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Artificial neural network model for the strength prediction of fully restrained RC slabs subjected to membrane action

Khandaker M. A. Hossain†, Mohamed Lachemi‡ and Said M. Easa‡†

Dept. of Civil Engineering, Ryerson University, 350 Victoria Street, Toronto, ON, M5B 2K3, Canada
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Abstract. This paper develops an artificial neural network (ANN) model for uniformly loaded restrained reinforced concrete (RC) slabs incorporating membrane action. The development of membrane action in RC slabs restrained against lateral displacements at the edges in buildings and bridge structures significantly increases their load carrying capacity. The benefits of compressive membrane action are usually not taken into account in currently available design methods based on yield-line theory. By extending the existing knowledge of compressive membrane action, it is possible to design slabs in building and bridge decks economically with less than normal reinforcement. The processes involved in the development of ANN model such as the creation of a database of test results from previous research studies, the selection of architecture of the network from extensive trial and error procedure, and the training and performance validation of the model are presented. The ANN model was found to predict accurately the ultimate strength of fully restrained RC slabs. The model also was able to incorporate strength enhancement of RC slabs due to membrane action as confirmed from a comparative study of experimental and yield line-based predictions. Practical applications of the developed ANN model in the design process of RC slabs are also highlighted.

Keywords : artificial neural network; membrane action; reinforced concrete slab; ultimate strength, yield-line method.

1. Introduction

The ultimate strength of a reinforced concrete (RC) slab horizontally restrained at the edges is affected by the development of compressive membrane forces. The effect of compressive membrane action has been recognized since the first half of the 20th century (Ockleston 1955). Since then many researchers have looked into compressive membrane action (Park 1964a-b, Powell 1956, Wood 1961, Kirkpatrick, *et al.* 1984, Rankin, *et al.* 1991, Eyre 1997). These researches have shown that slabs in buildings and bridge decks, which are restrained against lateral displacements at the edges, have ultimate strengths that are far in excess of those predicted by conventional design methods, which are based on flexural or yield-line theory (Johansen 1962). The enhancement in strength has been attributed to membrane action which is due to the in-plane forces developed at the supports. Hence by utilizing the advantage of compressive membrane action, it should be possible

† Adjunct Professor, Corresponding author, E-mail: ahossain@ryerson.ca; dr_k_m_hossain@hotmail.com
‡ Associate Professor and Canada Research Chair in Sustainable Construction
‡†Professor

to produce slabs in building or bridge decks with less than normal reinforcement.

While the existence of compressive membrane action is commonly acknowledged, the benefits of compressive membrane action are usually not taken into account in design or assessment methods (Das 2001). In recent times, research has been conducted over a wide range of building and bridge structures to understand and incorporate the beneficial effect of membrane action in structural design (Taylor 2000, Rankin, *et al*. 1999, Huang, *et al*. 2003a-b). More recently, the finite element method has also been used to model the membrane action in reinforced concrete slabs (Huang, *et al*. 2003a-b, Hossain and Olufemi 2004).

Artificial neural network (ANN) modeling has been applied to various civil engineering problems, such as structural damage detection (Wu, *et al.* 1992), structural system identification (Pal, *et al.* 1994), material behavior modeling (Ghaboussi, *et al.* 1991, Oh, *et al.* 1999, Nehdi, *et al.* 2001), structural optimization (Adeli, *et al.* 1995), structural control (Chen, *et al.* 1995, Lee, *et al.* 2001), and groundwater monitoring (Ranjithan, *et al.* 1993). Unlike traditional parametric models, an ANN does not have to assume a model form between input and output variables. The ANN consists of multiple layers of many interconnected linear or nonlinear processing units operating in a parallel fashion. Each processing unit receives multiple inputs through weighted connections from neurons in the previous layer to which it is connected, performs appropriate computation, and transmits its output to other processing units or as a network output using an assigned transfer function. The nonlinear nature of neural networks makes them suitable to perform functional approximation, classification, and pattern recognition.

The objective of this paper is to illustrate that a non-parametric approach based on ANN can effectively be used to predict more accurately the ultimate strength of fully restrained RC slabs where membrane action is prominent. The paper presents the development of an ANN model for the prediction of ultimate strength of uniformly loaded, fully restrained RC slabs based on available test results. The training and testing of the ANN model and its ability to incorporate membrane action are also described. The practical applications of reliable ANN predictions incorporating strength enhancement of RC slabs due to membrane action can lead to economical design and will be useful for professionals involved in the design and construction of building and bridge structures.

2. Basics of artifical neural network approach

2.1. General neural network analysis

A neural network consists of many simple processors that are called units, nodes, or neurons. Each of the units has a small amount of local memory. The units are connected by communication channels that are called connections; each usually carries numeric data, called strengths or weights, encoded by any of various means. The units operate only on their local data and on the inputs they receive via the connections. The restriction to local operations is often relaxed during training and self-organization to solve a problem. Neural networks are promising tools for those problems where the solution algorithm is unknown or so complicated that it is impossible to solve the problem directly. They learn from existing data to give reasonable answers, even to the problems that have never been learned.

Training of neural networks usually entails modifying connection weights by means of a learning rule (Rosenblatt 1962). In other words, neural networks learn from examples and exhibit some capability for generalization beyond the training data. Then, other testing data are used for checking the generalization. There are now many learning methods for neural networks.

