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Minimum cost design of RCMRFs based on consistent approximation method

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Abstract. In this paper, a procedure for automated optimized design of reinforced concrete frames has been presented. The procedure consists of formulation and solution of the design problem in the form of an optimization problem. The minimization of total cost of R/C frame has been taken as the objective of optimization problem. In this research, consistent approximation method is applied to explicitly formulate constraints and objective function in terms of the design variables. In the presented method, the primary optimization problem is replaced with a sequence of explicit sub-problems. Each sub-problem is efficiently solved using the Sequential Quadratic Programming (SQP) method. The proposed method is demonstrated through a four-story frame and an eight-story frame, and the optimum results are compared with those in the available literature. It is shown that the proposed method can be easily applied to obtain rational, reliable, economical and practical designs for Reinforced Concrete Moment Resisting Frames (RCMRFs) while it is converged after a few analyses.

Keywords: structural optimization; consistent approximation; sensitivity analysis; reinforced concrete frame

1. Introduction

Structural optimization is currently one of the most important topics in structural engineering and has a wide range of applicability. The objective of structural optimization is to find design variables for a structure that minimize cost and satisfy various design requirements. A large number of optimization techniques have been developed and used in structural optimization. Among optimization methods, the mathematical programming method is attractive due to its generality and rigorous theoretical basis. The main difficulty with the use of mathematical programming for structural optimization problems in which the structural form is specific is the formulation of constraints, such as displacement and stress limitations, as explicit functions of the design variables.

The material costs of reinforced concrete frames are dependent on dimensions, reinforcement ratios and formworks of structural elements, and the unit costs of concrete, steel reinforcement and formwork. Whilst trying to optimize the cost of a structure, certain conditions have to be met so that the equilibriums of the sections are maintained and the requirements of relevant standards are satisfied. Although developed various structural optimization methods, the minimum cost of reinforced concrete frames is difficult to achieve using existing design methods. There are an infinite number of alternative dimensions and reinforcement ratios for structural elements that can yield a similar force or moment of resistance. These elements are often the major components in concrete skeletal structures, and hence their economical design requires consideration as it is an important factor in achieving the overall cost reduction of a structure.

Reinforced concrete is used as the most common material in the world nowadays, so each improvement in optimization of its design procedure has many economical advantages (Habibi *et al*. 2016, Ozbay *et al*. 2010, Rahmanian *et al*. 2014, Franca *et al*. 2016, Guerra and Kiousis 2006). The design codes are based on traditional design methods and engineering economics topics are not considered in them seriously. Thus it is necessary to add an appropriate optimization method to codes. Although the first practical usage of reinforced concrete was at 150 years ago and structural optimization main idea returns to Galileo era (about 380 years ago) but optimization of the structures with this material is rather a new branch of structural optimization and returns to about 7 decades ago. A design optimization problem was formulated by Cohn *et al*. (1968) as a mathematical programming problem using LRFD method to design the reinforced concrete elements. In that study, a limit design method was presented in which load factors against yield of all critical sections are prescribed. Finally, optimal solution obtained in relation to given criteria such as minimum moment area or maximum economy versus elastic design by using the techniques of mathematical programming. Grierson (1968) formulated the problem of RC frames optimal design as a set of linear constraints and a linear objective function. Then the simplex algorithm for linear programming used for solving the optimization problem. Two-dimensional concrete frame formulation and it's solve was presented by Shunmagavel *et*

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al. (1974) as a nonlinear programming problem. It was observed that the convergence rate was low in that optimization method. Yang (1982) continued the studies of Shunmagavel *et al*. considering further design details. He considered cut point of the longitudinal reinforcement and shear reinforcement in addition to the width, height and the amount of longitudinal reinforcement sections, as design variables. Booz *et al*. (1984) studied on optimized design of concrete structures problem using a two-stage idea based on German codes. First, an optimization problem solved to obtain minimum amount of required reinforcement in individual member, for given sections characteristics using sequential quadratic algorithm. Then, the width and height of members of the sections obtained to minimize total cost in the whole structure using same algorithm. Sikiotis *et al*. (1987) reported performing a stressful design of concrete structures, they minimized total cost of structure includes concrete cost and reinforcement cost subjected to behavioral constraints and limitations of the code and architecture.

Another study was conducted based on Australia design codes considering width, height and longitudinal reinforcement area in member sections as design variables by Kanagasundaram and Karihaloo (1990). Spires and Arora (1990) optimized high-rise concrete frames, they analyzed an equivalent two-dimensional frame instead of the main three-dimensional frame. Sections dimensions and cross section area of beam and column reinforcement were considered as design variables. Structural costs include the cost of concrete, reinforcement and formworks. Optimal design of multi-storey concrete structures with shear walls, was studied by Saka (1992). He used a two-stage optimization method to minimize the cost includes the cost of concrete, reinforcement and formworks. Moharrami (1993) developed a computerized algorithm to optimize design of reinforced concrete frame structures by means of optimality criteria method. Width, height and longitudinal reinforcement area in beams and columns were considered as design variables. Design constraints include restrictions on members' resistance, deflection of the beam, lateral deformation, sections dimensions and reinforcement area according to ACI-318-89. Three-dimensional frames were optimized by Fadaee (1996). A computerized method for optimal design of reinforced concrete three-dimensional skeletal structures which has members under biaxial bending, biaxial shearing and axial loads was provided in his study. Rajeev and krishnamoorthy (1998) applied genetic algorithm to optimize design of two-dimensional frames. That method could provide a discrete distribution for optimal values of reinforcement area and sections dimensions.

