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Bending analysis of functionally graded porous plates via a refined shear deformation theory

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Abstract. In this investigation, study of the bending response of functionally graded (FG) porous plates is presented using a cubic shear deformation theory. The properties of the FG-plate vary according to a power-law distribution which is modified to approximate material characteristics for considering the effect of porosities. The equilibrium equations are derived by using the principle of virtual work and solved by using Navier's procedure. Various numerical results are discussed to demonstrate the influence of the variation of the power index, the porosity parameter and the geometric ratios on the bending response of FG porous plates.

Keywords: functionally graded plate; porosity; bending; shear deformation theory

1. Introduction

Functionally graded materials (FGMs) are novel types of composite materials, generally made from a mixture of ceramics and metals. These materials are microscopically inhomogeneous, their characteristics vary from one surface to another. This is done by varying the combination of these materials gradually within the thickness to obtain smooth distributions in the characteristics of the material and the best responses to the applied external loading. FGMs are widely used in many applications such as nuclear, spacecraft, civil engineering, mechanical, automotive and power generation industries. The idea of FGMs presented in 1984 in Japan (Yamanouchi *et al*. 1990).

This class of materials attracts the attention of scientists, for this purpose several works have been published dealing with the different behavior of structures in FGM (Abrate 2006, Ebrahimi and Rastgo 2008, Malekzadeh 2009, Hosseini-Hashemi *et al*. 2011, Koochi *et al*. 2014, Jabbari *et al*. 2014, Yaghoobi *et al*. 2015, Rouzegar and Abad 2015, Sedighi *et al*. 2016, 2017, Dong and Li 2017, Arshid and Khorshidvand 2018, Soliman *et al*. 2018a, b, Akbaş 2018, Faleh *et al*. 2018, Vu *et al*. 2018, Avcar 2019, Alizadeh and Fattahi 2019).

Several analytical and numerical models have been proposed to analyze the FG- structures responses. Ferreira *et al*. (2005) analyzed the static deformations of FG-plate using the multiquadric radial basis functions and TSDT model. By employing a discrete layer theory and the Ritz method, Ramirez *et al*. (2006) have examined the examined the Static behaviors of FG-elastic anisotropic-plates**.** Xiang and Kang (2013) studied the static response of the FG-plate using various HSDTs such as (Levinson 1980, Barati 2017a, Ebrahimi and Barati 2018, Barati and Shahverdi 2020). Based on FSDT model, Castellazzi *et al*. (2013) have developed a nodal integrated plate element (NIPE) formulation to study a static analysis of P FG-plate. Based on the extension of the Spline Finite-Strip Method, Beena and Parvathy (2014) have studied the bending analysis of the P-FG, S-FG and E-FG plate under uniform, central concentrated and line loads. Pradhan and Chakraverty (2015) examined the effects of the volume fraction of the FG plate constituents on the deflection, normal stresses and bending moment by employing a framework of CPT and Rayleigh-Ritz method. Mantari *et al*. (2016) developed a new non polynomial displacement field based on Carrera Unified Formulation to analyze the static behaviors of the single FG and sandwich-plate. Recently, Srividhya *et al*. (2018) analyzed the effect of the external static load on the FG-plate responses modeled via a nonlocal theory and TSDT model. Wu *et al*. (2018) investigated on dynamic analysis of the FG-structures based on EBT, FSDT and FEM. Also, many researchers are interested to studies the effect of the imperfections on behaviors of the FG-structures (Barati 2017b, Jouneghani *et al*. 2018, Eltaher *et al*. 2018, Chen *et al*. 2018, 2019, Sahmani *et al*. 2019a, b, c, Foroutan *et al*. 2019, Babaei *et al*. 2019, Thanh *et al*. 2019, Mirjavadi *et al*. 2019a, b, Fattahi *et al*. 2019a, Hamed *et al*.

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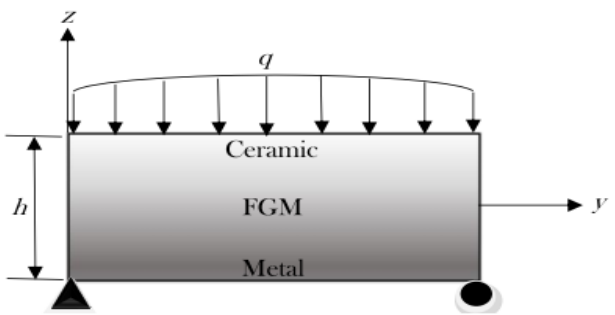

Fig. 1 The geometry of the simply supported functionally graded plate

2019, Forsat *et al*. 2020).

Through our reading in the literature, we have noticed that studies on the effect of variable porosity across the thickness of the FG-structure are rare. For this, the current research work deal, the effect of the porosity on the static behavior of the simply supported FG-plate modeled via a novel cubic shear deformation plate theory. The conditions of the zero-shear stresses on the free surface of the plate are ensured without introducing the shear correction factors. The analytical solutions of the static response of the perfect and imperfect FG-plate are determined via both virtual work principle and Navier method. Several numerical results of the in-plane and transverse displacements and normal and shear stresses are presented and compared with those existing in the literature. Finally, a parametric studies are investigated to shows a various parameter influencing the static analysis of the perfect and imperfect FG-plate under uniform loading.