The training process can occur in a supervised or unsupervised manner. Supervised training means that the network is provided with sets of training data that include the expected output for each set of input and the network is told what to learn. Unsupervised training is when the network must learn on its own the regularities and similarities among training patterns. There are no target outputs available in unsupervised training and the network must modify its weights and biases in response to the inputs only by categorizing the input patterns into a finite number of classes. Many of these learning methods are closely connected with a certain network topology. Well known examples in unsupervised learning are the Hopfield network (1982) and the competitive learning (Rumelhart and Zipser 1985) for feedback nets, and the fuzzy associative memory and the counter propagation for feed forward-only nets. In supervised learning, examples include the Boltzmann Machine and fuzzy cognitive map for feed-back nets, and the back-propagation and the perception for feed forward-only nets. In the present paper, the back-propagation network (Rumelhart, *et al.* 1986) which is most commonly used is implemented.

2.2. Back-propagation learning method

Back-propagation neural networks generally have a layered structure with an input layer, an output layer, and one or more hidden layers. Units in the input layer represent the possible influential factors that affect the network outputs and have no computation activities, while the output layer contains one or more processing units that produce the network outputs. Layers between the input and output layers are called hidden layers and may contain a large number of hidden processing units. As the name of this kind of network indicates, propagation takes place in a feed-forward manner from the input layer to the output layer, compares the network outputs with known targets, and propagates the error back to the network using a learning mechanism to adjust the weights and biases.

Fig. 1 shows a simple architecture of a back-propagation network that consists of an input layer, a hidden layer, an output layer, and connections between them. The learning mechanism of this back-propagation network is a generalized delta rule that performs a gradient-descent on the error space

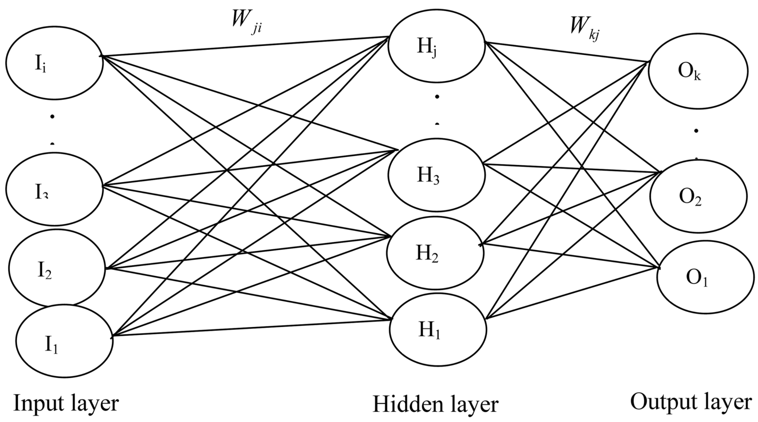

Fig. 1 Simple neural network

to minimize the total error between the calculated values and the desired values of the output layer during the modification of connection strengths. Although the training data contain various errors and uncertainties, it is no great matter to train and generalize the examples. In fact, it is reasonable to contain the errors and uncertainties in training data.

Training is accomplished in an iterative process. Each iteration cycle, called an epoch, involved a feed-forward computation followed by an error-backward propagation to modify their connection weights. The following six steps are involved in the process of training:

Step 1: Assign random values to the connection strengths W_{ji} and W_{kj}, and to the biases B_j and B_k

Step 2: Input values I_i become activations on the input neurons in the input layer

Step 3: The input values N_i and the activation values H_j of hidden neurons are given by

$$N_j=W_{ji}I_i+B_j \tag{1}$$

$$H_j=f(N_j) \tag{2}$$

where $f(\cdot)$ is the activation function, generally the sigmoid function: $f(x)=\dfrac{1}{1+e^{-x}}$

Step 4: The input values N_k and the activation values O_k of output neurons are given by

$$N_k = W_{kj}H_j + B_k;\ O_k = f(N_k); \tag{3}$$

Step 5: The error E between the calculated value O_k and the desired value T_k of output layer neurons may be defined as:

$$E=\frac{1}{2}\sum_{k=1}(O_k-T_k)^2 \tag{4}$$

In the back-propagation network, the error at the output neurons is propagated backward to the hidden layer neurons, and then to the input layer neurons modifying the connection weights and the biases between them by the generalized delta rule (Rumelhart, *et al.* 1986). The modification of the weights and the biases in the generalized delta rule is accomplished through the gradient descent of the error. From hidden to output, the relationships are:

$$\Delta W_{kj} = \eta\delta_k H_j \tag{5}$$

$$\Delta B_k = \eta\delta_k \tag{6}$$

$$\delta_k = (T_k-O_k)f'(N_j) \tag{7}$$

where η = the learning rate. From input to hidden, the relationships are:

$$\Delta W_{ji} = \eta\delta_j H_i \tag{8}$$

$$\Delta B_j = \eta\delta_j \tag{9}$$

$$\delta_j = W_{kj}\delta_k f'(N_j) \tag{10}$$

Step 6: Repeat steps (1) to (5) until error E goes below the specified error goal.

Convergence depends on the number of hidden layer neurons, the size of the learning rate parameters, and the amount of data necessary to create the proper results. Generally there is no structured algorithm to obtain optimal structure and parameters of neural network and the optimal ones should be found by trial and error. However, recently several methods proposed by some researchers to optimize the structures of ANN are found to be effective to some extent (Chen, *et al.* 2005).

3. Developing ANN model for RC slabs

Specialized computer software was used to develop the ANN model for RC slabs in this study (MATLAB 2004). Three important steps must be considered in constructing a successful artificial neural network: network architecture, training, and testing. The basic methodology for developing a successful ANN-based model for RC slab was to teach a neural network the relationship between inputs and outputs. In other words, the network will be trained on the relationship between inputs and outputs using existing data from the published literature. In this study, geometric and material parameters of RC slabs from tests are used as inputs in the feed forward-back propagation ANN to obtain the ultimate strength as output.

