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Combined effects of material properties and boundary conditions on the large deflection bending analysis of circular plates on a nonlinear elastic foundation

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Abstract. Geometrically nonlinear axisymmetric bending analysis of shear deformable circular plates on a nonlinear three-parameter elastic foundation was made. Plates ranging from "thin" to "moderately thick" were investigated for three types of material: isotropic, transversely isotropic, and orthotropic. The differential equations were discretized by means of the finite difference method (FDM) and the differential quadrature method (DQM). The Newton-Raphson method was applied to find the solution. A parametric investigation using seven unknowns per node was presented. The novelty of the paper is that detailed numerical simulations were made to highlight the combined effects of the material properties and the boundary conditions on (i) the deflection, (ii) the stress resultants, and (iii) the external load. The formulation was verified through comparison studies. It was observed that the results are highly influenced from the boundary conditions, and from the material properties.

Keywords: plate; nonlinear; Pasternak, Winkler; foundation; deflection

1. Introduction

Anisotropic plates and lightweight composites are widely used in modern engineering applications such as offshore structures, aircrafts, machines, pressure vessels, buildings, and automobiles (Chien and Chen 2005, Katsikadelis 2014). Due to their practical importance, they have received significant attention from the researchers (e.g., Jemielita and Kozyra 2018, Draiche *et al*. 2019, Abualnour *et al*. 2019, Addou *et al*. 2019). The studies involving anisotropy have generally dealt with orthotropic or transversely isotropic plates (e.g., Akgoz and Civalek 2011, Zenkour 2011, Temel and Sahan 2013, Bourad *et al*. 2016, Haciyev *et al*. 2019). This is partly because orthotropic plates are frequently encountered in civil infrastructure systems and other structural applications due to their advantageous features such as high ratio of stiffness and strength to weight, and partly because the computational effort required for modelling orthotropy is less than that of general anisotropy (Thai and Kim 2012).

Soil-structure interaction is a complex contact problem which arises when a structure or a machine is supported by a deformable medium (Levinson 1983, Bhardwaj *et al*. 2007, Kargaudas *et al*. 2019). Various mathematical models have been developed to understand the flexural response of structures on elastic foundation (Radeş 1971). Linear models have mostly been considered in the vast literature (e.g., Tajeddini *et al*. 2011, Yas and Tahouneh 2012, Jędrysiak and Kaźmierczak-Sobińska 2015, Guminiak and Knitter-Piątkowska 2018, Altekin 2019). However, nonlinear models have been investigated in the recent publications (Dumir 1985, Nassar and Labib 1988, Chien and Chen 2005, Jayachandran *et al*. 2008, Muradova and Stavroulakis 2012, Civalek 2013, Najafi *et al*. 2016, Sofiyev and Kuruoğlu 2017, Sofiyev *et al*. 2017).

The governing differential equations of plates are characterized by linear or nonlinear PDEs for which generally, it is difficult to find the exact solutions. "Linear bending theories are based on the assumption that the deflections are small compared to the thickness of the plate. However, in many engineering applications, the deflections and the thickness may be of the same order of magnitude" (Striz *et al*. 1988). Thus, in case of large deflections, the load and the displacements can no longer be assumed to be proportional (Zheng and Zhou 2007). Therefore, inevitably there is a necessity for nonlinear analysis (Alwar and Reddy 1979) for which very often closed form theoretical solutions are unavailable (Zong and Zhang 2009). Hence, numerous numerical methods have been used in the studies involving large deformation (e.g., Singh *et al*. 2008, Sepahi *et al*. 2010, Keleshteri *et al*. 2019).

Analysis of circular plates is one of the classic subjects in theory of elasticity (Wang *et al*. 2016). Besides, from engineering point of view circular plates are important and essential structural members (Temel and Noori 2020). Most of the publications in the extensive literature have focused on axisymmetric analysis (e.g., Li *et al*. 2008, Rad 2012, Lal and Ahlawat 2015, Temel and Noori 2020, Vivio and Vullo 2010, Wang *et al*. 2016, Yıldırım and Tutuncu 2018).