Leps *et al*. (2002) used hybrid optimization approach to design frames. In their study, discrete optimization of RC structures was conducted based on an effective combination of deterministic and stochastic optimization methods. A deterministic optimization algorithm was used to determine the details of a cross-section in internal forces combination mood. A stochastic optimization algorithm was used to optimize the structure, including materials, dimensions of members and reinforcements. Optimum design of reinforced concrete frame by means of damage control was performed by Lopez and Cruz (2004). In that study, a design approach was presented using damage control members and keeping members within the allowable range. They assumed linear behavior for structural elements under the influence of mild earthquakes and inelastic behavior under the influence of strong earthquakes. Maximum displacement of the structure and dissipated plastic energy in the structure were considered as control parameters. An integrated genetic algorithm complemented with direct search for optimum design of RC frames was provided by Kwak and Kim (2009). A conventional genetic algorithm occasionally has limitations due to a low convergence rate in spite of high computing times. The proposed method in their research uses a predetermined section database (DB) when determining trial sections for the next iteration and thus improved the convergence rate in genetic algorithm. A comparative study of two meta-heuristic algorithms for optimum design of reinforced concrete planar frames was provided by Kaveh and Sabzi (2011). They presented the application of two algorithms, heuristic big bang-big crunch (HBB-BC) and a heuristic particle swarm ant colony optimization (HPSACO) to discrete optimization of the frames subject to combinations of gravity and lateral loads based on ACI 318-08 code. Fatma (2014) proposed developed Genetic Algorithms (GAs) that enabled multi-objective optimization for scheduling a multi-storey building. However, a trade-off between time and cost for habitation projects was required to face limited fund and increasing population environment. This problem has an important position for developing countries' governments because it's related to low cost habitation projects. Multi-storey buildings are classified as special repetitive projects because of skeleton constraints. Activities can be classified into: repetitive and non-repetitive ones. The presented model in that research enabled construction planners to direct controlled time-cost construction plan by investigating optimal plans, generated from a set of feasible alternatives, which minimize project duration, total number of crews, and total work interruptions. Serpik *et al*. (2016) designed an evolutionary procedure for the optimal design of reinforced concrete structures of flat frames manufactured without reinforcement pre stress. The aim of their research was to minimize the planned production cost of the frame with restrictions on strength, hardness and crack resistance. The physically nonlinear behavior of concrete and armature, as well as the possibility of crack formation in cracked concrete was taken into consideration. The research was performed on discrete sets of design parameters. Kaveh and Bakhshpoori (2016) investigated the non-dominated sorting approach for extending the single objective Cuckoo Search (CS) into a multi-objective framework. The proposed approach used an archive composed of primary and secondary population to select and keep the non-dominated solutions at each generation instead of pairwise analogy used in the original Multi-objective Cuckoo Search (MOCS). Habibi *et al*. (2017) used a consistent approximation method to explicitly formulate design constraints of a multi-story RC frame. They applied the sequential quadratic programming for solving the optimization problem. The optimal solution of the frame was obtained by solving several optimization sub-problems produced by the approximation approach. Their optimization results for the studied frame were desirable but they did not present a general algorithm enabled for optimum design of such frames. Guilherme and Moacir (2017) presented a variant of the Harmony Search Algorithm (HS) and its application to discrete optimization. In order to investigate the efficiency of the algorithm, they applied it for obtaining optimal sections of concrete columns subjected to uniaxial flexural compression. The amount and diameters of the reinforcement bars and the dimensions of the columns cross sections were considered as design variables.

In this study, an efficient algorithm is developed based on the CONAP approach for optimum design of the reinforced concrete moment resisting frames. For this purpose, design of the RCMRFs is formulated as an optimization problem. Design variables are the dimensions of concrete sections and reinforcement areas. The objective function is the total cost of the frame which includes the cost of concrete, formwork and reinforcing steel for individual members of the frame. Design constraints are defined based on the requirements of design code requirements for concrete constructions. In the optimum design model, the primary optimization problem is replaced with a sequence of explicit problems by using the CONAP concept. Each sub-problem is first generated based on the analysis and sensitivity analysis results of the structure and then is efficiently solved by using the SQP method.

2. Optimization model

The solve of optimization problem for designing two-dimensional RC frames as well as any other optimization problem requires defining of the problem in the specific construction of optimization. This construction consists of three main parts of design variables, design constraints, and objective function. Each of these three parts discussed separately in the following.

The parameters that are selected for describing the design of a structure are called design variables. The sensible property of design variables is that a unique design of the structure could be presented by having their amount. In fact, the final aim of any design including optimized or classical designs is to present these variables. These variables must be chosen so that they influence the values of objective function and design constraints. In the RC frames, regarding that the structure cost and its responses are changed with variation of dimensions of the concrete sections and the amount of used reinforcements, thus, the width and height of beam and column sections and the tensile and compressive reinforcing bars in beams and the longitudinal reinforcing bars in columns are selected as design variable. The design variables can get any desired value in the adopted range. It could be defined upper and lower limits for the design variables and in each optimization step, each variable can get a value in this range. Design variables can be considered as a vector as follows

$$x = \left(x_1, x_2, x_3, \ldots, x_{n-1}, x_n\right) \tag{1}$$

where x_i can be the width or height of an element section or the rebar section. The value of n also depends on the structure dimension and the performed typing.

In structural engineering, design variables should be selected so that they satisfy special requirements, which are called design constraints. The constraints that represent the limitations of structural system behavior or performance such as the limitations applied on strength, displacement, and stiffness are so-called behavioral constraints. The constraints that are dependent to accessibility, construction or other physical limitations are known as side constraints. The design constraints are design limitations that are expressed in terms of mathematical logic. Constraints are generally divided into equal and unequal constraints. Finally, the presented design must be like that it satisfies all the design constraints. Design constraints could be including regulation constraints and in engineering practice, it could be including operational constraints and are often of unequal type of constraints.

