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Prediction of the critical buckling load of SWCNT reinforced concrete cylindrical shell embedded in an elastic foundation

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Abstract. Concrete is the most widely used substance in construction industry, so it's been required to improve its quality using new technologies. Nowadays, nanotechnology offers new frontiers for improving construction materials. In this paper, we study the stability analysis of the Single Walled Carbon Nanotubes (SWCNT) reinforced concrete cylindrical shell embedded in elastic foundation using the Donnell cylindrical shell theory. In this regard, we propose a new explicit analytical formula of the critical buckling load which takes into account the distribution of SWCNT reinforcement through the thickness of the concrete shell using the U, X, O and V forms and the elastic foundation using Winkler and Pasternak models. The rule of mixture is used to calculate the effective properties of the reinforced concrete cylindrical shell. The influence of diverse parameters on the stability behavior of the reinforced concrete shell is also discussed.

Keywords: stability analysis; SWCNT reinforced concrete cylindrical shell; elastic foundation; donnell shell theory

1. Introduction

Nanoscience and nanotechnology are a priority field for today's researchers. For example, the properties of micro/nanoshells make them an ideal tool for use in different sectors of industry, that drive the scientific research to study the micro/nanoshells as shown by several research works as (Taj *et al*. 2020, Karami *et al*. 2019, Shariati *et al*. 2020); Taj *et al*. (2020) examined the vibrational analysis in microtubules based on the nonlocal orthotropic elastic shell model, Karami *et al*. (2019) studied the elastic bulk wave characteristics of doubly curved nanoshell made of functionally graded anisotropic material and Shariati *et al*. (2020) presented a study on the stability analysis of cantilevered curved microtubules in axons regarding various size elements. Moreover, because of excellent properties of carbon nanotube (Thostenson *et al*. 2001, Esawi and Farag 2007, Rafiee *et al*. 2013), it is one of the best candidate materials in various industrial applications. The study of bending, buckling and vibration of single or multi-walled carbon nanotubes is the subject of numerous recent research works (Mohamed *et al*. 2020, Malikan *et al*. 2020, Rahmani and Antonov 2019, Xie *et al*. 2020, Asghar *et al*. 2020, Hussain *et al*. 2019, 2020). Nowadays, the carbon nanotubes have become a very attractive research area for the reinforcement of structures. In this research area of reinforcement materials, we can cite Wang and Shen (2011), Natarajan *et al*. (2014), Kiani (2016), Mohammadimehr and Alimirzaei (2016), Tornabene *et al*. (2016), Wu *et al*. (2016), Zhang and Selim (2017), Lei *et al*. (2018), Zghal *et al*. (2018), Mehar *et al*. (2018), Frikha *et al*. (2018), Chen *et al*. (2019), Qin *et al*. (2019) and Selmi (2019). The most concerned structures are the beams, columns and plates. In this context, several researchers are analyzed the different behaviors of beams and plates reinforced by carbon nanotubes (Selmi (2019), Yas and Samadi (2012), Wattanasakulpong and Ungbhakorn (2013), Lin and Xiang (2014), Wu and Li (2014), Alibeigloo and Liew (2015), Kamarian *et al*. (2015), Fantuzzi *et al*. (2017), Setoodeh and Shojaee (2017), Zarei *et al*. (2017)). Now we present some scientific research based on the beam theory, the nonlinear free-vibrational response of Functionally Graded Carbon Nanotubes (FG-CNT) reinforced composite beams is presented by Ke *et al*. (2010) using the Timoshenko's analytical model and nonlinearity geometric of Von Kármán, Arani *et al*. (2019) investigated on the stability of FG-CNT reinforced composite sandwich nano-beams under the mechanical, electrical and thermal effect, Bensaid and Kerboua (2019) examined the thermal stability analysis of the FG-CNT reinforced composite beams with temperature-dependent material properties, Bensattalah *et al*. (2019) studied a free vibrational behavior of chiral SWCNT basing nonlocal Timoshenko beam theory. Recently, Bousahla *et al*. (2020) studied the mechanical behavior of concrete beams using a developed version of Timoshenko's theory presented in these works (Yas and Samadi 2012, Wattanasakulpong and Ungbhakorn 2013, Arani and Kolahchi 2016), Barati and Shahverdi (2020) studied nanocomposite graphene platelet-reinforced beams by analyzing a forced vibration,

