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Finite element analysis of shear critical prestressed SFRC beams

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Abstract. This study reports the details of the finite element analysis of eleven shear critical partially prestressed concrete T-beams having steel fibers over partial or full depth. Prestressed concrete T-beams having a shear span to depth ratio of 2.65 and 1.59 and failing in the shear have been analyzed using 'ANSYS'. The 'ANSYS' model accounts for the nonlinear phenomenon, such as, bond-slip of longitudinal reinforcements, post-cracking tensile stiffness of the concrete, stress transfer across the cracked blocks of the concrete and load sustenance through the bridging of steel fibers at crack interface. The concrete is modeled using 'SOLID65'-eight-node brick element, which is capable of simulating the cracking and crushing behavior of brittle materials. The reinforcements such as deformed bars, prestressing wires and steel fibers have been modeled discretely using 'LINK8' – 3D spar element. The slip between the reinforcement (rebar, fibers) and the concrete has been modeled using a 'COMBIN39'-non-linear spring element connecting the nodes of the 'LINK8' element representing the reinforcement and nodes of the 'SOLID65' elements representing the concrete. The 'ANSYS' model correctly predicted the diagonal tension failure and shear compression failure of prestressed concrete beams observed in the experiment. The capability of the model to capture the critical crack regions, loads and deflections for various types of shear failures in prestressed concrete beam has been illustrated.

Keywords: analysis; finite element; concrete prestressed; fiber concrete; shear.

1. Introduction

The shear failures in reinforced concrete (RC) structures are highly brittle when compared with the flexural failures. The addition of chopped steel fibers in the concrete matrix is effective in mitigating the brittle failures of RC structures. The addition of fibers in the matrix improves the strength and post cracking tensile stiffness of the concrete. The chopped fibers induce confinement effect in concrete matrix, which contributes to the increase in the strength characteristics of concrete. The toughening mechanisms, such as, fiber pullout, fiber bridging or fiber fracture at crack interface improves the post cracking tensile stiffness of the matrix. Thus, the presence of fibers increases the strength and results in a relatively ductile type of failure of RC beams. In the literature, modeling the various effects due to the addition of fibers in RC structures has not been attempted extensively (Padmarajaiah and Ramaswamy 2002, Belletti, *et al.* 2003). The present study addresses this lacunae and reports the details of the finite element analysis of eleven shear critical partially prestressed concrete T-beams having steel fibers over partial or full depth. The finite

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element (FE) analysis of the T-beams has been carried out in 'ANSYS' package incorporating the bond-slip between the concrete and the reinforcements that are parallel to the longitudinal axis of the beam. The predicted results, namely, loads, deflections and cracking behavior using 'ANSYS' model have been compared with the corresponding test data.

Fanning (2001) analyzed reinforced and post tensioned concrete beams using the ANSYS package. Kawakami and Ito (2003) analyzed prestressed concrete columns and pre-cast segmental prestressed beams using the ADINA package. Kotosov and Pavlovic (2004) modeled short shear reinforced concrete beams having no fibers using 3D finite element approach to verify the possibility of the realistic prediction of its behavior observed in the test program. Zho, Kwan and He (2004) analyzed nonlinear behavior of reinforced concrete deep beams using a 2D finite element method. Perfect bond between the reinforcement and the surrounding concrete has been assumed in all these studies. The nonlinearities resulting from the bond slip between the reinforcement and the concrete has been accounted for in the present analysis.

2. Details of prestressed concrete T-beams

Eleven T-beams of 3.85 m long were cast varying the concrete strength (f'_{cu} = 35 MPa, 65 MPa and 85 MPa) and the presence or absence of fibers (V_f = 1.5 Percent) in the flange, web or entire section. The reinforcement details of the beams are given in Fig. 1. The designation of test beams is given in Fig. 2. Two beams (S65FFCWFC-A, S85FFCWFC-A) were tested over a shear span to depth ratio (a/d) of 1.59 and the remaining nine beams (S35FOCWOC, S65FOCWOC, S85FOCWOC, S35FFCWFC, S65FFCWFC, S85FFCWFC, S35FOCWFC, S65FOCWFC, S85FOCWFC) were tested over a shear span to depth ratio (a/d) of 2.65. The properties of concrete determined by testing the companion cylinder specimens cast and tested along with the T-beams are presented in Table 1.

3. FE analysis of the T-beams

The partially prestressed concrete T-beams have been analyzed using a finite element (FE) model in ANSYS. The 'ANSYS' model accounts for the non-lineraties, such as, bond-slip of longitudinal reinforcements, post-cracking tensile stiffness of the concrete, stress transfer across the cracked blocks of the concrete and load sustenance through the bridging of steel fibers at crack interface. The analysis was carried out in stages using Newton-Raphson technique. The load was applied in increments of 2 mm displacements corresponding to the actuator movement in the experiment and subsequently displacements were reduced to half of that in the previous iteration leading to convergence. In the last load-step, trial analyses were subsequently carried out with different magnitudes of displacements to ensure that no converged solution could be realized.

