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A mortar mix proportion design algorithm based on artificial neural networks

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Abstract. The concepts of four parameters of nominal water-cement ratio, equivalent water-cement ratio, average paste thickness, fly ash-binder ratio were introduced. It was verified that the four parameters and the mix proportion of mortar can be transformed each other. The behaviors (strength, workability, *et al.*) of mortar primarily determined by the mix proportion of mortar now depend on the four parameters. The prediction models of strength and workability of mortar were built based on artificial neural networks (ANNs). The calculation models of average paste thickness and equivalent water-cement ratio of mortar can be obtained by the reversal deduction of the two prediction models, respectively. A mortar mix proportion design algorithm was proposed. The proposed mortar mix proportion design algorithm is expected to reduce the number of trial and error, save cost, laborers and time.

Keywords: mortar mix proportion design; artificial neural network (ANN); nominal water-cement ratio; equivalent water-cement ratio; average paste thickness (APT); fly ash-binder ratio.

1. Introduction

Nowadays, research on cement matrix materials is focused on the inclusion of additives, such as polymer (Xiong, *et al.* 2004, Jenni, *et al.* 2006), expansive and shrinkage reducing admixtures (Maltese, *et al.* 2005), water-repellent admixture (Lu, *et al.* 2004) *et al.*, admixtures, such as fly ash (Chindaprasirt, *et al.* 2004, Li, *et al.* 2005), silica fume (Appa Rao 2003), *et al.* and short fibers, such as polypropylene fibers (Puertas, *et al.* 2003), carbon-fiber (Wu, *et al.* 2005), *et al.*, to improve certain physical and mechanical properties. Adherence, permeability, thermal and acoustical insulation, ductility, flexural strength, fire performance and viscous damping are some of the main research lines on cement matrix materials.

Cement mortar can be used as floor and bridge overlays, repairing mortars, bonding ceramic tile agents and precast elements joining material, *et al.* The traditional mortar mixture proportion algorithms are based on a generalization of previous experience, available as tables or empirical formula (Chinese Standard JGJ 98-2000 2001, European Standard EN 197-1 2000, ASTM C 150 1993). Due to the uncertain of mortar ingredients, such as sand, cement, chemical and mineral admixtures, the traditional mortar mixture proportion algorithms are a trial and error process, which results in the waste of cost, laborers and time.

Simple regression models and soft computing techniques (artificial neural networks, fuzzy logic

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and genetic programming), were already used in the literature for the prediction of compressive cement strength (CCS) (Because testing the strength of cement is ordinarily performed by mixing cement with sand and water (European Standard EN 197-1 2000), the cement mortar is also commonly referred to as cement.) However, the number of published papers on the subject is very small. Tsivilis and Parissakis (1995) and de Siquera Tango (1998) applied regression methods for CCS prediction. Akkurt, *et al*. (2003) used GA-based artificial neural networks (GA-ANNs) for the CCS prediction. They also analyzed effects of various parameters on the 28-day strength. Fa-Liang (1997) and Akkurt, *et al*. (2004) applied fuzzy logic (FL) to CCS prediction successfully. Baykasogùlu, *et al.* (2004) used genetic programming (GP) approaches on the prediction of CCS. However, due to the complexity between mortar behaviors (strength, workability, *et al.*) and mortar mix proportion, these soft computing techniques are still not used in the mix proportion design of mortar to reduce the number of trail and error.

ANNs have exceptional performance as regression tools, especially when used for pattern recognition and function estimation. They are highly nonlinear, and can capture complex interactions among input/output variables in a system without any prior knowledge about the nature of these interactions. The main advantage of ANNs is that one does not have to explicitly assume a model form, which is a prerequisite in regression methods. Indeed, in ANNs a relationship of possibly complicated shape between input and output variables is generated by the data points themselves. In comparison to regression methods, ANNs tolerate relatively imprecise or incomplete data, approximate results, and are less vulnerable to outliers. They are highly parallel, that is, their numerous independent operations can be executed simultaneously (Haykin 1994, Zupan, *et al.* 1993). Therefore, ANNs especially adapt to be used in the mix proportion design of mortar.

In this paper, the investigated ingredients of mortar only include cement, fly ash, sand and water. Other ingredients of mortar will be considered in further research. Based on the concepts of average paste thickness, nominal water-cement ratio, equivalent water-cement ratio, fly ash-binder ratio, the relation between the four parameters and mortar mix proportion is studied in detail. By combining artificial neural networks and mathematical models, a mix proportion design algorithm of mortar is proposed.

2. Basic concepts

2.1. Fly ash-binder ratio

The fly ash-binder ratio β_F is defined as

$$\beta_F = m_F /(m_C + m_F) \tag{1}$$

where m_C, m_F are the by weight contents of cement and fly ash in one cubic meter mortar, respectively, and m means by weight, not by volume.