The bending response of orthotropic plates is more complicated than that of isotropic plates due to the material anisotropy (Thai and Kim 2012), and the classical plate theory (CPT) does not yield sufficiently accurate results if the plate is not thin (Szilard 2004). The structure should be serviceable when subjected to design loads, and a part of serviceability can be achieved by imposing suitable

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limitations on the deflections (Szilard 2004). Geometrically nonlinear axisymmetric bending of moderately thick circular plates subjected to uniform transverse pressure was examined in the current study for three types of material: (i) isotropic, (ii) transversely isotropic, and (iii) orthotropic. A comprehensive parametric investigation was made to observe the combined effects of the material properties and the boundary conditions on circular plates in contact with a nonlinear three-parameter elastic foundation. First, for a given external load, the numerical solutions were obtained. Next, since the maximum deflection develops at the center of the plate, for a given central deflection, the external load was computed. Shear deformable circular plates were studied numerically by means of FDM and DQM on the basis of Mindlin plate theory (FSDT) with Von Karman strain field. The fundamentals and the background of FDM and DQM were given in Appendix A. The algorithm was coded by the author in Matlab. The accuracy of the numerical procedure was validated through comparison studies. Computer implementations were carried out to investigate the influence of the thickness parameter, the edge conditions, the material parameters, and the foundation parameters on the deflection, on the stress resultants, and on the external load. Numerical results were reported and graphical results were discussed.

2. Formulation

2.1 Geometry of the plate

The region occupied by the plate in cylindrical coordinates is given by (Draiche *et al*. 2019)

$$0 \le r \le a, \qquad -h/2 \le z \le h/2, \qquad 0 \le \theta \le 2\pi \quad (1)$$

2.2 Relations between stress resultants and displacements

Since "transverse isotropy" is a special case of orthotropy, using the kinematic and the constitutive relations of a cylindrically orthotropic axisymmetric plate (Dumir and Shingal 1986), the following expressions are obtained.

$$n_r = \frac{E_\theta h}{\beta - \nu_\theta^{\,2}}\left(u' + \frac{1}{2}\left(w'\right)^2 + \nu_\theta \frac{u}{r}\right) \quad (2)$$

$$n_\theta = \frac{E_\theta h}{\beta - \nu_\theta^{\,2}}\left(\nu_\theta u' + \frac{\nu_\theta}{2}\left(w'\right)^2 + \beta \frac{u}{r}\right) \quad (3)$$

$$m_r = D\left(\psi' + \nu_\theta \frac{\psi}{r}\right), \qquad m_\theta = D\left(\nu_\theta \psi' + \beta \frac{\psi}{r}\right) \quad (4)$$

$$q_r = \kappa^2 h G_{rz}\left(\psi + w'\right) \quad (5)$$

where $D = \dfrac{E_\theta h^3}{12\left(\beta - \nu_\theta^{\,2}\right)}$, and $\left(\ \right)' = \dfrac{d(\ )}{dr}$.

2.3 Equations of equilibrium

The equations of equilibrium for an axisymmetric plate undergoing large deflection are given by (Dumir and Shingal 1986)

$$\left(r n_r\right)' - n_\theta = 0, \qquad \left(r m_r\right)' - m_\theta - r q_r = 0 \quad (6)$$

$$\left(r w' n_r + r q_r\right)' + r\left(q - q_f\right) = 0 \quad (7)$$

where q_f is the foundation interface pressure defined by (Singh *et al*. 2008, Anh and Duc 2016)

$$q_f = kw + k_3 w^3 - g\left(w'' + \frac{1}{r} w'\right) \quad (8)$$

2.4 Nondimensional variables and parameters

The parameters and the nondimensional counterparts of the field variables ($w, u, \psi, n_r, q_r, m_r, m_\theta$) are introduced as follows

$$\frac{\nu_\theta}{\nu_r} = \frac{E_\theta}{E_r} = \beta, \qquad \frac{G_{rz}}{E_r} = \Gamma \quad (9)$$

$$k = \frac{E_\theta h^3}{a^4} K, \qquad g = \frac{E_\theta h^3}{a^2} G, \qquad k_3 = \frac{E_\theta h}{a^4} K_3 \quad (10)$$

$$c = \frac{a}{h}, \qquad r = \xi a, \qquad q = Q E_\theta \quad (11)$$

$$Q_c = Q c^4, \qquad \Lambda = \frac{W}{\beta Q_c} \quad (12)$$

$$w = Wh, \qquad u = Uh \quad (13)$$

$$q_r = Q_r E_\theta h, \qquad w = \mu \frac{q a^4}{D} \quad (14)$$

$$n_r = N_r E_\theta h, \qquad n_\theta = N_\theta E_\theta h \quad (15)$$

$$m_r = M_r E_\theta h^2, \qquad m_\theta = M_\theta E_\theta h^2 \quad (16)$$

2.5 Boundary conditions

The problem is regarded as a boundary value problem. The boundary conditions at r=a, and the regularity conditions at r=0 were satisfied exactly (Table 1).