3.1. Modeling of concrete and steel fibers

The concrete has been modeled using 'SOLID65' 8 node brick element capable of simulating the cracking and crushing of brittle materials. The test data of the cylinder compressive strength and split tensile strength based on the companion specimens cast and tested along with the T-beams were used for defining concrete ('CONCR') properties in 'ANSYS'. The shear transfer coefficients

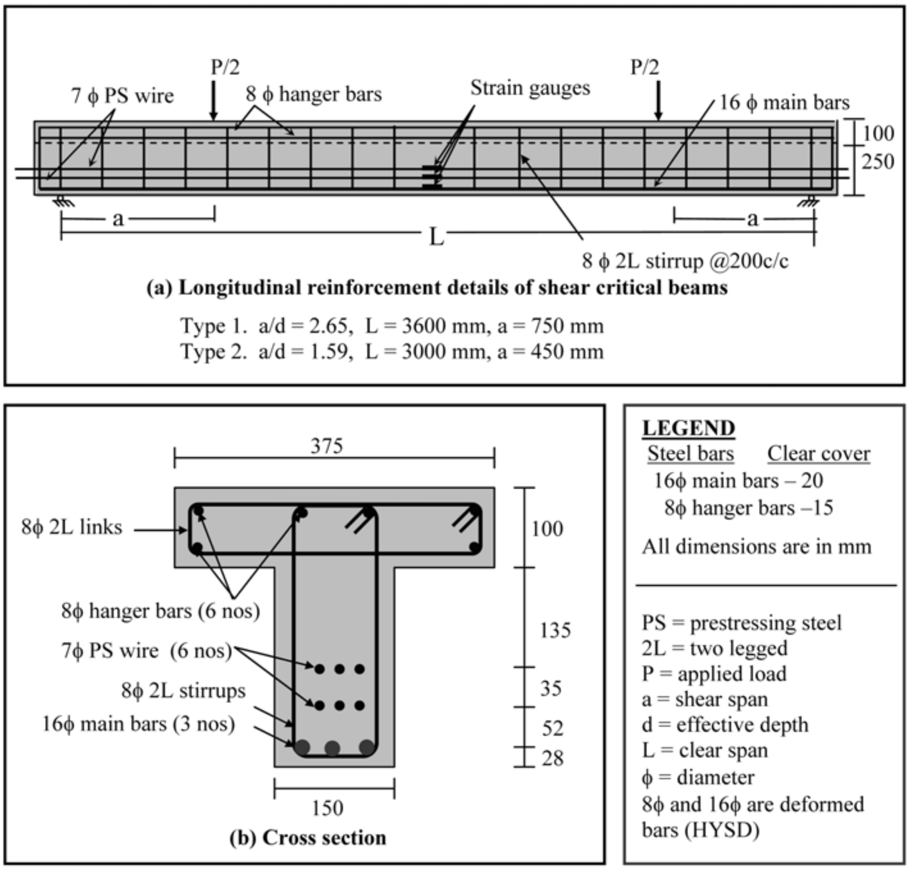

Fig. 1 Reinforcement details of the prestressed concrete T-beam

in opening (β_t) or closing (β_c) are assumed to take a value of 0.25 and 0.70 respectively, for plain concrete of all grades having no fibers as its influence is marginal except at very small values (Barzegar 1988). In fiber reinforced concrete, the shear transfer at the cracks depends on the matrix strength fiber interaction in the fiber pullout mechanism. To account for this fact, β_t has been assumed to take a value of 0.35, 0.40 and 0.65 and β_c 0.75, 0.80, and 0.90 for fiber reinforced concrete of (V_f = 1.5 percent) normal strength (35 MPa), moderately high strength (65 MPa) and high strength (85 MPa) concrete, respectively (Padmarajaiah and Ramaswamy 2002). In 'ANSYS', the failure surface of the concrete is computed based on the Willam and Warnke model (1975). In ANSYS, smeared representation of crack is used in 'SOLID65'. Before cracking or crushing, concrete is assumed to be an isotropic elastic material. After crushing, the concrete is assumed to have lost its stiffness in all directions. After cracking, concrete is assumed to be orthotropic having stiffness based on a bilinear softening stress-strain response. Fig. 3 shows the idealized stress-strain response of concrete in tension used in ANSYS. The model shown in Fig. 3 is one corresponding to plain concrete, in which the post-cracking tensile stiffness of the concrete is accounted for when the strain is less than six times the strain corresponding to the peak stress. In the present analysis, the mesh size was fixed based on the guidelines in the earlier literatures on objectivity (Bazant and Oh 1983) and considering the computational efforts involved. The size of the mesh along the

Table 1 Properties of concrete in T-beams based on the test data of companion cylinder specimens cast and tested along with the T-beams

Beam designation	V_f (%)	f'_{cy} (MPa)	f_{spc} (MPa)	E_c (MPa)	υ_c
S35FOCWOC	0.0	34.03	4.18	29002	0.185
S65FOCWOC	0.0	63.32	5.70	39557	0.205
S85FOCWOC	0.0	83.99	6.56	45559	0.215
S35FFCWFC	1.5	35.70	5.70	31687	0.272
S65FFCWFC	1.5	65.08	7.73	42915	0.294
S85FFCWFC	1.5	84.73	8.83	49000	0.306
S35FOCWFC	0.0	35.75	4.28	29724	0.186
	1.5	36.63	5.78	32103	0.273
S65FOCWFC	0.0	63.78	5.72	39703	0.205
	1.5	65.12	7.73	42929	0.294
S85FOCWFC	0.0	83.16	6.53	45335	0.214
	1.5	84.86	8.84	49037	0.306
S65FFCWFC-A	1.5	68.75	7.95	44116	0.297
S85FFCWFC-A	1.5	89.01	9.05	50226	0.308