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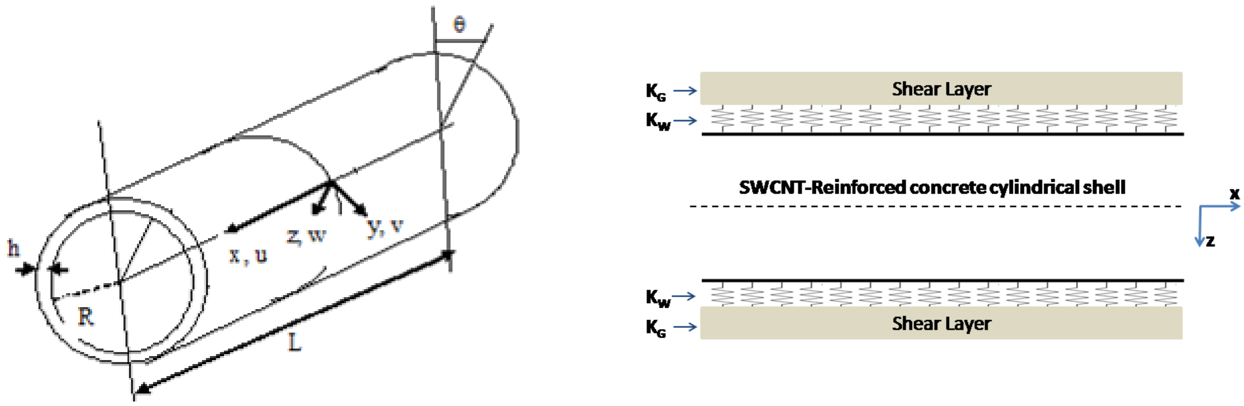


(a) Thin-walled circular cylindrical shell (b) Carbon nanotube embedded in elastic foundation

Fig. 1 Schematic for carbon nanotube embedded in elastic foundation

Alimirzaei *et al*. (2019) used finite element method (FEM) for the study of the nonlinear static, buckling and vibration analysis of viscoelastic micro-composite beam reinforced by various distributions of boron nitrid nanotube (BNNT) with initial geometrical imperfection by modified strain gradient theory (MSGT). We also present some scientific research based on plate and shell theories, the elastodynamic behavior of CNT reinforced composite plate is studied by Formica *et al.* (2010) using a theoretical model, the critical buckling load of SWCNTs laminated-composite plates is examined by Arani *et al.* (2011) employing both analytical (CPT and TSDT) and numerical (FEM) methods, Yazdani and Mohammadimehr (2019) used refined shear deformation theory and wave propagation solutions for double bonded Cooper-Naghdi micro sandwich cylindrical shells with CNT reinforced composite face sheets and porous, Medani *et al*. (2019) examined the static and dynamic behavior of FG-CNT reinforced porous sandwich polymer plate. The mechanical behavior of such structures and other ones can be studied using many High-order Shear Deformation Theories (HSDTs) or the first order shear deformation plate theory (FSDT), such as Xiang and Shi (2011), Faleh *et al.* (2018), Majeed and Sadiq (2018), Sayyad and Ghugal (2018), Barati *et al*. (2018), Fenjan *et al*. (2019), Mirjavadi *et al*. (2019), Avcar (2019), Semmah *et al*. (2019), Rahmani *et al*. (2020), Belbachir *et al*. (2020), Boussoula *et al*. (2020), Belbachir *et al*. (2019), Sahla *et al*. (2019) and Balubaid *et al*. (2019). We just present some recent research of reinforcement materials based on these theories, Draoui *et al*. (2019) studied the static and dynamic behavior of CNT reinforced composite sandwich plates, Abualnour *et al*. (2019) discussed the thermomechanical analysis of antisymmetric laminated reinforced composite plates using a new four variable trigonometric refined plate theory, Draiche *et al*. (2019) presented an analytical model to predict the static analysis of laminated reinforced composite plates subjected to sinusoidal and uniform loads by using a simple FSDT.

The theoretical researches on dynamic and stability of SWCNTs reinforced concrete beams, plates or shells are very interesting for researchers in the literature. The aim of this paper is to purpose a new explicit analytical formula for the critical buckling load of SWCNT reinforced concrete cylindrical shell embedded in elastic foundation for fixed aspect ratio without any assumption. This formula is obtained using a continuum model based on Donnell's cylindrical shell theory. The critical buckling load is derived by an analytical minimization procedure. The elastic foundation is modeled with Winkler and Pasternak models. The effects of different parameters on the buckling response of reinforced concrete cylindrical shell are discussed such as SWCNT percentage, diverse distribution types of carbon nanotube and elastic foundation parameters.

2. Kinematic and constitutive relations based on the Donnell shell theory

In this model based on Donnell shell (Donnell 1934, Timesli 2020, Asghar *et al*. 2020), the median surface of the shell are used to calculate the induced stresses and the effects of transverse shear and rotary inertia have been neglected using the thin shell assumption in the derivation.

We assume that the SWCNT reinforced concrete cylindrical shell is considered as a thin-walled circular cylindrical shell of a middle surface of radius R, wall thickness h which is much lower than R, and length L (see Fig. 1(a)). The young modulus E_u, shear modulus G_{12} and the poison ratios (ν_{12} and ν_{21}) of the SWCNT reinforced concrete shell are computed according to the mixture rule (Bousahla *et al*. 2020). The SWCNT-RC shell is embedded in elastic foundation as shown in Fig. 1(b), which allows us to study the effect of various elastic foundation parameters on the axial buckling load λ. These parameters are the lower spring modulus K_W called also Winkler modulus and the shear layer modulus K_G.

