-pile caps

The proposed model is an adaptable 3-dimensional strut-and-tie model, which can be used for the

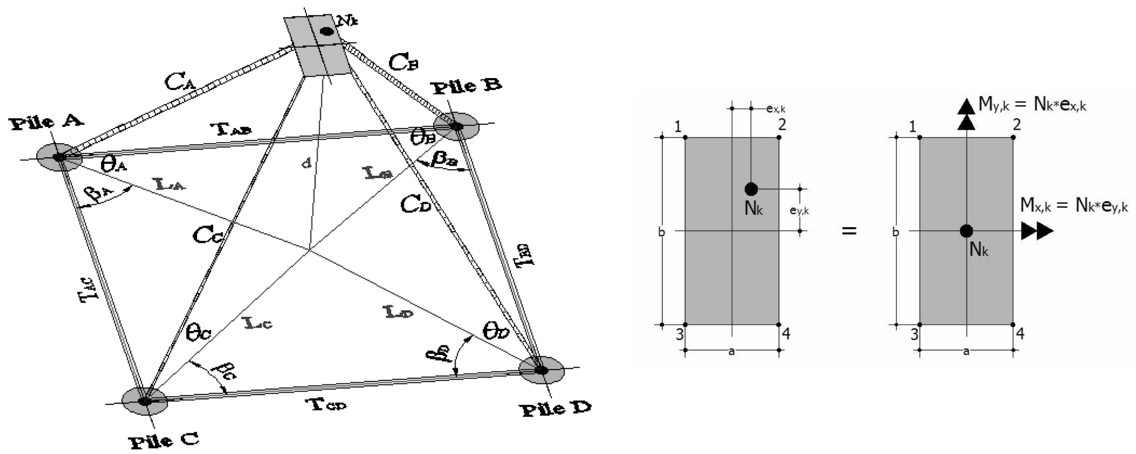

Fig. 1 Proposed strut-and-tie model for four-pile caps and positive signal convention for biaxial flexure

design or analysis of four-pile caps supporting square or rectangular columns subjected to the generic loading conditions of axial compression load and biaxial flexure. This model is presented in Fig. 1. In the proposed model, axial compression forces are taken as negative, tensile forces are taken as positive, and the net axial load acting from the column on the pile cap is always compressive.

The axial compression and biaxial flexure acting on the square or rectangular columns can be statically substituted by a single compressive axial load, which has the nominal eccentricities presented in Eqs. (1) and (2):

$$e_{x,k} = \frac{-M_{y,k}}{N_k} \leq \pm\frac{a}{2} \tag{1}$$

$$e_{y,k} = \frac{M_{x,k}}{N_k} \leq \pm\frac{b}{2} \tag{2}$$

Nominal reactions of the piles are calculated using Eqs. (3) to (6), and in order to keep the validity of the proposed model, no tensile piles are permitted in the present formulation:

$$R_{A,k} = R_{B,k}\frac{tg\theta_A \sin\beta_B}{tg\theta_B \sin\beta_A} \leq 0 \tag{3}$$

$$R_{B,k} = \frac{N_k}{1 + \frac{tg\theta_A \sin\beta_B}{tg\theta_B \sin\beta_A} + \frac{tg\theta_D \cos\beta_B\, tg\theta_C \cos\beta_B}{tg\theta_B \sin\beta_D\, tg\theta_D \cos\beta_C} + \frac{tg\theta_D \cos\beta_B}{tg\theta_B \sin\beta_D}} \leq 0 \tag{4}$$

$$R_{C,k} = R_{D,k}\frac{tg\theta_C \cos\beta_B}{tg\theta_D \cos\beta_C} \leq 0 \tag{5}$$

$$R_{D,k} = R_{B,k}\frac{tg\theta_D \cos\beta_B}{tg\theta_B \sin\beta_D} \leq 0 \tag{6}$$