V_f = volume fraction of fibers; f'_{cy} = compressive strength of concrete; f_{spc} = splitting tensile strength of concrete; E_c = modulus of elasticity of concrete; υ_c = Poisson's ratio of concrete

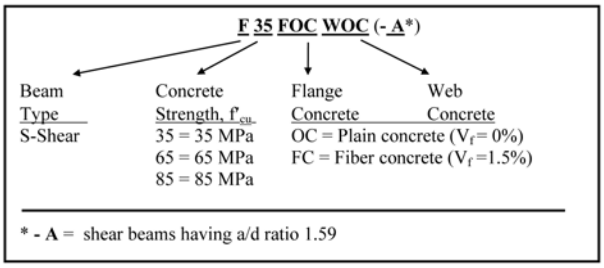

Fig. 2 Designation of prestressed concrete T-beam

longitudinal axis of the beam was fixed to 100 mm in the shear span and 50 mm in the constant moment zone. The details of the mesh used for the FE analysis of partially prestressed T-beams have been presented in Fig. 4.

As the compressive strength and tensile strength ascertained based on the test data of respective concrete specimens (plain or fiber reinforced concrete) is used in the analysis, the increase in the strength properties due to the addition of fibers has been accounted for in the FE analysis. Addition of fibers increases the post-cracking tensile stiffness of the concrete through the bridging of fibers across the crack interfaces. The fibers bridging across the crack interface induce resistance to the crack opening when the concrete strain is greater than '$6\varepsilon_{ot}$' (Fig. 3). To account for the effect of addition of steel fibers on the post cracking tensile stiffness of the concrete, the fibers have been modeled discretely. As the load sustenance against crack growth is mainly derived through the bridging of fibers orienting along the longitudinal axis of the beam, only this fraction of the fibers has been modeled as discrete reinforcement in the FE analysis. The fibers have been modeled using