2. Analytical formulations

2.1 Simply supported P-FG plate

In the current work, the plate is considered simply supported in the four edges and has the dimensions ($a\times b\times h$) in the Cartesian axis (x,y,z) is considered (as shown in Fig. 1). The simply supported rectangular FG-plate is composed from two components namely the ceramic and metal and the mixing between them is done according to the power-law function P-FGM. The young modulus and mass density of the set (ceramic-metal) are given as (Lal *et al*. 2017, Rezaiee-Pajand *et al*. 2018, Eltaher *et al*. 2018, Sahouane *et al*. 2019)

$$E(z)=E_m+\left(E_c-E_m\right)\left(\frac{1}{2}+\frac{z}{h}\right)^p \tag{1}$$

Where the index (m and c) corresponds to metal and ceramic respectively. "p" is the power index with "$p\geq 0$". For the simplification, the poison's ratio is taken "ν=0.3" for both ceramic and metal.

By taking into account the micro void that occurs in the FG plate during the manufacturing see (Fig. 2). The Young's modulus of the porous FG-plate becomes (Gupta and Talha 2018)

$$E(z)=\left(E_c-E_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p+E_m-\log\left(1+\frac{\xi}{2}\right)\left(E_c+E_m\right)\left(1-\frac{2|z|}{h}\right) \tag{2}$$

2.2 HSDT displacement field

In this work, the FG plate is modeled by using the higher order shear deformation theory hypothesis and applying some simplification to reduce the unknown displacement number. The refined displacement field of the present HSDT model is given as

$$\begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix}=\begin{Bmatrix} u_0(x,y) \\ v_0(x,y) \\ w_0(x,y) \end{Bmatrix}-z\begin{Bmatrix} \dfrac{\partial w_0}{\partial x} \\ \dfrac{\partial w_0}{\partial y} \\ 0 \end{Bmatrix}+f(z)\begin{Bmatrix} k_1\int\theta(x,y)dx \\ k_2\int\theta(x,y)dy \\ 0 \end{Bmatrix} \tag{3}$$

Where u_0; v_0; w_0 and θ are the four variables unknown in the mid-plane of the simply supported FG-plate. The constants k_1 and k_2 depend of the geometry of the simply supported FG-plate.

The warping shear function used in the present work is given as (Levinson 1980, Reddy 1984)

$$f(z)=z\left(1-\frac{4}{3}\left(\frac{z}{h}\right)^2\right) \tag{4}$$

2.3 Strains relations

By deriving the equations of displacement fields (HSDT model) of Eq. (3). The deformations of the simply supported FG-plates are obtained as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}=\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}+z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix}+f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix},$$
$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}=g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{5}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}=\begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x} \end{Bmatrix},\quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix}=\begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x\partial y} \end{Bmatrix}, \tag{6a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}=\begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\displaystyle\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\displaystyle\int\theta\,dy \end{Bmatrix}, \tag{6b}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta \, dy \\ k_1 \int \theta \, dx \end{Bmatrix}, \quad g(z) = \frac{df(z)}{dz} \tag{6c}$$

The terms of undetermined integrals appearing in the above equations are solved here by applying the current method of resolution (Navier method) and can be obtained as

$$\frac{\partial}{\partial y}\int \theta \, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta \, dy = B' \frac{\partial^2 \theta}{\partial x \partial y},$$
$$\int \theta \, dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta \, dy = B' \frac{\partial \theta}{\partial y} \tag{7}$$

With

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{8}$$

The terms α and β are defined in Eq. (23).

Based on stresses-strains linear relation according to Hook's formulations. The stresses expressions as function of the strains are given as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{9}$$

Where (σ and τ) and (ε and γ) are the stresses and strains respectively.

The rigidities coefficients C_{ij} are computed as function of the Young's modulus of Eqs. (1)-(2). The coefficient are given as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \frac{\nu \, E(z)}{1-\nu^2},$$
$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{10}$$

2.4 Virtual work principle

The virtual work principle is employed here for studying the static behaviors of the simply supported FG-plate under external load. The virtual work principle takes the analytical form as

$$0 = \int_V (\delta U + \delta V) \, dV \tag{11}$$

Where δU and δV are the variation of strain energy and virtual work done by external mechanical load, respectively.

The variation of strain energy associated to the current displacement field is given as

$$\delta U = \int_V \left[\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz} \right] dV$$
$$= \int_V \begin{bmatrix} N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_{xy} \delta \gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \\ + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta \gamma_{yz}^s + S_{xz}^s \delta \gamma_{xz}^0 \end{bmatrix} dA = 0 \tag{12}$$

Where A is the area.

The resulting efforts and moments N, M, and S are defined as

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} \left(1, z, f\right) \sigma_i dz, \quad \left(i = x, y, xy\right)$$
$$\text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{13}$$

The virtual work generated by the mechanical external load is given as

$$\delta V = \int_V q \, \delta w \, dV \tag{14}$$

By substituting the expressions of the variation of strain energy (Eq. (12)) and virtual work (Eq. (14)) into Eq. (11), integrating the results by part and collecting the terms δu_0, δv_0, δw_0.and $\delta\theta$. We obtain four governing equations

$$\delta u_0: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$
$$\delta v_0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0$$
$$\delta w_0: \quad \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q = 0 \tag{15}$$
$$\delta \theta: \quad -\mathrm{k}_1 M_x^s - \mathrm{k}_2 M_y^s - \left(k_1 A' + k_2 B'\right) \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^s}{\partial x}$$
$$+ k_2 B' \frac{\partial S_{yz}^s}{\partial y} = 0$$

The stresses as function of strain are obtained by replacing Eq. (5) into Eq. (9) and the results into Eq. (13). The resulting efforts and moments N, M, and S are obtained in the following form as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \quad S = A^s \gamma, \tag{16}$$