In order to calculate the angles between the idealized struts and ties, it is first necessary to calculate the projections of struts on the horizontal plane, as show in Eqs. (7) to (10):

$$L_A = \sqrt{(0,5e+e_{x,k})^2+(0,5e-e_{y,k})^2} \tag{7}$$

$$L_B = \sqrt{(0,5e-e_{x,k})^2+(0,5e-e_{y,k})^2} \tag{8}$$

$$L_C = \sqrt{(0,5e+e_{x,k})^2+(0,5e+e_{y,k})^2} \tag{9}$$

$$L_D = \sqrt{(0,5e-e_{x,k})^2+(0,5e+e_{y,k})^2} \tag{10}$$

The angles between struts and ties are calculated by Eqs. (11) to (18), as follows:

$$\theta_A = \tan^{-1}\frac{d}{L_A} \tag{11}$$

$$\theta_B = \tan^{-1}\frac{d}{L_B} \tag{12}$$

$$\theta_C = \tan^{-1}\frac{d}{L_C} \tag{13}$$

$$\theta_D = \tan^{-1}\frac{d}{L_D} \tag{14}$$

$$\beta_A = \tan^{-1}\frac{(0,5e+e_{x,k})}{(0,5e-e_{y,k})} \tag{15}$$

$$\beta_B = \tan^{-1}\frac{(0,5e-e_{x,k})}{(0,5e-e_{y,k})} \tag{16}$$

$$\beta_C = \tan^{-1}\frac{(0,5e+e_{y,k})}{(0,5e+e_{x,k})} \tag{17}$$

$$\beta_D = \tan^{-1}\frac{(0,5e+e_{y,k})}{(0,5e-e_{x,k})} \tag{18}$$

Nominal axial forces acting in concrete struts are calculated using Eqs. (19) to (22) while nominal axial forces acting on steel ties are determined using Eqs. (23) to (26), as follows:

$$C_{A,k} = \frac{R_{A,k}}{\sin\theta_A} \leq 0 \tag{19}$$

$$C_{B,k} = \frac{R_{B,k}}{\sin\theta_B} \leq 0 \tag{20}$$

$$C_{C,k} = \frac{R_{C,k}}{\sin\theta_C} \leq 0 \tag{21}$$

$$C_{D,k} = \frac{R_{D,k}}{\sin\theta_D} \leq 0 \tag{22}$$

$$T_{AB,k} = -C_{A,k}\cos\theta_A\sin\beta_A = -C_{B,k}\cos\theta_B\sin\beta_B \geq 0 \tag{23}$$

$$T_{CD,k} = -C_{C,k}\cos\theta_C\cos\beta_C = -C_{D,k}\cos\theta_D\cos\beta_D \geq 0 \tag{24}$$

$$T_{AC,k} = -C_{A,k}\cos\theta_A\cos\beta_A = -C_{C,k}\cos\theta_C\sin\beta_C \geq 0 \tag{25}$$

$$T_{BD,k} = -C_{B,k}\cos\theta_B\cos\beta_B = -C_{D,k}\cos\theta_D\sin\beta_D \geq 0 \tag{26}$$

Once the nominal forces acting in ties are known, the amount of reinforcement for each tie can be calculated by applying the necessary safety factors and taking into account the yielding of the reinforcement, as show in Eqs. (27) to (30):

$$A_{s,AB} = \frac{\gamma T_{AB,k}}{\phi f_y} \tag{27}$$

$$A_{s,AC} = \frac{\gamma T_{AC,k}}{\phi f_y} \tag{28}$$

$$A_{s,CD} = \frac{\gamma T_{CD,k}}{\phi f_y} \tag{29}$$

$$A_{s,BD} = \frac{\gamma T_{BD,k}}{\phi f_y} \tag{30}$$

Eqs. (27) to (30) give the minimum required amount of concentrated reinforcement for each tie. If biaxial moment is acting on the column, the amount of reinforcement is expected to be different for the ties. Taking in account the possibility of inaccurate positioning of these different reinforcements in the field, the largest calculated tie reinforcement in each direction may be provided for both ties in that direction, as shown in Eqs. (31) and (32):

$$A_{sx,tie} \geq \begin{cases} A_{s,AB} \\ A_{s,CD} \end{cases} \tag{31}$$

$$A_{sy,tie} \geq \begin{cases} A_{s,AC} \\ A_{s,BD} \end{cases} \tag{32}$$

Finally, in order to avoid a shear failure, herein represented by a longitudinal splitting of the compressive struts, the maximum compressive stress acting on the column should be limited to a certain portion of the concrete compressive strength. This additional verification can be made by evaluating the highest compressive stress (σ_{max}) acting in the corners of the column, as shown in Eq. (33). The stresses acting in the corners of the columns can be calculated by using Eqs. (34) to (37).