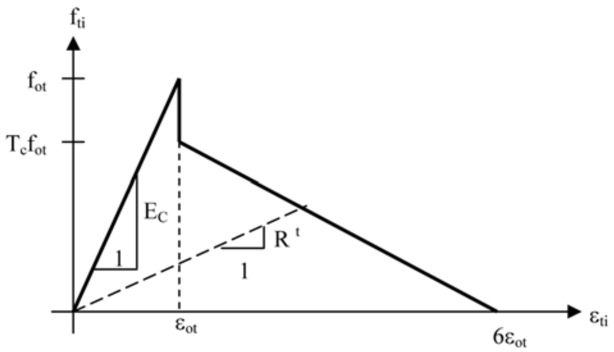

Fig. 3 Stress-strain response of concrete in tension -idealization used in ANSYS

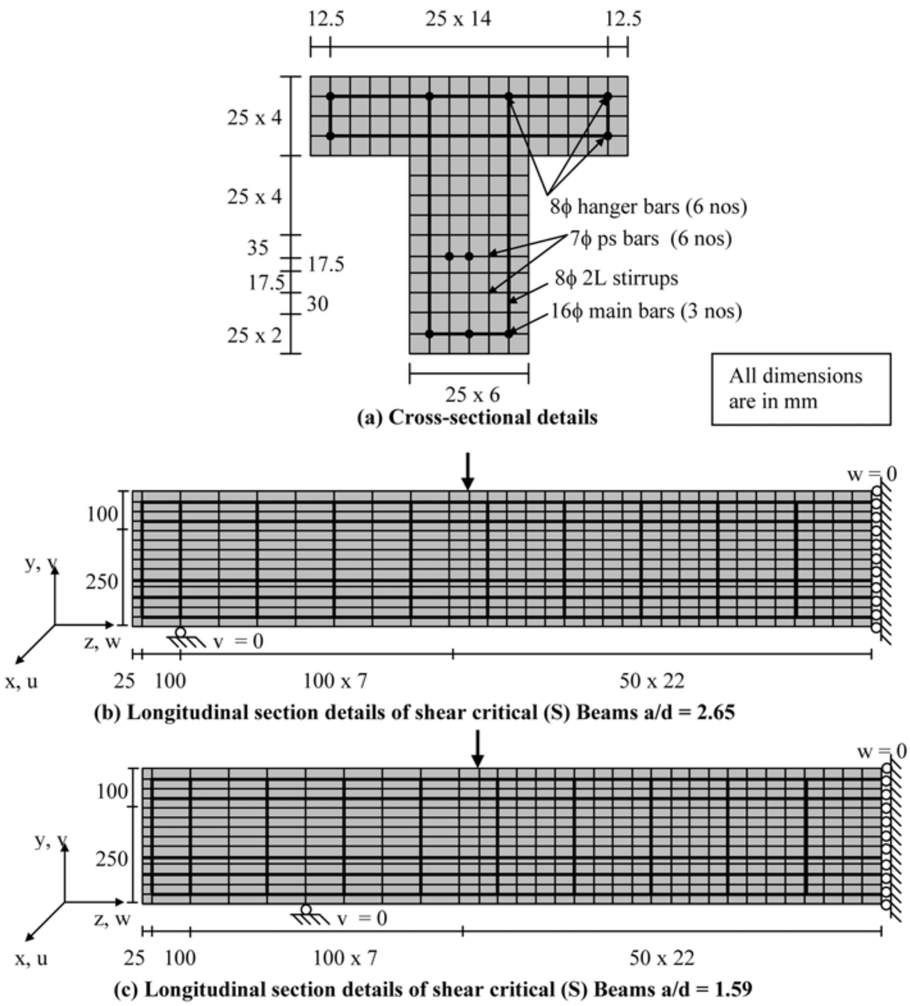

Fig. 4 Details of FE mesh used for the analysis of prestressed concrete T-beam

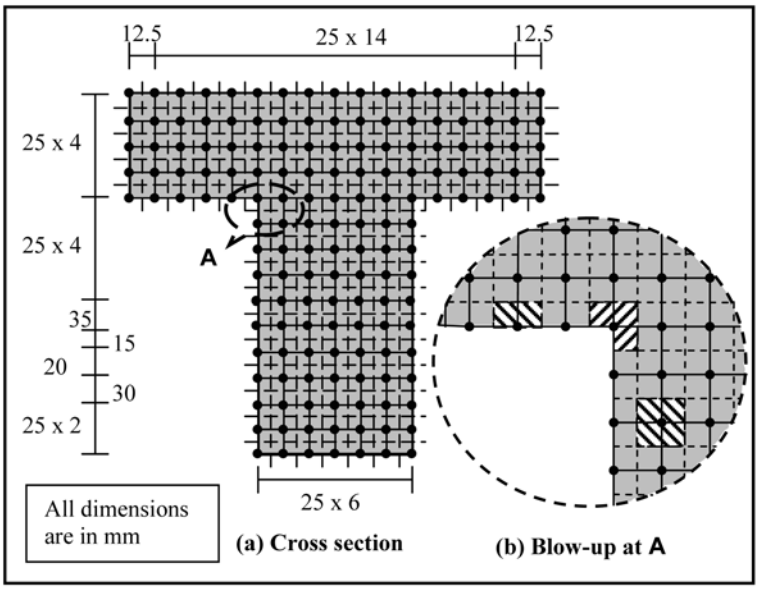

Fig. 5 Tributary area employed for computing the area of the discrete reinforcement representing the fiber

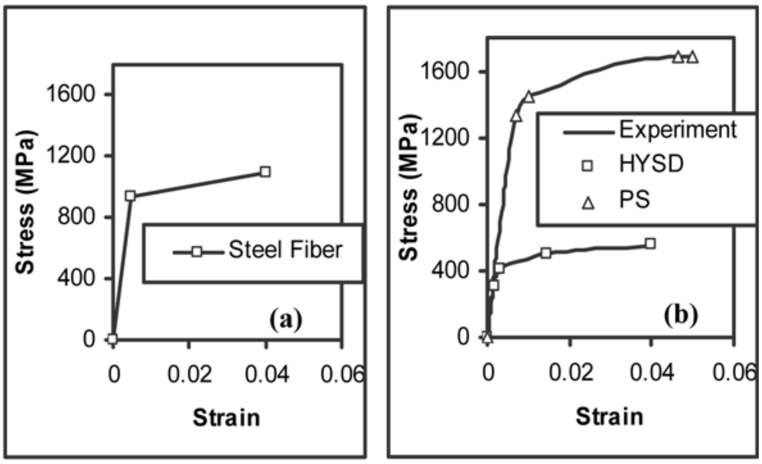

Fig. 6 Stress-strain response of steel reinforcements

'LINK8' three dimensional truss elements. Area of the 'LINK8' element representing the fiber has been computed based on the tributary area concept used by Padmarajaiah and Ramaswamy (2002). The area of discrete reinforcement representing the fiber (A_F) is computed by

$$A_F = \alpha' V_f A_{ct}$$

The orientation factor 'α'' is assumed to take a value of 0.64, which is the average of the values proposed by Souroushian and Lee (1990) for 2D and 3D orientation of fibers in a beam. 'V_f' is the volume fraction of the fiber. 'A_{ct}' is the tributary area of the concrete over which the fibers present is represented by a discrete reinforcement. The tributary area concept is illustrated in Fig. 5. The tributary area for the discrete reinforcement representing the fiber connecting to an edge node, a corner node and interior node has been shown as hatched area in Fig. 5(b). Rate independent multi-linear isotropic hardening option (MISO) with von-Mises yield criterion has been used to define the material property of steel fibers. An elasto-plastic stress-strain response as shown in Fig. 6(a) has been used for steel fibers. The Young's modulus of steel fibers is taken as 200 GPa and Poisson's ratio as 0.3.