2.2. Equivalent water-cement ratio

2.2.1. Equivalent water content $m_{W,E}$

The Equivalent water content $m_{W,E}$ is defined as:

$$m_{W,E} = m_W + m_{W,R} + m_{W,A} - m_{W,S} \tag{2}$$

where

$$m_{W,R} = m_R(1 - C_R) \tag{3}$$

$$m_{W,A} = m_S C_S \tag{4}$$

$$m_{W,S} = m_S C_{S0} \tag{5}$$

where m_W is the mixing water content in one cubic meter mortar; $m_{W,R}$ is the water content of superplasticizer (SP) in one cubic meter mortar; C_R is the ratio of the solid content of SP to the content of SP; $m_{W,A}$ is the real water content of sand in one cubic meter mortar; C_S is the absorption coefficients of water for in situ sand; $m_{W,S}$ is the water content of saturated surface-dry (SSD) sand; C_{S0} are the absorption coefficients of water for saturated surface-dry (SSD) sand; m_R is the content of SP in one cubic meter mortar. Herein, it is assumed that $m_{W,S}$ is independent of the workability and strength of mortar.

2.2.2. Equivalent cement content $m_{C,E}$

Including the content and activity of fly ash, the equivalent cement content $m_{C,E}$ is defined as:

$$m_{C,E} = \frac{m_C}{1 - \alpha\beta_F} \tag{6}$$

where α is the activity ratio of fly ash to cement, and can be taken as 0.55 for class two fly ash (Chinese Standard) and grade 32.5 Portland cement based on a great deal of test data. α has relationship with the strength of mortar, but independent of the workability of mortar.

2.2.3. The equivalent water-cement ratio is defined as Ji, et al. (2006)

$$(m_W/m_C)_E = \frac{m_{W,E}}{m_{C,E}} = \left(\frac{m_{W,E}}{m_C}\right)(1 - \alpha\beta_F) \tag{7}$$

$(m_W/m_C)_E$ has relationship with the strength of mortar, but independent of the workability of mortar. If β_F is taken as zero, $(m_W/m_C)_E$ is transform into the normal water-cement ratio $(m_{W,E}/m_C)$. Eq. (7) can be transformed into

$$\left(\frac{m_{W,E}}{m_C}\right) = \frac{(m_W/m_C)_E}{(1 - \alpha\beta_F)} \tag{8}$$

2.3. Nominal water-cement ratio

2.3.1. Nominal water W_N

The nominal water W_N is defined as:

$$m_{W,N} = \frac{m_{W,E} - m_{W,R}}{1 - \mu} = \frac{m_W + m_{W,A} - m_{W,S}}{1 - \mu} \tag{9}$$

where μ is the reducing-water ratio of SP, namely the ratio of water reduced by superplasticizer to primary total water. $m_{W,R}$ is included in μ, so $m_{W,R}$ should be subtracted from $m_{W,E}$. μ has relationship with the workability of mortar, but independent of the strength of mortar.

2.3.2. Water demand ratio λ_R

λ_R is the ratio of the by weight demand water of a paste composed by 300 g cement and 750 g premanufactured sand to that of a paste composed by 300 g fly ash and 750 g premanufactured sand when the flowing diameters of both the two pastes reach 125-135 mm (Chinese Standard GBJ 146-90 1990). λ_R has relationship with the workability of mortar, but independent of the strength of mortar.

2.3.3. Nominal cement $m_{C,N}$

The nominal cement $m_{C,N}$ is defined as:

$$m_{C,N} = m_C + \lambda_R m_F \tag{10}$$

2.3.4. Nominal water-cement ratio $(m_W/m_C)_N$

The nominal water-cement ratio $(m_W/m_C)_N$ is defined as Ji, *et al.* (2006):

$$(m_W/m_C)_N = \frac{m_{W,N}}{m_{C,N}} = \frac{\dfrac{m_{W,E} - m_{W,R}}{1-\mu}}{m_C + \lambda_R m_F} = \frac{\dfrac{m_W + m_{W,A} - m_{W,S}}{1-\mu}}{m_C + \lambda_R m_F} \tag{11}$$

$(m_W/m_C)_N$ has relationship with the workability of mortar, but independent of the strength of mortar. If μ and m_F are taken as zero (namely SP and fly ash are not included in mortar), $(m_W/m_C)_N$ is transform into the normal water-cement ratio $(m_{W,E}/m_C)$.

2.4. Average paste thickness

2.4.1. Basic assumptions

The theoretical models of sand are based on two assumptions as follows:

i) sand is perfect spheres

ii) sand is monosized

The two assumptions conflict with the practical combination of realistic sand. The two assumptions can be overcome by introducing characteristic diameters of sand d_{ST} and by using the measured eigenpacking degrees of sand ϕ_{ST}. These two parameters compensate the deviations from the two assumptions.

The characteristic diameter d_{ST} of sand can be chosen as the sieve size for which there is 36.8 percent residue (Goltermann, *et al.* 1997).