3.1. Development of database

The degree of success of the ANN model in prediction largely depends on how comprehensive the training data is. In other words, it depends on the availability of a large variety of pre-existing experimental data, capable of teaching the network all aspects of the relationship between inputs and outputs. A literature review had been conducted and test data on RC slabs tested under uniformly distributed load with restrained conditions on four sides were gathered from various research studies as illustrated in Table 1 (Powell 1956, Park 1964a-b, Hung and Nawy 1971, Wood 1961, Keenan 1969, Moy and Mayfield 1972, Skates 1986, Niblock 1986, Hossain and Iatipu 2000). The restrained slab-support conditions used in various research studies to facilitate the development of compressive membrane action are shown in Fig. 2. From the available data, a number of data sets were selected from different studies to train and test the network model, as summarized in Table 1.

3.2. Determination of network architecture

The first fundamental step in constructing a neural network model is to determine the network

Table 1 Database of test results on RC slabs for ANN study

Source of data	Training data		Testing data	
	No	Slab ID	No	Slab ID
Powell (1956)	11	S46, S48, S53, S54, S55, S57, S58, S59, S60, S63, S64	3	S47, S50, S56,
Park (1964a-b)	6	A2, A4, D1, D2,D4, D5	3	A1, A3, D3
Hung & Nawy (1971)	9	C1-1,C1-3,C1-4,C1-5,C1-6 C4-1,C4-2,C4-4, C4-5	3	C1-2, C1-7, C4-3
Wood (1961)	2	FS12, FS13, FS14	1	FS13
Keenan (1969)	4	3S1, 3S2, 3S4, 4.75S1	1	3S3
Moy & Mayfield (1972)	2	FEA1, FEA7	1	FEA4
Skates (1986)	1	S3	-	-
Niblock (1986)	2	S1, S4	1	S2
Hossain and Iatipu (2000)	8	H1, H3, H4, H5, H7, H8, H10, H11	3	H2, H6, H9
Total	45		15	

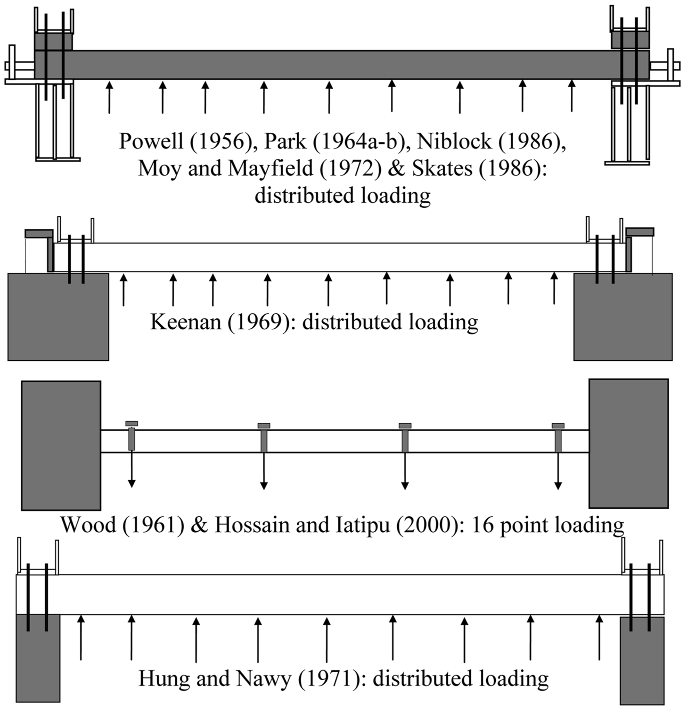

Fig. 2 Diagrams showing edge restraints of tested RC slabs in various studies

architecture. To date, there are no established rules to determine the architecture of a back-propagation neural network that best suits a certain problem. Therefore, a trial-and-error approach was adopted. The basic aspects of network architecture consist of the number of hidden layers between the input and output layers, the number of processing units in each layer, the pattern of connectivity among the processing units, and the activation (transfer) function employed for each processing unit. For a given neural network architecture, it is the connection strengths (weights) between the processing units that determine the network performance. In general, initial weights are set randomly and modified through network training until the network stabilizes. Upon successful completion of the training process, a well-trained neural network is not only capable of computing the expected output of any input set of data used in the training stage, but should also be able to predict with an acceptable accuracy the outcome of any unfamiliar set of input located within the range of the training data.

After trying several network architectures, the network model shown in Fig. 3 was selected for the present study. The input layer of this network model consists of an external input vector of 10 elements consisting of geometric and material parameters which represent adequately RC slabs fully restrained on four sides as shown in Fig. 4. They are: aspect ratio - ratio of long (L_y) to short (L_x) span (L_y/L_x), breadth (L_x) to total depth (h) ratio (L_x/h), percentage of long span top steel (ρ_{yt}), percentage of long span bottom steel (ρ_{yb}), percentage of short span top steel (ρ_{xt}), percentage of short span bottom steel (ρ_{xb}), effective depth (d_x) to total depth (h) ratio for steel along x axis (d_x/h), effective depth (d_y) to total depth (h) ratio for steel along y axis (d_y/h), compressive cylinder strength of concrete (f'_c) and yield strength of steel (f_y). The output layer contains one processing unit that represents the network's output for each input vector. In this study, uniformly distributed