3.2. Modeling of rebar

The longitudinal and transverse reinforcements (HYSD bars and PS wires) have been modeled as discrete reinforcements using 'LINK8' elements. Rate independent multi-linear isotropic hardening option (MISO) with von-Mises yield criterion has been used to define the material property of steel rebar. The tensile stress-strain response of steel based on the test data Fig. 6(b) has been used in the present analysis. An initial strain (corresponding to the effective prestress) has been defined for the discrete 'LINK8' element representing the prestressing steel wires in order to simulate the prestressing effect.

3.3. Modeling of bond-slip of reinforcements

The bond-slip between the reinforcements (fibers, deformed bars and prestressing steel wires) has been modeled using 'COMBIN39' non-liner spring element. The'COMBIN39' having very small dimension connecting the nodes of 'LINK8' elements and 'SOLID65' elements has been used to model bond-slip of the reinforcements in the present analysis. The slip test data reported in the literature has been used for the load-deformation characteristics of the 'COMBIN39' element. The slip test data of Mirza and Houde (1979) was used for HYSD bars (Fig. 7a). For smooth PS wires, the test data for mild steel having smooth finish by Edward and Yannopoulos (1979) was used Fig. 7(a). A linear variation with out tension cutoff as used by Padmarajaiah and Ramaswamy based on

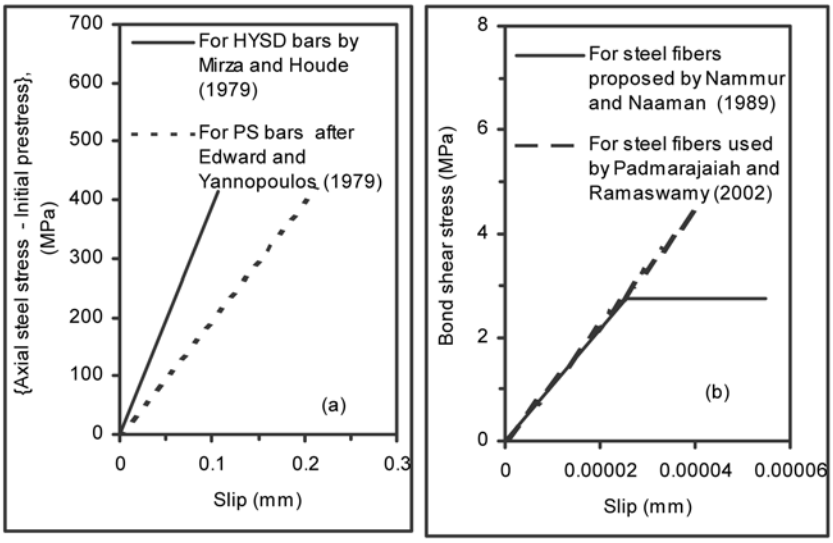

Fig. 7 Slip response of steel reinforcements

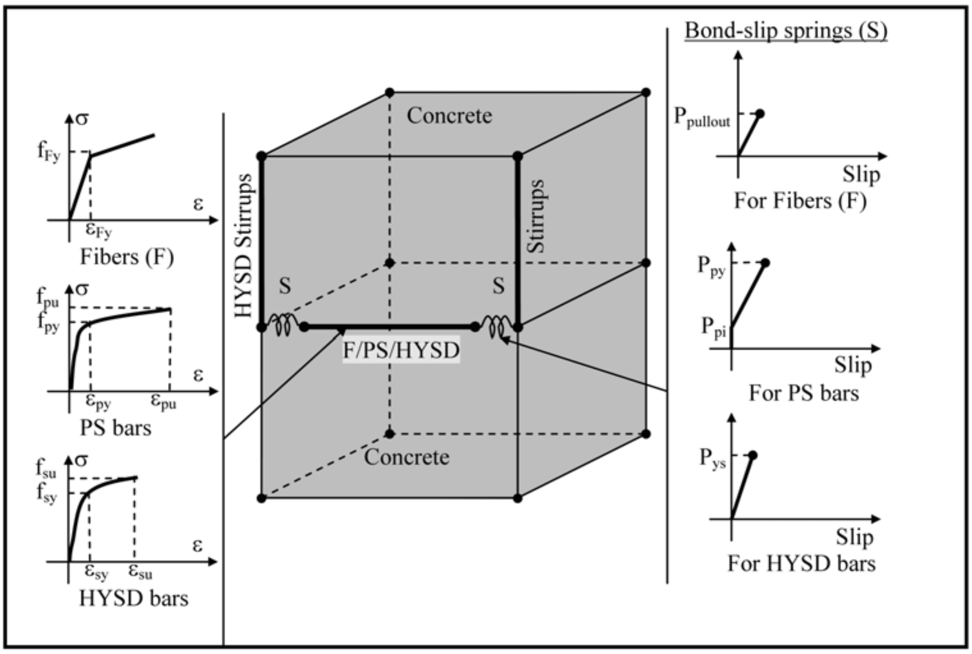

Fig. 8 Details of the FE model

the test data of Nammur and Namman (1989) has been used for steel fibers (Fig. 7b). The transverse reinforcements (stirrups) were assumed as perfectly bonded to the surrounding concrete in the present analysis. The 'COMBIN39' having very small dimension connecting the nodes of 'LINK8' elements and 'SOLID65' elements was used to model bond-slip of the reinforcements in the present analysis (Fig. 8).