Using the Donnell model of the thin-walled circular cylindrical shell and the foundation model (He *et al*. 2005), the nonlinear equilibrium equation can be written in the following form

$$\frac{\partial^2 M_{xx}}{\partial x^2}+\frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial x\partial\theta}+\frac{1}{R^2}\frac{\partial^2 M_{\theta\theta}}{\partial\theta^2}+\frac{N_{\theta\theta}}{R}+N_{xx}\frac{\partial^2 w}{\partial x^2} +2\frac{N_{x\theta}}{R}\frac{\partial^2 w}{\partial x\partial\theta}+\frac{N_{\theta\theta}}{R^2}\frac{\partial^2 w}{\partial\theta^2}-f=0 \tag{1}$$

where N_{xx} and $N_{\theta\theta}$ are the normal forces, $N_{x\theta}$ is the internal shear force, M_{xx} and $M_{\theta\theta}$ are the bending moments and $M_{x\theta}$ is the twisting moment, w is the transverse displacement of the reference surface and f external load which is related to foundation model of elastic foundation. We can express the Hooke's law for the strain and stress relation by

$$\begin{cases} \sigma_{xx} = \dfrac{E_{11}}{1-\nu_{12}\nu_{21}}\varepsilon_{xx} + \dfrac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}}\varepsilon_{\theta\theta} \\ \sigma_{\theta\theta} = \dfrac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}}\varepsilon_{xx} + \dfrac{E_{22}}{1-\nu_{12}\nu_{21}}\varepsilon_{\theta\theta} \\ \sigma_{x\theta} = G_{12}\gamma_{x\theta} \end{cases} \tag{2}$$

The constitutive relations of Donnell shell are given as follows

$$\begin{cases} N_{xx} = C_{11}\varepsilon_{xx} + C_{12}\varepsilon_{\theta\theta} \\ N_{\theta\theta} = C_{22}\varepsilon_{\theta\theta} + C_{12}\varepsilon_{xx} \\ N_{x\theta} = C_{66}\gamma_{x\theta} \end{cases} \tag{3}$$

where C_{ij} are the extensional stiffness coefficients of the shell which are given by

$$\begin{cases} C_{11} = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E_{11}}{1-\nu_{12}\nu_{21}}dz \\ C_{22} = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E_{22}}{1-\nu_{12}\nu_{21}}dz \\ C_{12} = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}}dz \\ C_{66} = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} G_{12}dz \end{cases} \tag{4}$$

and

$$\begin{cases} M_{xx} = -D_{11}\dfrac{\partial^2 w}{\partial x^2} - \dfrac{D_{12}}{R^2}\dfrac{\partial^2 w}{\partial\theta^2} \\ M_{\theta\theta} = -D_{12}\dfrac{\partial^2 w}{\partial x^2} - \dfrac{D_{22}}{R^2}\dfrac{\partial^2 w}{\partial\theta^2} \\ M_{x\theta} = -\dfrac{D_{66}}{R}\dfrac{\partial^2 w}{\partial x\partial\theta} \end{cases} \tag{5}$$

where D_{ij} are the bending stiffness coefficients of the shell, they are given by

$$\begin{cases} D_{11} = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2\frac{E_{11}}{1-\nu_{12}\nu_{21}}dz \\ D_{22} = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2\frac{E_{22}}{1-\nu_{12}\nu_{21}}dz \\ D_{12} = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2\frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}}dz \\ D_{66} = \displaystyle\int_{-\frac{h}{2}}^{\frac{h}{2}} z^2 G_{12}dz \end{cases} \tag{6}$$

Replacing the bending stiffness coefficients given by (5) in the Eq. (1) and we obtain

$$\begin{aligned} &-D_{11}\frac{\partial^4 w}{\partial x^4} - \frac{2}{R^2}(D_{12}+D_{66})\frac{\partial^2 w}{\partial x^2\partial\theta^2} - \frac{D_{22}}{R^4}\frac{\partial^4 w}{\partial\theta^4} + \frac{N_{\theta\theta}}{R} \\ &+N_{xx}\frac{\partial^2 w}{\partial x^2} + 2\frac{N_{x\theta}}{R}\frac{\partial^2 w}{\partial x\partial\theta} + \frac{N_{\theta\theta}}{R^2}\frac{\partial^2 w}{\partial\theta^2} - f = 0 \end{aligned} \tag{7}$$

According to the shell theory, the membrane forces are connected to the stress function ϕ and we can write them as follows $N_{xx} = \frac{1}{R^2}\frac{\partial^2\phi}{\partial\theta^2}$, $N_{\theta\theta} = \frac{\partial^2\phi}{\partial x^2}$ and $N_{x\theta} = \frac{1}{R}\frac{\partial^2\phi}{\partial x\partial\theta}$. We use the adjacent equilibrium criterion (Brush and Almroth 1975) to study the possible existence of an adjacent equilibrium configurations. We consider that the indices 0 and b indicate respectively pre-buckling and post-buckling quantities and we neglect the terms of second order in index b to obtain the following equation

$$\begin{aligned} &-D_{11}\frac{\partial^4 w_b}{\partial x^4} - \frac{2}{R^2}(D_{12}+D_{66})\frac{\partial^2 w_b}{\partial x^2\partial\theta^2} - \frac{1}{R^4}D_{22}\frac{\partial^4 w_b}{\partial\theta^4} + \\ &\frac{1}{R}\frac{\partial^2\phi}{\partial x^2} + N_{xx0}\frac{\partial^2 w_b}{\partial x^2} + \frac{2}{R}N_{x\theta0}\frac{\partial^2 w_b}{\partial x\partial\theta} + \frac{1}{R^2}N_{\theta\theta0}\frac{\partial^2 w_b}{\partial\theta^2} - f_b = 0 \end{aligned} \tag{8}$$