$$\sigma_{max} \leq \sigma_{1,C};\ \ \sigma_{2,C};\ \ \sigma_{3,C};\ \ \sigma_{4,C} \tag{33}$$

$$\sigma_{1,C} = \left(\frac{N_k}{a.b} + \frac{M_{x,k}}{a.b^2/6} + \frac{M_{y,k}}{b.a^2/6}\right) \tag{34}$$

$$\sigma_{2,C} = \left(\frac{N_k}{a.b} + \frac{M_{x,k}}{a.b^2/6} - \frac{M_{y,k}}{b.a^2/6}\right) \tag{35}$$

$$\sigma_{3,C} = \left(\frac{N_k}{a.b} - \frac{M_{x,k}}{a.b^2/6} + \frac{M_{y,k}}{b.a^2/6}\right) \tag{36}$$

$$\sigma_{4,C} = \left(\frac{N_k}{a.b} + \frac{M_{x,k}}{a.b^2/6} - \frac{M_{y,k}}{b.a^2/6}\right) \tag{37}$$

Once the maximum stress acting on the column is found, this value is checked against the maximum permissible stress, proposed in order to avoid the possibility of longitudinal splitting before reinforcement yielding. The additional recommended verification is proposed in Eq. (38) and the maximum admissible pressure for the column, based on the factor λ, is discussed latter.

$$|\sigma_{\max}| \le \sigma_{\lim} = \lambda f_c \tag{38}$$

3. Application of NLFEA to available experimental data of pile caps

Prior to the use of a NLFEA program for evaluating this adaptable strut-and-tie model, it is first necessary to evaluate the ability of this program to predict the behavior of tested pile caps. This was completed using the experimental data from four-pile caps tested by Suzuki, *et al.* (1998). Table 1 presents the dimensions of these pile caps as well as the measured cracking, yielding, and ultimate strengths.

Table 1 Properties and average results of the four-pile caps tested by Suzuki, *et al* (1998)

Specimen	Layout	L (m)	d (m)	e (m)	$a=b$ (m)	f_c (MPa)	f_y (MPa)	A_{sx} A_{sy}	$\|N_{crack}\|$ (kN)	$\|N_{yield}\|$ (kN)	$\|N_{\max}\|$ (kN)
BP-20-30-1,2	Grid	0,8	0,15	0,50	0,30	29,45	413	6ϕ10 mm	215,50	475,00	482,50
BPC-20-30-1,2	Bunched	0,8	0,15	0,50	0,30	29,80	413	6ϕ10 mm	230,00	490,00	497,50
BP-30-25-1,2	Grid	0,8	0,25	0,50	0,30	28,60	413	8ϕ10 mm	377,50	784,00	759,50
BPC-30-25-1,2	Bunched	0,8	0,25	0,50	0,30	29,15	413	8ϕ10 mm	363,00	833,00	862,50
BP-30-30-1,2	Grid	0,8	0,25	0,50	0,25	27,90	413	8ϕ10 mm	441,00	907,00	911,50
BPC-30-30-1,2	Bunched	0,8	0,25	0,50	0,25	29,90	413	8ϕ10 mm	411,50	1029,00	1034,00

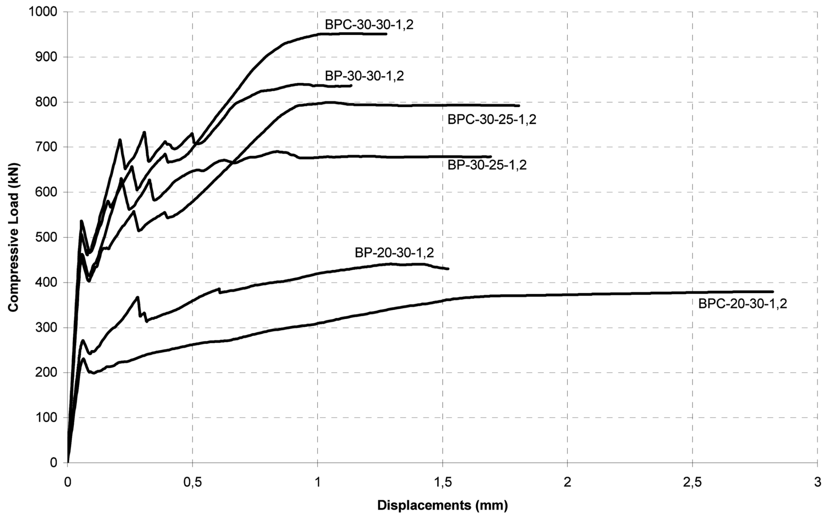

Fig. 2 Numerical load-displacement behavior obtained for the four-pile caps tested by Suzuki, *et al* (1998)