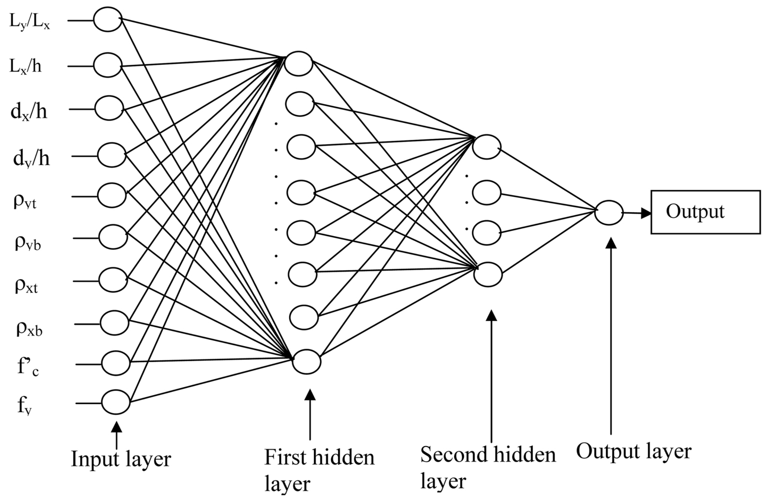

Fig. 3 Adopted architecture of the neural network model

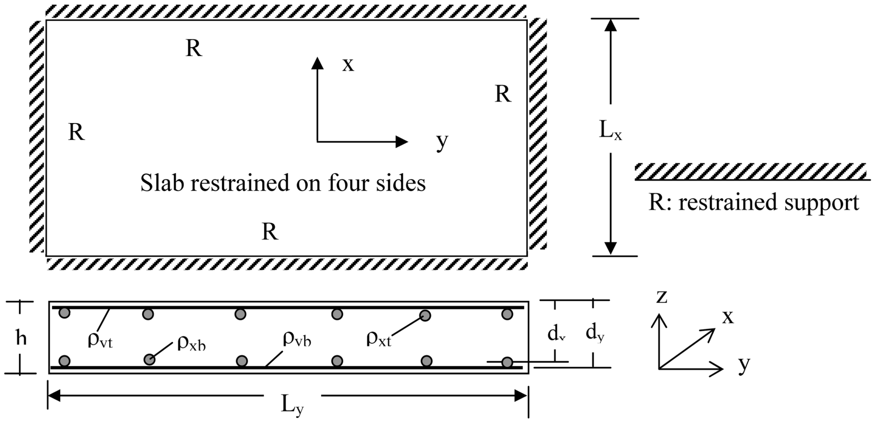

Fig. 4 A schematic diagram showing full details of fully restrained RC slabs

ultimate strength (P) of RC slabs is considered as a single output. The range, mean value, and standard deviation of all input data and output results as extracted from the database of RC slab tests are presented in Table 2.

In addition to the input and output layers, the final network contains two hidden layers. The first hidden layer has 8 processing units, while the second layer has only 4 processing units. A sigmoid function (logsig) was employed as an activation function for all processing units with full connection adopted between units in different layers within the network (Fig. 3).

3.3 Training process of ANN Model

Training a back-propagation neural network is an iterative process; involving the presentation of experimental data as pairs (input/target) and having the network modify its weights by the invocation of learning rules until it stabilizes. Each training pair consists of an input vector

Table 2 Range, average and standard deviation of measured input and output RC slab variables

Variables	Training data			Testing data		
	Range	Average	Stdev.	Range	Avg.	St. Dev.
L_y/L_x	1.00- 2.00	1.35	0.332	1.00-1.75	1.36	0.31
L_x/h	15.20- 40.32	22.34	6.367	16.00-41.00	22.33	6.93
d_x/h	0.63-0.68	0.65	0.017	0.64-0.68	0.66	0.02
d_y/h	0.75-0.81	0.77	0.019	0.75-0.81	0.78	0.02
ρ_{yt} (%)	0.00-1.53	0.395	0.340	0.00-1.53	0.46	0.36
ρ_{yb} (%)	0.00-1.53	0.379	0.343	0.00-1.53	0.43	0.37
ρ_{xt} (%)	0.00-2.42	0.456	0.463	0.00-1.53	0.52	0.43
ρ_{xb} (%)	0.00-1.53	0.411	0.361	0.00-1.53	0.46	0.37
f'_c (N/mm^2)	21.9 - 56.2	34.49	6.339	28.4-50.0	36.57	5.76
f_y (kN/mm^2)	0.211-0.511	0.361	0.103	0.25-0.475	0.34	0.10
P (N/mm^2)	0.027- 0.464	0.232	0.129	0.05-0.41	0.24	0.13

L_y/L_x: Aspect ratio of slab: ratio of long (L_y) to short (L_x) span; L_x/h: breadth (L_x) to total depth (h) ratio, ρ_{yt}: percentage of long span top steel, ρ_{yb}: percentage of long span bottom steel, (ρ_{xt}): percentage of short span top steel, ρ_{xb}: percentage of short span bottom steel, d_x/h: effective depth (d_x) to total depth (h) ratio for steel along x axis, d_y/h: effective depth (d_y) to total depth (h) ratio for steel along y axis (d_y/h), f'_c: compressive cylinder strength of concrete, f_y; yield strength of steel, P: uniformly distributed ultimate strength of RC slabs

containing RC slab variables and a target representing the actually measured ultimate strength of that slab. The network is presented with the data in the first input vector, carries out the appropriate computation and activation through the processing units in the hidden layers and then produces an output through the unit in the output layer. The network compares its output to the corresponding target which is provided in the training pair. The difference between the network output and the target is calculated and stored. After this procedure is done with the first training pair, called the training pattern, the network is presented with a second training pair and so on until the network has gone through all the data available for training; that completes the first epoch. After each epoch, the network calculates the mean squared of all errors it calculated and stored after each training pattern and back-propagates it using the network learning algorithm to adjust the weights and biases for all units in the network. The training continues until either the network converges and reaches its goal for the minimum error between the predicted RC slab behavior and the desired target provided for training, or the maximum number of epochs specified for early stopping is reached.