In this study, some constraints are also considered for design of beam sections. The most important one is the constraint related to beam's flexural strength. This constraint is controlled by the following relation

$$g_{mb}^{i}\left(x\right) = M_{ub}^{i} - M_{rb}^{i} \leq 0 \tag{2}$$

where, "i" is the number of the intended beam, M_{rb}^{i} is the beam ultimate flexural strength, M_{ub}^{i} is the maximum bending moment created along the beam under the effect of the regulation's load combination, and $g_{mb}^{i}(x)$ is the constraint of beam's flexural strength. The other constraint is the limitation related to the maximum and minimum tensile rebar in beams, which are controlled by the following relations:

$$g_{\rho b\max}^{i}(x) = \rho_b - \rho_{\max} \leq 0 \tag{3}$$

$$g_{\rho b\min}^{i}(x) = \rho_{\min} - \rho_b \leq 0 \tag{4}$$

where, i is the number of the intended beam, ρ_b is the ratio of tensile rebar existing in the beam section, and $\rho_{\max}$ and $\rho_{\min}$ are the maximum and minimum ratios of the beam tensile rebars, respectively. $g_{\rho b\max}^{i}$ and $g_{\rho b\min}^{i}$ are also the constraints related to the maximum and minimum amounts of tensile rebar in the beam section.

The constraints of controlling flexural and axial strengths are among the most important design constraints for column sections that are obtained according to the following relations

$$g_{n}^{i}\left(x\right) = N_{u}^{i} - N_{r}^{i} \leq 0 \tag{5}$$

$$g_{mc}^{i}\left(x\right) = M_{uc}^{i} - M_{rc}^{i} \leq 0 \tag{6}$$

where, i is the number of the intended column, M_{rc}^{i} and N_{r}^{i} are respectively the ultimate flexural and axial strengths of the column, M_{uc}^{i} and N_{u}^{i} are respectively the maximum bending moment and axial force created along the column under the effect of the regulation's load combination and

$g_{mc}^{i}(x)$ and $g_{n}^{i}(x)$ are the constraints of columns' flexural and axial strengths. Other constraints where are considered for the columns are the constraints related to the maximum and minimum amounts of allowable longitudinal rebar in the column section that are expressed in the form of following relations

$$g_{\rho c\max}^{i}(x) = \rho_c - \rho_{c\max} \leq 0 \tag{7}$$

$$g_{\rho c\min}^{i}(x) = \rho_{c\min} - \rho_c \leq 0 \tag{8}$$

where, "i" is the number of the intended column, ρ_c is the ratio of tensile rebar existing in the column section, and $\rho_{c\max}$ and $\rho_{c\min}$ are the maximum and minimum ratios of the column tensile rebars, respectively. $g_{\rho c\max}^{i}$ and $g_{\rho c\min}^{i}$ are also the constraints related to the maximum and minimum amounts of tensile rebar in the column section.

In the present research, other limitations are also considered for the design of beam and column sections in the form of constraints. For example, the limitation of minimum number of rebars in beam or column sections, which is four in the corners, or the minimum distance between longitudinal rebars that is considered equal to 40 mm.

The last considered constraints are corresponded to the limitation of relative lateral displacement of stories (drift) that is controlled as follows

$$\Delta_s - \Delta_M \leq 0,\ s=1,2,\ldots,n_s \tag{9}$$

where, n_s is the number of stories, Δ_s is the story drift and Δ_M is the allowable drift.

An objective function is mostly known as a cost or a performance criterion and is expressed in terms of design variables. An optimal design provides the best value of the objective function while it satisfies all design constraints. Therefore, choosing an appropriate objective function in an optimization problem is very important. In RC structure, since the structure materials are not of the same kind, it could not be possible to define the objective function based on weight or volume. In this study, objective function is defined based on the structure construction cost that includes the costs of concreting, reinforcing, and forming.

The objective function is considered equal to the summation of two minor functions that specify the costs of beam and column elements, separately. The cost of each element is including the costs of concreting, reinforcing, and forming. As in the practical cases, in which three faces of beams and four faces of column elements are forming, it is also respected in the following functions. Cost coefficients for unit volume of concreting, steel unit weight (steel density), and unit surface of concrete forming are applied in the intended objective function. According to the assumptions, the cost of beams could be determined from the following relation

$$F_b = \sum \left(C_c . b_i . h_i . L_i + C_s . A_{st_i} . L_i . \gamma_s + C_f . L_i . (b_i + 2h_i) \right),\quad i = 1,2,\ldots,n_b \tag{10}$$

The cost related to columns could also be determined by

$$F_c = \sum \left(C_c . b_i . h_i . L_i + C_s . A_{st_i} . L_i . \gamma_s + C_f . L_i . 2(b_i + h_i) \right),\quad i = 1,2,\ldots,n_c \tag{11}$$

The total cost is the sum of costs of beams and column, which is defined as

$$F = F_b + F_c \tag{12}$$

where in the above relations, n_b is the number of beams, n_c is the number of the columns, C_c is the cost of concrete unit volume, C_s is the cost of steel unit weight, C_f is cost of unit surface of forming, b, h, and L are respectively the element width, height, and length, γ_s is the steel weight per unit volume, A_{st} is the steel cross section area, and F is the objective function.