For modeling the concrete behavior, a fracture-plastic model based on the classical orthotropic smeared crack formulation (CC3 Non Lin Cementitious 2) implemented by Cervenka, *et al.* (2005) was applied. Reinforcements were modeled using an embedded formulation and the Newton-Raphson solution method was applied for the solution scheme. Boundary conditions and material properties were defined in order to accurately represent the described experimental setup and the overall response was recorded using monitoring points for loading (at the top of the column) and displacements (at the center bottom of the pile caps)

Fig. 2 presents the predicted load-displacement behavior for the simulated four-pile caps using NLFEA. Table 2 presents in details some comparisons between the experimental results obtained by

Table 2 Comparison between experimental data obtained by Suzuki, *et al.* (1998) and numerical predictions

Specimen	Cracking Loads (kN)			Yielding Loads (kN)			Maximum Loads (kN)		
	$\|N_{exp}\|$ (1)	$\|N_{num}\|$ (2)	$\|N_{exp}\|/\|N_{num}\|$ (3)	$\|N_{exp}\|$ (4)	$\|N_{num}\|$ (5)	$\|N_{exp}\|/\|N_{num}\|$ (6)	$\|N_{exp}\|$ (7)	$\|N_{num}\|$ (8)	$\|N_{exp}\|/\|N_{num}\|$ (9)
BP-20-30-1,2	215,50	243,40	0,89	475,00	381,90	1,24	482,50	440,60	1,10
BPC-20-30-1,2	230,00	195,30	1,18	490,00	312,00	1,57	497,50	379,60	1,31
BP-30-25-1,2	377,50	380,10	0,99	784,00	654,60	1,20	759,50	690,00	1,10
BPC-30-25-1,2	363,00	382,30	0,95	833,00	709,40	1,17	862,50	799,20	1,08
BP-30-30-1,2	441,00	437,30	1,01	907,00	780,30	1,16	911,50	839,70	1,09
BPC-30-30-1,2	411,50	465,20	0,88	1029,00	835,50	1,23	1034,00	951,90	1,09
		Mean	0,98		Mean	1,26		Mean	1,13
		S.D.	0,11		S.D.	0,15		S.D.	0,09
		C.V.	0,11		C.V.	0,12		C.V.	0,08

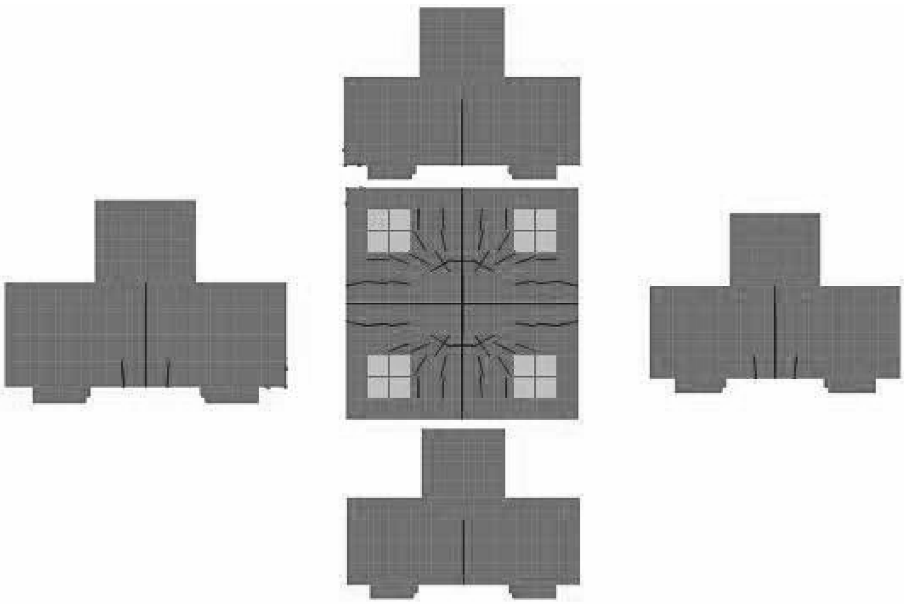

Fig. 3 Predicted crack pattern at failure for pile cap BPC-30-30-1,2 (only crack widths over 0,2 mm)

Suzuki, *et al.* (1998) and the numeric predictions from the NLFEAs. The non-linear predictions were reasonably close to the measured experimental results, leading to coefficients of variations that were less than 15%.