The measured eigenpacking degree ϕ_{ST} of sand is defined as the ratio of sand bulk density ρ_{ST} to the sand grain density ρ_S:

$$\phi_{ST} = \rho_{ST}/\rho_S \tag{12}$$

The sand is poured into a steel barrel. Then the steel barrel is vibrated for 30 seconds on the vibrating platform of a vibration device. The bulk density ρ_{ST} can be obtained by dividing the by weight content of sand by the volume of the steel barrel occupied by the sand.

2.4.2. Average paste thickness APT

The average paste thickness wrapping sand can be calculated as follows (Ji, *et al.* 2006):

$$APT = \frac{(V_P - V_{P0})}{(\pi \cdot d_{ST}^2 \cdot n_S)} = \frac{(V_P - V_{P0}) d_{ST}}{6 m_S / \rho_S} \tag{13}$$

$$V_P = \frac{m_C}{\rho_C} + \frac{m_F}{\rho_F} + \frac{m_{W,E}}{\rho_W} + \theta \tag{14}$$

$$V_{P0} = \frac{m_S}{\rho_{ST}} \cdot \left(1 - \frac{\rho_{ST}}{\rho_S}\right) \tag{15}$$

$$n_S = \frac{m_S}{\frac{1}{6}\pi d_{ST}^3 \cdot \rho_S} \tag{16}$$

where V_P is the total paste volume; V_{P0} is the paste volume filling the void of sand; n_S are theoretical numbers of sand; ρ_C, ρ_W, ρ_F are the specific densities of cement, water, fly ash, respectively; θ is the by volume air content.

3. Experimental program

3.1. Material properties

All materials used in the experiments are produced in China. The cement is a grade 32.5 Portland cement. A class two fly ash (Chinese Standard) and nature river sand were used. The fineness modulus of sand is 2.04. Material properties can be found in Table 1.

3.2. Test methods

3.2.1. Workability

The workability of mortar mixture was assessed by consistency test. The consistency test was regarded as the most convenient basis for classifying the workability of the mortar mixes and was carried out in accordance with JGJ 70.7 (Chinese Standard JGJ70-90 1990).

3.2.2. Compressive strength

Standard metallic cube moulds (70.7 mm) were used for preparation of the mortar specimens for compressive strength. A table vibrator was used for compaction of the mortar filled cubes. The specimens were demoulded after 24 h and subsequently immersed in water till the time of testing. Three cube specimens were used for the determination of average compressive strength.

Table 1 Material properties

ρ_C (kg/m^3)	ρ_F (kg/m^3)	ρ_W (kg/m^3)	ρ_S (kg/m^3)	ρ_{ST} (kg/m^3)	θ
2990	2290	1000	2630	1549	2%
d_{ST} (mm)	C_R	C_{S0}	C_S	λ_R	μ
0.578	0.4	0.6%	0	0.8	0

3.3. Test results

Sixteen-group mortar mix proportions were designed as listed in Table 2. Herein, for simplification, no superplasticizer (SP) was used in this test. The 28d compressive strength $f_{c,28}$ and consistency values

Table 2 Mix proportion of mortar

No.	m_C (kg/m^3)	m_W (kg/m^3)	m_F (kg/m^3)	m_S (kg/m^3)	V (m^3)
1	559.64	233.15	0	1549	1.02
2	476.18	294.47	0	1459.8	1.02
3	418.88	343.38	0	1380.2	1.02
4	376.42	384.27	0	1308.9	1.02
5	385.99	240.35	192.99	1459.8	1.02
6	282.92	263.92	141.46	1549	1.02
7	286.58	351.74	143.29	1308.9	1.02
8	231.49	355.51	115.74	1380.2	1.02
9	302.46	250.25	302.46	1380.2	1.02
10	254.83	313.65	254.83	1308.9	1.02
11	173.35	286.64	173.35	1549	1.02
12	160.57	329.9	160.57	1459.8	1.02
13	208.62	258.19	417.23	1308.9	1.02
14	157.99	292.66	315.98	1380.2	1.02
15	123.33	304.76	246.67	1459.8	1.02
16	97.682	302.34	195.36	1549	1.02

Table 3 Calculated four parameters and test results

No.	$(m_W/m_C)_N$	$(m_W/m_C)_E$	APT (10^{-6} m)	β_F	ϖ (mm)	$f_{c,28}$ (MPa)
1	0.40	0.40	3.27	0	34	19.29
2	0.60	0.60	13.47	0	26	30.98
3	0.80	0.80	23.67	0	62	23.94
4	1.00	1.00	33.87	0	67	13.46
5	0.42	0.49	13.47	1/3	26	20.43
6	0.64	0.73	3.27	1/3	30	26.89
7	0.85	0.98	33.87	1/3	117	13.11
8	1.07	1.22	23.67	1/3	105	7.32
9	0.44	0.58	23.67	1/2	29	25.39
10	0.66	0.87	33.87	1/2	112	15.01
11	0.88	1.16	3.27	1/2	34	8.35
12	1.11	1.45	13.47	1/2	88	5.07
13	0.46	0.76	33.87	2/3	23	16.15
14	0.69	1.14	23.67	2/3	64.5	8.7
15	0.92	1.52	13.47	2/3	62.5	5.08
16	1.15	1.90	3.27	2/3	36.5	2.98

ϖ (coming from the consistency test) of the sixteen-group mortars C_S were listed in Table 3.