4. Comparison between predicted and experimental results

The predicted load of T-beams at first crack and at ultimate stage was compared with the corresponding test data (Table 2). The average value of the ratio of the predicted load at first crack to the corresponding load observed in the experiment was found to be 0.90 with a standard deviation of 4 percent. The average value of the ratio of the predicted load at ultimate to the corresponding load observed in the experiment was found to be 0.98 with a standard deviation of 4 percent. This indicates that, 'ANSYS' model predicted the load at various stages, namely, at cracking and at ultimate, quite accurately.

The load-deflection response of T-beams predicted using 'ANSYS' model was compared with the corresponding experimental results (Fig. 9). In the initial stage of loading, the predicted load-deflection response of various beams agrees with the corresponding test data. However, 'ANSYS' model predicted slightly softer deflection results in the post-cracking regime when compared with the corresponding test data. This variation in the predicted results may be attributed to the difference in the bond-slip model of reinforcements used in the analysis when compared with that present in the test. The sensitivity of the FE model to the variation in the bond-slip input to predict the load-

Table 2 Comparison of predicted load at various stages with the corresponding experimental data

Beam Designation	At first crack			At ultimate		
	P_{cr_expt} (kN)	P_{cr_FE} (kN)	$\frac{P_{cr\_FE}}{P_{cr\_expt}}$	P_{u_expt} (kN)	P_{u_FE} (kN)	$\frac{P_{cr\_FE}}{P_{cr\_expt}}$
S35FOCWOC	142.75	136.00	0.95	380.20	367.43	0.97
S65FOCWOC	162.90	148.00	0.91	423.60	428.84	1.01
S85FOCWOC	174.36	156.00	0.89	441.00	449.56	1.02
S35FFCWFC	167.99	152.00	0.90	423.20	408.07	0.96
S65FFCWFC	193.54	172.00	0.89	498.40	503.18	1.01
S85FFCWFC	207.39	184.00	0.89	531.80	500.00	0.94
S35FOCWFC	168.94	148.00	0.88	404.20	380.00	0.94
S65FOCWFC	193.57	172.00	0.89	464.60	487.79	1.05
S85FOCWFC	207.47	181.00	0.87	503.60	500.00	0.99
S65FFCWFC-A	327.13	280.00	0.86	728.60	657.14	0.90
S85FFCWFC-A	350.30	342.00	0.98	829.90	801.73	0.97
Average	-	-	0.90	-	-	0.98
Standard Deviation	-	-	0.04	-	-	0.04

P_{cr}, P_u = load at first crack and ultimate respectively
Suffix: _expt, _FE = observed in the experiment and predicted using FE analysis respectively

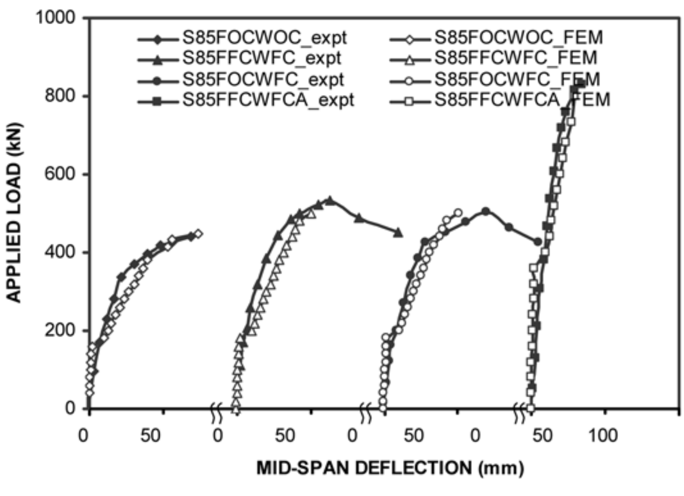

Fig. 9 Comparison of predicted load-deflection response of T-beams (85 MPa) with the corresponding experimental data

deflection response of the beam S85FFCWFC has been presented in Fig. 10. The load-deflection response of beams with the bond-slip springs having a stiffness equal to 0.5 times and 0.1 times of the stiffness of the corresponding springs in the beam S85FFCWFC is shown in Fig. 10. The load-deflection response predicted using ANSYS for beam S85FFCWFC with the reinforcements assumed to be perfectly bonded to the surrounding concrete is also shown in Fig. 10. Fig. 10 indicates that the bond-slip model influences the prediction of load-deflection response of shear

