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A generalized 4-unknown refined theory for bending and free vibration analysis of laminated composite and sandwich plates and shells

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Abstract. This research is devoted to investigate the bending and free vibration behaviour of laminated composite/sandwich plates and shells, by applying an analytical model based on a generalized and simple refined higher-order shear deformation theory (RHSDT) with four independent unknown variables. The kinematics of the proposed theoretical model is defined by an undetermined integral component and uses the hyperbolic shape function to include the effects of the transverse shear stresses through the plate/shell thickness; hence a shear correction factor is not required. The governing differential equations and associated boundary conditions are derived by employing the principle of virtual work and solved via Navier-type analytical procedure. To verify the validity and applicability of the present refined theory, some numerical results related to displacements, stresses and fundamental frequencies of simply supported laminated composite/sandwich plates and shells are presented and compared with those obtained by other shear deformation models considered in this paper. From the analysis, it can be concluded that the kinematics based on the undetermined integral component is very efficient, and its use leads to reach higher accuracy than conventional models in the study of laminated plates and shells.

Keywords: bending; free vibration; laminated composite; sandwich; shell

1. Introduction

A shell is defined as a type of structural element which is characterized by the curved geometry of its middle surface and by its thickness at any point on this surface, being a fully three-dimensional solid object whose thickness is very small when compared with other two dimensions. Usually, these lightweight elements have many superior mechanical properties (e.g., high specific strength and stiffness and stability), are assembled to form large structures able to absorb an additional amount of energy as compared to flat structures during their service life. Moreover, the number of research articles and books published over the past four decades indicates that the increasing and rapid demand for shell structures made of composite materials, appears to be the driving force behind the recent technological development observed in the various branches of engineering, especially in civil and architectural engineering (varieties of shell roofs, curved bridges, silos, storage tanks, vaulted dams, containment shells of nuclear power plants and cooling towers). They are, also applied in aeronautical construction (rockets, propellers aircraft wings). Recently, the continuous development of material science and engineering, has led to the use of shell structures in mechanical engineering (ships shell, vehicle body, turbine blades), and in all branches of the chemical and petroleum industries. Many technical publications on this topic can be found throughout the archival literature. On the other hand the design of shell structures requires the ability to combine aesthetic knowledge to give the construction an attractive appearance, as well as structural analysis to dimension the structure is needed to insure precision, safety and economical design. Since shell structures are more complicated as compared to flat plates due to their curvature effect (cylindrical, spherical, ellipsoidal, conical shells, …etc.), various numerical models have been proposed by practicing structural engineers using the finite element method and/or analytical solution procedures have been established by scientific researchers using many shell theories to analyze their structural behaviour. Nevertheless, they have the particularity of being among the most delicate structures to investigate.

In general, the analysis of the dynamic and static behaviour of thin to thick laminated composite/sandwich plates and shells in the past has been classified according to one of the different approaches derived from the equations of elasticity, such as equivalent single layer (ESL) theories,

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zigzag theories, three-dimensional elasticity theory and/or multiple model methods. It should be recalled that the ESL theoretical approaches are derived from the 3D theory of elasticity by making suitable assumptions on the form of the displacement field or the stress state through the thickness of laminated plates and shells. In the field of thin shells, the first admissible theory was originally formulated by Love in 1888 (Love 1888). It is developed on the same simplest ESL model based on the well-known Kirchhoff hypotheses (Kirchhoff 1850) of the classical plate theory (CPT) for thin homogeneous plate structures. However, the application of this theory for moderately thick or thick plates and shells leads to considerable errors due to neglect of the effects of transverse shear deformation and rotary inertia. Whereas Reissner (1941) developed the first-order approximation theory of shells, in which some inadequacies of Love's theory were removed without complicating the system of equations, as well as the strain-displacement relationships and stress resultants expressions are acquired in the context of the 3D theory of elasticity. Sanders (1959) also proposed a modified first-order approximation shell theory by using the Kirchhoff-Love assumptions and the principle of virtual work. For simplicity this theory has been developed almost entirely as a two-dimensional model. After extensive research and controversy, the validity of the simple theory of Love was finally generalized and confirmed only much later by Koiter (1961), as he derived a 2D model for thin linearly elastic shells. Koiter's model is in fact one of the most currently used for numerical calculations, it includes both membrane and bending effects coupled at different order of magnitudes. Shortly after Koiter's work, other developments which also utilize another 2D linear shell model are founded under the Reissner-Mindlin plate theory and are based on some kinematic assumptions of moderately thick plate theory. Among these we mention the different mathematical models for shells proposed by Naghdi (1963, 1972). However, these models are described in curvilinear coordinates defined on the middle surface. It is also possible to express these models in Cartesian coordinates. Tessler *et al*. (2010) utilized a consistent refinement of first-order shear deformation theory for laminated composite and sandwich plates using improved zigzag kinematics. Versino *et al*. (2013) developed C° triangular elements based on the Refined Zigzag Theory for multilayer composite and sandwich plates. Using a Reissner's mixed variational principle; Tessler (2015) presented a refined zigzag theory for homogeneous, laminated composite, and sandwich beams. Iurlaro *et al*. (2015) employed a Refined Zigzag Theory for laminated composite and sandwich plates derived from Reissner's Mixed Variational Theorem. Kefal *et al*. (2017) presented an enhanced inverse finite element method for displacement and stress monitoring of multilayered composite and sandwich structures. Kefal *et al*. (2019) proposed a novel isogeometric beam element based on mixed form of refined zigzag theory for thick sandwich and multilayered composite beams. Madenci and Özütok (2020) presented a variational approximate for high order bending analysis of laminated composite plates.

Furthermore, a general small deflection thermoelastic theory of thick laminated composite shells subjected to mechanical and arbitrary temperature distribution is presented by Kant (1981). In thus investigation the material of each layer is assumed to have its planes of symmetry coincident with the orthogonal shell coordinates. Whitney (1984) presented an analytical solution to study the buckling of anisotropic laminated cylindrical plates subjected to arbitrary combinations of axial compression, internal pressure and in-plane shear loads, utilizing the classical laminated shell theory, in conjunction with Galerkin's method based on a variational principle of displacements. Reddy and Liu (1985) have developed a simple higher-order shear deformation theory (HSDT) for bending and free vibration analysis of laminated spherical and cylindrical shells. This theory involves the same dependent unknowns as in the first-order shear deformation theory (FSDT) and takes into account the parabolic distribution of transverse shear stresses through the shell thickness. Barbero and Reddy (1990) presented a general 2D shear deformation theory of laminated cylindrical shells, in which geometric nonlinearity in the von-Karman sense is also considered. Various computational models available in the open literature were used by Noor and Burton (1992) for predicting the thermal and thermo-mechanical responses of multilayered plates and shells while Leissa and Chang (1996) derived a rigorous and comprehensive theory which governs the linearly elastic deformation, including the effects of shear deformation and rotary inertia to solve the static and dynamic problems of laminated composite shallow shells, having an arbitrary curvature and a constant thickness. The Closed-form formulations of 2D HSDT are provided by Khare *et al*. (2003) for the bending analysis of simply supported cross-ply laminated composite and sandwich shallow shells under thermo-mechanical loading conditions. Furthermore, the effect of the variation of geometry, shallowness, lamination scheme and the other parameters on transverse central deflection is examined in detail in this analysis. In another study, Panda and Singh (2009) studied the thermal post-buckling response of laminated composite cylindrical/hyperboloid shallow shell panels subjected to uniform temperature field, using nonlinear finite element model based on the HSDT for different geometric parameters.

Since the composite plates and the curvature of the shells pose new and delicate problems compared to the case of conventional plates, several higher-order shear deformation theories (HSDTs) were subsequently developed to optimize the analysis of different types of laminated/sandwich plates and shells responses and are extensively used by many researchers. Kumar *et al*. (2013) applied a new finite element model based on HSDT to solve many problems for the static response of laminated composite skew shells considering different geometries, boundary conditions, loadings and other shell parameters. In the same year, Viola *et al*. (2013) proposed a 2D HSDT for free vibration analysis of moderately thick laminated doubly-curved shells and panels with different curvatures, by using the generalized differential quadrature technique and a displacement field having a fixed nine degrees of freedom. Sayyad and Ghugal (2015) reviewed the various methods carried out in the available literature for the free vibration analysis of multilayered laminated composite and sandwich plates using different HSDTs. In the following year, new shear deformation theories have been proposed by Sarangan and Singh (2016) to study the bending, buckling and free vibration analysis of laminated composite and sandwich plates. Further, the generalized form of governing differential equations is derived by employing the principle of virtual work and solved by Navier's closed-form solution technique. Afterwards, Tornabene (2016) proposed a higher-order layer-wise theory, in which the stretching effect is included for each layer using a general displacement field based on the Carrera unified formulation for free vibration analysis of thick doubly-curved laminated composite shells and panels. An analytical solution for the thermoelastic static problem of simply supported laminated composite plates under bi-sinusoidal thermal load was presented by Ramos *et al.* (2016), using a modified non-polynomial displacement field based on Carrera unified formulation. Abed and Majeed (2020) analyzed the effect of boundary conditions on harmonic response of laminated plates.

Recently, many other researchers have worked on development of theory of laminated orthotropic plates and shells (Chien *et al.* 2016, Swain *et al.* 2017, Sayyad and Ghugal 2017, Yarasca *et al.* 2017, Thakur *et al.* 2017, Biswal *et al.* 2017, Hirwani *et al.* 2018, Benhenni *et al.* 2018, Katariya and Panda 2019, Monge *et al.* 2019, Cuba *et al.* 2019, Bakhshi and Taheri-Behrooz 2019, Sayyad and Ghugal 2020). The main objective of the present work is to investigate the bending and free vibration behaviour of laminated composite and sandwich plates and shells, using the generalized and simple refined higher-order shear deformation theory (RHSDT), which account for the effects of the transverse shear stresses through the plate/shell thickness and without requiring the shear correction factor. The governing equations and its boundary conditions are derived by employing the principle of virtual works. Analytical solutions are obtained for bending and free vibration response of simply supported laminated composite and sandwich plates and shells applying Navier's solution procedure. Several numerical examples are presented and compared with other shear deformation theories to verify the validity and applicability of the present theory.