3. Design algorithm

To propose design algorithm used in this study, first, the design problem is formulated. The objective of designing is to minimize the cost under the design conditions such as the limitations of members' strength, members' deformations, the structure lateral displacement, side constraints and operational constraints. According to the definition of design variables based on section 3-1, the formulation of design constraints based on section 3-2, and the formulation of cost function as the objective function based on section 3-3, the problem of optimal design of RC frames can be formulated as follows:

Minimize $F = f(x_1, x_2, \ldots, x_{n-1}, x_n)$

Subjected to:

$$\begin{gathered}
g_n^i(x) = N_u^i - N_r^i \leq 0, i = 1,2,\ldots,n_c \\
g_{mc}^i(x) = M_{uc}^i - M_{rc}^i \leq 0, i = 1,2,\ldots,n_c \\
g_{mb}^i(x) = M_{ub}^i - M_{rb}^i \leq 0, i = 1,2,\ldots,3n_b \\
g_{db}^i(x) = h^i - h_{all}^i \leq 0, i = 1,2,\ldots,n_b \\
g_d^i(x) = Dr^i - Dr_{all} \leq 0, i = 1,2,\ldots,n_s \\
g_{\rho bmax}^i(x) = \rho_b - \rho_{max} \leq 0, i = 1,2,\ldots,n_b \\
g_{\rho bmin}^i(x) = \rho_{min} - \rho_b \leq 0, i = 1,2,\ldots,n_b \\
g_{\rho cmax}^i(x) = \rho_c - \rho_{cmax} \leq 0, i = 1,2,\ldots,n_c \\
g_{\rho cmin}^i(x) = \rho_{cmin} - \rho_c \leq 0, i = 1,2,\ldots,n_c \\
b_i^l \leq b_i \leq b_i^u, i = 1,2,\ldots,n_b + n_c \\
h_i^l \leq h_i \leq h_i^u, i = 1,2,\ldots,n_b + n_c
\end{gathered} \tag{12}$$

In the above-presented formulation, x is the vector of design variables, F is objective function (cost), g_n^i is the constraint of controlling columns axial force, g_{mc}^i is the constraint of controlling columns bending moment, and g_{mb}^i is the constraint of controlling beams bending moment. Moreover, g_{db}^i is the constraint of controlling beams deflection, g_d^i the constraint of controlling relative lateral displacement of building stories, $g_{\rho b\max}^i$ is the constraint of controlling the maximum tensile rebar of beams, and $g_{\rho b\min}^i$ is the constraint of controlling the minimum amount of tensile rebar of beams. In addition, $g_{\rho c\max}^i$ is the constraint of controlling the maximum longitudinal rebar of column and $g_{\rho c\min}^i$ is the constraint of controlling the minimum amount of longitudinal rebar of the columns.

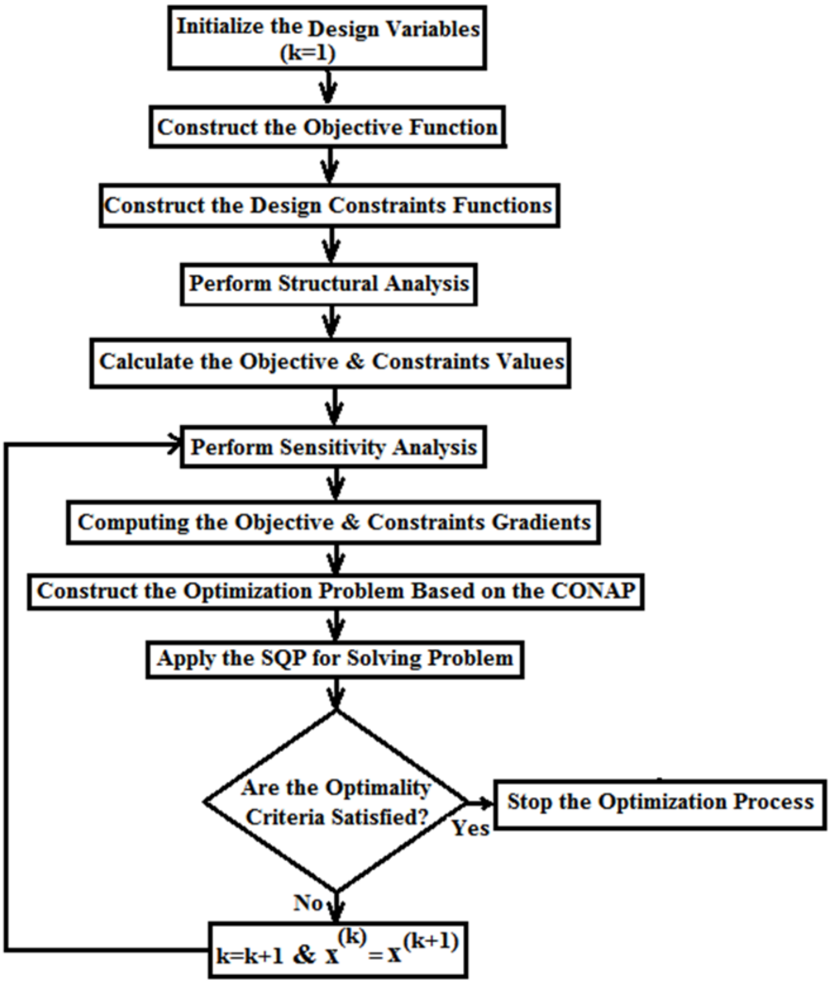

Fig. 1 Flowchart of the proposed design procedure

In this study, the consistent approximation method (CONAP) that is one of the new methods in structural optimization (Habibi 2012) is used for obtaining optimal design of reinforced concrete moment frames. First modeling manner will be expressed and be formulated based on CONAP method. Then optimization algorithm will be presented and the application of the proposed method will be discussed. Based on the CONAP, each design constraint is approximated by the following equation

$$f(x) = f(x^0) + \sum_i \frac{1}{\alpha_i} \left(x_i^0\right)^{1-\alpha_i} f_i^0 \left[(x_i)^{\alpha_i} - \left(x_i^0\right)^{\alpha_i}\right] \quad (13)$$