However, the mortar volumes V of the sixteen-group mix proportions as shown in Eq. (17) are not equal to one cubic meter, as shown in Table 2.

$$V=\frac{m_C}{\rho_C}+\frac{m_F}{\rho_F}+\frac{m_{W,E}}{\rho_W}+\frac{m_S}{\rho_S}+\theta \tag{17}$$

So an algorithm is used to modify the sixteen-group mortar mix proportions as follows:

1. From Eq. (2), $m_{W,E}$ can be obtained.
2. From Eq. (17), V can be obtained.

If V is not equal to 1, the old mortar mix proportions dividing by V are the new mortar mix proportions, namely the new mortar mix proportions m_C/V, m_F/V, m_W/V and m_S/V can be obtained, and then return to step (1). If V is equal to 1, the calculated process ended.

The modified sixteen-group mortar mix proportions are listed in Table 4.

4. Equivalency between four parameters and mix proportion of mortar

4.1. From mix proportion of mortar to four parameters

According Eqs. (1), (7), (11), (13), the four parameters of the sixteen-group mortars can be calculated as listed in Table 3.

4.2. From four parameters to mix proportion of mortar

For a mix proportion of mortar, the grade of cement and the nature of sand *et al.* are constant. If the four parameters β_F, $(m_W/m_C)_E$, $(m_W/m_C)_N$, APT are given, the mix proportion of mortar can be calculated. The algorithm is provided as follows:

1) From Eq. (12), ϕ_{ST} can be obtained.
2) The total paste volume of mortar mixtures corresponding to a unit volume of sand V'_P can be obtained according to Eq. (18).

$$V'_P=APT\cdot\pi\cdot d_{ST}^2\cdot n_S+V'_{P0}=\frac{6\cdot APT}{d_{ST}}+\frac{1-\phi_{ST}}{\phi_{ST}} \tag{18}$$

$$V'_{P0}=\frac{m_S}{\rho_{ST}}\cdot e_S=\frac{\rho_S}{\rho_{ST}}\cdot e_S=\frac{1-\phi_{ST}}{\phi_{ST}} \tag{19}$$

$$n_S=\frac{m_S}{\frac{1}{6}\cdot\pi\cdot d_{ST}^3\cdot\rho_S}=\frac{\rho_S}{\frac{1}{6}\cdot\pi\cdot d_{ST}^3\cdot\rho_S} \tag{20}$$

where V'_{P0} and S' are the void volume and the total surface area of a unit volume of sand, respectively. Because the volume of sand V_S=1, the mass of sand $m_S=\rho_S\cdot V_S=\rho_S\cdot 1=\rho_S$.

3. The total paste volume corresponding to a unit volume of mortar mixture can be obtained according to Eq. (21).

$$V_P = V'_P/(1+V'_P) \tag{21}$$

4) The by weight content of sand corresponding to a unit volume of mortar mixture is shown in Eqs. (22).

$$m_S = \rho_S/(1+V'_P) \tag{22}$$

5) From Eq. (8), $(m_{W,E}/m_C)$ can be obtained.
6) From Eqs. (1) and (14), m_C can be obtained.

$$m_C = (V_P - \theta)/\left(\frac{1}{\rho_C} + \frac{\beta_F}{\rho_F(1-\beta_F)} + \left(\frac{m_{W,E}}{m_C}\right)\cdot\frac{1}{\rho_W}\right) \tag{23}$$

7) From Eq. (1), m_F can be obtained.

$$m_F = \frac{\beta_F m_C}{1-\beta_F} \tag{24}$$

8) According to Eq. (25), $m_{W,E}$ can be obtained.

$$m_{W,E} = m_C\cdot\left(\frac{m_{W,E}}{m_C}\right) \tag{25}$$

9) According to Eqs. (4) and (5), $m_{W,A}$ and $m_{W,S}$ can be obtained.
10) According to Eqs. (3) and (11) and the test data provided by SP companies, μ and m_R can be obtained. Herein, because no SP is used in this test, m_R and $m_{W,R}$ are zero.
11) According to Eq. (2), m_W can be obtained.

Table 4 Mix proportion of mortar calculated from four parameters or the modified mix proportion of mortar

No.	m_C (kg/m^3)	m_W (kg/m^3)	m_F (kg/m^3)	m_S (kg/m^3)	V (m^3)
1	548.5	228.5	0	1518.0	1
2	466.6	288.6	0	1430.6	1
3	410.5	336.5	0	1352.6	1
4	368.9	376.6	0	1282.7	1
5	378.3	235.5	189.1	1430.6	1
6	277.3	258.6	138.6	1518.0	1
7	280.9	344.7	140.4	1282.8	1
8	226.9	348.4	113.4	1352.6	1
9	296.4	245.2	296.4	1352.6	1
10	249.7	307.4	249.7	1282.8	1
11	169.9	280.9	169.9	1518.0	1
12	157.4	323.3	157.4	1430.6	1
13	204.5	253.0	408.9	1282.7	1
14	154.8	286.8	309.7	1352.6	1
15	120.9	298.7	241.7	1430.6	1
16	95.7	296.3	191.5	1518.0	1