2. Formulation of the problem

A generalized higher-order shear deformation theory for laminated composite and sandwich plates and shells is developed. The theory takes into account for a hyperbolic distribution of transverse shear stresses through the plate/shell thickness, and satisfies exactly the zero shear stress conditions on the top and bottom surfaces of the plate/shell without requiring any shear correction factor. Moreover, the proposed model is easy to implement since it contains a smaller number of unknowns and governing equations than the other higher-order theories. The main idea of the present theory arises from the conventional HSDT models developed by several authors for plates to the bending and free vibration analysis of shells. The original version of the earlier HSDT assumes the following displacement field

$$
\begin{aligned}
\overline{u}(\xi_1,\xi_2,\xi_3,t) &= \left(1+\frac{\xi_3}{R_1}\right)u - \xi_3\frac{\partial w}{a_1\,\partial\xi_1} + f(\xi_3)\phi_1,\\
\overline{v}(\xi_1,\xi_2,\xi_3,t) &= \left(1+\frac{\xi_3}{R_2}\right)v - \xi_3\frac{\partial w}{a_2\,\partial\xi_2} + f(\xi_3)\phi_2,\\
\overline{w}(\xi_1,\xi_2,t) &=
\end{aligned} \tag{1}
$$

where $u(\xi_1,\xi_2,t)$, $v(\xi_1,\xi_2,t)$, $w(\xi_1,\xi_2,t)$, $\phi_1(\xi_1,\xi_2,t)$ and $\phi_2(\xi_1,\xi_2,t)$ are the well known displacement components of the middle surface of the panel, while $f(\xi_3)$ represents shape function identifying the distribution of the transverse shear strains and stresses across the thickness of the plate/shell. The generalized displacement model under discussion is a four independent variables theory defined as follows

$$
\begin{aligned}
\overline{u}(\xi_1,\xi_2,\xi_3,t) &= \left(1+\frac{\xi_3}{R_1}\right)u - \xi_3\frac{\partial w}{a_1\,\partial\xi_1} + k_1 f(\xi_3)\int\theta\,d\xi_1\\
\overline{v}(\xi_1,\xi_2,\xi_3,t) &= \left(1+\frac{\xi_3}{R_2}\right)v - \xi_3\frac{\partial w}{a_2\,\partial\xi_2} + k_2 f(\xi_3)\int\theta\,d\xi_2\\
\overline{w}(\xi_1,\xi_2,t) &= w
\end{aligned} \tag{2}
$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the displacement components of any point in the laminate domain in the ξ_1, ξ_2 and ξ_3 directions, respectively. In this study, the shape function is chosen based on the hyperbolic function proposed by Soldatos (1992) as

$$
f(\xi_3) = \hbar\sinh\left(\frac{\xi_3}{\hbar}\right) - \xi_3\cosh\left(\frac{1}{2}\right) \tag{3}
$$

In the derivation of the necessary equations, small elastic deformations are supposed, (i.e., displacements and rotations are small, and comply with Hooke's law), and the shell structure is made up of a number of orthotropic layers, which are supposed to be perfectly bonded together.

The basis of the present laminated shell theory is the 3D elasticity theory, expressed in general curvilinear (reference) surface-parallel coordinates, whereas the thickness coordinate is normal with respect to the reference surface as indicated in Fig. 1. The strain-displacement

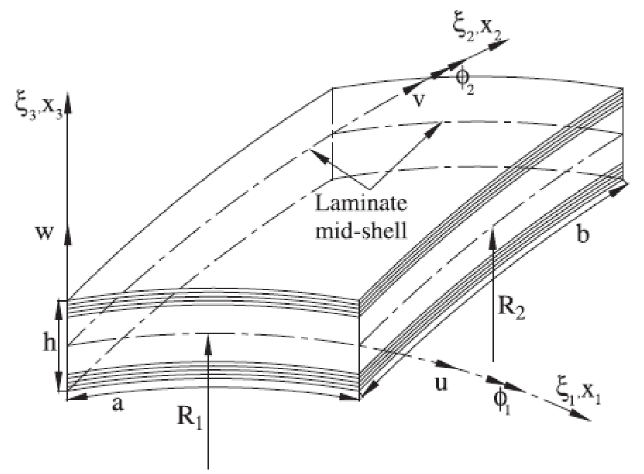

Fig. 1 Geometry and notations for generic laminated shells with positive set of layer/laminated reference axes, displacement components and fiber orientation

relations in the curvilinear coordinate system are given as follows (Reddy 2004)

$$\begin{gathered}\varepsilon_1=\frac{1}{A_1}\left(\frac{\partial\bar{u}}{\partial\xi_1}+\frac{1}{a_2}\frac{\partial a_1}{\partial\xi_2}\bar{v}+\frac{a_1}{R_1}\bar{w}\right),\\ \varepsilon_2=\frac{1}{A_2}\left(\frac{\partial\bar{v}}{\partial\xi_2}+\frac{1}{a_1}\frac{\partial a_2}{\partial\xi_1}\bar{u}+\frac{a_2}{R_2}\bar{w}\right)\\ \varepsilon_4=\frac{1}{A_2}\frac{\partial\bar{w}}{\partial\xi_2}+A_2\frac{\partial}{\partial\xi_3}\left(\frac{\bar{v}}{A_2}\right),\varepsilon_5=\frac{1}{A_1}\frac{\partial\bar{w}}{\partial\xi_1}+A_1\frac{\partial}{\partial\xi_3}\left(\frac{\bar{u}}{A_1}\right)\\ \varepsilon_6=\frac{A_2}{A_1}\frac{\partial}{\partial\xi_1}\left(\frac{\bar{v}}{A_2}\right)+\frac{A_1}{A_2}\frac{\partial}{\partial\xi_2}\left(\frac{\bar{u}}{A_1}\right)\end{gathered}\tag{4}$$

in which $A_1=a_1\left(1+\frac{\xi_3}{R_1}\right)$, $A_2=a_2\left(1+\frac{\xi_3}{R_2}\right)$ and $\xi_i(i=1,\dots.,6)$ denote strain components, while a_1 and a_2 are scalar values associated to the type of shells. Substituting the expressions of displacements considered by Eq. (2) into the relations given in Eq. (4) of a moderately shallow and deep shell provides the following strain-displacement equations, valid for an arbitrary point of a double curvature panel under study

$$\begin{gathered}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\end{Bmatrix}=\begin{Bmatrix}\varepsilon_1^0\\ \varepsilon_2^0\\ \varepsilon_6^0\end{Bmatrix}+z\begin{Bmatrix}\varepsilon_1^1\\ \varepsilon_2^1\\ \varepsilon_6^1\end{Bmatrix}+f(\xi_3)\begin{Bmatrix}\varepsilon_1^2\\ \varepsilon_2^2\\ \varepsilon_6^2\end{Bmatrix},\\ \begin{Bmatrix}\varepsilon_4\\ \varepsilon_5\end{Bmatrix}=g(\xi_3)\begin{Bmatrix}\varepsilon_4^3\\ \varepsilon_5^3\end{Bmatrix}\end{gathered}\tag{5}$$

where

$$\begin{Bmatrix}\varepsilon_1^0\\ \varepsilon_2^0\\ \varepsilon_6^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u}{\partial x_1}+\dfrac{w}{R_1}\\ \dfrac{\partial v}{\partial x_2}+\dfrac{w}{R_2}\\ \dfrac{\partial u}{\partial x_2}+\dfrac{\partial v}{\partial x_1}\end{Bmatrix},\begin{Bmatrix}\varepsilon_1^1\\ \varepsilon_2^1\\ \varepsilon_6^1\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w}{\partial x_1^2}\\ -\dfrac{\partial^2 w}{\partial x_2^2}\\ -2\dfrac{\partial^2 w}{\partial x_1\partial x_2}\end{Bmatrix},\tag{6a}$$

$$\begin{Bmatrix}\varepsilon_1^2\\ \varepsilon_2^2\\ \varepsilon_6^2\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial x_2}\displaystyle\int\theta dx_1+k_2\dfrac{\partial}{\partial x_1}\int\theta dx_2\end{Bmatrix},$$

$$\begin{Bmatrix}\varepsilon_4^3\\ \varepsilon_5^3\end{Bmatrix}=\begin{Bmatrix}k_2\displaystyle\int\theta dx_2\\ k_1\displaystyle\int\theta dx_1\end{Bmatrix}\tag{6b}$$

and

$$g(\xi_3)=\frac{df(\xi_3)}{d\xi_3}\tag{6c}$$

The integrals adopted in the previous relations shall be resolved by a Navier solution and can be written as

$$\begin{gathered}\frac{\partial}{\partial x_2}\int\theta dx_1=A'\frac{\partial^2\theta}{\partial x_1\partial x_2},\quad\frac{\partial}{\partial x_1}\int\theta dx_2=B'\frac{\partial^2\theta}{\partial x_1\partial x_2},\\ \int\theta dx_1=A'\frac{\partial\theta}{\partial x_1},\quad\int\theta dx_2=B'\frac{\partial\theta}{\partial x_2}\end{gathered}\tag{7}$$

where A' and B' are determined according to the type of solution employed, in this case via Navier procedure. Thus, the coefficients A',B',k_1 and k_2 are expressed by

$$A'=-\frac{1}{\alpha^2}\quad,B'=-\frac{1}{\beta^2},\quad k_1=\alpha^2,\qquad k_2=\beta^2\tag{8}$$

where $\alpha=r\pi/a$; $\beta=s\pi/b$ and x_i denote the Cartesian coordinates $(dx_1=a_i d\xi_i, i=1,2)$. Assuming linearly elastic behavior, the constitutive stress-strain relations in the orthotropic local coordinate system can be summarized in the following matrix form

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\\ \sigma_4\\ \sigma_5\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\ Q_{12}&Q_{22}&0&0&0\\ 0&0&Q_{66}&0&0\\ 0&0&0&Q_{44}&0\\ 0&0&0&0&Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\\ \varepsilon_4\\ \varepsilon_5\end{Bmatrix}\tag{9}$$

in which $(\sigma_1,\sigma_2,\sigma_6,\sigma_4,\sigma_5)$ are the normal and shear stresses and $(\varepsilon_1,\varepsilon_2,\varepsilon_6,\varepsilon_4,\varepsilon_5)$ are the normal and shear strains components in the orthotropic local coordinate system. The stiffness coefficients Q_{ij} are calculated in the conventional manner from the terms of engineering constants given as follows

$$\begin{gathered}Q_{11}=\frac{E_{11}}{1-\nu_{12}\nu_{21}},\ Q_{22}=\frac{E_{22}}{1-\nu_{12}\nu_{21}},Q_{12}=\frac{\nu_{12}E_{11}}{1-\nu_{12}\nu_{21}},\\ Q_{66}=G_{12},\ Q_{44}=G_{23},Q_{55}=G_{13}\end{gathered}\tag{10}$$

in which $E_{11},E_{22},\nu_{12},\nu_{21},G_{12},G_{23}$ and G_{13} are the material properties of the layer. By performing the transformation rule of stress-strain between the local coordinate system of the layer and the global coordinate system of the laminated plate/shell, the stress-strain relations in the global (ξ_1,ξ_2,ξ_3) coordinate system can be obtained as

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\\ \sigma_4\\ \sigma_5\end{Bmatrix}^{(k)}=\begin{bmatrix}\bar{Q}_{11}&\bar{Q}_{12}&\bar{Q}_{16}&0&0\\ \bar{Q}_{12}&\bar{Q}_{22}&\bar{Q}_{26}&0&0\\ \bar{Q}_{16}&\bar{Q}_{26}&\bar{Q}_{66}&0&0\\ 0&0&0&\bar{Q}_{44}&0\\ 0&0&0&0&\bar{Q}_{55}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\\ \varepsilon_4\\ \varepsilon_5\end{Bmatrix}^{(k)}\tag{11}$$

where $\bar{Q}_{ij}^{k}$ are the transformed elastic coefficients given by Reddy (2004), which are calculated according to the fibers orientation angle of each layer with respect to the global coordinate system.