Where

$$N = \left\{N_x, N_y, N_{xy}\right\}^t, \quad M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t,$$
$$M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}^t, \tag{17a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t,$$
$$k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t, \tag{17b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix},$$
$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{17c}$$

$$B^s=\begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix},\quad D^s=\begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix},$$
$$H^s=\begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{17d}$$

$$S=\left\{S_{xz}^s, S_{yz}^s\right\}^t,\quad \gamma=\left\{\gamma_{xz}^0, \gamma_{yz}^0\right\}^t,\quad A^s=\begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{17e}$$

Where the terms of rigidity (A, B, D, B^S, D^S, H^S and A^S) are given as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix}=$$
$$\int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\,f(z), f^2(z)\right)\begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{18a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right)=\left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{18b}$$

$$A_{44}^s=A_{55}^s=\int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz, \tag{18c}$$

The governing equations as function of displacements term can be obtained by substituting the Eq. (16) into Eq. (15). The governing equations becomes

$$\begin{aligned} &A_{11}d_{11}u_0+A_{66}\,d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0-B_{11}d_{111}w_0 \\ &-\left(B_{12}+2B_{66}\right)d_{122}w_0+\left(B_{66}^s\left(k_1A'+k_2\,B'\right)\right)d_{122}\theta \\ &+\left(B_{11}^sk_1+B_{12}^s\,k_2\right)d_1\theta=0 \end{aligned} \tag{19a}$$

$$\begin{aligned} &A_{22}\,d_{22}v_0+A_{66}\,d_{11}v_0+\left(A_{12}+A_{66}\right)d_{12}u_0-B_{22}\,d_{222}w_0 \\ &-\left(B_{12}+2B_{66}\right)d_{112}w_0+\left(B_{66}^s\left(k_1A'+k_2\,B'\right)\right)d_{112}\theta \\ &+\left(B_{22}^sk_2+B_{12}^sk_1\right)d_2\theta=0 \end{aligned} \tag{19b}$$

$$\begin{aligned} &B_{11}\,d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0+\left(B_{12}+2B_{66}\right)d_{112}v_0 \\ &+B_{22}\,d_{222}v_0-D_{11}d_{1111}w_0-2\left(D_{12}+2D_{66}\right)d_{1122}w_0 \\ &-D_{22}\,d_{2222}w_0+\left(D_{11}^s\,k_1+D_{12}^s\,k_2\right)d_{11}\theta \\ &+2\left(D_{66}^s\left(k_1A'+k_2B'\right)\right)d_{1122}\theta+\left(D_{12}^sk_1+D_{22}^sk_2\right)d_{22}\theta+q=0 \end{aligned} \tag{19c}$$

$$\begin{aligned} &-\left(B_{11}^sk_1+B_{12}^sk_2\right)d_1u_0-\left(B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{122}u_0 \\ &-\left(B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{112}v_0-\left(B_{12}^sk_1+B_{22}^sk_2\right)d_2v_0 \\ &+\left(D_{11}^sk_1+D_{12}^sk_2\right)d_{11}w_0+2\left(D_{66}^s\left(k_1A'+k_2B'\right)\right)d_{1122}w_0 \\ &+\left(D_{12}^sk_1+D_{22}^sk_2\right)d_{22}w_0-H_{11}^s\,k_1^2\;\theta-H_{22}^s\;k_2^2\,\theta \\ &-2\,H_{12}^s\,k_1\,k_2\theta-\left(\left(k_1A'+k_2B'\right)^2H_{66}^s\right)d_{1122}\theta \\ &+A_{44}^s\left(k_2B'\right)^2d_{22}\theta+A_{55}^s\left(k_1\,A'\right)^2d_{11}\theta=0 \end{aligned} \tag{19d}$$

Where d_{ij}, d_{ijl} and $d_{ijlm}$are:

$$d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\quad d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},$$
$$d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},\quad d_i=\frac{\partial}{\partial x_i},\quad (i,j,l,m=1,2). \tag{20}$$

2.5 Navier procedure

In the present research work, the FG-plate is considered simply supported in the all four edges. The analytical solutions for flexural behavior of the FG-plate are derived by employing the Navier's procedure. The displacements components (u_0, v_0, w_0.and θ) can be expressed in the trigonometric form as (Hadji *et al*. 2019, Safa *et al*. 2019)

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\cos(\alpha\,x)\sin(\beta\,y) \\ V_{mn}\sin(\alpha\,x)\cos(\beta\,y) \\ W_{mn}\sin(\alpha\,x)\sin(\beta\,y) \\ X_{mn}\sin(\alpha\,x)\sin(\beta\,y) \end{Bmatrix} \tag{21}$$

And the mechanical load is expressed as

$$\{q\}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{q_0\,\sin(\alpha x)\sin(\beta y)\right\} \tag{22}$$

with

$$\alpha=m\pi/a\quad\text{and}\quad\beta=n\pi/b \tag{23}$$

By replacing the Navier method of Eq. (21) into governing equations (Eq. (19)).we obtain the following matrix system

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}\right)\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ q \\ 0 \end{Bmatrix} \tag{24}$$