where f_i implies on the first derivations of function *f(x)* relative to x_i variables. Since there are no explicit functions of design constraints, these derivations are not determinable in structural engineering problems. Hence, the sensitivity analysis theory should be used. The sign $\sum$ denotes the sum of all design parameters (Habibi 2012). By normalizing the design parameters x_i relative to current design parameters and the definitions of $x_i' = x_i / x_i^0$ and $f_i'' = f_i' x_i^0 / \alpha_i$ by using Eq. (2), we have (Habibi 2012)

$$f(x'') = f(x^0) + \sum_i f_i'' \left[\left(x_i''\right)^{\alpha_i} - 1\right] + f_0$$

$$= \sum_i f_i'' \left(x_i''\right)^{\alpha_i} + f_0 - \sum_i f_i'' \quad (14)$$

where f_0 is the amount of function in the initial design point. Eq. (14) expresses the basis of consistent approximation strategy for the formation of optimization problem for designing an engineering structure. In the present research, this strategy is used for constitution of the optimization sub-problem of frames in each design cycle.

The problem of optimizing RC frame is one of difficult and complex problems due to the large numbers of design constraints and their complexity as well as their high nonlinearity and their solution requires special measures in order to make compatible the existing algorithms with the problem. The consistent approximation optimization method is also not excepted from this general rule. The first issue that has to be dealt is being implicit the design constraints defined based on design variables, which is solved by employing compatible approximation. For this purpose, central finite difference method is used to calculate design sensitivities that are used in the consistent approximation method. The other problem that is dealt with due to the structural analysis in the regain of elastic and linear behavior, is being zero the sensitivity of some constraints relative to some design variables. In this regards, an effective and practical solution was proposed and utilized, based on which the design variables with zero related sensitivities are going out from the design cycle.

The proposed algorithm of the present research for the optimal design of RC frames can be expressed in the following steps:

1- Setting k=1 and assuming proper initial values for the design variables.

2- Constructing the objective function.

3- Constructing the design constraints functions.

4- Performing structural analysis.

5- Calculating the values of the objective function and design constraints at the current design point.

6- Performing sensitivity analysis of the structure.

7- Computing gradient vectors of the objective and constraint functions at the current design point.

8- Constructing the optimization sub-problem based on the consistent approximation approach at the current design point

9- Solving the sub-problem using the SQP method and modifying the current design variables.

10- Controlling the optimality criteria. If the optimality criteria are satisfied, stop the optimization process; otherwise, set k=k+1 and $x^{(k)}$=$x^{(k+1)}$ and then go back to Step 4.

Based on the above steps, flowchart of the proposed design process is shown in Fig. 1.

In step 9 of the proposed algorithm, each sub-problem is efficiently solved by using SQP method. In each iteration of this technique, by a quadratic approximation of Lagrangian function and a linear approximation of constraints, the optimization sub-problems are formulated and solved as a quadratic programming problem. The solution of this quadratic programming sub-problem involves determining a search direction and a suitable step-length. The step-length parameter is used to have a global convergence, i.e., when starting from an arbitrary design, the final solution be a design that satisfies the Karush-Kuhn-Tucker (KKT) optimality conditions. The main convergence criteria in SQP are based on numerically satisfying the KKT conditions. When the KKT conditions are satisfied with a desired tolerance, the algorithm will stop. More details about this technique have been presented by Khaledy *et al*. (2018).

4. Case studies

In this section, two concrete frames are designed by using the proposed design algorithm. The studied structures consist of a four-story frame and eight-story frame. Based on the strength-based design method, the following load combinations have been used for designing the frames (ACI-318-08).

$$U = 1.2D + 1.6L \tag{15a}$$

$$U = 1.2D + 1.0L \pm 1.4E \tag{15b}$$

$$U = 0.9D \pm 1.4E \tag{15c}$$

The results of the proposed method are compared with the results from two methods of Big Bang-Big Crunch and the Ant Colony algorithms developed by Kaveh and Sabzi (2011). To ensure the correctness of such a comparison, the following assumptions are considered for these numerical examples:

- The objective function is including the costs of concrete, formwork and steel for the both studies (this study and Kaveh and Sabzi's study).
- In the both studies, the specified compressive strength of concrete and yield strength of reinforcement bars are considered to be 23.5 and 392 MPa, respectively.
- In the both studies, the geometric parameters, loading and grouping details of the frames are the same.
- For rectangular sections of beams and columns, two databases are created by Kaveh and Sabzi (2011). In construction of these databases, some limitations and rules are imposed. In this study, similar to Kaveh and Sabzi's study, rectangular sections are used for beams and columns but no database is needed to be considered for them. That is, their dimensions and reinforcements are not limited by specific databases. Although these values are limited between lower and upper bounds. In the numerical examples, for the purpose of comparison, the lower and upper bounds are assumed based on the minimum and maximum corresponding values of the databases produced by Kaveh and Sabzi.
- Most of regulatory requirements such as flexural strength of beams, flexural and axial strength of columns, the minimum and maximum of steel area for beams and columns, and lower and upper bounds of dimensions of beams and columns, used in this study as the design constraints are similar to the requirements used by Kaveh and Sabzi (2011). Only one constraint is different. That is the limitation of relative lateral displacement of stories (drift) which is considered in this study but it is not considered the article by Kaveh and Sabzi (2011). Since the drift constraint is not active for the design of the studied frames, it can be concluded that the active constraints of the considered examples in this study are as same as the Kaveh and Sabzi's study.
- In the both studies, the unit costs of the concrete, steel, formwork and the density of rebar are estimated to be 105 $/m^3, 0.9 $/kg, 92 $/m^2 and 7850 kg/m3; respectively.
- In the both studies, the dead and live loads, which are applied on beams as distributed loading, are equal to 22.3 and 10.7 kN/m, respectively. The earthquake forces are applied on the structure as concentrated loads in the floor levels.