The governing equations and associated boundary conditions of the present generalized shear deformation theory are derived using the dynamic version of the principle of virtual work stated by the following analytical form (Abdelmalek *et al.* 2017, Ebrahimi and Barati 2017a, b, Eltaher *et al.* 2018, 2020, Fenjan *et al.* 2019, Safa *et al.* 2019, Zouatnia and Hadji 2019a, Rachedi *et al.* 2020, Hamed *et al.* 2020)

$$\begin{gathered}0=\int_0^t\left[\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_\Omega\begin{pmatrix}\sigma_1\delta\,\varepsilon_1^{(k)}+\sigma_2\delta\,\varepsilon_2^{(k)}+\sigma_6\delta\,\varepsilon_6^{(k)}+\\ \sigma_4\delta\,\varepsilon_4^{(k)}+\sigma_5\delta\,\varepsilon_5^{(k)}\end{pmatrix}\right.\\ \left.dx_1dx_2d\xi_3\right]dt-\int_0^t\int_\Omega q\,\delta w dx_1dx_2dt\\ -\int_0^t\delta\left\{\int_{-h/2}^{h/2}\int_\Omega\rho[(\dot{u})^2+(\dot{v})^2+(\dot{w})^2]\,dx_1dx_2d\xi_3\right\}dt\end{gathered}\tag{12}$$

$$\begin{gathered}=\int_0^t\left\{\int_\Omega(N_1\delta\varepsilon_1^0+N_2\delta\varepsilon_2^0+N_6\delta\varepsilon_6^0+M_1\delta\varepsilon_1^1+M_2\delta\varepsilon_2^1+\right.\\ M_6\delta\varepsilon_6^1+P_1\delta\varepsilon_1^2+P_2\delta\varepsilon_2^2+P_6\delta\varepsilon_6^2+Q_4\delta\varepsilon_4^3+Q_5\delta\varepsilon_5^3-q\delta w\end{gathered}\tag{13}$$

$$+\left(\left(I_1+\frac{2I_2}{R_1}\right)\ddot{u}-\left(I_2+\frac{I_3}{R_1}\right)\frac{\partial\ddot{w}}{\partial x_1}+k_1A'\left(I_4+\frac{I_5}{R_1}\right)\frac{\partial\ddot{\theta}}{\partial x_1}\right)\delta u$$
$$+\left(\left(I_1+\frac{2I_2}{R_2}\right)\ddot{v}-\left(I_2+\frac{I_3}{R_2}\right)\frac{\partial\ddot{w}}{\partial x_2}+k_2B'\left(I_4+\frac{I_5}{R_2}\right)\frac{\partial\ddot{\theta}}{\partial x_2}\right)\delta v$$
$$+\left(\begin{array}{l}\left(I_2+\frac{I_3}{R_1}\right)\frac{\partial\ddot{u}}{\partial x_1}+\left(I_2+\frac{I_3}{R_2}\right)\frac{\partial\ddot{v}}{\partial x_2}-I_3\left(\frac{\partial^2\ddot{w}}{\partial x_1^2}+\frac{\partial^2\ddot{w}}{\partial x_2^2}\right)\\+I_5\left(k_1A'\frac{\partial^2\ddot{\theta}}{\partial x_1^2}+k_2B'\frac{\partial^2\ddot{\theta}}{\partial x_2^2}\right)+I_1w\end{array}\right)\delta w$$
$$+\left(\begin{array}{l}k_1A'\left(I_4+\frac{I_5}{R_1}\right)\frac{\partial\ddot{u}}{\partial x_1}+k_2B'\left(I_4+\frac{I_5}{R_1}\right)\frac{\partial\ddot{v}}{\partial x_1}\\-I_5\left(k_1A'\frac{\partial^2\ddot{w}}{\partial x_1^2}+k_2B'\frac{\partial^2\ddot{w}}{\partial x_2^2}\right)\\+I_6\left(k_1^2A'^2\frac{\partial^2\ddot{\theta}}{\partial x_1^2}+k_2^2B'^2\frac{\partial^2\ddot{\theta}}{\partial x_2^2}\right)\end{array}\right)\delta\theta\Bigg)dx_1dx_2\Bigg\}dt$$

where δ denotes the variational operator, q is the transverse load, ρ is the density of the plate or shell under consideration, N_i, M_i, P_i and Q_i are the stress resultants can be determined in the usual form as

$$(N_i,M_i,P_i)=\sum_{k=1}^{n}\int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}}\sigma_i^{(k)}\left(1,\xi_3,f(\xi_3)\right)d\,\xi_3,\quad i=1,2,6,$$
$$Q_i=\sum_{k=1}^{n}\int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}}\sigma_i^{(k)}g(\xi_3)d\,\xi_3,\quad i=4,5 \tag{14}$$

and the coefficients $I_i\,(i=1,2,3,4,5,6)$ are defined by the following equations

$$(I_1,I_2,I_3,I_4,I_5,I_6)=\sum_{k=1}^{n}\int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}}\rho^{(k)}Q_{ij}^{(k)}\left(1,\xi_3,\xi_3{}^2,f(\xi_3),\xi_3\,f(\xi_3),[f(\xi_3)]^2\right)d\,\xi_3 \tag{15}$$

The generalized governing equations of motion are derived from Eq. (13) by integrating the displacement gradients by parts and setting the coefficients of $\delta u, \delta v, \delta w$ and $\delta\theta$ equal to zero, individually. The generalized equations obtained are as follows

$$\begin{array}{l}\delta\,u:\ \frac{\partial N_1}{\partial x_1}+\frac{\partial N_6}{\partial x_2}=\left(I_1+\frac{2I_2}{R_1}\right)\ddot{u}-\left(I_2+\frac{I_3}{R_1}\right)\frac{\partial\ddot{w}}{\partial x_1}\\\qquad+k_1A'\left(I_4+\frac{I_5}{R_1}\right)\frac{\partial\ddot{\theta}}{\partial x_1}\\\delta\,v:\ \frac{\partial N_2}{\partial x_2}+\frac{\partial N_6}{\partial x_1}=\left(I_1+\frac{2I_2}{R_2}\right)\ddot{v}-\left(I_2+\frac{I_3}{R_2}\right)\frac{\partial\ddot{w}}{\partial x_2}\\\qquad+k_2B'\left(I_4+\frac{I_5}{R_2}\right)\frac{\partial\ddot{\theta}}{\partial x_2}\\\delta\,w:\ -\frac{N_1}{R_1}-\frac{N_2}{R_2}+\frac{\partial^2M_1}{\partial x_1^2}+2\frac{\partial^2M_6}{\partial x_1\partial x_2}+\frac{\partial^2M_2}{\partial x_2^2}\\\qquad+q=I_1\ddot{w}+\left(I_2+\frac{I_3}{R_1}\right)\frac{\partial\ddot{u}}{\partial x_1}\\\qquad+\left(I_2+\frac{I_3}{R_2}\right)\frac{\partial\ddot{v}}{\partial x_2}-I_3\left(\frac{\partial^2\ddot{w}}{\partial x_1^2}+\frac{\partial^2\ddot{w}}{\partial x_2^2}\right)\\\qquad+I_5\left(k_1A'\frac{\partial^2\ddot{\theta}}{\partial x_1^2}+k_2B'\frac{\partial^2\ddot{\theta}}{\partial x_2^2}\right)\\\delta\,\theta:\ -k_1P_1-k_2P_2-(k_1A'+k_2B')\frac{\partial^2P_6}{\partial x_1\partial x_2}\\\qquad+k_1A^{\frac{\partial Q_4}{\partial x_1}}+k_2B^{\frac{\partial Q_5}{\partial x_2}}=\\\qquad-k_1A^{'\left(I_4+\frac{I_5}{R_1}\right)\frac{\partial\ddot{u}}{\partial x_1}}-k_2B^{'\left(I_4+\frac{I_5}{R_2}\right)\frac{\partial\ddot{v}}{\partial x_2}}\\\qquad+I_5\left(k_1A'\frac{\partial^2\ddot{w}}{\partial x_1^2}+k_2B'\frac{\partial^2\ddot{w}}{\partial x_2^2}\right)\\\qquad-I_6\left((k_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x_1^2}+(k_2B')^2\frac{\partial^2\ddot{\theta}}{\partial x_2^2}\right)\end{array} \tag{16}$$