While calculated displacements were generally lower than experimental displacements, the cracking patterns and failure modes were quite well predicted. Fig. 3 presents the typical radial crack pattern predicted for four-pile caps with bunched reinforcement, which is a very similar pattern to that observed by Suzuki, *et al.* (1998).

4. Validation of the proposed strut-and-tie model using NLFEA

Based on the accurate quantitative as well as qualitative performance obtained in the previous simulations, the necessary confidence in this NLFEA program was obtained for reliably predicting the behavior of four-pile caps designed with the proposed methodology. For these additional investigations, four-pile caps subjected to the same loading conditions and different heights (0,38 <= *c/d* <= 0,70), as shown in Fig. 4, were designed using the proposed model and further analyzed using the nonlinear potentialities of DIANA software (Cervenka, *et al.* (2000)).

In order to evaluate the performance of the proposed model, the amount of reinforcement were obtained by setting load factors and material reduction factors to 1.0. The same methodology of not applying safety factor was used when defining material properties in the selected commercial finite element software, i.e., characteristics strength for steel and concrete were defined.

The NLFEA differs from what was done for the model validation in that eccentric displacements were applied at the top of the column rather than a constant displacement applied in the centroid of the column. The constant eccentricities of the load condition were defined as e_x = 0,046 m and e_y = 0,092 m, obtained as a result of the division of the biaxial flexure loads ($M_{x,k}$ = -57,1 kN.m and $M_{y,k}$

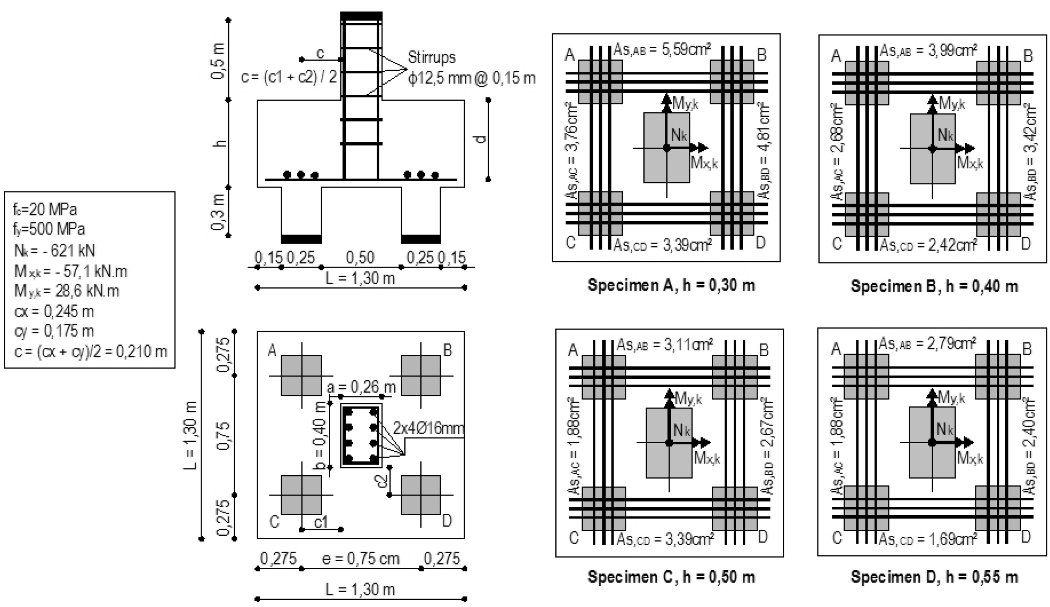

Fig. 4 Four-pile caps designed using the proposed methodology and analyzed using NLFEA