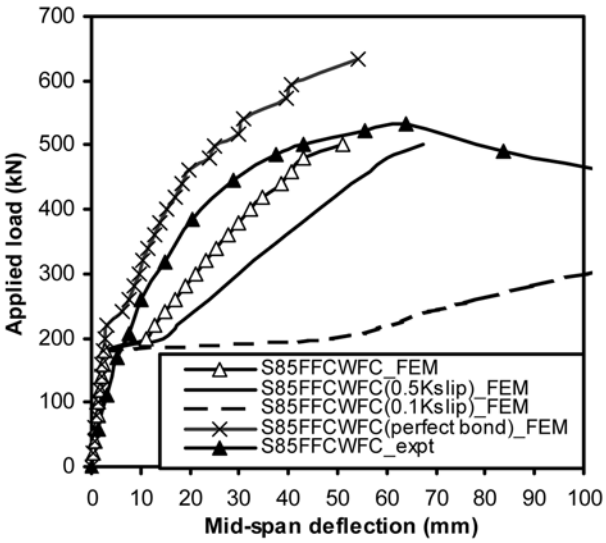

Fig. 10 Analytical prediction of load-deflection response of S85FFCWFC indicating the influence of bond-slip model. 'S85FFCWFC (0.5Kslip)_FEM' indicates the case of beam analyzed with bond-slip springs having a stiffness equal to half of the stiffness of the corresponding spring element in beam 'S85FFCWFC_FEM'

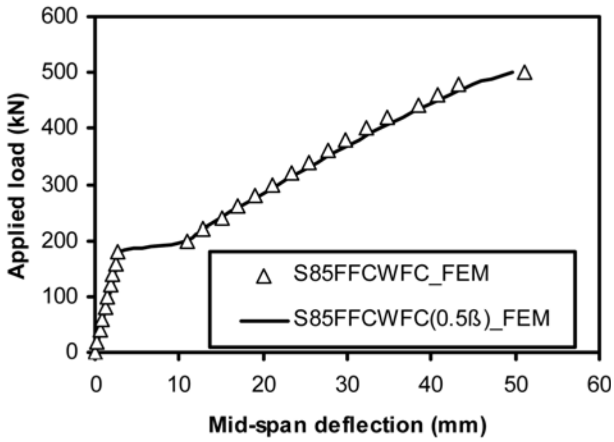

Fig. 11 Analytical prediction of load-deflection response of S85FFCWFC indicating the influence of shear transfer coefficient of concrete. 'S85FFCWFC (0.5β)_FEM' indicates the case of beam analyzed with shear transfer coefficients of concrete (β_c and β_t) assumed to take a value equal to half of that in beam 'S85FFCWFC_FEM'

critical partially prestressed concrete T-beam significantly. It is expected that with the use of a correct bond-slip model, ANSYS model would predict the post-cracking regime of the load-deflection response of T-beams accurately.

The influence of the shear transfer coefficient of concrete (β_c and β_t) on the load-deflection response has been studied. The load-deflection response of the beam S85FFCWFC predicted using FE analysis is compared with the load-deflection response of a beam in which the shear transfer

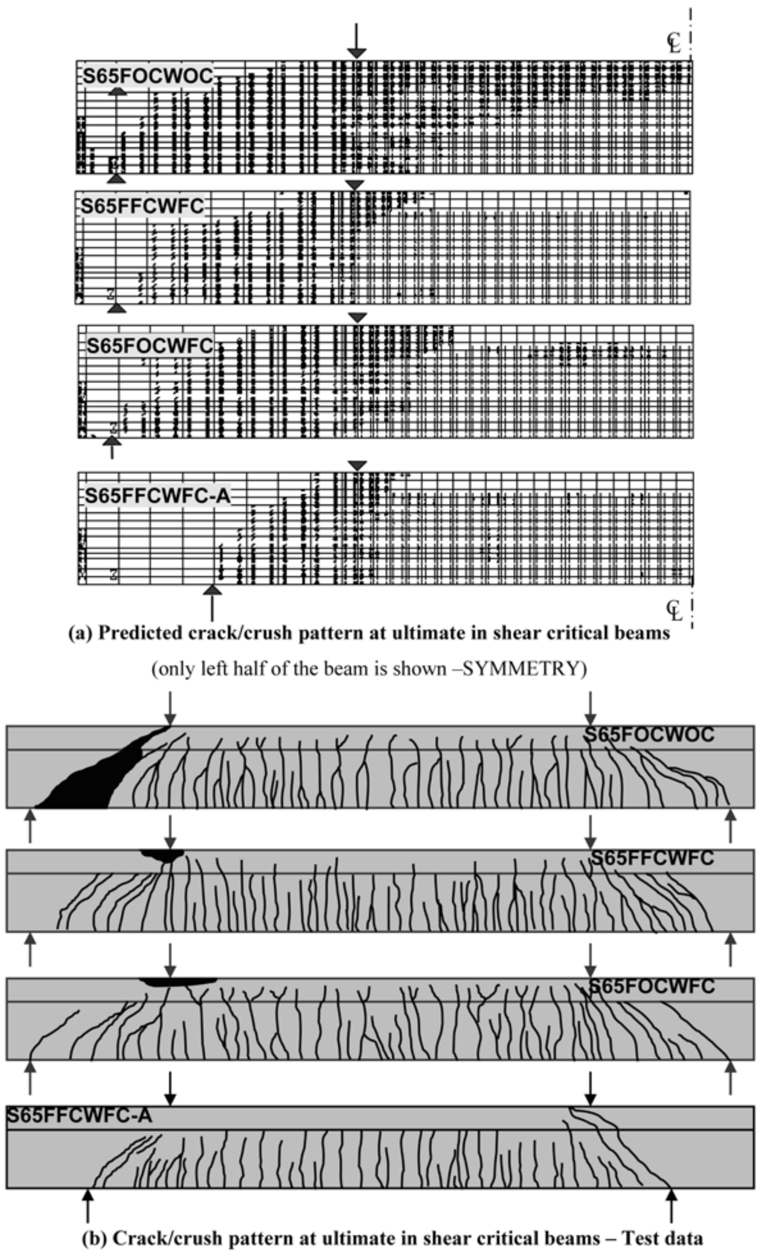

Fig. 12 Comparison of predicted crack/crush pattern with the corresponding test data