Substituting Eq. (5) into Eq. (11) and the subsequent results into Eq. (14), the stress resultants of the proposed analytical model can be expressed in terms of virtual strains, using the following constitutive equations

$$\begin{array}{c}N_1=A_{11}\varepsilon_1^0+A_{12}\varepsilon_2^0+B_{11}\varepsilon_1^1+B_{12}\varepsilon_2^1+E_{11}\varepsilon_1^2+E_{12}\varepsilon_2^2,\\N_2=A_{12}\varepsilon_1^0+A_{22}\varepsilon_2^0+B_{12}\varepsilon_1^1+B_{22}\varepsilon_2^1+E_{12}\varepsilon_1^2+E_{22}\varepsilon_2^2,\\N_6=A_{66}\varepsilon_6^0+B_{66}\varepsilon_6^1+E_{66}\varepsilon_6^2\end{array} \tag{17a}$$

$$\begin{array}{c}M_1=B_{11}\varepsilon_1^0+B_{12}\varepsilon_2^0+D_{11}\varepsilon_1^1+D_{12}\varepsilon_2^1+F_{11}\varepsilon_1^2+F_{12}\varepsilon_2^2,\\M_2=B_{12}\varepsilon_1^0+B_{22}\varepsilon_2^0+D_{12}\varepsilon_1^1+D_{22}\varepsilon_2^1+F_{12}\varepsilon_1^2+F_{22}\varepsilon_2^2,\\M_6=B_{66}\varepsilon_6^0+D_{66}\varepsilon_6^1+F_{66}\varepsilon_6^2\end{array} \tag{17b}$$

$$\begin{array}{c}P_1=E_{11}\varepsilon_1^0+E_{12}\varepsilon_2^0+F_{11}\varepsilon_1^1+F_{12}\varepsilon_2^1+H_{11}\varepsilon_1^2+H_{12}\varepsilon_2^2,\\P_2=E_{12}\varepsilon_1^0+E_{22}\varepsilon_2^0+F_{12}\varepsilon_1^1+F_{22}\varepsilon_2^1+H_{12}\varepsilon_1^2+H_{22}\varepsilon_2^2,\\P_6=E_{66}\varepsilon_6^0+F_{66}\varepsilon_6^1+H_{66}\varepsilon_6^2\end{array} \tag{17c}$$

$$Q_4=A_{44}^s\varepsilon_4^3,\quad Q_5=A_{55}^s\varepsilon_5^3 \tag{17d}$$

in which the components of the stiffnesses are defined by

$$\{A_{ij},B_{ij},D_{ij},E_{ij},F_{ij},H_{ij}\}=\sum_{k=1}^{n}\int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}}\bar{Q}_{ij}^{(k)}\left(1,\xi_3,\xi_3{}^2,f(\xi_3),\xi_3\,f(\xi_3),[f(\xi_3)]^2\right)d\,\xi_3,\quad i,j=1,2,6 \tag{18a}$$

$$A_{ij}^s=\sum_{k=1}^{n}\int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}}\bar{Q}_{ij}^{(k)}[g(z)]^2d\xi_3,\quad i,j=4,5 \tag{18b}$$

The following simply supported boundary conditions are assumed at all four edges of the laminated composite and sandwich plates and shells

$$\begin{array}{c}u(x_1,0)=u(x_1,b)=v(0,x_2)=v(a,x_2)=0,\\w(x_1,0)=w(x_1,b)=w(0,x_2)=w(a,x_2)=0,\\N_2(x_1,0)=N_2(x_1,b)=N_1(0,x_2)=N_1(a,x_2)=0,\\M_2(x_1,0)=M_2(x_1,b)=M_1(0,x_2)=M_1(a,x_2)=0,\\P_2(x_1,0)=P_2(x_1,b)=P_1(0,x_2)=P_1(a,x_2)=0,\\\theta(x_1,0)=\theta(x_1,b)=\theta(0,x_2)=\theta(a,x_2)=0\end{array} \tag{19}$$

3. Solution procedure

The governing differential equations given in Eq. (16) for the bending and free vibration analysis of laminated composite and sandwich plates and shells can be solved by using the Navier's solution procedure in the form of double trigonometric series. In the case of antisymmetric cross-ply laminated plates and shells, it should be noted that the following stiffness components are identically zero

$$\begin{array}{c}A_{i6}=B_{i6}=D_{i6}=E_{i6}=F_{i6}=H_{i6}=0;\quad i=1,2\\B_{12}=E_{12}=B_{66}=E_{66}=A_{45}^s=A_{54}^s=0\end{array} \tag{20}$$

The Navier's solution procedure is applied to determine the analytical solutions for a simply supported laminated/sandwich plate and shell. The solution is supposed to be of the form (Zouatnia and Hadji 2019b)

$$\begin{Bmatrix}u(x_1,x_2,t)\\v(x_1,x_2,t)\\w(x_1,x_2,t)\\\theta(x_1,x_2,t)\end{Bmatrix}=\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\begin{Bmatrix}U_{rs}\cos(\alpha\,x_1)\sin(\beta\,x_2)\,e^{i\,\omega\,t}\\V_{rs}\sin(\alpha\,x_1)\cos(\beta\,x_2)\,e^{i\,\omega\,t}\\W_{rs}\sin(\alpha\,x_1)\sin(\beta\,x_2)\,e^{i\,\omega\,t}\\\Theta_{rs}\sin(\alpha\,x_1)\sin(\beta\,x_2)\,e^{i\,\omega\,t}\end{Bmatrix} \tag{21}$$

Table 1 Comparison of non-dimensional displacements and stresses for two-layer (0°/90°) cross-ply laminated plate under sinusoidal load, (b=a, Material 1)

a/h	Theory	$\bar{u}$, $(-h/2)$	$\bar{w}$, (0)	$\bar{\sigma}_x$, $(-h/2)$	$\bar{\sigma}_y$ $(-h/2)$	$\bar{\tau}_{xy}$, $(-h/2)$	$\bar{\tau}_{xz}$, (0)	$\bar{\tau}_{yz}$, (0)
4	Exact 3D [(a)]	-	2.0670	0.8410	0.1090	0.0591	0.120	0.135
	Reddy (1984)	0.0113	1.9985	0.9060	0.0891	0.0577	0.1251	0.1251
	Mindlin (1951)	0.0088	2.1492	0.7157	0.0843	0.0525	0.1091	0.1091
	Kirchhoff (1850)	0.0088	1.0636	0.7157	0.0843	0.0525	-	-
	Present	0.0112	2.0003	0.9052	0.0891	0.0577	0.1249	0.1249
10	Exact 3D [(a)]	-	1.2250	0.7302	0.0886	0.0535	0.121	0.125
	Reddy (1984)	0.0092	1.2161	0.7468	0.0851	0.0533	0.1276	0.1276
	Mindlin (1951)	0.0088	1.2373	0.7157	0.0843	0.0525	0.1091	0.1091
	Kirchhoff (1850)	0.0088	1.0636	0.7157	0.0843	0.0525	-	-
	Present	0.0092	1.2163	0.7466	0.0851	0.0533	0.1273	0.1273
20	Reddy (1984)	0.0089	1.1018	0.7235	0.0845	0.0527	0.1280	0.1280
	Mindlin (1951)	0.0088	1.1070	0.7157	0.0843	0.0525	0.1091	0.1091
	Kirchhoff (1850)	0.0088	1.0636	0.7157	0.0843	0.0525	-	-
	Present	0.0089	1.1019	0.7235	0.0845	0.0527	0.1277	0.1277
100	Reddy (1984)	0.0088	1.0651	0.7161	0.0843	0.0525	0.1281	0.1281
	Mindlin (1951)	0.0088	1.0653	0.7157	0.0843	0.0525	0.1091	0.1091
	Kirchhoff (1850)	0.0088	1.0636	0.7157	0.0843	0.0525	-	-
	Present	0.0088	1.0651	0.7161	0.0843	0.0525	0.1280	0.1280

[(a)] Results taken from reference of Zenkour (2007)

where U_{rs}, V_{rs}, W_{rs} and Θ_{rs} are the unknown coefficients, while ω is the natural frequency of the system. Substituting Eqs. (17)-(21) into Eq. (16), the following equations are obtained

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{12} & M_{22} & M_{23} & M_{24} \\ M_{13} & M_{23} & M_{33} & M_{34} \\ M_{14} & M_{24} & M_{34} & M_{44} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{rs} \\ \ddot{V}_{rs} \\ \ddot{W}_{rs} \\ \ddot{\Theta}_{rs} \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix} \begin{Bmatrix} U_{rs} \\ V_{rs} \\ W_{rs} \\ \Theta_{rs} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{rs} \\ 0 \end{Bmatrix} \tag{22}$$

The transverse distributed load $q$is also chosen as the following form

$$q(x,y) = \sum_{r=1}^{\infty} \sum_{s=1}^{\infty} Q_{rs} \, sin(\alpha \, x) \, sin(\beta \, y) \tag{23}$$

Where the coefficients Q_{rs} are given below for certain typical loads

$$Q_{rs} = \begin{cases} q_0 & \text{For sinusoidal loads} \\ \frac{16q_0}{rs\pi^2} & \text{For uniform loads} \\ \frac{4P}{ab} sin\left(\frac{r\pi x_0}{a}\right) sin\left(\frac{s\pi y_0}{b}\right) & \text{For point loads} \end{cases} \tag{24}$$

The elements of stiffness matrix $[K_{ij}]$ and mass matrix $[M_{ij}]$ implicated in Eq. (22) are given as the following form

$$\begin{gathered} K_{11} = -(\alpha^2 A_{11} + \beta^2 A_{66}), \;\; K_{12} = -\alpha\beta(A_{12} + A_{66}), \\ K_{13} = \alpha\left(\frac{A_{11}}{R_1} + \frac{A_{12}}{R_2} + \alpha^2 B_{11}\right), \;\; K_{14} = \alpha k_1 E_{11}, \\ K_{22} = -(\alpha^2 A_{66} + \beta^2 A_{22}), \\ K_{23} = \beta\left(\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2} + \beta^2 B_{22}\right), \\ K_{24} = \beta k_2 E_{22}, \\ K_{33} = -\frac{1}{R_2}\left(\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2} + 2\beta^2 B_{22}\right) \\ -2\alpha^2\beta^2(D_{12} + 2D_{66}) \\ -\frac{1}{R_1}\left(\frac{A_{11}}{R_1} + \frac{A_{12}}{R_2} + 2\alpha^2 B_{11}\right) - (\alpha^4 D_{11} + \beta^4 D_{22}), \\ K_{34} = -\left(k_1\frac{E_{11}}{R_1} + k_2\frac{E_{22}}{R_2}\right) - (k_2\alpha^2 + k_1\beta^2)F_{12} \\ +2\alpha^2\beta^2(k_1 A' + k_2 B')F_{66} - k_1\alpha^2 F_{11} - k_2\beta^2 F_{22}, \\ K_{44} = -k_1^2 H_{11} - k_2^2 H_{22} - 2k_1 k_2 H_{12} \\ -(k_1 A' + k_2 B')(k_2 B'\alpha^2\beta^2 H_{66} + k_1 A'\alpha^2\beta^2 H_{66}) \\ -k_2^2 B'^2\beta^2 A_{44}^s - k_1^2 A'^2\alpha^2 A_{55}^s \end{gathered} \tag{25}$$