Three important factors were determined before the training process of a back-propagation neural network was initiated: network parameter, validation of experimental data available for training, and the learning algorithm employed.

In a back-propagation neural network, the parameters (including learning rate, minimum gradient, and the desired minimum error between the network output and the measured targets) must be set to selected values. In this study, the following values were used: learning parameter = 0.05, minimum gradient = 1E-10, and desired error at the output layer = 1E-5. Weights and biases are often initialized randomly.

The experimental data used for network training and testing contain sets of pairs. Each pair consists of an input vector of 10 elements (L_y/L_x, L_x/h, ρ_{yt}, ρ_{yb}, ρ_{xt}, ρ_{xb}, d_x/h, d_y/h, f'_c and f_y) and an output vector of one element (P). There was no need for normalization/validation of the elements of input vector considering the range of their values (Table 2). Parametric studies conducted on this aspect suggested no significant effect of normalization on the prediction of ultimate strength of slabs. The element in the output vector ranges between 0 and 1 to be compatible with the limits of the sigmoid function assigned as an activation function for the processing unit in the output layer and hence, there was no need for normalization/validation.

4. Results and discussion

4.1. Validation of ANN model using training data

The neural network model shown in Fig. 3 was trained by using test results of 45 RC slabs from different research studies as listed in Table 1. As previously mentioned, each training pair consists of an input vector containing RC slab variables and a target/output representing the ultimate strength for that slab (Fig. 3 and Table 2).

A successfully trained network was characterized by its ability to predict the ultimate strength values (P) for the data it was trained on. Therefore, the trained network was used to predict the ultimate strength of RC slabs already used in the training process and the results are shown in Fig. 5. It can be observed that the ANN model adequately predicted the ultimate strength of training slabs with an average absolute error of 8.2%. The ratio of experimental to ANN value ranges between 0.83 and 1.15 with a mean and standard deviation of 0.987 and 0.096, respectively. This illustrates that the network has learned the relationship between RC slab variables and respective ultimate strength effectively, and the model performance on the training data is satisfactory.

4.2. Performance validation of ANN model using test data

The performance of the developed ANN model was tested by the ability of the network to predict the ultimate strength of new RC slabs unfamiliar to the network but comparable with slabs used in the training process. Therefore, a total of 15 RC slabs collected from different sources were

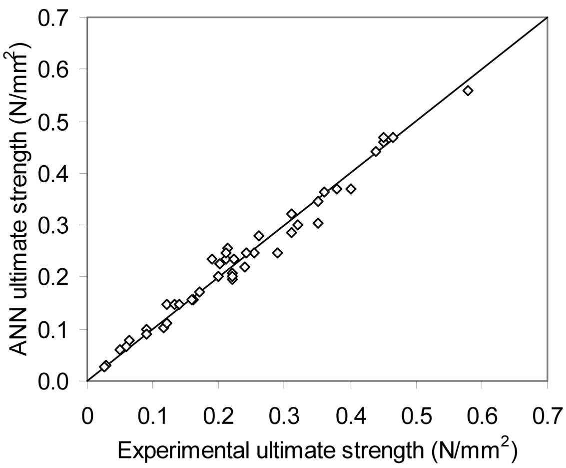

Fig. 5 Experimental versus ANN model predicted ultimate strength (training process)

Table 3 Experimental data of RC slabs used to test predictive ability of ANN model

Slab No.	Geometric parameters				Reinforcing steel (%)				Material properties		Ultimate strength (P)	
	L_y/L_x	L_x/h	d_y/h	d_x/h	ρ_{xt}	ρ_{xb}	ρ_{yt}	ρ_{yb}	f'_c	f_y	Expt.	ANN
S50	1.75	16.0	0.77	0.65	0.45	0.45	0.45	0.45	37.2	0.21	0.33	0.32
S47	1.75	16.0	0.77	0.65	0.25	0.25	0.25	0.25	44.8	0.21	0.27	0.28
S56	1.75	16.0	0.77	0.65	0.00	0.00	0.00	0.00	38.2	-	0.26	0.25
FS13	1.00	30.2	0.81	0.68	0.26	0.26	0.26	0.26	26.5	0.23	0.09	0.09
3S3	1.00	24.0	0.76	0.64	0.82	0.82	0.82	0.82	28.4	0.33	0.24	0.21
FEA4	1.50	30.6	0.76	0.64	0.49	0.49	0.49	0.49	31.7	0.39	0.05	0.07
S2	1.00	19.0	0.76	0.64	0.26	0.26	0.26	0.26	37.0	0.51	0.40	0.42
D3	1.50	40.8	-	-	0.00	0.00	0.00	0.00	35.5	0.00	0.03	0.03
A1	1.50	20.0	0.75	0.63	0.38	0.19	0.41	0.20	33.0	0.33	0.22	0.20
A3	1.50	20.0	0.75	0.63	1.44	0.72	0.45	0.22	34.4	0.33	0.26	0.25
H2	1.75	16.0	0.80	0.67	1.53	1.53	1.53	1.53	38.0	0.25	0.45	0.43
H6	1.00	19.0	0.80	0.67	0.34	0.34	0.34	0.34	50.0	0.45	0.32	0.30
H9	1.38	18.8	0.80	0.67	0.58	0.58	0.58	0.58	33.0	0.30	0.41	0.38
C1-2	1.00	26.0	0.80	0.67	0.36	0.36	0.36	0.36	38.6	0.48	0.13	0.15
C1-7	1.00	26.0	0.80	0.67	0.58	0.58	0.58	0.58	39.0	0.29	0.16	0.16
C4-3	1.38	18.8	0.80	0.67	0.58	0.58	0.58	0.58	39.8	0.47	0.22	0.26

Cylinder compressive strength of concrete (f'_c) in N/mm^2;
Yield strength of steel (f_y) in kN/mm^2; Ultimate strength (P) in N/mm^2

presented to the ANN model and the network was required to predict the strength associated with each of the slabs. The details of 15 RC slabs including information on geometric, reinforcing steel and material properties are presented in Table 3. The ANN predicted ultimate strength values are compared with those obtained from experiments in Figs. 6-7.