Where

$$S_{11}=-\left(A_{11}\alpha^2+A_{66}\beta^2\right),\quad S_{12}=-\alpha\beta\left(A_{12}+A_{66}\right)$$
$$S_{13}=\alpha\left(B_{11}\alpha^2+B_{12}\beta^2+2B_{66}\beta^2\right)$$
$$S_{14}=\alpha\left(k_1B_{11}^s+k_2B_{12}^s-\left(k_1A'+k_2B'\right)B_{66}^s\beta^2\right),$$
$$S_{22}=-\left(A_{66}\alpha^2+A_{22}\beta^2\right)$$
$$S_{23}=\beta\left(B_{22}\beta^2+B_{12}\alpha^2+2B_{66}\alpha^2\right)$$
$$S_{24}=\beta\left(k_2B_{22}^s+k_1B_{12}^s-\left(k_1A'+k_2B'\right)B_{66}^s\alpha^2\right)$$
$$S_{33}=-\left(D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4\right)$$

Table 1 Effectives properties of FG-plate

Properties	Metal (Aluminium) *Al*	Ceramic (Alumina) Al_2O_3
$E(z)$ [N/m^2]	70×10^9	380×10^9
ν	0.3	

$$
\begin{aligned}
S_{34} &= -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1A' + k_2B')D_{66}^s\alpha^2\beta^2 \\
&\quad -k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right) \\
S_{44} &= -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1A' + k_2B')^2 H_{66}^s\alpha^2\beta^2 \\
&\quad -k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - \left(k_1A'\right)^2 A_{55}^s\alpha^2 - \left(k_2B'\right)^2 A_{44}^s\beta^2
\end{aligned}
\tag{25}
$$

3. Numerical results and discussions

In this part, the numerical results of flexural behavior of the simply supported porous FG-plate under transversal mechanical load are presented. The effective's properties of materials composing the FG-plate are abstracted in the Table 1.

The obtained results are presented in the following non-dimensional form

$$
\begin{aligned}
&\bar{u} = \frac{100\,h^3E_c}{a^4q_0}u\left(\frac{a}{2},\frac{b}{2},\bar{z}\right);\ \bar{v} = \frac{100\,h^3E_c}{a^4q_0}v\left(\frac{a}{2},\frac{b}{2},\bar{z}\right); \\
&\bar{w} = \frac{10\,h^3E_c}{a^4q_0}w\left(\frac{a}{2},\frac{b}{2},\bar{z}\right);\bar{\sigma}_x = \frac{h}{a\,q_0}\sigma_x\left(\frac{a}{2},\frac{b}{2},\bar{z}\right); \\
&\bar{\sigma}_y = \frac{h}{a\,q_0}\sigma_y\left(\frac{a}{2},\frac{b}{2},\bar{z}\right);\ \bar{\sigma}_z = \frac{1}{q_0}\sigma_z\left(\frac{a}{2},\frac{b}{2},\bar{z}\right); \\
&\bar{\tau}_{xz} = \frac{h}{a\,q_0}\tau_{xz}\left(0,\frac{b}{2},\bar{z}\right);\bar{\tau}_{yz} = \frac{h}{a\,q_0}\tau_{yz}\left(\frac{a}{2},0,\bar{z}\right); \\
&\bar{\tau}_{xy} = \frac{h}{a\,q_0}\tau_{xy}\left(0,0,\bar{z}\right);\quad \bar{z} = \frac{z}{h}.
\end{aligned}
\tag{26}
$$

3.1 Comparison and validation

In this section, the comparisons of the obtained results with those found in the literature are presented.

The Table 2 shows the effect of the material parameter "p" and geometry ratio "a/h" on dimensionless deflection "$\bar{w}$" and axial stress "$\bar{\sigma}_x$" of the simply supported square FG-plate. The obtained results are compared with the other models found in the literature such as (Zenkour 2013, Zenkour and Alghanmi 2018). From the computed results, it can be seen that the current results are in good agreement with those given by Zenkour and Alghanmi (2018) in the case of thick, moderately thick and thin plate, and the results of CPT developed by Zenkour (2013) are not precise because the omission of the transverse shear deformation effect. It can be also noted that the dimensionless deflection "$\bar{w}$" is in direct correlation relation with material parameter "p" and inverse relation with length to thickness ratio "a/h".

The Table 3 presents the comparisons of the values of the dimensionless displacements "$\bar{u}$, $\bar{v}$ and $\bar{w}$" and stresses "$\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}_{xz}$, $\bar{\tau}_{yz}$ and $\bar{\tau}_{xy}$" with those given by Zenkour (2006) using a generalized shear deformation theory and Zenkour and Alghanmi (2018) using the RPT model. From the table, it is clear that the all current results (dimensionless displacement "$\bar{u}$, $\bar{v}$ and $\bar{w}$" and stresses "$\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}_{xz}$, $\bar{\tau}_{yz}$ and $\bar{\tau}_{xy}$") computed using the cubic shear deformation theory are almost the same with those given by Zenkour and Alghanmi 2018 using the Refined Plate Theory. We can confirm again that the dimensionless deflection "$\bar{w}$" increase with the increase of the material parameter "p".