And

$$\begin{gathered} M_{11} = -\left(I_1 + 2\frac{I_2}{R_1}\right), \;\; M_{12} = 0, \; M_{13} = \left(I_2 + \frac{I_3}{R_1}\right)\alpha, \\ M_{14} = -k_1 A'\left(I_4 + \frac{I_5}{R_1}\right)\alpha, M_{22} = -\left(I_1 + 2\frac{I_2}{R_2}\right), \\ M_{23} = \left(I_2 + \frac{I_3}{R_2}\right)\beta, \;\; M_{24} = -k_2 B^{\left(I_4 + \frac{I_5}{R_2}\right)}\beta, \\ M_{33} = -I_1 - I_3(\alpha^2 + \beta^2), \\ M_{34} = I_5(k_1 A'\alpha^2 + k_2 B'\beta^2), \\ M_{44} = -I_6(k_1^2 A'^2\alpha^2 + k_2^2 B'^2\beta^2) \end{gathered} \tag{26}$$

4. Numerical results and discussion

In order to confirm the accuracy and efficacy of the present refined higher-order shear deformation theory (RHSDT) with four unknown variables, a number of numerical examples are investigated for the static bending and free vibration analysis of simply supported laminated composite, and sandwich plates and shells. For this purpose, the results are compared with those obtained by existing theories in the literature to demonstrate the validity of the proposed model. The following problems are considered for the detailed numerical study

1. Bending analysis of antisymmetric (0°/90°)$_n$ cross-

Table 2 Comparison of non-dimensional displacements and stresses for four-layer (0°/90°)$_2$ cross-ply laminated plate under sinusoidal load, ($b=a$, Material 1)

a/h	Theory	$\bar{u}\ (-h/2)$	$\bar{w}\ (0)$	$\bar{\sigma}_x\ (h/2)$	$\bar{\sigma}_y\ (h/2)$	$\bar{\tau}_{xy}\ (-h/2)$	$\bar{\tau}_{xz}\ (0)$	$\bar{\tau}_{yz}\ (0)$
	Exact 3D [(a)]	0.0087	1.9581	-	0.7444	0.0457	0.2325	0.2410
	Reddy (1984)	0.0087	1.6093	0.0495	0.6970	0.0350	0.1358	0.1358
4	Mindlin (1951)	0.0061	1.5921	0.0357	0.4868	0.0250	0.1091	0.1091
	Kirchhoff (1850)	0.0061	0.5065	0.0357	0.4868	0.0250	-	-
	Present	0.0087	1.6092	0.0494	0.6959	0.0349	0.1355	0.1355
	Exact 3D [(a)]	0.0066	0.7624	-	0.5308	0.0292	0.2713	0.2712
	Reddy (1984)	0.0065	0.6865	0.0380	0.5211	0.0266	0.1386	0.1386
10	Mindlin (1951)	0.0061	0.6802	0.0357	0.4868	0.0250	0.1091	0.1091
	Kirchhoff (1850)	0.0061	0.5065	0.0357	0.4868	0.0250	-	-
	Present	0.0065	0.6865	0.0380	0.5209	0.0266	0.1382	0.1382
	Exact 3D [(a)]	0.0062	0.5717	-	0.4979	0.0260	0.2781	0.2781
	Reddy (1984)	0.0062	0.5517	0.0363	0.4954	0.0254	0.1390	0.1390
20	Mindlin (1951)	0.0061	0.5500	0.0357	0.4868	0.0250	0.1091	0.1091
	Kirchhoff (1850)	0.0061	0.5065	0.0357	0.4868	0.0250	-	-
	Present	0.0062	0.5516	0.0363	0.4954	0.0254	0.1386	0.1386
	Exact 3D [(a)]	0.0061	0.5091	-	0.4872	0.0250	0.2803	0.2803
	Reddy (1984)	0.0061	0.5083	0.0358	0.4872	0.0250	0.1391	0.1391
100	Mindlin (1951)	0.0061	0.5083	0.0357	0.4868	0.0250	0.1091	0.1091
	Kirchhoff (1850)	0.0061	0.5065	0.0357	0.4868	0.0250	-	-
	Present	0.0061	0.5083	0.0358	0.4872	0.0250	0.1387	0.1387

[(a)] Results taken from reference of Zenkour (2007)

Table 3 Comparison of non-dimensional displacements and stresses for eight-layer (0°/90°)$_4$ cross-ply laminated plate under sinusoidal load, ($b=a$, Material 1)

a/h	Theory	$\bar{u}\ (-h/2)$	$\bar{w}\ (0)$	$\bar{\sigma}_x\ (h/2)$	$\bar{\sigma}_y\ (h/2)$	$\bar{\tau}_{xy}\ (-h/2)$	$\bar{\tau}_{xz}\ (0)$	$\bar{\tau}_{yz}\ (0)$
	Exact 3D [(a)]	0.0081	1.7903	-	0.6867	0.0347	0.2220	0.2266
	Reddy (1984)	0.0088	1.5168	0.0417	0.6996	0.0311	0.1335	0.1335
4	Mindlin (1951)	0.0062	1.5335	0.0296	0.4950	0.0221	0.1091	0.1091
	Kirchhoff (1850)	0.0062	0.4479	0.0296	0.4950	0.0221	-	-
	Present	0.0088	1.5173	0.0417	0.6986	0.0311	0.1332	0.1332
	Exact 3D [(a)]	0.0066	0.6698	-	0.5247	0.0244	0.2430	0.2433
	Reddy (1984)	0.0066	0.6229	0.0316	0.5285	0.0236	0.1366	0.1366
10	Mindlin (1951)	0.0062	0.6216	0.0296	0.4950	0.0221	0.1091	0.1091
	Kirchhoff (1850)	0.0062	0.4479	0.0296	0.4950	0.0221	-	-
	Present	0.0066	0.6229	0.0316	0.5283	0.0236	0.1362	0.1362
	Exact 3D [(a)]	0.0063	0.5037	-	0.5024	0.0227	0.2467	0.2467
	Reddy (1984)	0.0063	0.4918	0.0301	0.5034	0.0225	0.1371	0.1371
20	Mindlin (1951)	0.0062	0.4913	0.0296	0.4950	0.0221	0.1091	0.1091
	Kirchhoff (1850)	0.0062	0.4479	0.0296	0.4950	0.0221	-	-
	Present	0.0063	0.4918	0.0301	0.5033	0.0225	0.1367	0.1367
	Exact 3D [(a)]	0.0062	0.4504	-	0.4956	0.0214	0.2481	0.2481
	Reddy (1984)	0.0062	0.4496	0.0297	0.4953	0.0221	0.1372	0.1372
100	Mindlin (1951)	0.0062	0.4496	0.0296	0.4950	0.0221	0.1091	0.1091
	Kirchhoff (1850)	0.0062	0.4479	0.0296	0.4950	0.0221	-	-
	Present	0.0062	0.4496	0.0297	0.4953	0.0221	0.1368	0.1368

[(a)] Results taken from reference of Zenkour (2007)

ply laminated composite plates.
2. Bending analysis of three-layer symmetric sandwich (0°/*Core*/0°) plates.
3. Bending analysis of two-layer antisymmetric cross-ply laminated spherical shells.
4. Free vibration analysis of antisymmetric (0°/90°)$_n$ cross-ply laminated composite plates.
5. Free vibration analysis of cross-ply laminated cylindrical and spherical shells.

The numerical results of these problems are illustrated in Tables 1-10 and graphically shown in Figs. 2-7 followed by subsequent discussions. The material properties used in the numerical studies are as follows

Material 1 (Reddy 1984)

Table 4 Comparison of non-dimensional transverse maximum displacement and stresses of symmetric $(0°/Core/0°)$ sandwich square plate under uniform load (a/h=10, Material 2)

R	Theory	$\bar{w}(0)$	$\bar{\sigma}_x^1(-h/2)$	$\bar{\sigma}_x^2(-2h/5)$	$\bar{\sigma}_y^1(-h/2)$	$\bar{\sigma}_y^2(-2h/5)$	$\bar{\tau}_{xz}(0)$
5	Exact 3D [(b)]	258.970	60.353	46.623	38.491	30.097	4.364
	Pandya and Kant HSDT (1988)	258.740	62.380	46.910	38.930	30.330	3.089
	Pandya and Kant FSDT (1988)	236.100	61.870	49.500	36.650	29.320	3.313
	Kirchhoff CPT (1850)	216.940	61.141	48.623	36.622	29.297	4.590
	Ferreira *et al*. HSDT (2003)	257.110	60.366	47.003	38.456	30.242	4.548
	Xiang *et al.* (Levinson) (2009)	253.724	59.950	46.655	38.191	30.018	3.637
	Xiang *et al.* (Touratier) (2009)	253.989	60.123	47.097	38.249	30.187	3.707
	Xiang *et al.* (Karama) (2009)	253.638	60.124	46.703	38.242	30.020	3.764
	Mantari *et al.* HSDT (2011)	256.706	60.525	46.969	38.493	30.207	5.135
	Present	250.856	61.861	48.660	37.113	29.124	3.486
10	Exact 3D [(b)]	159.380	65.332	48.857	43.566	33.413	4.096
	Pandya and Kant HSDT (1988)	152.330	64.650	51.310	42.830	33.970	3.147
	Pandya and Kant FSDT (1988)	131.095	67.800	54.240	40.100	32.080	3.152
	Kirchhoff CPT (1850)	118.870	65.332	48.857	40.099	32.079	4.367
	Ferreira *et al*. HSDT (2003)	154.658	65.381	49.973	43.240	33.637	3.528
	Xiang *et al.* (Levinson) (2009)	152.664	65.008	49.684	42.945	33.394	3.450
	Xiang *et al.* (Touratier) (2009)	153.139	65.050	50.206	43.015	33.653	3.641
	Xiang *et al.* (Karama) (2009)	153.357	65.100	49.499	43.059	33.379	3.843
	Mantari *et al.* HSDT (2011)	155.498	65.542	49.708	43.385	33.591	4.814
	Present	149.146	68.074	52.951	40.869	31.665	3.162
15	Exact 3D [(b)]	121.720	66.787	48.299	46.424	34.955	3.964
	Pandya and Kant HSDT (1988)	110.430	66.620	51.970	44.920	35.410	3.035
	Pandya and Kant FSDT (1988)	90.850	70.040	56.030	41.390	33.110	3.091
	Kirchhoff CPT (1850)	81.768	69.135	55.308	41.410	33.128	4.283
	Ferreira *et al*. HSDT (2003)	114.644	66.920	50.323	45.623	35.170	3.021
	Xiang *et al.* (Levinson) (2009)	113.088	66.539	50.043	45.293	34.903	3.254
	Xiang *et al.* (Touratier) (2009)	113.964	66.544	50.679	45.431	35.278	3.472
	Xiang *et al.* (Karama) (2009)	114.585	66.621	49.663	45.546	34.919	3.706
	Mantari *et al.* HSDT (2011)	116.609	67.043	49.741	45.953	35.149	4.581
	Present	109.669	70.605	54.392	42.414	32.503	2.916