It is found that the ANN model adequately predicted the ultimate strength of the testing slabs with an average absolute error of 8%. The ratio of experimental to ANN value ranges between 0.73 and 1.16 with a mean and standard deviation of 0.998 and 0.11, respectively. This illustrates that the ANN model is capable of predicting the ultimate strength of an arbitrary RC slabs whose geometric and material parameters fall within the range of the training slabs.

5. Modeling of membrane action

The membrane action due to in-plane forces, inherent in the case of fully restrained RC slabs under consideration but which the yield-line (*YL*) analysis cannot account for, as illustrated in Tables 4-5 and Fig. 8. Tables 4-5 re-state the experimental and ANN model predicted ultimate strength values, as well as the equivalent predictions based on the yield-line method for most of the slabs used in the training and testing of the ANN model. The yield-line analysis was based on the use of a moment per unit width (m), calculated from the equation first proposed by Whitney (1937) and later used by Leet and Bernal (1996), conforming to the 1995 ACI code. This is expressed as:

$$m = \rho f_y d^2 (1–0.59\, \rho f_y / f'_c) \tag{11}$$

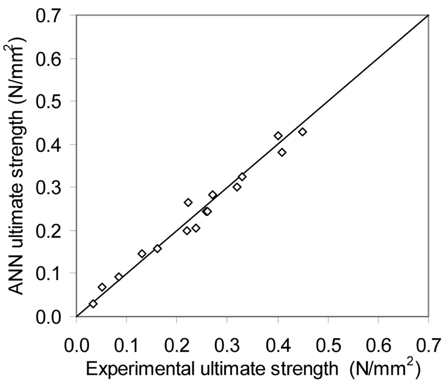

Fig. 6 Performance of ANN model with testing slabs

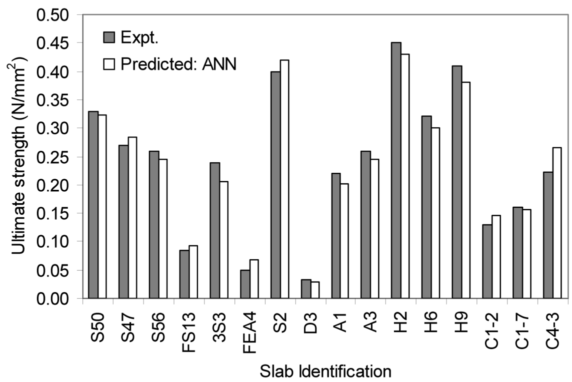

Fig. 7 Comparison of ultimate strength from ANN model and experiments (testing stage)

Table 4 Comparative study of ultimate strength prediction (Training data)

Slab ID	Ultimate strength (P) N/mm^2			$P_{Exp.}/P_{ANN}$	$P_{Exp.}/P_{YL}$	Slab ID	Ultimate strength (P) N/mm^2			$P_{Exp.}/P_{ANN}$	$P_{Exp.}/P_{YL}$
	P_{exp}	P_{ANN}	P_{YL}				P_{exp}	P_{ANN}	P_{YL}		
S54	0.36	0.37	0.11	0.99	3.40	H1	0.31	0.32	0.07	0.97	4.77
S59	0.35	0.35	0.17	1.01	2.03	H3	0.12	0.11	0.03	1.09	3.87
S63	0.46	0.47	0.26	0.99	1.76	H4	0.06	0.07	0.04	0.92	1.46
S46	0.31	0.28	0.04	1.09	8.20	H5	0.20	0.20	0.07	1.00	2.78
S55	0.38	0.37	0.11	1.02	3.59	H7	0.32	0.30	0.12	1.07	2.67
S58	0.44	0.44	0.17	1.00	2.56	H8	0.24	0.22	0.05	1.09	4.62
FS12	0.12	0.10	0.02	1.13	5.58	H10	0.40	0.37	0.11	1.08	3.81
3S1	0.22	0.20	0.12	1.12	1.80	H11	0.22	0.20	0.11	1.10	2.00
3S4	0.22	0.20	0.12	1.13	1.82	C1-1	0.16	0.16	0.12	1.02	1.34
4.75S1	0.58	0.56	0.35	1.04	1.66	C1-3	0.12	0.15	0.06	0.83	2.08
FEA1	0.09	0.10	0.05	0.93	1.69	C1-4	0.12	0.15	0.05	0.83	2.29
FEA7	0.05	0.06	0.03	0.85	1.57	C1-5	0.13	0.15	0.08	0.90	1.76
S3	0.35	0.30	0.13	1.15	2.76	C1-6	0.14	0.15	0.08	0.95	1.77
S4	0.45	0.47	0.19	0.96	2.34	C4-1	0.21	0.25	0.11	0.84	2.03
A2	0.22	0.21	0.08	1.07	2.75	C4-2	0.19	0.24	0.08	0.81	2.25
A4	0.26	0.28	0.10	0.93	2.60	C4-4	0.21	0.25	0.11	0.86	1.94
A2	0.22	0.21	0.08	1.07	2.75	C4-5	0.20	0.23	0.11	0.89	1.88
A4	0.26	0.28	0.10	0.93	2.60						

Statistics	Ratio	
	$P_{Exp.}/P_{ANN}$	$P_{Exp}.-/P_{YL}$
Range	0.83 – 1.15	1.34 – 8.20
Mean	0.99	2.71
St. deviation	0.10	1.40

$P_{Exp.}$: Experimental strength, P_{YL}: Yield-line strength; P_{ANN}: ANN strength – by using training data on the trained ANN model

where ρ = reinforcement ratio and d = effective depth of slab.