The stress function $\phi(x,\theta)$ verifies the following compatibility condition

$$C_{22}^*\frac{\partial^4\phi}{\partial x^4} + \frac{1}{R^2}\left(C_{66}^* + 2C_{12}^*\right)\frac{\partial^2\phi}{\partial x^2\partial\theta^2} + \frac{1}{R^4}C_{11}^*\frac{\partial^4\phi}{\partial\theta^4} + \frac{1}{R}\frac{\partial^2 w}{\partial x^2} = 0 \tag{9}$$

where

$$\begin{cases} C_{11}^* = \dfrac{C_{11}}{\Delta} \\ C_{22}^* = \dfrac{C_{22}}{\Delta} \\ C_{12}^* = \dfrac{C_{12}}{\Delta} \\ C_{66}^* = \dfrac{1}{C_{66}} \end{cases} \tag{10}$$

with

$$\Delta = C_{11}C_{22} - C_{12}^2 \tag{11}$$

If the shear membrane forces are neglected $N_{x\theta0} = 0$, the axial compression is $N_{xx0} = \lambda$ and the circumferential membrane force is $N_{\theta\theta0} = 0$, the system (8)-(9) for representing Donnell equations becomes:

$$\begin{cases} -D_{11}\dfrac{\partial^4 w_b}{\partial x^4} - \dfrac{2}{R^2}(D_{12}+D_{66})\dfrac{\partial^2 w_b}{\partial x^2\partial\theta^2} - \dfrac{D_{22}}{R^4}\dfrac{\partial^4 w_b}{\partial\theta^4} \\ \quad + \dfrac{1}{R}\dfrac{\partial^2\phi}{\partial x^2} + \lambda\dfrac{\partial^2 w_b}{\partial x^2} - f_b = 0 \\ C_{22}^*\dfrac{\partial^4\phi}{\partial x^4} + \dfrac{1}{R^2}(C_{66}^* + 2C_{12}^*)\dfrac{\partial^2\phi}{\partial x^2\partial\theta^2} + \\ \quad \dfrac{1}{R^4}C_{11}^*\dfrac{\partial^4\phi}{\partial\theta^4} + \dfrac{1}{R}\dfrac{\partial^2 w}{\partial x^2} = 0 \end{cases} \tag{12}$$

3. CNT reinforced concrete cylindrical shell

The Young's moduli E_{11} and E_{22}, shear modulus G_{12}, Poisson's ratio ν_{12} of the SWCNT reinforced concrete shell

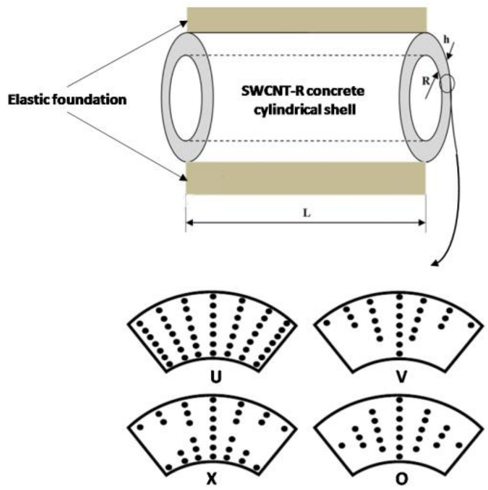


Fig. 2 Configurations of SWCNT distributions in a concrete cylindrical shell

are computed according to the mixture rule (Ke *et al*. 2010, Yas and Samadi 2012, Han and Elliott 2007)

$$\begin{cases} E_{11} = \eta_1 C_r E_{11}^r + (1 - C_r)\,E^c \\ \dfrac{\eta_2}{E_{22}} = \dfrac{C_r}{E_{22}^r} + \dfrac{1 - C_r}{E^c} \\ \dfrac{\eta_3}{G_{12}} = \dfrac{C_r}{G_{12}^r} + \dfrac{1 - C_r}{G^c} \\ \nu_{12} = C_r \nu_{r12} + (1 - C_r)\nu_c \\ \rho = C_r \rho_r + (1 - C_r)\rho_c \end{cases} \tag{13}$$

where C_r is the volume fraction of SWCNT (such as $(1-C_r)$ is the volume fraction of the concrete). E_{11}^r, E_{22}^r, G_{12}^r, ν_{r12} and ρ_r are respectively the Young'is moduli, shear modulus, Poison'is ratio and mass density of the reinforcement. E^c, G^c, ν_c and ρ_c are respectively the Young'is and shear modulus, Poison'is ratio and mass density of the concrete. η_1, η_2 and η_3 are the efficiency parameters associated to the volume fractions S_r^*. The four SWCNT distributions (*U*, *O*, *X* and *V*) are presented in Fig. 2.