[(b)] Results taken from reference of Srinivas (1973)

$$E_1/E_2 = 25,\ \ E_3/E_2 = 1,\ \ G_{12}/E_2 = G_{13}/E_2 = 0.5,$$
$$G_{23}/E_2 = 0.2, \nu_{12} = \nu_{13} = \nu_{23} = 0.25,\ \ \rho = 1 \tag{27}$$

Material 2 (Srinivas 1973)

$$\begin{aligned}&\bar{Q}_{11}^{core} = 0.999781,\ \ \bar{Q}_{12}^{core} = \bar{Q}_{21}^{core} = 0.231192,\\ &\bar{Q}_{22}^{core} = 0.524886,\ \ \bar{Q}_{44}^{core} = 0.266810,\\ &\bar{Q}_{55}^{core} = 0.159914,\ \ \bar{Q}_{66}^{core} = 0.262931 \text{ (for core)}\\ &\bar{Q}_{ij}^{skins} = R\ \bar{Q}_{ij}^{core} \quad \text{(for skins)}\end{aligned} \tag{28}$$

Material 3 (Noor 1973)

$$\begin{aligned}&E_1/E_2 = open,\ \ G_{12}/E_2 = G_{13}/E_2 = 0.6,\\ &G_{23}/E_2 = 0.5,\quad \nu_{12} = \nu_{13} = 0.25,\quad \rho = 1\end{aligned} \tag{29}$$

For the simplicity, the results obtained for displacements, stresses and fundamental frequencies are presented in the following non-dimensional forms

Laminated plates and shells

$$\begin{aligned}&\bar{w}\left(\frac{a}{2},\frac{b}{2},0\right) = \frac{100h^3E_2}{q_0a^4}w,\quad \bar{\sigma}_x\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right) = \frac{h^2}{q_0a^2}\sigma_x,\\ &\bar{\sigma}_y\left(\frac{a}{2},\frac{b}{2},\frac{h}{6}\right) = \frac{h^2}{q_0a^2}\sigma_y,\ \ \bar{\tau}_{xy}\left(0,0,\frac{h}{2}\right) = \frac{h^2}{q_0a^2}\tau_{xy},\\ &\bar{\tau}_{xz}\left(0,\frac{b}{2},0\right) = \frac{h}{q_0a}\tau_{xz},\ \ \bar{\tau}_{yz}\left(\frac{a}{2},0,0\right) = \frac{h}{q_0a}\tau_{yz},\\ &\bar{\omega} = \omega\frac{a^2}{h}\sqrt{\rho/E_2}\end{aligned} \tag{30}$$

Sandwich plates

$$\begin{aligned}&\bar{w}\left(\frac{a}{2},\frac{b}{2},0\right) = \frac{\bar{Q}_{11}^{core}}{hq_0}w,\quad \bar{\sigma}_x^1\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{1}{q_0}\sigma_x,\\ &\bar{\sigma}_x^2\left(\frac{a}{2},\frac{b}{2},-\frac{2h}{5}\right) = \frac{1}{q_0}\sigma_x,\ \bar{\sigma}_y^1\left(\frac{a}{2},\frac{b}{2},-\frac{h}{2}\right) = \frac{1}{q_0}\sigma_y,\\ &\bar{\sigma}_y^2\left(\frac{a}{2},\frac{b}{2},-\frac{2h}{5}\right) = \frac{1}{q_0}\sigma_y,\ \ \bar{\tau}_{xz}\left(0,\frac{b}{2},0\right) = \frac{1}{q_0}\tau_{xz},\\ &\bar{\omega} = \omega\frac{a^2}{h}\sqrt{\rho/E_2}\end{aligned} \tag{31}$$

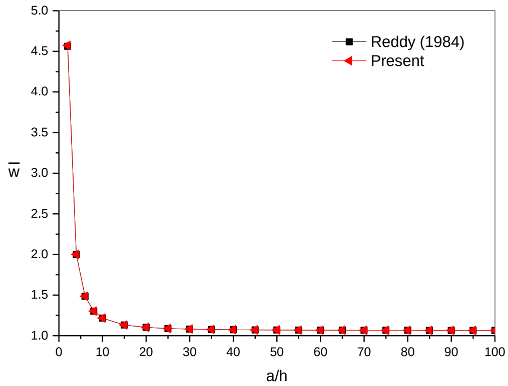

Fig. 2 Effect of side-to-thickness ratio (*a/h*) on non-dimensional transverse displacement ($\bar{w}$) of a two-layer (0°/90°) cross-ply laminated square plate under sinusoidal load

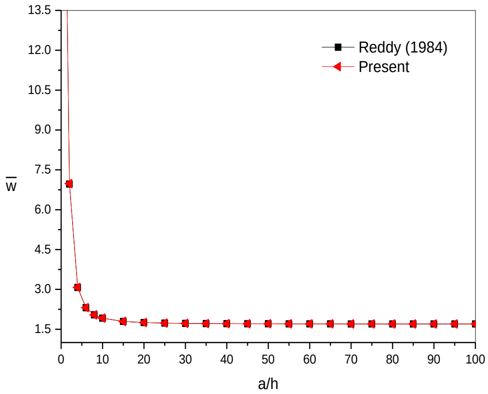

Fig. 3 Effect of side-to-thickness ratio (*a/h*) on non-dimensional transverse displacement ($\bar{w}$) of a two-layer (0°/90°) cross-ply laminated square plate under uniform load.

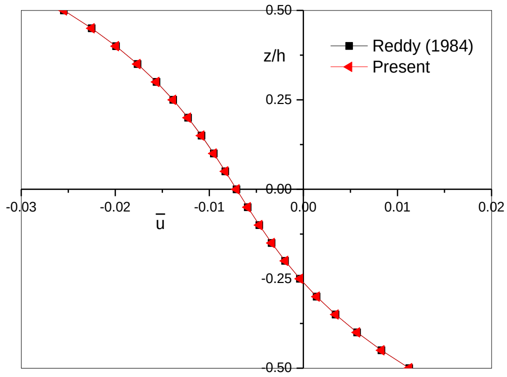

Fig. 4 Variation of in-plane displacement ($\bar{u}$) through the thickness of two-layer (0°/90°) cross-ply laminated composite square plate under sinusoidal load (*a/h* = 4)

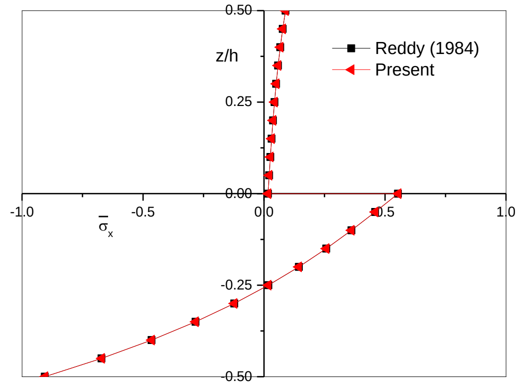

Fig. 5 Variation of in-plane normal stress ($\bar{\sigma}_x$) through the thickness of two-layer (0°/90°) cross-ply laminated composite square plate under sinusoidal load (*a/h* = 4)

4.1 Bending analysis of antisymmetric cross-ply laminated composite plates

The first problem is carried out for simply supported multilayered $(0°/90°)_n$ antisymmetric cross-ply laminated composite square plate under sinusoidal load. The number of layers is varied from 2 to 8 with the same thickness and made up of Material 1 defined by Eq. (27). The numerical results of non-dimensional displacements and stresses for different values of side-to-thickness ratio (*a/h*=4, 10, 20, 100) are presented in Tables 1-3, respectively. These results are compared with the corresponding results of three-dimensional elasticity solutions provided by Zenkour (2007), CPT of Kirchhoff, FSDT of Mindlin (1951) with a correction factor *k*=5/6 and HSDT of Reddy (1984). Examination of Tables 1-3 reveals that the numerical results of non-dimensional displacements and stresses obtained by using the present formulations are in excellent agreement with those calculated according to Reddy's theory whereas the transverse maximum displacement and in-plane shear stress may be identical to those of the exact 3D solution in the case of moderately thick laminated plates. It can be seen that the in-plane and transverse displacements $\bar{u}, \bar{w}$ are decreased with increased in thickness ratio (*a/h*) for all lamination schemes (i.e., (0°/90°), $(0°/90°)_2$, $(0°/90°)_4$); it means that the effect of transverse shear deformation is more pronounced in thick laminated plate not in thin laminated plate. It is also observed that the results for the non-dimensional in-plane normal stresses $\bar{\sigma}_x, \bar{\sigma}_y$ and in-plane shear stress $\bar{\tau}_{xy}$ decrease with increasing value of thickness ratio (*a/h*), moreover the CPT and FSDT underestimate these stresses compared to those obtained by the present generalized theory and HSDT of Reddy for all thickness ratios. However, all theories agree well with each other for thin laminated plates.