For the training data, the range, mean, and standard deviation of $P_{Exp.}/P_{YL}$ are 1.34-8.20, 2.71, and 1.40, respectively compared to 0.83-1.15, 0.99, and 0.10 of $P_{Exp.}/P_{ANN}$. For the testing data the range,

Table 5 Comparative study of ultimate strength prediction (Testing data)

Slab ID	Ultimate strength (P); N/mm^2			Ratio	
	P_{exp}	P_{ANN}	P_{YL}	$P_{Exp.}/P_{ANN}$	$P_{Exp.}/P_{YL}$
S50	0.33	0.32	0.07	1.02	4.88
S47	0.27	0.28	0.04	0.95	7.12
FS13	0.09	0.09	0.02	0.91	4.11
3S3	0.24	0.21	0.12	1.16	1.93
FEA4	0.05	0.07	0.04	0.73	1.31
S2	0.40	0.42	0.10	0.95	4.04
A1	0.22	0.20	0.07	1.10	3.14
A3	0.26	0.25	0.09	1.04	2.89
H2	0.45	0.43	0.17	1.05	2.65
H6	0.32	0.30	0.12	1.07	2.67
H9	0.41	0.38	0.10	1.08	3.94
C1-2	0.13	0.15	0.08	0.87	1.71
C1-7	0.16	0.16	0.07	1.00	2.16
C4-3	0.22	0.26	0.17	0.85	1.29
$P_{Exp.}$: Experimental strength, P_{YL}: Yield-line strength; P_{ANN}: ANN strength			Range	0.85 – 1.16	1.31 – 7.12
			Mean	0.98	3.13
			St. deviation	0.12	1.59

mean, and standard deviation of $P_{Exp.}/P_{YL}$ are 1.31-7.12, 3.13, and 1.59, respectively compared to 0.85-1.16, 0.98, and 0.12 of $P_{Exp.}/P_{ANN}$.

The values of $P_{Exp.}/P_{ANN}$ in close proximity of 1.0 (as shown in Fig. 8) suggest the ability of the ANN model to take care of the strength enhancement of fully restrained RC slabs due to membrane action. The inability of the design oriented yield-line method is demonstrated by the scattered values of $P_{Exp.}/P_{YL}$ far in excess of 1.0 (Fig. 7). From Tables 4-5 and Fig. 8, it is noted that the yield-line theory under-predicts the strength of RC slabs by a factor as much as 8.0 in comparison with experimental tests and ANN predictions.

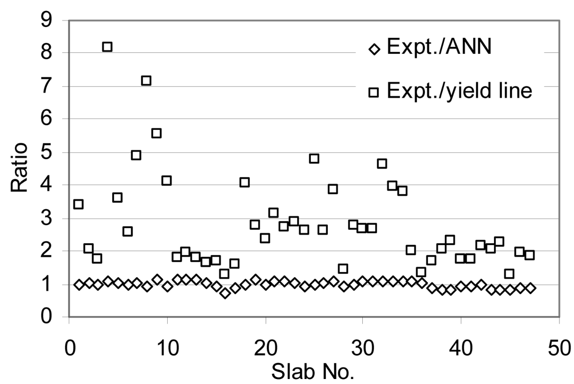

Fig. 8 Comparative simulation of membrane action

6. Practical applications of ANN model

6.1. Development of design aids for RC slabs

Achieving reliable predictions for a class of slabs subjected to membrane action in concrete structures based on ANN modeling opens the avenue for numerous practical applications. Hundreds of 'ANN model slabs' of various geometric and strength properties could be analyzed with greater confidence to provide a database of ultimate strength values. This database could be used as the basis for the development of charts and equations, which can be used for an easy and quick strength determination, for any proposed reinforced concrete slab structure. This task is done without the need for the traditional extensive physical testing of a typical slab, which is time consuming and laborious.

A total of 864 "ANN-model slabs" with fully restrained conditions resulting from various combinations of geometric and strength properties as summarized in Table 6, are used to create a database for the ultimate strength, and hence, to develop design aids in the form of design charts for practical applications. The uniformly loaded 'ANN-model' slabs are assumed to be isotropically reinforced (with equal top and bottom reinforcement ratios) having d_x/h and d_y/h ratio of 0.65 and 0.8, respectively (Fig. 4).

From the results of ANN modeling of 864 ANN-model slabs, design charts (best fit polynomial of 3rd degree) are produced for strength prediction purposes. A typical chart is shown in Fig. 9. The concrete cylinder strength (f'_c), steel yield strength (f_y) and aspect ratio (L_y/L_x) of slab are shown at the top of the chart. The design curves for reinforcement ratios varying from 0.2 to 1.5% and the design or ultimate loads/strengths are expressed as a function of breadth/depth ratios (L_x/h) varying from 15 to 40. The estimation of procedure of ultimate loads for slabs having L_x/h of 17 and 20 and reinforcement ratios of 0.5 and 1.25%, respectively are shown by lines with arrows in Fig. 9. For the slabs with reinforcement ratio of 1.25%, a linear interpolation between 1.0 and 1.5% is adopted.