3.2 Parametric studies

The Tables 4 and 5 shows the Effect of the porosity parameter "ξ", geometry ratio "a/h" and material index "p" on the dimensionless displacements "$\bar{u}$, $\bar{v}$ and $\bar{w}$" and stresses "$\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}_{xz}$, $\bar{\tau}_{yz}$ and $\bar{\tau}_{xy}$" of the simply supported square FG-plate under uniform external load "q". From the tables computed using the present cubic shear deformation theory, it can be observed that the values of the parameters of porosity "ξ", geometry "a/h" and material index "p" have an important roles on the dimensionless displacements "$\bar{u}$, $\bar{v}$ and $\bar{w}$" and stresses "$\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}_{xz}$, $\bar{\tau}_{yz}$ and $\bar{\tau}_{xy}$". We can see from the results that that the non-

Table 2 Effect of material index "p" and geometry ratio "a/h" on dimensionless deflection "$\bar{w}$" and axial stress "$\bar{\sigma}_x$" of the simply supported square FG-plate

p	Theory	$\bar{w}(0)$			$\bar{\sigma}_x(1/3)$		
		a/h=4	a/h=10	a/h=100	a/h=4	a/h=10	a/h=100
1	Zenkour (2013) CPT	0,5623	0,5623	0,5623	0,8060	2,0150	20,1500
	Zenkour (2013) FPT	0,7291	0,5889	0,5625	0,8060	2,0150	20,1500
	Zenkour and Alghanmi (2018)	0,7984	0,5889	0,5625	0,5819	1,4898	14,9680
	Present	0.7284	0.5889	0.5625	0.5812	1.4898	14.9675
4	Zenkour (2013) CPT	0,8281	0,8281	0,8281	0,642	1,6049	16,0490
	Zenkour (2013) FPT	1,1125	0,8736	0,8281	0,642	1,6049	16,0490
	Zenkour and Alghanmi (2018)	1,1598	0,8815	0,8287	0,4449	1,1794	11,9210
	Present	1.1599	0.8815	0.8287	0.4449	1.1794	11.9209
10	Zenkour (2013) CPT	0,9354	0,9354	0,9354	0,4796	1,1990	11,9900
	Zenkour (2013) FPT	1,3178	0,9966	0,9360	0,4796	1,1990	11,9900
	Zenkour and Alghanmi (2018)	1,3909	1,0087	0,9362	0,3259	0,8785	8,9059
	Present	1.3909	1.0087	0.9362	0.3259	0.8785	8.9060

Table 3 Comparison of dimensionless displacements and stresses of simply supported square FG plate

p	Theory	$\bar{u}\left(-\frac{1}{4}\right)$	$\bar{v}\left(-\frac{1}{6}\right)$	$\bar{w}(0)$	$\overline{\sigma_x}\left(\frac{1}{2}\right)$	$\overline{\sigma_y}\left(\frac{1}{3}\right)$	$\overline{\tau_{yz}}\left(\frac{1}{6}\right)$	$\overline{\tau_{xz}}(0)$	$\overline{\tau_{xy}}\left(-\frac{1}{3}\right)$
Ceramic	Zenkour (2006)	0,2309	0,1539	0,296	1,9955	1,3121	0,2132	0,2462	0,7065
	Zenkour and Alghanmi (2018)	0,2183	0,1451	0,2961	1,9943	1,3124	0,2121	0,2386	0,7067
	Present	0.2183	0.1451	0.2961	1.9943	1.3124	0.2121	0.2386	0.7067
1	Zenkour (2006)	0,6626	0,5093	0,5889	3,0870	1,4894	0,2622	0,2642	0,611
	Zenkour and Alghanmi (2018)	0,6414	0,4944	0,5889	3,0850	1,4898	0,2608	0,2386	0,6111
	Present	0.6414	0.4944	0.5889	3.0850	1.4898	0.2608	0.2386	0.6111
2	Zenkour (2006)	0,9281	0,7311	0,7573	3,6094	1,3954	0,2763	0,2265	0,5441
	Zenkour and Alghanmi (2018)	0,8984	0,7102	0,7573	3,6067	1,3960	0,2737	0,2186	0,5442
	Present	0.8983	0.7101	0.7573	3.6067	1.3960	0.2737	0.2186	0.5442
4	Zenkour (2006)	1,0941	0,8651	0,8819	4,0693	1,1783	0,258	0,2029	0,5667
	Zenkour and Alghanmi (2018)	1,0502	0,8342	0,8815	4,0655	1,1794	0,2537	0,1944	0,5669
	Present	1.0502	0.8342	0.8815	4.0655	1.1794	0.2537	0.1944	0.5669
10	Zenkour (2006)	1,1372	0,8756	1,0089	5,089	0,8775	0,2041	0,2198	0,5894
	Zenkour and Alghanmi (2018)	1,0766	0,8329	1,0087	5,0849	0,8785	0,2014	0,2114	0,5896
	Present	1.0766	0.8329	1.0087	5.0849	0.8785	0.2014	0.2114	0.5896
Metal	Zenkour (2006)	1,2534	0,8356	1,607	1,9955	1,3121	0,2132	0,2462	0,7065
	Zenkour and Alghanmi (2018)	1,1851	0,7875	1,6072	1,9943	1,3124	0,2121	0,2386	0,7067
	Present	1.1851	0.7875	1.6072	1.9943	1.3124	0.2121	0.2386	0.7067