The calculation results of the sixteen-group mortars based on the four parameters listed in Table 3 are just the same as the modified sixteen-group mortar mix proportions as shown in Table 4. In order to further verify the correctness of the proposed algorithm above, another exact algorithm is proposed, as follows:

1) For one cube meter mortar, there is

$$\frac{m_C}{\rho_C}+\frac{m_F}{\rho_F}+\frac{m_{W,E}}{\rho_W}+\frac{m_S}{\rho_S}+\theta=1 \tag{26}$$

2) From Eqs. (13), (14), (15) and (26), m_S can be obtained.

$$m_S=\frac{1}{\dfrac{6APT}{d_{ST}\cdot\rho_S}+\dfrac{1}{\rho_{ST}}} \tag{27}$$

3) From Eqs. (1), (8) and (26), m_C, m_F, $m_{W,F}$ can be obtained.

$$\beta_W=\frac{m_{W,E}}{m_C+m_F}=\left(\frac{m_{W,E}}{m_C}\right)(1-\beta_F)=\frac{(m_W/m_C)_E}{(1-\alpha\beta_F)}(1-\beta_F) \tag{28}$$

$$D=-\frac{(m_S+\theta\cdot\rho_S-\rho_S)\cdot\rho_C\cdot\rho_F\cdot\rho_W}{\rho_S(\rho_F\cdot\rho_W+\rho_C\cdot\rho_F\cdot\beta_W+\rho_C\cdot\beta_F\cdot\rho_W-\rho_W\cdot\rho_F\cdot\beta_F)} \tag{29}$$

$$m_C=D\cdot(1-\beta_F) \tag{30}$$

$$m_F=D\cdot\beta_F \tag{31}$$

$$m_{W,E}=D\cdot\beta_W \tag{32}$$

4) From Eq. (2), m_W can be obtained

Making a program according to the above proposed exact algorithm, the calculated mix proportions of mortar are the same as that of Table 4. Therefore, the correctness of the proposed algorithms above is verified, and the equivalency between the four parameters and the mix proportion of mortar is also confirmed.

5. Application of ANNs

5.1. Neural networks methodology

A neural network is an information processing system whose architecture essentially mimics the biological system of the brain. The neural network technique is a relatively new computational tool that is particularly useful for evaluating systems with a multitude of nonlinear variables. A neural network consists of a number of interconnected processing units. These units are commonly referred to as neurons. Each neuron receives an input signal from neurons to which it is connected. Each of these connections has numerical weights associated with them. These weights determine the nature and strength of the influence between the interconnected neurons. The signals from each input are then processed through a weighted sum on the inputs. The processed output signal is then transmitted to another neuron via a transfer function. The transfer function adopted in this study is $f(x)=1-2/(1+e^{2x})$. The transfer function modulates the weighted sum of the inputs so that the output approaches unity when the input gets larger and approaches zero when the input gets smaller. The architecture of a typical neural network consists three layers of interconnected neurons. Each neuron is connected to the neurons in the next layer. There is an input layer where data is presented to the neural network, and an output layer that holds the response of the network to the input. It is the intermediate layers, also known as hidden layers, which enable these networks to represent and compute complicated associations between patterns. Currently, there is no rule for determining the optimal number of neurons in the hidden layer or the number of hidden layers, except through experimentation. A single hidden layer has been found to be satisfactory for many problems.

Training of the neural network is essentially carried out through the presentation of a series of example patterns of associated input and observed output values. The neural network "learns" what it is to compute through the modification of the weights of the interconnected neurons. The most commonly used learning system is the back-propagation model. To simplify the learning process of the back-propagation neural network and to reduce the time required for training, the learning algorithm adopted to train the network model in this study is the Levenberg-Marquardt algorithm. The learning algorithm processes the patterns in two stages. In the first stage, the input pattern generates a forward flow of signals from the input layer to the output layer. The error of each output neuron is then determined from the difference between the computed values and the observed (experimental) values. The second stage involves the readjustment of the weights and biases in the hidden and output layers to reduce the difference between the computed and desired outputs. The modification of the weights is carried out using a "generalized delta rule" through the gradient descent on the error. Training is carried out iteratively until the average sum-squared errors over all training patterns are minimized.

On the satisfactory completion of the training phase, verification of the performance of the neural network is then carried out using patterns that were not included in the training set. This determines whether the neural network can generalize correct responses for patterns that only broadly resemble the data in the training set. This is often called the testing phase. Since no additional learning or connection weight changes occur during this phase, the run time is almost instantaneous (Haykin 1994, Zupan, *et al.* 1993).