coefficients of the concrete (β_c and β_t) assume to take a value half of that of the beam S85FFCWFC_FEM is shown in Fig. 11 indicates that the influence of the shear transfer coefficients in the load-deflection response of shear critical partially prestressed concrete T-beam is marginal.

As observed in the experiment, the ANSYS model predicted the first crack in the constant moment zone. In the later stages, ANSYS model predicted propagation of existing cracks, more cracks in the constant moment zone and new cracks in the shear span. The predicted orientation of the crack in the T-beam was vertical in the constant moment zone and inclined in the shear span (Fig. 12). The cracks predicted using 'ANSYS' model were found to be in good agreement with the experimental observation. At ultimate load, one of the inclined cracks in the shear span widened and concrete near the tip of the crack close to the loading point crushed. 'ANSYS' model predicted crushing of concrete at the ultimate, which was indicated by large deformation at the node. Four T-beams, namely, S65FOCWOC, S85FOCWOC, S65FFCWFC-A and S85FFCWFC-A, failed in diagonal tension forming a through crack Fig. 12(b). The comparison of the crack/crush pattern predicted to that observed in the experiment indicated that the 'ANSYS' model predicts the zones of critical cracks quite accurately.

5. Conclusions

Based on the comparison of the predicted results of partially prestressed beams having steel fibers over partial or full depth with the corresponding experimental data, following conclusions were drawn.

- The predicted load of T-beams at various stages was found to be in good agreement with the test data.
- The proposed model predicted slightly softer results in post-cracking regime of the load-deflection response of T-beams. This variation is due to the difference in the bond-slip model of reinforcements used in the analysis when compared with that present in the test.
- The bond-slip model used for the reinforcements influences the load-deflection response of the beam predicted using FE model significantly (Fig. 10).
- 'ANSYS' model correctly predicted the diagonal tension failure and shear compression failure of prestressed concrete beams observed in the experiment.

It is expected that the modeling strategy for the finite element analysis proposed in this study will be useful for designing/ analyzing SFRC reinforced and prestressed concrete members.

References

Barzegar, F., (1988) "Analysis of RC membrane elements with anisotropic reinforcements", *J. Struct. Eng.*, ASCE, **115**(3), 647-665.

Bazant, Z. P. and Oh, B. H., (1983) "Crack band theory for fracture of concrete", *Materials and Structures,* **16**, 155-177.

Belletti, B., Bernardi, P., Cerioni, R., and Iori, I. (2003), "On a fiber reinforced concrete constitutive model for NLFE analysis", *Studies and Researc*hes, Italy **24**, 23-49.

Edward, A.D. and Yannopoulos, P.J. (1979), "Local bond-stress to slip relationship of hot rolled deformed bars and mild steel plain bars", *ACI J.*, **76**(1), 405-420.

Fanning, P., (2001), "Non-linear models for reinforced and post-tensioned concrete beams", *Electronic J. Struct. Eng.*, **2**, 111-119.

Kotosov, M. D. and Pavlovic, M. N. (2004), "Size effects in beams with smaller shear span to depth ratios", *Comp. Struct.*, **82**, 143-156.

Kawakami, M. and Ito, T., (2003), "Non-linear finite element analysis of prestressed concrete members using ADINA," *Comput. Struct.*, **81**, 727-734

Mirza, S.M. and Houde, J. (1979), "Study of bond stress-slip relationships in reinforced concrete", *ACI J.*, **76**(1), 19-45.

Nammur, Jr. G. and Namman, A. E. (1989), "Bond stress model for fibre reinforced concrete based on bond stress slip relationship", *ACI Materials J.*, **86**(1), 45-56.

Padmarajaiah, S. K. and Ramaswamy, A. (2002), "A finite element assessment of flexural strength of prestressed concrete beams with fiber reinforcement", *Cement Conc. Com.*, **24**, 229-241.

Soroushian, P., and Lee, D. N., (1990) "Distribution and orientation of fibers in steel fiber reinforced concrete", *ACI Materials J.*, **87**(5), 433-439.

William, K. J. and Warnke, E. D. (1975), "Constitutive model for tri-axial behavior of concrete", *Proceedings of the International Association for Bridge and Structural Engineering*, ISMES, Bergamo, Italy, 19/1-30.

Zhao, Z. Z., Kwan, A. K. H., and He, X. G., (2004), "Non-linear finite element analysis of deep reinforced concrete coupling beams", *Eng. Struct.* **26**, 13-25.

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