To test the accuracy of developed ANN-charts, the ultimate strengths of 45 experimental slabs from training and testing data (those are closely related to ANN-model slabs) are compared to those obtained from ANN-charts (Fig. 10). The mean value of the ratios of experimental to ANN-chart predicted loads is found to be equal to 1.025, showing a mean accuracy value to within 2.5% with a standard deviation of 0.17, thus showing reasonably good agreement. The close correlation between experimental and chart predicted values suggests that the ANN-charts could be used as a basis for design.

Table 6 Parameters of theoretical ANN-model slabs

Aspect ratio L_y/L_x	Breadth to depth ratio, L_x/h	Concrete cylinder strength, f'_c N/mm^2	Steel yield strength, f_y, N/mm^2	Percentage of steel, ρ
2.0	15	25	250	0.2
1.5	20	30	460	0.5
1.0	25	40	550	1.0
	30	60		1.5
	35			
	40			

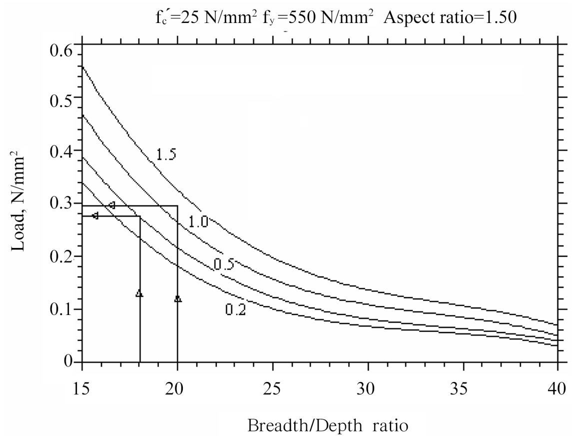

Fig. 9 Typical ANN design chart showing the prediction of ultimate load/strength

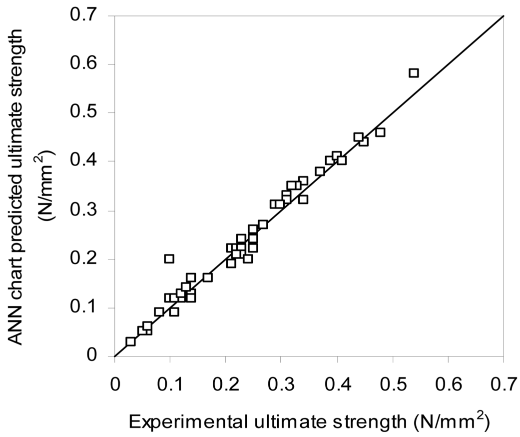

Fig. 10 Comparison of ultimate strength from ANN chart and experiments (testing stage)

6.2. Other applications for ANN model predictions

Another avenue for the application of the proposed ANN modeling is in the design of RC structures. Existing design practices use Codes that adopt either partial or global factors of safety for the applied loads and materials. These factors limit, to varying degrees (even for the limit-state design methods), the structural performance to within the elastic range. This could be the case because in most cases, the method of analysis that precedes the design does not adequately account for the nonlinearities that are inherent in concrete structures, thus the design tends to be overly conservative. The structural integrity is well known to be preserved even when the structure is in the nonlinear range. Since accurate determination of the ultimate strength has been demonstrated as achievable with the ANN modeling, the designer is able to know when the structure may be assumed to have failed. Knowing the actual point of failure is synonymous to the imposition of a global factor of safety whose choice would lead to optimum use of material while not compromising safety requirements. Knowledge of the actual capacity of the slab will increase the confidence of the designer enabling him/her to choose special factor of safety, thereby optimizing the construction cost.

The ANN simulation of RC slabs through optimization of membrane action (as presented in this paper) is the first step towards a comprehensive research on floor slabs in buildings and bridge decks under various loading conditions including fire. Research is currently conducted in these directions.

7. Conclusions

The development of compressive membrane action in slabs in building and bridge structures due to the presence of horizontal end restraints significantly increases their load carrying capacity. The incorporation of strength enhancement due to membrane action can lead to the economical design of such structures. This study has described the artificial neural network (ANN) modeling of restrained reinforced concrete (RC) slabs subjected to uniformly load through optimization of membrane action. The developed ANN model is trained by using test results of 45 RC slabs from previous research studies and its performance is validated through test results of additional 15 slabs. The study clearly demonstrates that the developed ANN model can be used for reliable prediction of ultimate strength of restrained RC slabs with reasonable accuracy. The weakness of the yield-line method of analysis (which is the basis for most Code-based design procedures) in not adequately accounting for membrane action has also been demonstrated. The ANN model is able to predict the ultimate strength of RC slabs tested by various researchers, with an average absolute error of 8%.

The developed ANN model is used to simulate 864 ANN-model slabs resulting from various combinations of geometric dimensions, material strengths and reinforcement ratios. The database generated is used to develop ANN-design charts for the strength prediction of arbitrary fully restrained slabs, falling within the range of predictions. A comparative study involving 45 experimental slabs, reveals that ANN-charts are able to predict the ultimate strength with an average absolute error of 2.5%. The accuracy of the ANN-charts, and that of the ANN model from which they are developed has many implications. Such charts may be used for design purposes (with appropriate choice of safety factor), thus optimising the materials required for construction. The charts may prove useful in assisting design engineers in estimating the global load factor in their code-of-practice-based design. Such designs incorporate partial factors of safety for material and loading without giving any indication of the global factor of safety.

However, the performance of the developed ANN model can be further improved by increasing the scope of the database. Work is underway to further develop the model by incorporating slabs with different material properties as well as varying support and loading conditions.

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