5.2. Prediction model for the workability of mortar

The architecture of prediction model for the workability of mortar consists three layers as shown in Fig. 1. The input layer includes three neurons $(m_W/m_C)_N$, β_F, APT. The output layer includes one neurons ϖ. The hidden layer consists of three neurons. It has been verified that the four parameters β_F, $(m_W/m_C)_E$, $(m_W/m_C)_N$, APT and the mix proportion of mortar m_C, m_F, m_W, m_S, m_R can be transformed each other. The behaviors (strength, workability, *et al.*) of mortar primarily determined by the mix proportion of mortar now depend on the four parameters. It is also explained above that the workability of mortar has no relationship namely $(m_W/m_C)_N$, β_F, APT, with $(m_W/m_C)_E$. So the workability of mortar is now determined only by other three parameters, as Fig. 1 shows. Training of the neural network is carried out through the example patterns in Table 3 (Nørgaard 2000). The average sum squared error after 500 cycles is 4.6×10^{-6}. The connection weights and biases of neurons after training are shown in Table 5 and Fig. 1. The solid lines in Fig.

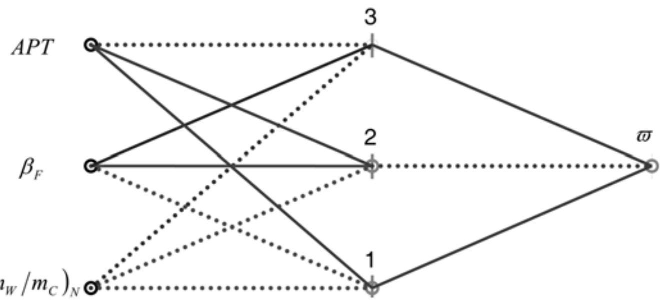

Fig. 1 Architecture of ANN used to predict the workability of mortar

Table 5 Connection weights and biases used to predict the workability of mortar

Neuron	Connection weights			Biases
	$(m_W/m_C)_N$	β_F	*APT*	
1	-0.1286	-0.0110	0.8577	-0.3492
2	-0.1761	0.0368	0.8222	-0.4216
3	-1.4642	1.9501	-0.5459	1.9247
Neuron	1	2	3	Biases
ϖ	67.0931	-67.3102	5.1859	-6.3534

1 represent the positive values of connection weights in Table 5, while the dashed lines in Fig. 1 represent the negative values of connection weights. The verification phase is omitted in this study due to the less example patterns. However, the correctness of the neural work has been verified by theoretical analysis and calculation.

Other parameters also affect the workability of mortar, such as the grade of cement and the nature of sand. However, for a ready-mix mortar company, these parameters are often kept constant. They have been included in the connection weights and biases of the prediction model for the workability of mortar. If these parameters change, another prediction model based on ANNs should be built.

5.3. Calculation model for APT

From the prediction model for the workability of mortar, the calculation model for *APT* can be

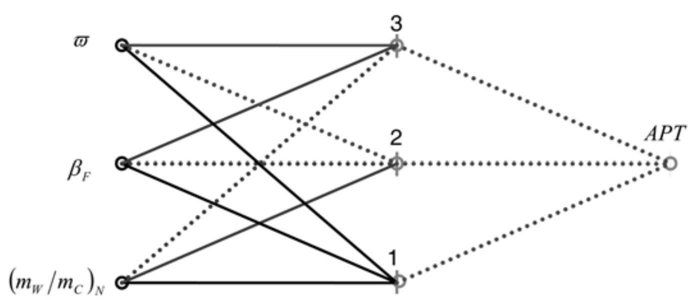

Fig. 2 Architecture of ANN used to calculate *APT*

Table 6 Connection weights and biases used to obtain *APT*

Neuron	Connection weights			Biases
	$(m_W/m_C)_N$	β_F	ϖ	
1	10.4846	25.4244	4.0939	-14.8669
2	0.6156	-0.6210	-0.4086	0.1456
3	-0.6049	0.5618	0.2316	0.3680
Neuron	1	2	3	Biases
APT	-0.4418	-5.3308	-5.2482	1.7600

obtained, as Fig. 2 shows. Training of the neural network is carried out through the example patterns in Table 3. The average sum squared error after 500 cycles is 2.3×10^{-3}. The connection weights and biases of neurons after training are shown in Table 6 and Fig. 2.

5.4. Prediction model for the 28d compressive strength of mortar

The architecture of prediction model for the strength of mortar consists three layers as shown in Fig. 3. The input layer includes three neurons $(m_W/m_C)_E$, β_F, *APT*. The output layer includes one neurons $f_{c,29}$. The hidden layer consists of three neurons. It has been explained above that the strength of mortar has no relationship with $(m_W/m_C)_N$. So the architecture of prediction model for the strength of mortar doesn't include $(m_W/m_C)_N$. Training of the neural network is carried out through the example patterns in Table 3. The average sum squared error after 500 cycles is 0.0122. The connection weights and biases of neurons after training are shown in Table 7 and Fig. 3.