Table 4 Effect of porosity parameter "ξ" on dimensionless deflection "$\bar{w}$" and axial stress "$\bar{\sigma}_x$" of the simply supported square FG-plate

p	Theory	ξ	$\bar{w}(0)$			$\bar{\sigma}_x$ (1/3)		
			a/h=4	a/h=10	a/h=100	a/h=4	a/h=10	a/h=100
0.5	Present	0	0.5655	0.4537	0.4325	0.5716	1.4588	14.6446
		0.1	0.5837	0.4661	0.4438	0.5666	1.4479	14.5389
		0.2	0.6025	0.4787	0.4552	0.5614	1.4367	14.4296
		0.3	0.6219	0.4916	0.4669	0.5560	1.4250	14.3164
1	Present	0	0.7284	0.5889	0.5625	0.5812	1.4898	14.9675
		0.1	0.7620	0.6133	0.5851	0.5737	1.4737	14.8115
		0.2	0.7980	0.6392	0.6091	0.5656	1.4562	14.6420
		0.3	0.8367	0.6668	0.6346	0.5568	1.4371	14.4571
5	Present	0	1.2207	0.9113	0.8527	0.4134	1.1041	11.1745
		0.1	1.3494	0.9864	0.9175	0.3853	1.0456	10.6123
		0.2	1.5143	1.0782	0.9954	0.3503	0.9739	9.9255
		0.3	1.7375	1.1957	1.0927	0.3048	0.8826	9.0556
10	Present	0	1.3909	1.0087	0.9362	0.3259	0.8785	8.9060
		0.1	1.5627	1.0926	1.0034	0.2965	0.8184	8.3306
		0.2	1.8034	1.1983	1.0833	0.2597	0.7470	7.6557
		0.3	2.1826	1.3425	1.1825	0.2100	0.6578	6.8314

dimensional displacements "$\bar{u}$, $\bar{v}$ and $\bar{w}$" and stresses "$\bar{\sigma}_x$ and $\bar{\tau}_{yz}$" increase with increasing of the porosity parameter, but for the stresses "$\bar{\sigma}_y$, $\bar{\tau}_{xy}$ and $\bar{\tau}_{xz}$" this is reversed.

Fig. 2 plots the Non-dimensional deflection "$\bar{w}$" versus the geometry and aspect ratios "*a/h* and *a/b*" of simply supported square FG-plate (with material index *p*=1) under transversal mechanical load "*q*". From the plotted curves, it is clear that the deflection "$\bar{w}$" is in inverse relation with the both geometry and aspect ratios "*a/h* and *a/b*". It can be also seen that the values of the dimensionless deflection "$\bar{w}$" is reduced in the case of the perfect FG-plate "ξ=0". The biggest values of the deflection "$\bar{w}$" are obtained for the FG-plate with porosity parameter "ξ=0.3".

Figs. 3 and 4 presents the variation of the dimensionless axial stress "$\bar{\sigma}_x$ and $\bar{\sigma}_y$" across the thickness "$\bar{z}$" versus the porosity parameter "ξ" of the rectangular FG-plate with (*p*=5 and *a/h*=10) under uniform mechanical load "*q*". From the graphs, it can be noted that the stresses "$\bar{\sigma}_x$ and $\bar{\sigma}_y$" compress through the thickness of the FG- plate for "$\bar{z} \succ 0.1$" and then turn out to be tensile in the case "$\bar{z} \leq 0.1$".

It can be concluded that the porosity parameter "ξ" has a significant role on the axial stresses "$\bar{\sigma}_x$ and $\bar{\sigma}_y$".

Fig. 5 illustrate the variation of the non-dimensional tangential stress "$\bar{\tau}_{xy}$" across the thickness "$\bar{z}$" versus the porosity parameter "ξ" of square FG-plate with (*p*=5 and

Table 5 Effect of porosity parameter on dimensionless displacements and stresses of simply supported square FG plate

p	Theory	ξ	$\bar{u}\left(-\frac{1}{4}\right)$	$\bar{v}\left(-\frac{1}{6}\right)$	$\bar{w}(0)$	$\bar{\sigma}_x\left(\frac{1}{2}\right)$	$\bar{\sigma}_y\left(\frac{1}{3}\right)$	$\bar{\tau}_{yz}\left(\frac{1}{6}\right)$	$\bar{\tau}_{xz}(0)$	$\bar{\tau}_{xy}\left(-\frac{1}{3}\right)$
0.5	Present	0	0.2933	0.2179	0.4418	3.9059	2.1930	0.2424	0.2441	1.0381
		0.1	0.3038	0.2264	0.4536	3.9790	2.1769	0.2439	0.2378	1.0332
		0.2	0.3146	0.2353	0.4655	4.0511	2.1603	0.2456	0.2311	1.0286
		0.3	0.3259	0.2445	0.4777	4.1222	2.1431	0.2474	0.2240	1.0243
1	Present	0	0.4287	0.3306	0.5741	4.6014	2.2405	0.2609	0.2387	0.9178
		0.1	0.4531	0.3511	0.5975	4.7137	2.2168	0.2641	0.2303	0.9090
		0.2	0.4795	0.3733	0.6223	4.8271	2.1910	0.2678	0.2212	0.9005
		0.3	0.5082	0.3976	0.6487	4.9417	2.1629	0.2718	0.2112	0.8925
5	Present	0	0.7143	0.5659	0.8784	6.3172	1.6674	0.2387	0.1931	0.8657
		0.1	0.7971	0.6374	0.9477	6.5081	1.5815	0.2446	0.1651	0.8597
		0.2	0.9004	0.7273	1.0318	6.6986	1.4765	0.2529	0.1273	0.8574
		0.3	1.0340	0.8441	1.1379	6.8876	1.3432	0.2654	0.0728	0.8607
10	Present	0	0.7213	0.5584	0.9680	7.5710	1.3279	0.2015	0.2115	0.8868
		0.1	0.8040	0.6295	1.0426	7.7995	1.2399	0.2010	0.1800	0.8789
		0.2	0.9085	0.7203	1.1338	8.0078	1.1361	0.2007	0.1323	0.8757
		0.3	1.0463	0.8412	1.2528	8.1826	1.0080	0.2009	0.0501	0.8799