4.1 Three bay, eight-story reinforced concrete frame

A three bay, eight-story reinforced concrete frame, as shown in Fig. 2, is considered as the first numerical example. Geometrical properties, typing and the frame lateral loading are presented in Fig. 2. The frame is composed of 56 elements, 24 beams and 32 columns, which are divided into three beam groups and four column groups (Habibi *et al*. 2017). After applying the CONAP and the proposed design algorithm on this example, which have been discussed in details in the previous sections, the

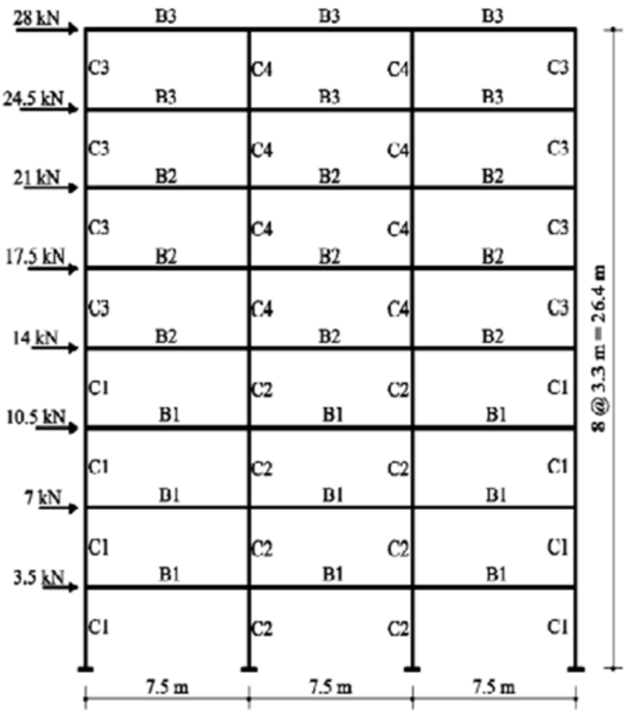

Fig. 2 Eight-story RC moment resisting frame

Table 1 The obtained results of CONAP for the eight-story frame

Element type	Type No.	Width (mm)	Height (mm)	Tensile reinforcement	Compression reinforcement
Beam	B1	300	450	3Φ19	6Φ22
	B2	300	450	3Φ19	6Φ22
	B3	350	500	3Φ19	5Φ22
Column	C1	450	450	8 Φ25	
	C2	500	500	10 Φ25	
	C3	350	350	10Φ25	
	C4	300	300	8Φ25	
Frame cost (dollars)			47594		

obtained results of optimal design were as presented in Table 1.

A structural design optimization considering the same properties and conditions of the example has been conducted by Kaveh and Sabzi (2011) by using two methods of Big Bang-Big Bang Crunch and ant colony search algorithms. In their research, a database including predetermined section properties has been used. Cost coefficients of the objective function are also assumed to be the coefficients used in the present study.

Tables 2 and 3 summarize the results of optimal design of both methods of Big Bang-Big Bang Crunch and ant colony search algorithm for the intended frame.

Comparison of the optimization results from the method proposed in this study with those of Big Bang-Big Crunch and Ant Colony algorithms shows that the optimum design parameters obtained from the proposed method are in good agreement with the two aforementioned methods. The design obtained from the CONAP is cheaper than Big Bang-Big Crunch and Ant Colony methods by 1.38% and 1.89%, respectively.

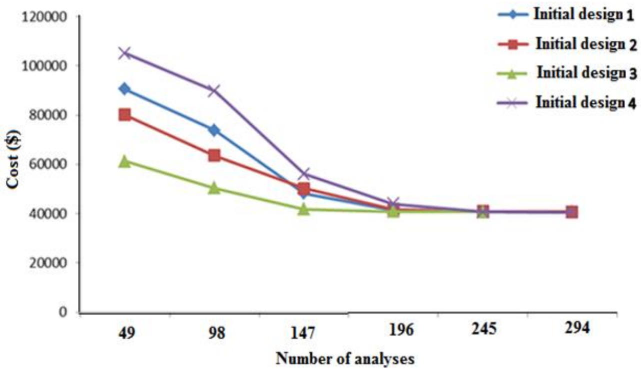

Fig. 3 Iteration history for the eight-story frame

Table 2 The obtained results of HBB-BC for the eight-story frame

Element type	Type No.	Width (mm)	Height (mm)	Tensile reinforcement	Compression reinforcement
Beam	B1	300	500	3Φ19	6Φ22
	B2	300	500	3Φ19	6Φ22
	B3	300	500	3Φ19	5Φ22
Column	C1	400	400	8 Φ25	
	C2	450	450	12 Φ25	
	C3	350	350	8 Φ25	
	C4	350	350	8 Φ25	
Frame cost (dollars)			48263		

Table 3 The obtained results of HPSACO for the eight-story frame

Element type	Type No.	Width (mm)	Height (mm)	Tensile reinforcement	Compression reinforcement
Beam	B1	300	500	3Φ19	6Φ22
	B2	300	500	3Φ19	6Φ22
	B3	300	500	3Φ19	5Φ22
Column	C1	400	400	8 Φ25	
	C2	500	500	8 Φ25	
	C3	350	350	8 Φ25	
	C4	350	350	8 Φ25	
Frame cost (dollars)			48514		

Fig. 3 presents the design convergence process for the 8-story frame with 4 different initial designs. As is seen in this figure, the number of analyses required for convergence of the design is slightly dependent on the initial design. However, generally it could be said that the more different the initial design with the optimal design is, the more analyses for optimization is required. It can be seen from Fig. 3 that the objective function variation in the initial design may be very high due to the high difference between the initial and optimal designs; however, only after a few analyses, the results are converged together. This properly reveals the ability of the optimization algorithm used for concrete frames.