Figs. 2 and 3 show the effect of side-to-thickness ratio (*a/h*) on non-dimensional transverse displacement $\bar{w}$ of a two-layer (0°/90°) cross-ply laminated square plate under sinusoidal and uniform loads, respectively. It can be seen an excellent agreement between the present theory and HSDT of Reddy for both loading cases. It is also pointed out from Figs. 2 and 3 that the increase in the thickness ratio has a significant effect on the decrease of the transverse displacement. The graphical results obtained by using the present theory and Reddy's theory, which corresponds to the variations of in-plane displacement $\bar{u}$, in-plane stresses $\bar{\sigma}_x$, $\bar{\tau}_{xy}$ and transverse shear stress $\bar{\tau}_{xz}$ through the thickness of two-layer (0°/90°) cross-ply laminated composite square plate under sinusoidal load are also

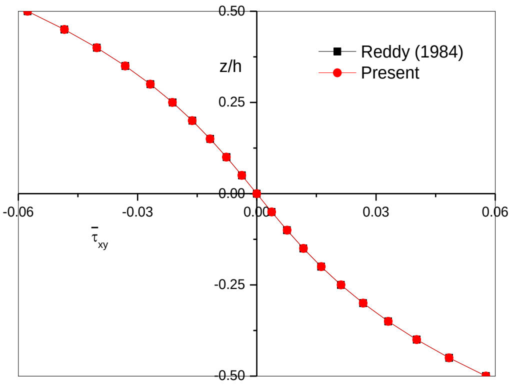

Fig. 6 Variation of in-plane shear stress $(\bar{\tau}_{xy})$ through the thickness of two-layer (0°/90°) cross-ply laminated composite square plate under sinusoidal load (a/h = 4)

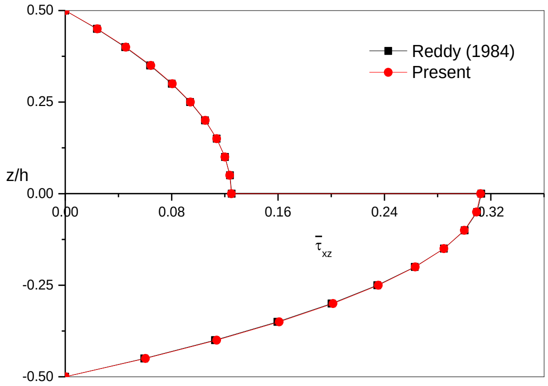

Fig. 7 Variation of transverse shear stress $(\bar{\tau}_{xz})$ through the thickness of two-layer (0°/90°) cross-ply laminated composite square plate under sinusoidal load (a/h = 4)

Table 5 Comparison of non-dimensional transverse displacement of cross-ply laminated spherical shells (0°/90°) under sinusoidal load, (b=a, R_1=R_2=R, Material 1)

R/a	Theory	$\frac{a}{h}=10$	$\frac{a}{h}=100$
5	Reddy and Liu FSDT (1985)	11.4290	1.1948
	Reddy and Liu HSDT (1985)	11.1660	1.1937
	Mantari *et al.* HSDT (2011)	11.1080	1.1940
	Present	11.1542	1.1935
10	Reddy and Liu FSDT (1985)	12.1230	3.5760
	Reddy and Liu HSDT (1985)	11.8960	3.5733
	Mantari *et al.* HSDT (2011)	11.8296	3.5751
	Present	11.8945	3.5729
20	Reddy and Liu FSDT (1985)	12.3090	7.1270
	Reddy and Liu HSDT (1985)	12.0940	7.1236
	Mantari *et al.* HSDT (2011)	12.0249	7.1295
	Present	12.0952	7.1232
50	Reddy and Liu FSDT (1985)	12.3620	9.8717
	Reddy and Liu HSDT (1985)	12.1500	9.8692
	Mantari *et al.* HSDT (2011)	12.0807	9.8800
	Present	12.1526	9.8690
100	Reddy and Liu FSDT (1985)	12.3700	10.4460
	Reddy and Liu HSDT (1985)	12.1580	10.4440
	Mantari *et al.* HSDT (2011)	12.0887	10.4562
	Present	12.1609	10.4442
Plate	Reddy and Liu FSDT (1985)	12.3730	10.6530
	Reddy and Liu HSDT (1985)	12.1610	10.6510
	Mantari *et al.* HSDT (2011)	12.0914	10.6635
	Present	12.1636	10.6511

plotted in Figs. 4 through 7 from which it is observed that the results obtained by the proposed model have good accuracy with Reddy's theory.

4.2 Bending analysis of three-layer symmetric sandwich (0°/Core/0°) plates

For this problem, efficiency of proposed theory is checked for the bending response of a simply supported moderately thick (a/h=10) sandwich square plate under uniform load. In this analysis, the symmetric sandwich plate is constituted by the orthotropic properties given by relations (28) and is composed of two outside layers (skins) of thickness h_1=h_3=0.1h and one middle layer (core) of thickness h_2=0.8h. Thus, the skin orthotropic properties are assumed as 'R' times the orthotropic properties of core, i.e., R is a factor that defines the degree of anisotropy in a sandwich laminate. Numerical results of non-dimensional displacements and stresses for three factor values (R=5, 10, 15) are depicted in Table 4 and compared with the following theories: exact 3D elasticity solution of Srinivas (1973), finite element evaluations of Pandya and Kant (1988) which have been determined based on a higher-order displacement model, HSDT solution with multiquadrics method reported

Table 6 Comparison of non-dimensional transverse displacement of cross-ply laminated spherical shells (0°/90°) under uniform load, ($b=a$, $R_1=R_2=R$, Material 1)

R/a	Theory	$a/h=10$	$a/h=100$
5	Reddy and Liu FSDT (1985)	19.9440	1.7535
	Reddy and Liu HSDT (1985)	17.5660	1.7519
	Mantari *et al.* HSDT (2011)	17.4886	1.7523
	Present	17.5557	1.7517
10	Reddy and Liu FSDT (1985)	19.0650	5.5428
	Reddy and Liu HSDT (1985)	18.7440	5.5388
	Mantari *et al.* HSDT (2011)	18.6543	5.5414
	Present	18.7511	5.5383
20	Reddy and Liu FSDT (1985)	19.3650	11.2730
	Reddy and Liu HSDT (1985)	19.0640	11.2680
	Mantari *et al.* HSDT (2011)	18.9699	11.2775
	Present	19.0752	11.2676
50	Reddy and Liu FSDT (1985)	19.4520	15.7140
	Reddy and Liu HSDT (1985)	19.1550	15.7110
	Mantari *et al.* HSDT (2011)	19.0601	15.7281
	Present	19.1681	15.7108
100	Reddy and Liu FSDT (1985)	19.4640	16.6450
	Reddy and Liu HSDT (1985)	19.1680	16.6420
	Mantari *et al.* HSDT (2011)	19.0731	16.6611
	Present	19.1813	16.6419
Plate	Reddy and Liu FSDT (1985)	19.4690	16.9800
	Reddy and Liu HSDT (1985)	19.1720	16.9770
	Mantari *et al.* HSDT (2011)	19.0774	16.9968
	Present	19.1858	16.9769

Table 7 Comparison of non-dimensional transverse displacement of cross-ply laminated spherical shells (0°/90°) under point load, ($b=a$, $R_1=R_2=R$, Material 1)

R/a	Theory	$a/h=10$	$a/h=100$
5	Reddy and Liu FSDT (1985)	7.1015	-
	Reddy and Liu HSDT (1985)	5.8953	-
	Mantari *et al.* HSDT (2011)	5.7174	-
	Present	5.8890	0.8219
10	Reddy and Liu FSDT (1985)	7.3836	-
	Reddy and Liu HSDT (1985)	6.1913	-
	Mantari *et al.* HSDT (2011)	6.0098	-
	Present	6.1894	1.8358
20	Reddy and Liu FSDT (1985)	7.4692	-
	Reddy and Liu HSDT (1985)	6.2714	-
	Mantari *et al.* HSDT (2011)	6.0888	-
	Present	6.2708	3.2779
50	Reddy and Liu FSDT (1985)	7.4909	-
	Reddy and Liu HSDT (1985)	6.2943	-
	Mantari *et al.* HSDT (2011)	6.1115	-
	Present	6.2941	4.3831
100	Reddy and Liu FSDT (1985)	7.4940	-
	Reddy and Liu HSDT (1985)	6.2976	-
	Mantari *et al.* HSDT (2011)	6.1147	-
	Present	6.2974	4.6142
Plate	Reddy and Liu FSDT (1985)	7.4853	-
	Reddy and Liu HSDT (1985)	6.2987	-
	Mantari *et al.* HSDT (2011)	6.1158	-
	Present	6.2984	4.6973

by Ferreira *et al*. (2003). The present theory is also compared with the various shear deformation theories used by Xiang *et al*. (2009), new HSDT developed by Mantari *et al*. (2011) and CPT. According to the Table 4, it can be seen again that the present computations are in good concordance with the analytical results reported by Xiang *et al*. (2009). In this analysis, CPT and FSDT are reflected by high percentage error in the results of moderately thick square sandwich. On the other hand, it can be pointed out that the increase of R values can reduce the transverse maximum displacement of the symmetric sandwich plate.

4.3 Bending analysis of two-layer antisymmetric cross-ply laminated spherical shells

Table 8 Comparison of non-dimensional fundamental frequencies $(\bar{\omega})$ of multilayered $(0°/90°)_n$antisymmetric cross-ply laminated composite square plates, (*a/h*=5, Material 3)

E_1/E_2	Theory	Lamination scheme			
		$(0/90)_1$	$(0/90)_2$	$(0/90)_3$	$(0/90)_5$
3	Exact 3D [c]	6.2578	6.5455	6.6100	6.6458
	Sayyad and Ghugal (2017)	6.2190	6.5012	6.5567	6.5854
	Thai and Kim (RPT1) (2010)	6.2169	6.5008	6.5558	6.5842
	Thai and Kim (RPT2) (2010)	6.2167	6.5008	6.5558	6.5842
	Reddy (1984)	6.2169	6.5008	6.5558	6.5842
	Mindlin (1951)	6.2085	6.5043	6.5569	6.5837
	Kirchhoff (1850)	6.7705	7.1690	7.2415	7.2415
	Present	6.2168	6.5008	6.5558	6.5842
10	Exact 3D [c]	6.9845	8.1445	8.4143	8.5625
	Sayyad and Ghugal (2017)	6.9967	8.1929	8.4065	8.5156
	Thai and Kim (RPT1) (2010)	6.9887	8.1954	8.4052	8.5126
	Thai and Kim (RPT2) (2010)	6.9836	8.1949	8.4052	8.5126
	Reddy (1984)	6.9887	8.1954	8.4052	8.5126
	Mindlin (1951)	6.9392	8.2246	8.4183	8.5132
	Kirchhoff (1850)	7.7420	9.7192	10.053	10.053
	Present	6.9881	8.1958	8.4053	8.5126
20	Exact 3D [c]	7.6745	9.4055	9.8398	10.0843
	Sayyad and Ghugal (2017)	7.8385	9.6205	9.9210	10.0740
	Thai and Kim (RPT1) (2010)	7.8210	9.6265	9.9181	10.0674
	Thai and Kim (RPT2) (2010)	7.8011	9.6252	9.9181	10.0671
	Reddy (1984)	7.8210	9.6265	9.9181	10.0614
	Mindlin (1951)	7.7060	9.6885	9.9427	10.0638
	Kirchhoff (1850)	8.8555	12.476	13.058	13.0585
	Present	7.8197	9.6272	9.9181	10.0671
30	Exact 3D [c]	8.1763	10.1650	10.6958	11.0027
	Sayyad and Ghugal (2017)	8.5320	10.5268	10.8603	11.0309
	Thai and Kim (RPT1) (2010)	8.5050	10.5348	10.8547	11.0197
	Thai and Kim (RPT2) (2010)	8.4646	10.5334	10.8547	11.0186
	Reddy (1984)	8.5050	10.5348	10.8547	11.0197
	Mindlin (1951)	8.3211	10.6198	10.8828	11.0058
	Kirchhoff (1850)	9.8337	14.7250	15.4907	15.4907
	Present	8.5028	10.5358	10.8546	11.0191
40	Exact 3D [c]	8.5625	10.6789	11.2728	11.6245
	Sayyad and Ghugal (2017)	9.1246	11.1628	11.5100	11.6893
	Thai and Kim (RPT1) (2010)	9.0871	11.1716	11.5012	11.6730
	Thai and Kim (RPT2) (2010)	9.0227	11.1705	11.5009	11.6705
	Reddy (1984)	9.0871	11.1716	11.5012	11.6730
	Mindlin (1951)	8.8383	11.2708	11.5264	11.6444
	Kirchhoff (1850)	10.721	16.6725	17.5897	17.5897
	Present	9.0841	11.1728	11.5010	11.6721