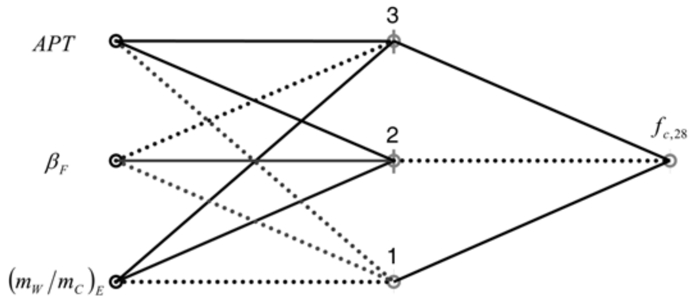

Fig. 3 Architecture of ANN used to predict the 28d compressive strength of mortar

Table 7 Connection weights and biases used to predict the 28d compressive strength of mortar

Neuron	Connection weights			Biases
	$(m_W/m_C)_N$	β_F	*APT*	
1	-2.9049	-0.1944	-0.7072	1.3755
2	2.9021	0.7518	5.1195	2.6848
3	7.8258	-2.1806	15.8237	8.3786
Neuron	1	2	3	Biases
$f_{c,28}$	0.6712	-73.3616	72.9730	-0.1424

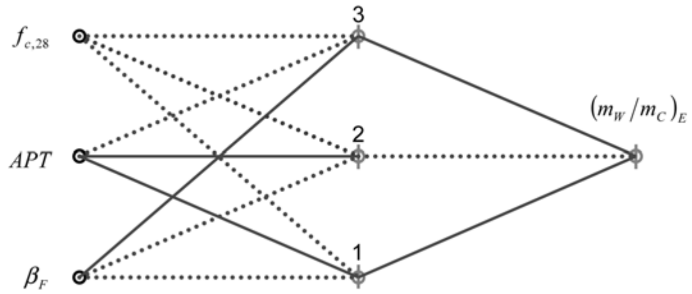

Fig. 4 Architecture of ANN used to calculate $(m_W/m_C)_E$

Table 8 Connection weights and biases used to obtain $(m_W/m_C)_E$

Neuron	Connection weights			Biases
	β_F	APT	$f_{c,28}$	
1	-0.2995	0.4423	-0.4917	0.4794
2	-0.2935	0.4356	-0.6338	0.6018
3	0.2489	-0.3277	-0.0225	-0.9642
Neuron	1	2	3	Biases
$(m_W/m_C)_E$	23.8832	-18.9935	14.5442	9.5961

5.5. Calculation model for $(m_W/m_C)_E$

From the prediction model for the strength of mortar, the calculation model for $(m_W/m_C)_E$ can be obtained, as Fig. 4 shows. Training of the neural network is carried out through the example patterns in Table 3. The average sum squared error after 500 cycles is 2.1×10^{-6}. The connection weights and biases of neurons after training are shown in Table 8 and Fig. 4.

6. Mortar mix proportion design algorithm

The mortar mix proportion design process is shown in Fig. 5. The design algorithm of mortar mix proportion based ANNs is provided in detailed as follows:

(1) Given $f_{c,28}$, ϖ, β_F, μ, θ.
(2) Given C_{S0}, C_S, ρ_C, ρ_F, ρ_W, ρ_S, ρ_{ST}, d_{ST}, λ_R, C_R, α.
(3) Assume $(m_W/m_C)_N$.
(4) Solve APT through the calculation model of APT as shown in Fig. 2 and Table 6.
(5) Solve $(m_W/m_C)_E$ through the calculation model of $(m_W/m_C)_E$ as shown in Fig. 4 and Table 8.
(6) The total paste volume corresponding to a unit volume of mortar mixture can be obtained according to Eq. (21).
(7) From Eq. (8), $(m_{W,E}/m_C)$ can be obtained.
(8) The by weight contents of sand corresponding to a unit volume of mortar mixture m_S can be obtained according to Eqs. (22).