The other point that could be expressed about the consistent approximation algorithm, is that this algorithm significantly increases the amount of the objective function in the first design cycle in the case which the primary design violates a large numbers of the design constraints (in this case, the amount of objective function is mostly less

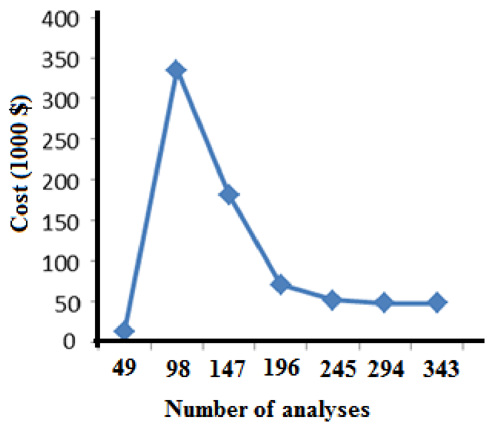

Fig. 4 Iteration history for the eight-story frame with illogical initial design

that the optimal objective function). This is presented in Fig. 4 in more details. In this figure, the start point of optimization process is a point that gives the objective function less than the amount of optimal objective function. This also confirms the results of the study performed by Spires and Arora (1990). Fig. 4 shows that the proposed algorithm achieved to the optimum design after 343 analyses while the HBB-BC and HPSACO algorithms achieved to the optimum design after 39500 and 52500 analyses, respectively. Thus the computational effort of the CONAP is significantly less than the HBB-BC and HPSACO**.**

In comparison with the other two methods of Big Bang-Big Bang Crunch and ant colony algorithm, the other advantage of the consistent approximation method in optimizing RC frames is the lack of need for database including different structural sections. This fact will reduce the computational effort.

The diagram of violating design constraints for the 8-story frame is presented in Fig. 5. As it is seen, there is no violated constraint in the optimal design and all the constraints are less than or equal to zero. In this figure, the active constraints are the points that are placed on the zero line or close to it. The maximum stress resulted from load combinations considered for all structural members are shown in the optimal design in Fig. 6. As it is observed in this figure, the consistent approximation algorithm has led to the more stressful design of beams relative to columns. The obtained result leads to relatively stronger columns than beams, which is a desirable design.

4.2 Three bay, four-story reinforced concrete frame

As the second numerical example, a three bay, four-story reinforced concrete frame is considered. The geometry, loading and grouping details of this frame are