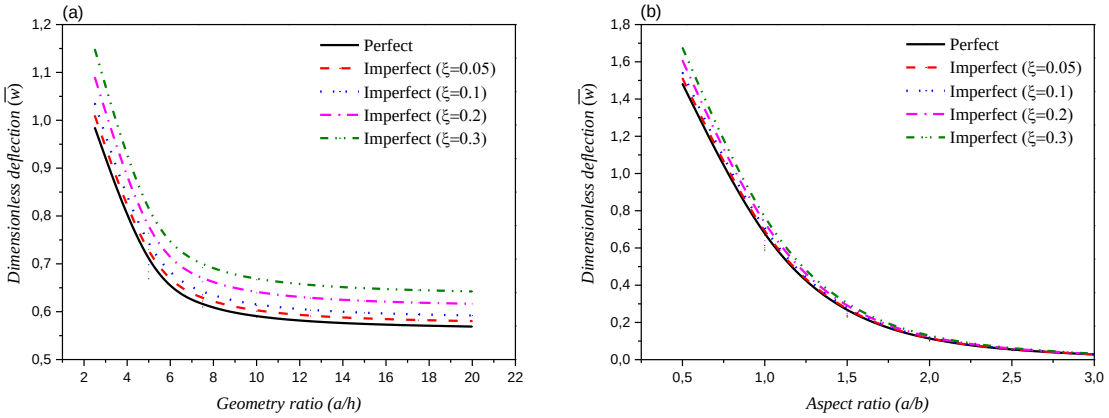

Fig. 2 Non-dimensional displacement "$\bar{w}$" versus: (a) geometry ratio "*a/h*", (b) aspect ratio "*a/b*" with the material index "*p*=1"

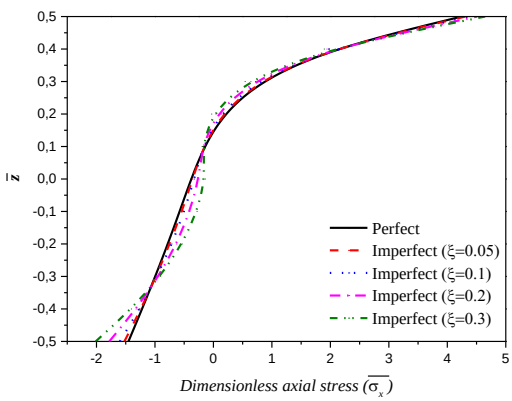

Fig 3 The dimensionless axial stress "$\bar{\sigma}_x$" across the thickness of a square FG plate with (*p*=5; *a/b*=2 and *a/h*=10)

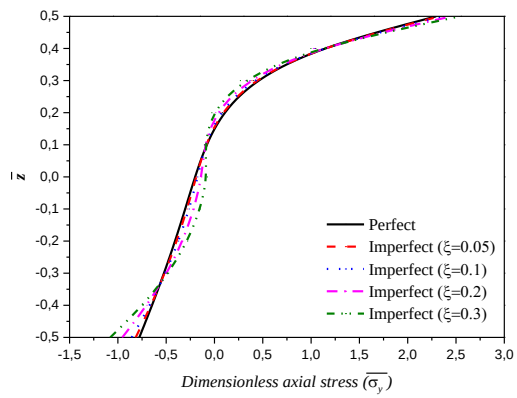

Fig. 4 The dimensionless axial stress "$\bar{\sigma}_y$" across the thickness of a rectangular FG plate with (*p*=5; *a/b*=2 and *a/h*=10)

a/h=10) under uniform load "*q*". From the plotted curves, it can be seen that the tangential stress "$\bar{\tau}_{xy}$" is the tensile when "$\bar{z} \succ 0.1$" and is compressive when "$\bar{z} \leq 0.1$". We can conclude again that the presence of the porosity "$\xi \neq 0$" in the material has a significant role on the stresses.

Fig. 6 plots the effect of the porosity parameter "ξ" on

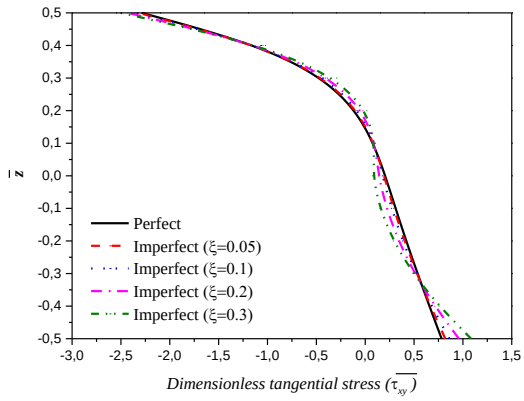

Fig. 5 The dimensionless tangential stress "$\bar{\tau}_{xy}$" across the thickness of a FG plate with (p=5 and a/h=10)