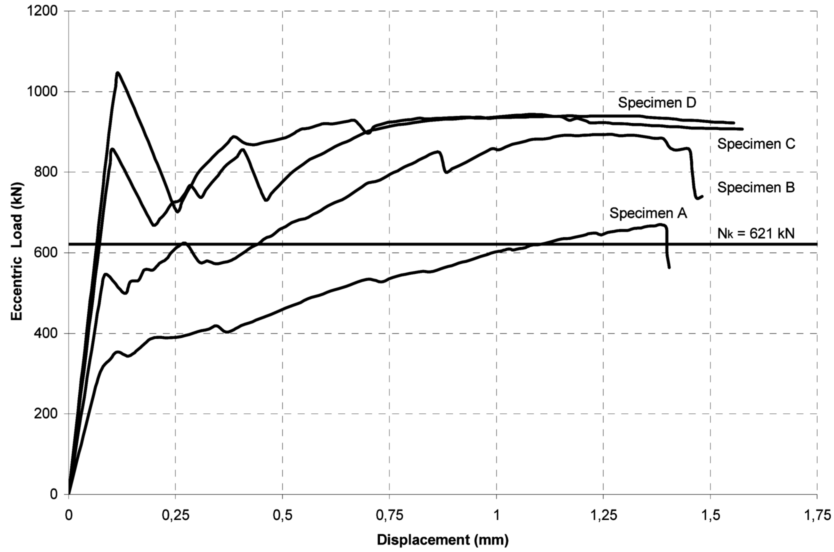

Fig. 5 Load-displacement behavior for the four-pile caps designed using the proposed methodology

Table 3 Numerical results of the four-pile caps designed using the proposed model

Specimen	c/d	h (m)	d (cm)	Struts Angles	N_{crack} (kN)	N_{yield} (kN)	N_{max} (kN)	$\sigma_{c,c}$ (MPa)	$\sigma_{c,y}$ (MPa)	$\sigma_{c,u}$ (MPa)	Failure Mode
A	0,70	0,30	0,25	θ_A = 26,23° θ_B = 29,95° θ_C = 21,68° θ_D = 23,64°	287,40	-	669,50	$0{,}48f_c$	-	$1{,}11f_c$	Shear
B	0,53	0,40	0,35	θ_A = 34,60° θ_B = 38,89° θ_C = 29,10° θ_D = 31,50°	514,40	785,30	894,10	$0{,}85f_c$	$1{,}30f_c$	$1{,}48f_c$	Flexure
C	0,42	0,50	0,45	θ_A = 41,57° θ_B = 46,04° θ_C = 35,59° θ_D = 38,23°	702,60	795,00	943,20	$1{,}16f_c$	$1{,}32f_c$	$1{,}56f_c$	Flexure
D	0,38	0,55	0,50	θ_A = 44,58° θ_B = 49,04° θ_C = 38,49° θ_D = 41,20°	896,10	-	1044,00	$1{,}48f_c$	-	$1{,}73f_c$	Shear

= 28,6 kN.m) by the compressive axial load (N_k = - 621 kN).

Fig. 5 presents the load-displacement behavior obtained by using monitoring points at the top of the columns and at the middle bottom of the pile caps. As can be seen, all designed pile caps using the proposed methodology present a maximum eccentric load at fixed eccentricities higher than the nominal eccentric load used for the design of the longitudinal reinforcements.

Table 3 presents the primary results obtained for the designed pile caps including the predicted cracking, yielding and maximum eccentric loads. This table also presents the angle of the initial

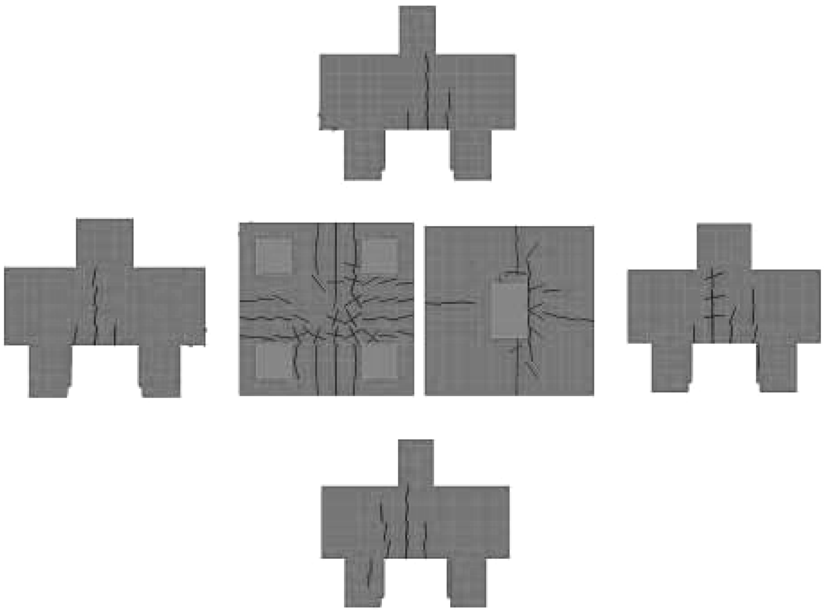

Fig. 6 Crack pattern at maximum load for the Specimen C (only crack widths over 0,1 mm)