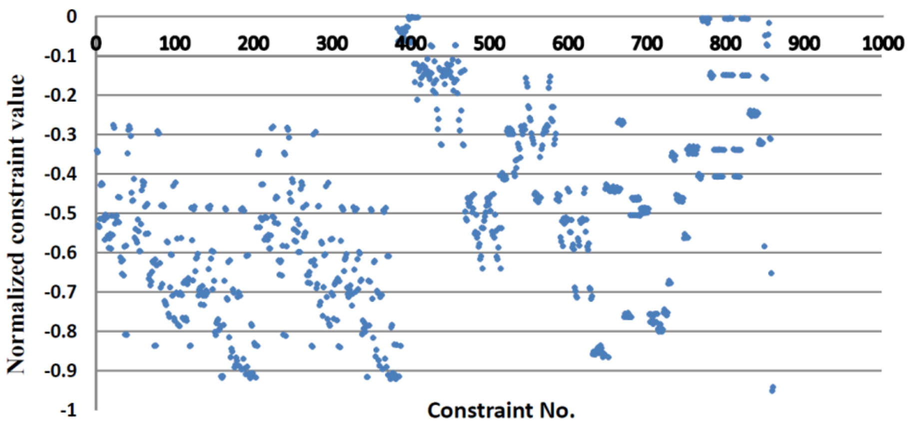

Fig. 5 Constraint violation graph for the eight-story frame

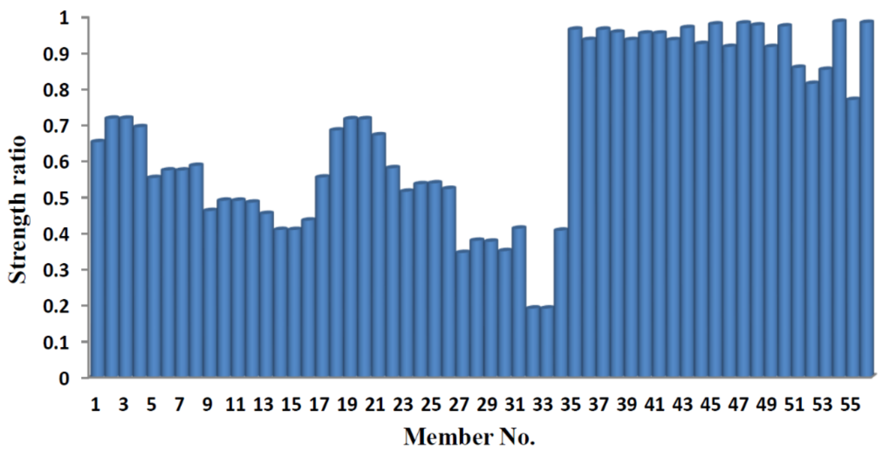

Fig. 6 Strength ratio of members for the eight-story frame

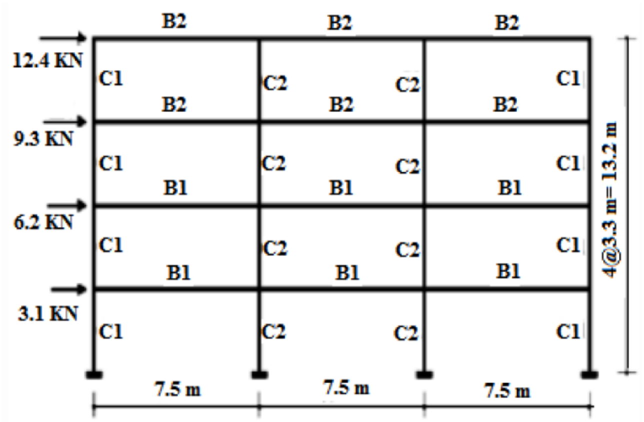

Fig. 7 Four-story RC moment resisting frame

Table 4 The optimization results for the four-story frame

Optimization algorithm	Element type	Type No.	Width (mm)	Height (mm)	Tensile reinforcement	Compression reinforcement	Frame cost ($)
CONAP	Beam	B1	300	500	4Φ19	5Φ22	21235
		B2	300	450	4Φ19	5Φ22	
	Column	C1	300	300	8 Φ25		
		C2	300	300	6 Φ25		
HPSACO	Beam	B1	300	500	3Φ19	5Φ22	22207
		B2	300	500	4Φ19	5Φ22	
	Column	C1	350	350	8 Φ25		
		C2	300	300	6 Φ25		
HBB-BC	Beam	B1	300	500	3Φ19	5Φ22	22207
		B2	300	500	4Φ19	5Φ22	
	Column	C1	350	350	8 Φ25		
		C2	300	300	6 Φ25		

shown in Fig. 7. The frame has a total of 28 members including 12 beams and 16 columns. Two groups for beams and two groups for columns are considered. This frame is subjected to gravity and lateral loads.

Optimum design of this frame has been previously presented by Kaveh and Sabzi (2011) using HBB-BC and HPSACO algorithms. The optimization of the frame is optimized by using the CONAP algorithm and the achieved results are compared with those of the aforementioned algorithms. The comparison of the results obtained by the three algorithms is given in Table 4. The optimum cost obtained by the CONAP is 21235 $ and the HBB-BC and HPSACO algorithms have achieved an optimum cost of 22207 $ for the frame. It can be concluded that the cost of the optimum design resulting from the CONAP algorithm is 4.3% cheaper than the HBB-BC and HPSACO algorithms.

Fig. 8 shows the iteration history of the frame. As can be observed in this figure, the CONAP algorithm has converged to the optimal solution with four different initial designs. It is obvious that the maximum number of analyses required for achieving the optimum design is 174 that is very less than the number of analyses of the HBB-BC and HPSACO (9250 & 8500, respectively). This is due to using the design sensitivities for improving the design in CONAP algorithm that considerably speed up the optimization process compared to other optimization algorithms. This result demonstrates the ability of the algorithm to obtain the

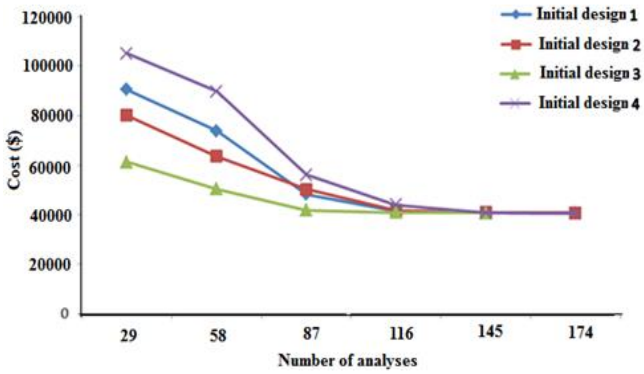

Fig. 8 Iteration history for the four-story frame

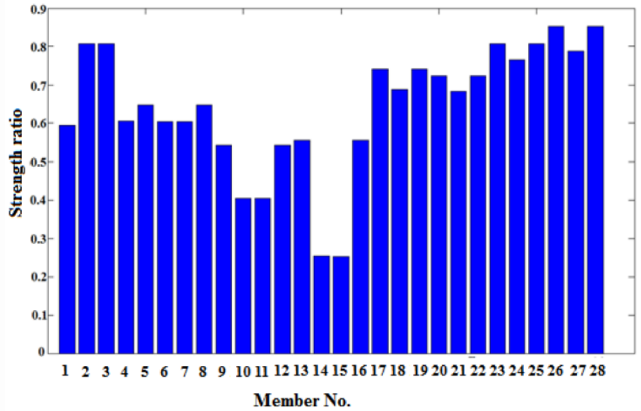

Fig. 9 Strength ratio of members for the four-story frame

optimum design through a few analyses. Also, it can be concluded that the optimal design is independent on the specific initial guess designs. That is, assuming four different initial designs for the four-story frame has led to a specific optimal design.

The optimum strength ratios of the members obtained by the CONAP for the four-story frame is given in figure 9. As can be seen in Fig. 9, the strength ratios for some of the members are high values. That is, the using of section capacity for these members is high.

5. Conclusions

In the present research, a new design algorithm was developed based on the consistent approximation method to achieve optimal design of RC frames. In order to show the efficiency and ability of the method developed in this research, design of a four-story frame and an eight-story frame was investigated. It was shown that the proposed method is capable to obtain optimal and automatic design of RC frames. The numerical results indicate that the consistent approximation method is more efficient as compared to the other two methods of Big Bang-Big Bang Crunch and ant colony algorithms and leads to economical designs while meeting all design constraints. It was shown that the proposed algorithm is converged to the optimal design after passing a few analyses, which this property is related to involving design sensitivities in the optimization process.

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