From Fig. 2, we can note that the distributions of carbon nanotubes, of *U*, *O*, *X* and *V* shapes, vary continuously across shells thickness. So these different shapes are some kind of functionally graded shells where the SWCNT distributions (*U*, *O*, *X* and *V*) are expressed in the mathematical form as:

• U-CNT distribution (Uniform distribution)

$$S_r = S_r^* \tag{14}$$

• O-CNT distribution

$$S_r = 2\left(1 - \frac{2|z|}{h}\right)S_r^* \tag{15}$$

• X-CNT distribution

$$S_r = 4\frac{|z|}{h}S_r^* \tag{16}$$

Table 1 Efficiency parameters η_1, η_2 and η_3 associated to the volume fractions S_r^*

S_r^*	0.12	0.17	0.28
η_1	1.2833	1.3414	1.3228
η_2	1.0566	1.7101	1.7380
η_3	1.0566	1.7101	1.7380

• V-CNT distribution

$$S_r = \left(1 + \frac{2z}{h}\right)S_r^* \tag{17}$$

where the volume fraction S_r^* of the CNT is obtained from the following expression

$$S_r^* = \frac{w_r}{w_r + \left(\dfrac{\rho_r}{\rho}\right)(1 - w_r)} \tag{18}$$

where w_r is the mass fraction of CNTs. The value of the efficiency parameters of the SWCNT (η_1, η_2 and η_3) are presented in the Table 1 (Yas and Samadi 2012, Wattanasakulpong and Ungbhakorn 2013).

4. Elastic foundation models

Winkler *et al*. (1867) assume that the reaction force of carbon nanotube is proportional to the foundation deflection at each point in the foundation which amounts to modeling the foundation by a juxtaposition of elastic springs as follows

$$f(x) = K_W w \tag{19}$$

where K_W is the modulus of the reaction force of carbon nanotube. Winkler foundation model (19) combines well with numerical and analytical methods and it is simple and efficient. The disadvantage of this model is to neglect the interaction between the springs which represents the shearing effect in the elastic foundation. Consequently, a displacement discontinuity is created between the loaded area and the unloaded area under the foundation. Based on the Winkler model, Pasternak (1954) also proposed a model which assumes that there is a shear interactions between the springs (see Fig. 1(b)). According to Pasternak module K_G, the relationship between contact pressure and DWCNT can be expressed as follows

$$f(x) = K_W w - K_G \nabla^2 w \tag{20}$$

Several research works are based on these two models. Recently, Refrafi *et al*. (2020) used a novel shear deformation theory to study the hygrothermal and mechanical buckling responses of simply supported functionally graded sandwich plate seated on elastic foundation, Bellal *et al*. (2020) used nonlocal four-unknown integral model to analyze the buckling behavior of a single-layered graphene sheet embedded in visco-Pasternak'is medium, Tounsi *et al*. (2020) proposed a simple four-variable trigonometric integral shear deformation model for the static behavior of advanced functionally graded ceramic-metal plates supported by a elastic foundation and subjected to a nonlinear hygro-thermo-mechanical load, Boukhlif *et al*. (2019) based on a simple quasi-3D higher shear deformation theory to present a dynamic investigation of functionally graded plates resting on elastic foundation. There are many other recent references in the literature which use the Winkler and Pasternak models such as Boulefrakh *et al*. (2019), Chaabane *et al*. (2019), Mahmoudi *et al*. (2019), Zaouia *et al*. (2019) and Karami *et al*. (2019). In addition, there is Kerr model (Kerr 1965) which provides more flexibility for foundation continuity between loaded and unloaded area of the shell elastic, where the surrounding elastic medium consists of lower and upper spring beds sandwiching shear layer. Using the Kerr foundation model, Kaddari *et al*. (2020) proposed a new type of quasi-3D hyperbolic shear deformation theory to discuss the statics and free vibration of functionally graded porous plates resting on Kerr-type elastic foundation. Other researchers have discussed the effect of all models as Addou *et al*. (2019), in this work a simple quasi-3D hyperbolic theory is used to investigate the effect of porosity on dynamic behavior of functionally graded plates and all types of elastic foundation cited above.

5. Buckling analysis of SWCNT-RC shell embedded in an elastic foundation

Using the Eqs. (12) and (20), the transverse displacement $w(x,\theta)$ and the corresponding stress functions $\phi(x,\theta)$ are solutions of the following nonlocal Donnell shell equilibrium equations

$$\begin{cases} -D_{11}\dfrac{\partial^4 w_b}{\partial x^4}-\dfrac{2}{R^2}(D_{12}+D_{66})\dfrac{\partial^2 w_b}{\partial x^2\partial\theta^2}-\dfrac{D_{22}}{R^4}\dfrac{\partial^4 w_b}{\partial\theta^4} \\ +\dfrac{1}{R}\dfrac{\partial^2\phi}{\partial x^2}+\lambda\dfrac{\partial^2 w_b}{\partial x^2}-K_W w-K_G\nabla^2 w=0 \\ C_{22}^*\dfrac{\partial^4\phi}{\partial x^4}+\dfrac{1}{R^2}(C_{66}^*+2C_{12}^*)\dfrac{\partial^2\phi}{\partial x^2\partial\theta^2}+\dfrac{1}{R^4}C_{11}^*\dfrac{\partial^4\phi}{\partial\theta^4}+\dfrac{1}{R}\dfrac{\partial^2 w}{\partial x^2}=0 \end{cases} \tag{21}$$