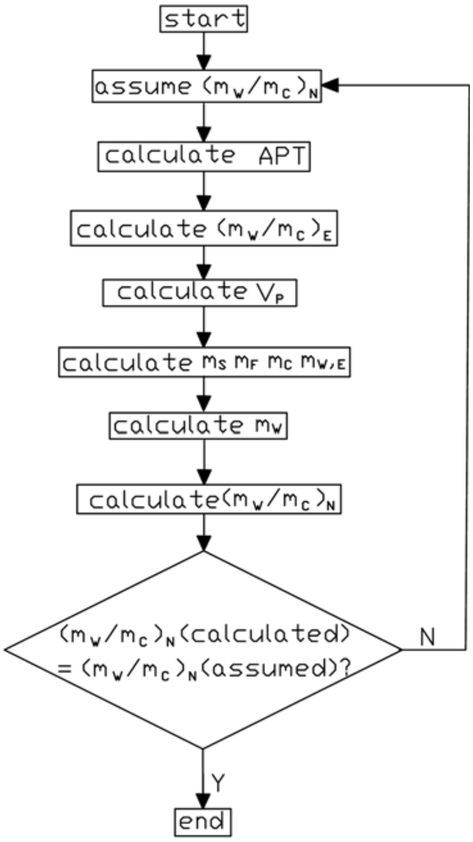

Fig. 5 Flow-chart of mix proportion design algorithm

(9) From Eq. (23), m_C can be obtained.
(10) From Eq. (24), m_F can be obtained.
(11) According to Eq. (25), $m_{W,E}$ can be obtained.
(12) According to Eqs.(4) and (5), $m_{W,A}$, and $m_{W,S}$ can be obtained.
(13) According to m_C, μ and the test data provided by SP companies, m_R can be obtained.
(14) According to Eq. (3), $m_{W,R}$ can be obtained.
(15) According to Eq.(2), m_W can be obtained.
(16) According to Eq.(11), $(m_W/m_C)_N$ can be obtained.
(17) Check if the calculated $(m_W/m_C)_N$ in the step (16) agrees with the assumed $(m_W/m_C)_N$. If not, assume the new $(m_W/m_C)_N$ as the mean of the calculated $(m_W/m_C)_N$ and the assumed $(m_W/m_C)_N$, and return to step (4).

7. Example

The 28d compressive strength $f_{c,28}$, the consistency value ϖ and the fly ash-binder ratio β_F are selected as shown in Table 3. No SP is used (namely m_R=0). The other parameter values are the same as that of Table 1. According to the results of the previous tests on sixteen mix-design

Table 9 Mix proportion of mortar calculated from 28d compressive strength and workability of mortar

No.	$(m_W/m_C)_N$	$(m_W/m_C)_E$	APT (10^{-6} m)	m_C (kg/m^3)	m_W (kg/m^3)	m_F (kg/m^3)	m_S (kg/m^3)
1	0.399	0.398	2.51	545.99	226.49	0	1525.5
2	0.599	0.598	13.44	467.44	287.99	0	1431.4
3	0.804	0.808	24.13	408.57	338.19	0	1349.9
4	0.999	0.999	33.67	368.77	375.94	0	1284.6
5	0.43	0.494	12.6	374.32	234.85	187.16	1438.2
6	0.587	0.669	4.55	296.69	252.01	148.34	1507
7	0.854	0.981	33.68	280.26	344.37	140.13	1284.5
8	1.068	1.219	23.59	227.4	347.66	113.7	1353.8
9	0.464	0.608	25.48	292.35	253.02	292.35	1340.2
10	0.663	0.87	33.96	249.82	307.32	249.82	1282.7
11	0.884	1.158	3.36	170.06	280.84	170.06	1517.8
12	1.111	1.45	13.79	157.62	323.84	157.62	1428.6
13	0.447	0.734	32.28	206.28	246.72	412.56	1293.7
14	0.693	1.145	24.42	155.01	288.39	310.02	1347.8
15	0.922	1.522	12.81	120.14	297.32	240.28	1436.5
16	1.155	1.91	2.82	94.94	295.46	189.88	1522.8

mortars, two artificial neural networks (ANNs) are built as shown in Fig. 2 and Fig. 4 before being able to obtain the intended mortar in terms of workability and 28d compressive strength. The parameter values of $(m_W/m_C)_N$, $(m_W/m_C)_E$, APT, and the mix proportion of mortar m_C, m_W, m_F, m_S can be obtained according to the designed process provided in Chapter 6, and are listed in Table 9. The mix proportions of mortars shown in Table 9 are consistent with the modified mix proportions of mortars as shown in Table 4.

8. Conclusions

1. A mortar mix proportion design algorithm based on artificial neural networks (ANNs) was proposed. The proposed mortar mix proportion design algorithm is expected to reduce the number of trial and error, save cost, laborers and time.

2. The four parameters of nominal water-cement ratio, equivalent water-cement ratio, average paste thickness, fly ash-binder ratio and the mix proportion of mortar can be transformed each other. The behaviors (strength, workability, *et al.*) of mortar primarily determined by the mix proportion of mortar now depend on the four parameters when other parameters, such as the grade of cement and the nature of sand *et al.*, are kept constant.

3. The prediction models of strength and workability of mortar were built based on artificial neural networks (ANNs). The calculation models of average paste thickness and equivalent water-cement ratio can be obtained by the reversal deduction of the two prediction models, respectively. And other parameters, such as the grade of cement and the nature of sand, *et al.*, are considered in the connection weights and biases of these ANN models as long as these parameters are kept constant. If these parameters change, another prediction model based on ANNs should be built.

4. Artificial neural networks of workability, strength, equivalent water-cement ratio and average paste thickness should be built in each ready-mixed mortar plants of different areas. Test data (namely, example patterns of ANNs) should be accumulated in order to update the connection weights and biases of ANNs. Then the prediction precision of the artificial neural networks can be improved.

5. To complement this type of approach, more research is needed to predict the bonding strength, shrinkage and durability of mortar, and to design the mix proportion of high performance mortar (HPM) including ingredients of silica fume, polypropylene fibres, nanophase materials, polymer, expansive and shrinkage reducing admixtures *et al*., whose strength, workability and durability satisfy specific requirements.

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