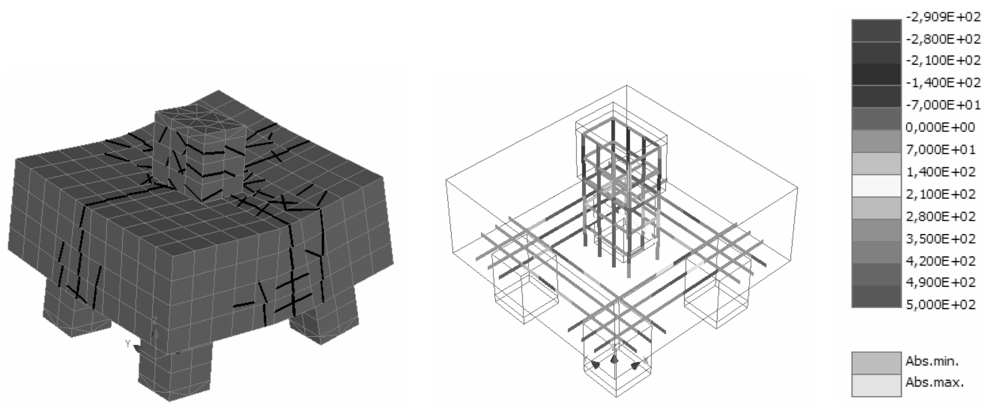

Fig. 7 Deformed shape at maximum eccentric load and stress in the reinforcements ties for the Specimen C

struts obtained using the proposed model as well as the calculated maximum stresses acting in the column, by application of the Eqs. (33) to (37) using the numerically predicted capacity.

Typical failure cracks for the designed four-pile caps supporting a rectangular column subjected to biaxial flexure and compression load are shown in Fig. 6. Typical deformed shapes at maximum load, as well as, the principal stress acting in the ties are shown in Fig. 7 for Specimen C.

In order to assess the concrete contribution on the capacity of the pile caps, additional analyses were conducted of unreinforced pile caps. Fig. 8 present the eccentric load-displacement behavior of non-reinforced pile caps of different heights. As can be seen, for pile caps with heights over 50 cm, no longitudinal reinforcement would be necessary to support the design loads. These results show that concrete tensile strength, often neglected in structural codes, is a critically important factor in

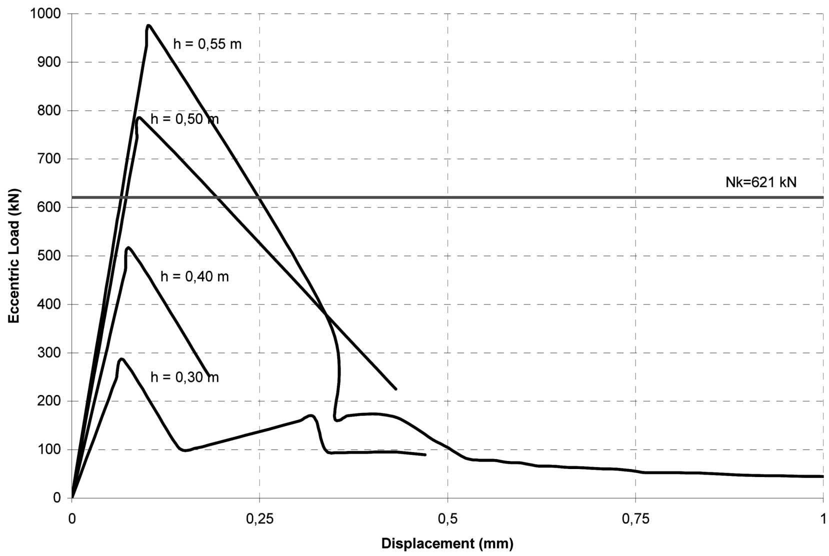

Fig. 8 Load-displacement behavior for the four-pile caps without adopting longitudinal reinforcement

the design of stocky member such as pile caps. Taking into account that safety factors are additionally applied to the design, it is very clear that a large portion of some pile caps will be reinforced.

5. Concluding remarks

Due to the lack of a generic strut-and-tie model for the design of pile caps to support realistically complex loadings from columns, designers commonly use engineering beam theory or very simplified strut-and-tie models for the design of pile caps. In the latter approach, knowing the piles reactions due to the simultaneous action of biaxial flexure and axial compression, the highest reaction is multiplied by the number of piles, in a manner that an equivalent compressive axial load is found. This equivalent load is then used for the design of the pile cap, by using the available strut-and-tie models developed for pile caps supporting square columns under the basic situation of axial compression.