The solution of the problem (21) is sought in the following form

$$\begin{cases} w(x,\theta)=A\,e^{(i\frac{m\pi}{L}x)}\cos(n\,\theta)+cc \\ \phi(x,\theta)=a\,e^{(i\frac{m\pi}{L}x)}\cos(n\,\theta)+cc \end{cases} \tag{22}$$

where A and a are arbitrary complex constants, n and m are respectively the circumferential and axial half wavenumbers of the cylindrical shell and cc represents the complex conjugate. After the substitution of the solution (22) in the problem (21), the equations of the system gives

$$\begin{cases} -D_{11}p^4A-2(D_{12}+D_{66})p^2q^2A-D_{22}q^4A-\rho p^2a \\ +\lambda p^2A-K_WA-K_G(p^2+q^2)A=0 \\ C_{22}^*p^4a+(C_{66}^*+2C_{12}^*)p^2q^2a+C_{11}^*q^4a-\rho p^2A=0 \end{cases} \tag{23}$$

where $\rho=1/R$ is the curvature and $p=m\pi/L$ and $q=n/R$ are the wave numbers in axial and circumferential direction, respectively. The second equation of the system (23) leads to determine the constant a

$$a=\frac{\rho p^2A}{C_{22}^*p^4+(C_{66}^*+2C_{12}^*)p^2q^2+C_{11}^*q^4} \tag{24}$$

Using the value of the constant a in the first equation of the system (23), we can rewrite this equation in the following form

$$\begin{aligned} &(-D_{11}-2(D_{12}+D_{66})\beta^2-D_{22}\beta^4)p^4 \\ &+\big(\lambda-K_G(1+\beta^2)\big)p^2-K_W \\ &-\frac{\rho^2}{C_{22}^*+(C_{66}^*+2C_{12}^*)\beta^2+C_{11}^*\beta^4}=0 \end{aligned} \tag{25}$$

where $\beta=q/p$ is the aspect ratio. So we can determine the expression of the buckling load λ according to β and p as follows

$$\begin{aligned} \lambda(\beta,p)=&\left(D_{11}+2\left(D_{12}+D_{66}\right)\beta^2+D_{22}\beta^4\right)p^2+K_G\left(1+\beta^2\right) \\ &+\left(\frac{\rho^2}{C_{22}^*+\left(C_{66}^*+2C_{12}^*\right)\beta^2+C_{11}^*\beta^4}+K_W\right)\frac{1}{p^2}=0 \end{aligned} \tag{26}$$

We can obtain the critical buckling load λ_{cr} by minimizing the buckling load $\lambda(\beta,p)$ given by the Eq. (26) with respect to the axial wave number p

$$\left.\frac{\partial\lambda(\beta,p)}{\partial p}\right|_{\beta\text{ fixed}}=0 \tag{27}$$

The Equ. (27) leads to the following polynomial of degree 4 in p

$$\begin{aligned} &(D_{11}+2(D_{12}+D_{66})\beta^2+D_{22}\beta^4)p^4+K_W \\ &-\frac{\rho^2}{C_{22}^*+(C_{66}^*+2C_{12}^*)\beta^2+C_{11}^*\beta^4}=0 \end{aligned} \tag{28}$$

So we can conclude the critical axial wave number p_{cr} which is equal

$$p_{cr}=\left(\frac{\frac{\rho^2}{C_{22}^*+(C_{66}^*+2C_{12}^*)\beta^2+C_{11}^*\beta^4}+K_W}{D_{11}+2(D_{12}+D_{66})\beta^2+D_{22}\beta^4}\right)^{\frac{1}{4}} \tag{29}$$

Finally the critical buckling load λ_{cr} of SWCNT reinforced concrete cylindrical shell is obtained according to the critical axial wave number p_{cr} for fixed value of aspect ratio β as follows $\lambda_{cr}=\lambda(p=p_{cr})$.