[c] Results taken from reference of Noor and Burton (1990)

In this problem, the proposed theory is applied for bending analysis of two-layer antisymmetric cross-ply laminated spherical shells under sinusoidal, uniform and point loads, respectively. For this section, both layers have the same thickness and are made up of the same material properties defined by Eq. (27). Tables 5-7 show the comparison of non-dimensional transverse displacements for various values of curvature ratios (*R/a*=5, 10, 20, 50, 100). The obtained results are compared with those predicted by the FSDT and HSDT derived from Reddy and Liu (1985) and the new HSDT shell models developed by Mantari *et al.* (2011) based on a new displacement field with five unknown functions. It can be observed that, the transverse displacements obtained by present refined theory (RHSDT) and Reddy's theory (HSDT) is in good agreement with each other for all curvature ratios (*R/a*). In addition, it can be found from Tables 5-7 that the transverse displacements of cross-ply laminated spherical shells are decreased with the increase in the thickness ratio (*a/h*) values and decrease in the curvature ratios (*R/a*) for the

Table 9 Comparison of non-dimensional fundamental frequencies ($\bar{\omega}$) of cross-ply cylindrical shells (0°/90°), Material 1

R/a	Theory	$a/h=10$	$a/h=100$
5	Reddy and Liu FSDT (1985)	8.9082	16.6680
	Reddy and Liu HSDT (1985)	9.0230	16.6900
	Mantari *et al.* HSDT (2011)	9.1254	16.7030
	Present	9.0957	16.7037
10	Reddy and Liu FSDT (1985)	8.8879	11.8310
	Reddy and Liu HSDT (1985)	8.9790	11.8400
	Mantari *et al.* HSDT (2011)	9.0453	11.8440
	Present	9.0144	11.8440
20	Reddy and Liu FSDT (1985)	8.8900	10.2650
	Reddy and Liu HSDT (1985)	8.9720	10.2700
	Mantari *et al.* HSDT (2011)	9.0207	10.2707
	Present	8.9895	10.2705
50	Reddy and Liu FSDT (1985)	8.8951	9.7816
	Reddy and Liu HSDT (1985)	8.9730	9.7830
	Mantari *et al.* HSDT (2011)	9.0109	9.7843
	Present	8.9797	9.7840
100	Reddy and Liu FSDT (1985)	8.8974	9.7108
	Reddy and Liu HSDT (1985)	8.9750	9.7120
	Mantari *et al.* HSDT (2011)	9.0085	9.7127
	Present	8.9773	9.7123
Plate	Reddy and Liu FSDT (1985)	8.8998	9.6873
	Reddy and Liu HSDT (1985)	8.9760	9.6880
	Mantari *et al.* HSDT (2011)	9.0065	9.6886
	Present	8.9753	9.6882

Table 10 Comparison of non-dimensional fundamental frequencies ($\bar{\omega}$) of cross-ply spherical shells (0°/90°), Material 1

R/a	Theory	$a/h=10$	$a/h=100$
5	Reddy and Liu FSDT (1985)	9.2309	28.8250
	Reddy and Liu HSDT (1985)	9.3370	28.8400
	Mantari *et al.* HSDT (2011)	9.3654	28.8391
	Present	9.3408	28.8412
10	Reddy and Liu FSDT (1985)	8.9841	16.7060
	Reddy and Liu HSDT (1985)	9.0680	16.7100
	Mantari *et al.* HSDT (2011)	9.0980	16.7121
	Present	9.0685	16.7128
20	Reddy and Liu FSDT (1985)	8.9212	11.8410
	Reddy and Liu HSDT (1985)	8.9990	11.8400
	Mantari *et al.* HSDT (2011)	9.0295	11.8442
	Present	8.9987	11.8442
50	Reddy and Liu FSDT (1985)	8.9034	10.0630
	Reddy and Liu HSDT (1985)	8.9800	10.0600
	Mantari *et al.* HSDT (2011)	9.0101	10.0647
	Present	8.9790	10.0644
100	Reddy and Liu FSDT (1985)	8.9009	9.7826
	Reddy and Liu HSDT (1985)	8.9770	9.7840
	Mantari *et al.* HSDT (2011)	9.0074	9.7840
	Present	8.9762	9.7836
Plate	Reddy and Liu FSDT (1985)	8.8998	9.6873
	Reddy and Liu HSDT (1985)	8.9760	9.6880
	Mantari *et al.* HSDT (2011)	9.0065	9.6886
	Present	8.9753	9.6882

different loading cases. However, the difference between the solution predicted by FSDT and HSDT is more pronounced, especially in the case of the point load at the center, as pointed out by Reddy and Liu (1985).

4.4 Free vibration analysis of antisymmetric (0°/90°)$_n$ cross-ply laminated plates

In order to illustrate the accuracy of the present theory, the fundamental natural frequencies of multilayered (0°/90°)$_n$ antisymmetric cross-ply laminated square plates with simply supported boundary conditions are also calculated by using the generalized formulation of the proposed model. It is considered that the number of layers is varied from 2 to 10 with the same thickness and made up of Material 3 defined by Eq. (29). Table 8 shows the numerical results of non-dimensional fundamental natural frequencies for various values of modular ratios $(E_1/E_2 = 3, 10, 20, 30, 40)$ and are obtained for the fundamental flexural mode $(m = n = 1)$.

The results of this problem are compared with those achieved by exact elasticity solution given by Noor and Burton (1990), four variable trigonometric shear deformation theory developed by Sayyad and Ghugal (2017), two variable refined plate theory (RPT) presented by Thai and Kim (2010), HSDT of Reddy (1984), FSDT and CPT. It can be seen that the present model shows the best accuracy and agree well with those reported by Reddy (1984) based on HSDT and to those cited by Thai and Kim (2010) using a two variable refined plate theory. However, the CPT overestimates the natural frequencies as compared to the results of other theories due to neglect of transverse shear strains. It is also apparent that the natural frequencies are increasing with the increase of the number of layers (see Table 8).

4.5 Free vibration analysis of cross-ply laminated cylindrical and spherical shells

The last problem is performed for free vibration analysis of two-layer (0°/90°) antisymmetric cross-ply laminated cylindrical and spherical shells to investigate the accuracy and applicability of the present theory. For this section, the material properties of each layer are given by Eq. (27). Numerical results of non-dimensional fundamental natural frequencies with respect to the several values of curvature ratios $(R/a = 5, 10, 20, 50, 100)$ are listed in Tables 9 and 10. It can be seen again that the present analytical method gives more accurate results in predicting the natural frequencies for cross-ply laminated cylindrical and spherical shells when compared to HSDTs provided by Reddy and Liu (1985) and Mantari *et al.* (2011). in addition, the fundamental frequencies follow a decreasing trend for the decrease in the thickness ratio (*a/h*) values and increase in the curvature ratios (*R/a*). However, in the both cases treated in this example for the free vibration analysis of cross-ply laminated shells, it should be clearly pointed out that the FSDT underestimates the natural frequency as compared to the results of the other HSDTs.

5. Conclusions

In this study, the bending and free vibration analysis of simply supported laminated composite and sandwich plates and shells is presented by using a generalized and simple refined higher-order shear deformation theory with four unknown variables. The present generalized displacement model was developed based on an undetermined integral component and hyperbolic shape function to include the effects of the transverse shear stresses through the plate/shell thickness without requiring the shear correction factor. The governing equations and its boundary conditions are derived by applying the dynamic version of the principle of virtual work and the analytical solutions of displacements, stresses and fundamental natural frequencies were obtained using the Navier's solution procedure. The effect of some parameters such as the side-to-thickness ratio and the curvature ratios is discussed by presenting several problems and comparing them with the previously published results. From this study, it is found that the present model is in excellent agreement while predicting the bending and free vibration analysis of laminated composite and sandwich plates and shells. Consequently, it is concluded that the proposed computational method can be applied for the bending analysis of thick laminated plates and shells with other boundary conditions and for different fiber orientations. An improvement of the present formulation will be considered in the future work to consider other type of materials (Sofiyev *et al*. 2008, Setoodeh *et al*. 2011, Sedighi and Shirazi 2012, Iurlaro *et al*. 2013, Sedighi *et al*. 2012, 2013, Avcar 2014, Cerracchio *et al*. 2015, Benferhat *et al*. 2016, Daouadji 2017, Lal *et al*. 2017, Ebrahimi and Barati 2017c, Ayat *et al*. 2018, Belmahi, *et al*. 2018, Ebrahimi and Barati 2018, Dihaj *et al*. 2018, Avcar and Mohammed 2018, Hamidi *et al*. 2018, Lal and Markad 2018, Faleh *et al*. 2018, Panjehpour *et al*. 2018, Bensattalah *et al*. 2018, 2019, Abrishambaf *et al*. 2019, Rajabi and Mohammadimehr 2019, Avcar 2019, Fadoun 2019, Selmi 2019, Tabrizi *et al*. 2019, Madenci 2019, Barati *et al*. 2019, Al-Maliki *et al*. 2019, Belmahi *et al*. 2019, Eltaher *et al*. 2019a, b, Fládr *et al*. 2019, Hadji *et al*. 2019, Kossakowski and Uzarska 2019, López-Chavarría *et al*. 2019, Nikkhoo *et al*. 2019, Sahouane *et al*. 2019, Shokrieh and Kondori 2020, Singh and Kumari 2020, Faleh *et al*. 2020, Ghannadpour and Mehrparvar 2020, Al-Maliki *et al*. 2020, Ghadimi 2020, Forsat *et al*. 2020).

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