To encourage the use of more appropriate design procedures for pile caps, an adaptable 3-dimensional strut-and-tie model was presented in this paper. The main strength of the proposed model is that it provides a clear methodology for calculating the design forces and capacity of four-pile caps supporting columns subjected to axial compressive load and biaxial flexure. The proposed methodology is shown by analyses to result in safe and economical design solutions.

The performance of the proposed model was evaluated using non-linear analyses. The results show that the predicted capacities are greater than those calculated from the adaptable strut-and-tie model. As was presented in Fig. 5, the lower the shear span-to-depth ratio, c/d, the higher was the failure load. However, as the same strut-and-tie model was applied for the same loading condition, the same capacity at reinforcement yielding and at failure would have been expected for all specimens. The differences in the predicted behavior can be explained by the significant influence of the concrete tensile strength in the bottom region of the pile caps, which is not considered in the present formulation and in most codes of practice.

In order to obtain ductile behavior at the ultimate state, two conditions should be considered in the use of the presented model. Firstly, based on the results shown in Table 3, a maximum compressive stress for the column under $1{,}0f_c$ as proposed by Adebar, *et al.* (1990), helps to ensure a safe design and can be introduced in the present formulation by adopting λ = 1,0 in Eq. (38). Secondly, a minimum angle for the struts should be provided in order to increase the final shear strength of the pile caps. Numerical results showed that yielding were only possible when the inclination of the struts were between 29º and 46º. If the design intention is to yield the reinforcement (flexure failure) before longitudinal splitting of the concrete struts (shear failure), then the two previous recommendations should be followed.

The proposed adaptable strut-and-tie model is considered to provide a more rational basis for the design and analysis of four-pile caps. Even so, it should be noted that the proposed model may lead to the use of more than necessary amounts of longitudinal tension reinforcement. The numerical simulations illustrated the capacity provided by the concrete alone would support most service loads. This implies that field experience should not provide a good indication of the appropriateness of design practice.

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Notation

N_k = Nominal axial loading acting on the column;
$M_{x,k}$; $M_{y,k}$ = Nominal flexure loading acting from the column on the pile cap about the x and y-axes;
$e_{x,k}$; $e_{y,k}$ = Nominal eccentricities of the load from the x and y-axes;
a, b = Column dimensions;
$R_{A,k}$; $R_{B,k}$; $R_{C,k}$; $R_{D,k}$ = Nominal pile reactions for piles A, B, C and D, respectively;
θ_A = Angle between projection LA and Strut A;
θ_B = Angle between projection LB and Strut B;
θ_C = Angle between projection LC and Strut C;
θ_D = Angle between projection LD and Strut D;
β_A = Angle between projection LA and Tie AC;
β_B = Angle between projection LB and Tie BD;
β_C = Angle between projection LC and Tie CD;
β_D = Angle between projection LD and Tie CD;
L_A, L_B, L_C, L_D = Horizontal projections of the struts A, B, C and D, respectively;
e = Pitch between center of piles;
c = Average distance between face column and pile centers;
$c1$, $c2$ = Distance between face columns and pile center in x and y-directions, respectively;
c/d = Shear span-to-depth ratio;
L = Pile cap length and width;
h = Pile cap height;
d = Effective height;
p = pile diameter or width;
$C_{A,k}$; $C_{B,k}$; $C_{C,k}$; $C_{D,k}$ = Nominal forces acting in the struts A, B, C and D, respectively;
$T_{AC,k}$; $T_{BD,k}$; $T_{CD,k}$; $T_{AB,k}$ = Nominal forces acting in the ties AC, BD, CD and AB, respectively;
γ, ϕ = Safety factor for loads and strength reduction factor for materials, respectively;

A_{sAC}, $A_{s,BD}$, $A_{s,CD}$, $A_{s,AB}$ = Demanded reinforcement for ties AC, BD, CD and AB, respectively;
$\sigma_{1,c}$; $\sigma_{2,c}$; $\sigma_{3,c}$; $\sigma_{4,c}$; = Stresses acting at the corners of the column;
σ_{max} = Maximum compressive stress acting at the corners of the column;
$\sigma_{c,c}$; $\sigma_{c,y}$; $\sigma_{c,u}$ = Maximum compressive stress at the corners of column for the crack, yield and ultimate (maximum) loads;
f_c = Concrete compression strength;
f_y = Steel yielding strength.

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