6. Numerical analysis and discussion

In this investigation, we consider a concrete cylindrical shell with the following geometrical and mechanical proprieties: the inner diameter R=100 mm, the thickness H=20 mm, the length L=10R, the aspect ratio β=0.6, the Young's modulus E_c=20 GPa, the Poison's ratio υ_c=0.27 and the density ρ_c=2600 kg/m^3. This cylindrical shell is reinforced with SWCNT (armchair (10,10)) of the following proprieties: E_{11}^r=600 GPa, E_{22}^r=10 GPa, ρ_r=1400 kg/m^3, υ_{r12}=0.19 and G_{12}^r=17.2 GPa (Wattanasakulpong and Ungbhakorn 2013). In the buckling analysis of the concrete cylindrical shell reinforced with carbon nanotubes presented in this work, we compute the results by adimensional form of the critical buckling load parameter as

Table 2 Critical buckling load ($\bar{\lambda}$) of SWCNT reinforced concrete cylindrical shell

(β_W, β_G)	S_r^*	U	V	X	O
(0.0, 0.0)	0.12	0.1174	0.1175	0.1295	0.1052
	0.17	0.1811	0.1811	0.1984	0.1641
	0.28	0.2060	0.2071	0.2305	0.1863
(0.5, 0.0)	0.12	0.1180	0.1181	0.1301	0.1058
	0.17	0.1816	0.1817	0.1990	0.1646
	0.28	0.2066	0.2076	0.2311	0.1868
(0.5, 0.05)	0.12	0.1860	0.1861	0.1981	0.1738
	0.17	0.2496	0.2497	0.2670	0.2326
	0.28	0.2746	0.2756	0.2991	0.2548

Table 3 The value of aspect ratio β corresponding to minimum value of $\bar{\lambda}_{cr}$

S_r^*	U	O
0.12	0.46	0.54
0.17	0.51	0.62
0.28	0.48	0.61

$$\bar{\lambda} = \frac{\lambda}{C_{110}} \tag{30}$$

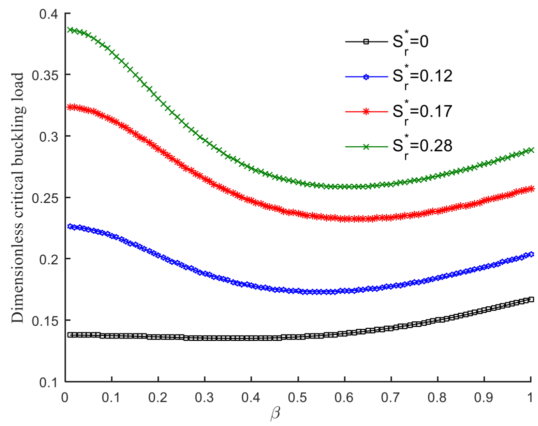
and also by adimensional forms of the Winkler and Pasternak constants as

$$\begin{cases} \beta_W = \dfrac{K_W L^2}{C_{110}} \\ \beta_G = \dfrac{K_G}{C_{110}} \end{cases} \tag{31}$$

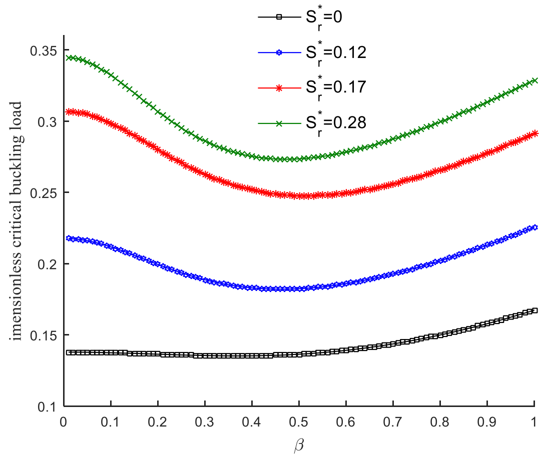
where C_{110} is the extension stiffness of the concrete only given by

$$C_{110} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E_c}{\left(1-\upsilon_c^2\right)} dz \tag{32}$$

In Table 2, we present the dimensionless critical buckling load ($\bar{\lambda}$) of different types of distributions (U, V, X and O) of reinforced concrete cylindrical shell with and without elastic foundation versus the volume fraction of the CNT (S_r^*). The results show that the critical buckling load increases with the increasing the values of the elastic foundation parameters (β_W, β_G) which shows that the shell rigidity increases due to the presence of the elastic foundation. This table shows also an increase in CNT volume fraction S_r^* leads to higher critical buckling load ($\bar{\lambda}$) for both uniform distribution and functionally graded SWCNT reinforced concrete cylindrical shell, so there is a good correlation between the critical buckling load and SWCNT percentage. Moreover, we can conclude that the distribution of carbon nanotubes of "O" shape gives the most flexible shell where the critical load is lower compared to other shapes. We can also conclude that the largest values of the critical buckling load are obtained by a distribution of CNTs through the thickness of the "X" shape.



(a) OD-CNT



(b) UD-CNT

Fig. 4 The effect of SWCNT percentage on critical buckling load $\bar{\lambda}_{cr}$ of reinforced concrete cylindrical shell

Now the question is whether these observations will remain valid regardless of the aspect ratio value β.

In the following tests, the parameters of elastic foundation are taken β_W=0.5 and β_W=0.05. In Fig. 3, it is seen that the critical buckling load without reinforcement presents a minimum versus the aspect ratio β and decreases monotonically from β=0.37. We can observe the same remark for the critical buckling load of reinforced concrete cylindrical shell as shown Fig. 4, but the value of β corresponding to minimum value of $\bar{\lambda}_{cr}$ depends on the distribution shape and SWCNT percentage as you can see in Table 3. We can conclude that there is a critical value of β, depends on the distribution shape and SWCNT percentage, which must be avoided to increase the shell rigidity.