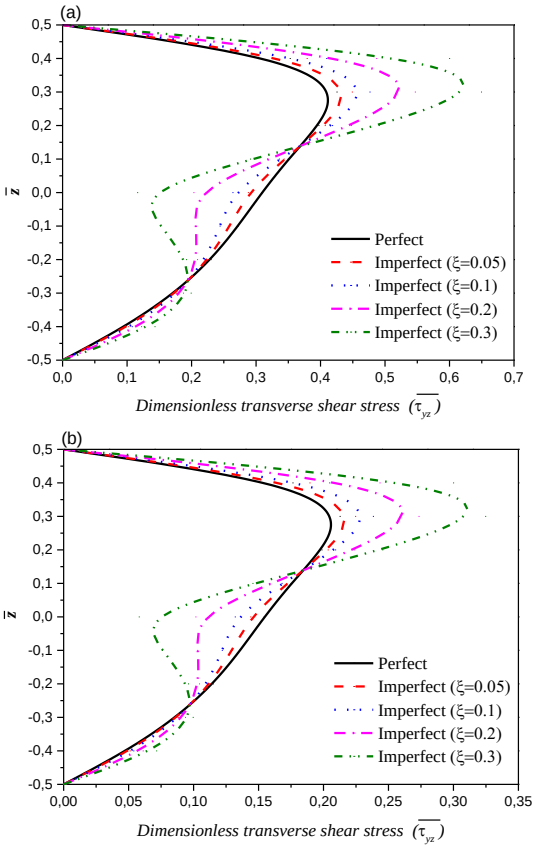

Fig. 6 The dimensionless transverse shear stresses "$\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$" across the thickness of a FG plate: (a) $\bar{\tau}_{xz}$ (b) $\bar{\tau}_{yz}$ with (p=5; a/h=10 and a/b=0.5)

the variations of the non-dimensional transverse shear stresses "$\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$" across the thickness "$\bar{z}$" of simply supported rectangular FG-plate under uniform loading with (p=5; a/h=10 and a/b=0.5). From the curves presented in the Fig. 6, it is clear that the imperfect FG-plate with porosity parameter "ξ=0.3" gives the maximum transverse shear stresses "$\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$".

The variation of the non-dimensional in-plane displacements "$\bar{u}$ and $\bar{v}$" across the thickness "$\bar{z}$" of the simply supported rectangular perfect "ξ=0" and imperfect "ξ≠0" FG-plate with (p=5; a/h=10 and a/b=2) is presented

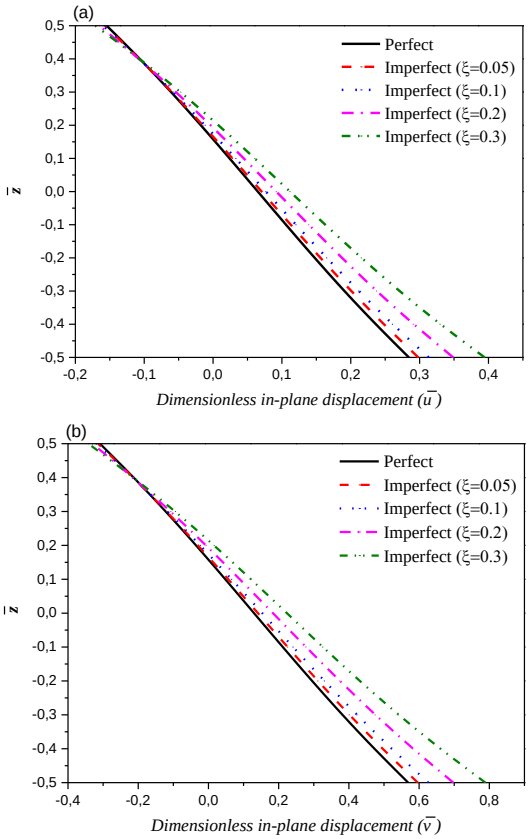

Fig. 7 The dimensionless in-plane displacements "$\bar{u}$ and $\bar{v}$" across the thickness of a FG plate: (a) $\bar{u}$, (b) $\bar{v}$, *with* (p=5; a/h=10 and a/b=2)

in the Fig. 7. From the obtained graphs, it can be seen that the variations of the in-plane displacements "$\bar{u}$ and $\bar{v}$" through the thickness "$\bar{z}$" are almost linear. It is clear also that the presence of the porosity "ξ" has an important role on the values of the in-plane displacements "$\bar{u}$ and $\bar{v}$".

4. Conclusions

This work deals with static analysis for simply supported perfect and imperfect FG-plate employing a cubic shear deformation theory where the shear correction factors are not necessary. The presence of the micro vide in the structure has been taken into consideration. The stability equations of the system have been derived using the virtual works principle and solved via Navier solution. The accuracy and efficiency of the current models have been verified by comparing the obtained results with those found in the literature. Finally we can conclude that the porosity parameter has a significant role on the displacements and stresses of the FG- plate. An improvement of the developed formulation will be considered in the future work to consider other type of structures and materials (Avcar 2014, Arani and Kolahchi 2016, Kolahchi *et al*. 2017, Fattahi and Sahmani 2017, Sedighi and Sheikhanzadeh 2017, Motezaker and Kolahchi 2017, Kolahchi and Cheraghbak 2017, Abdelmalek *et al*. 2017, Sahmani and Fattahi 2017, 2018, Ghodrati *et al*. 2018, Dihaj *et al*. 2018, Ayat *et al*.

2018, Hamidi *et al*. 2018, Hajmohammad *et al*. 2019, YaylacI *et al*. 2019, Fenjan *et al*. 2019, López-Chavarría *et al*. 2019, Mohamed *et al*. 2019, Fládr *et al*. 2019, Abedini *et al*. 2019, Jamali *et al*. 2019, Selmi 2019, Akbaş 2019a, b, Rajabi and Mohammadimehr 2019, Karami *et al*. 2019, Fattahi *et al*. 2019b, Safaei *et al*. 2019, Karami and Janghorban 2019, Oyarhossein *et al*. 2020, Al-Maliki *et al*. 2019, Eltaher *et al*. 2019, 2020, Ghannadpour and Mehrparvar 2020, Faleh *et al*. 2020, Lee *et al*. 2020, Shokrieh and Kondori 2020, Moayedi *et al*. 2020, Hamed *et al*. 2020).

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