Now, the value of SWCNT percentage is fixed to $S_r^* = 0.12$ and we draw the curves of different distributions in the same figure (see Fig. 5). For small values of aspect ratio β, the distribution of carbon nanotubes of "X" shape gives the most flexible concrete cylindrical shell then the other shapes in the following order "V", "U" and "O". This arrangement is reversed for high values of aspect ratio β where the distribution of carbon nanotubes of "O" shape gives the most flexible concrete cylindrical shell. Moreover, for high values of β the two distributions carbon nanotubes

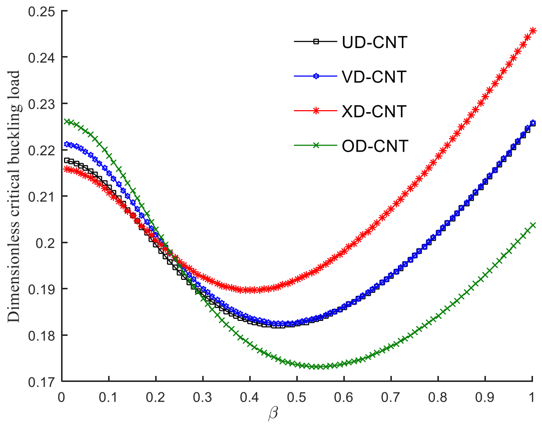


Fig. 5 The effect of distribution carbon nanotubes on critical buckling load $\bar{\lambda}_{cr}$ of SWCNT reinforced concrete cylindrical shell

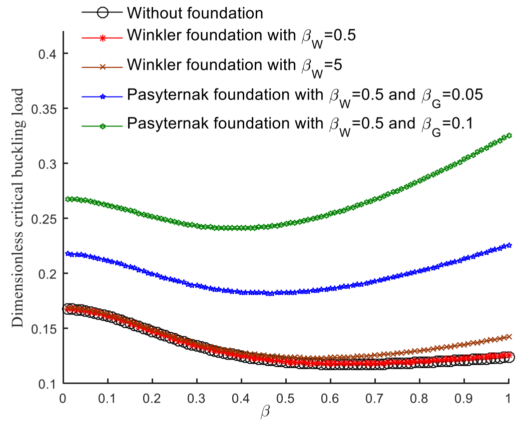


Fig. 6 The effect of elastic foundations parameter (β_W, β_G) on critical buckling load $\bar{\lambda}_{cr}$ of UD-CNT reinforced concrete cylindrical shell with $S_r^* = 0.12$

of "*U*" and "*V*" shapes give almost the same values of the critical buckling load $\bar{\lambda}_{cr}$. Note that, the choice of the most rigid material depends on the distribution shape of CNT and the aspect ratio value β.

The Fig. 6 presents the effect of elastic foundations parameters β_W and β_G on critical buckling load $\bar{\lambda}_{cr}$ of UD-CNT reinforced concrete cylindrical shell with $S_r^* = 0.12$. We can conclude that the shell without elastic foundation is less rigid than a shell on Winkler elastic foundation type especially for high aspect ratio values (β>0.5) and Pasternak foundation type is the more rigid.

7. Conclusions

An exact explicit analytical formula of the critical buckling load of SWCNT reinforced concrete cylindrical shell embedded in an elastic foundation for fixed value of aspect ratio has been established. Using this analytical formula, the stability analysis of SWCNT reinforced concrete cylindrical shell is investigated. The analytical solution have been determined via the Donnel shell theory. To illustrate the influences of the reinforcement percentage, aspect ratio, parameters of the elastic foundation and SWCNT distributions on the critical buckling load of SWCNT reinforced concrete cylindrical shells, many parametric studies have been performed. The main results were obtained from this study:

- Increasing of CNT volume fraction S_r^* leads to higher critical buckling load ($\bar{\lambda}_{cr}$) for both UD- and FG-CNT reinforced concrete cylindrical shell where there is a good correlation between the SWCNT percentage and the critical buckling load.
- The distribution of carbon nanotubes of "*O*" shape gives the most rigid concrete cylindrical shell then the other shapes in the following order "*U*", "*V*" and "*X*" for high values of aspect ratio β.
- The distribution of carbon nanotubes of "*X*" shape gives the most rigid concrete cylindrical shell then the other shapes in the following order "*V*", "*U*" and "*O*" for small values of aspect ratio β.
- The shell without elastic foundation is less rigid than a shell on Winkler elastic foundation type especially for high aspect ratio values (β>0.5) and Pasternak foundation type is the more rigid.
- There is a critical value of aspect ratio β equivalent to a minimum value of the critical load, the shell rigidity increases if this critical value is avoided.
- The choice of the most rigid material depends on the distribution shape of CNT and the aspect ratio value β.

The proposed formulation, of SWCNT reinforced concrete cylindrical shell embedded in an elastic foundation, can be applied to other type of materials as poly methyl methacrylate (PMMA) (Wattanasakulpong and Ungbhakorn 2013, Belbachir *et al*. 2019).

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