3. Solution method

Upon reorganizing Eqs. (2)-(8) together with the boundary conditions presented in Table 1 in terms of the nondimensional variables and parameters defined in Eqs. (9)-(16), a system of nonlinear ordinary differential equations was obtained. Since there is no analytical solution due to nonlinearity, the problem was attacked numerically. After performing pointwise discretization by means of FDM and DQM, the Newton-Raphson method was employed to obtain the solution (Eftekhari 2016). The grids for both FDM and DQM were constructed along the radial coordinate in the computational domain.

3.1 Finite difference method (FDM)

FDM is the oldest while still a widely used approach in applied mathematics (e.g., Rajasekaran and Varghese 2005) because of its simplicity (Zhao and Wei 2009). Equally spaced grid points defined by

$$r_i = a - \frac{(i-1)}{(N-1)} a \quad \text{for} \quad i = 1, 2, ..., N \tag{17}$$

were used via forward and backward difference formulations given by (Mathews 1992)

$$f_i' = \frac{d(f(r_i))}{dr} \cong \frac{-3f_i + 4f_{i+1} - f_{i+2}}{2s_s} \tag{18}$$

$$f_i'' = \frac{d^2(f(r_i))}{dr^2} \cong \frac{2f_i - 5f_{i+1} + 4f_{i+2} - f_{i+3}}{{s_s}^2} \tag{19}$$

$$f_i' = \frac{d(f(r_i))}{dr} \cong \frac{f_{i-2} - 4f_{i-1} + 3f_i}{2s_s} \tag{20}$$

$$f_i'' = \frac{d^2(f(r_i))}{dr^2} \cong \frac{-f_{i-3} + 4f_{i-2} - 5f_{i-1} + 2f_i}{{s_s}^2} \tag{21}$$

3.2 Differential quadrature method (DQM)

DQM is a simple and efficient technique which can yield highly accurate solutions to boundary value problems with relatively little computational effort (Shu 2000, Wu and Ren 2007). The m^{th}-order derivative of a function with respect to r at point r_i can be approximated as a linear sum of weighted function values at all of the discrete points in the domain of r (Hsu 2007, Alibeigloo and Simintan 2011).

$$\frac{d^m(f(r_i))}{dr^m} \cong \sum_{j=1}^{N} C_{ij}^{(m)} f(r_j)$$
$$\text{for } i = 1, 2, ..., N \text{ and } m = 1, 2, ..., N-1 \tag{22}$$

Here, r_i denotes the location of the i^{th} sampling point in the domain, and $C_{ij}^{(m)}$ are the r_i -dependent weight coefficients (Hsu 2007, Alibeigloo and Simintan 2011). After selecting the sampling points, the weight coefficients can be obtained from the equations given by (Hsu 2007)

$$C_{ij}^{(1)} = \frac{M_1(r_i)}{(r_i - r_j) M_1(r_j)} \quad \text{for} \quad i \neq j \quad \text{and} \quad i, j = 1, 2, ..., N \tag{23}$$

$$C_{ii}^{(1)} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} C_{ij}^{(1)} \quad \text{for} \quad i = 1, 2, ..., N \tag{24}$$

$$M_1(r_i) = \prod_{\substack{j=1 \\ j \neq i}}^{N} (r_i - r_j) \quad \text{for} \quad i = 1, 2, ..., N \tag{25}$$

For the second order and the higher order derivatives, the weight coefficients are obtained by using the following recurrence relations given by (Hsu 2007)

$$C_{ij}^{(m)} = m\left(C_{ii}^{(1)} C_{ij}^{(m-1)} - \frac{C_{ij}^{(m-1)}}{(r_i - r_j)} \right)$$
$$\text{for} \quad i \neq j \quad \text{and} \quad i, j = 1, 2, ..., N \tag{26}$$

$$C_{ii}^{(m)} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} C_{ij}^{(m)} \quad \text{for} \quad i = 1, 2, ..., N \tag{27}$$

As it was shown by Shu and Du (1997) the choice of the grid points affects the efficiency in DQM. C-G-L grid points given by

$$r_i = a - \frac{1}{2}\left(1 - \cos\left(\frac{(i-1)}{(N-1)}\pi\right)\right) a \quad \text{for} \quad i = 1, 2, ..., N \tag{28}$$

were considered in the study (Han and Liew 1997, Han and Liew 1999, Hsu 2007).

4. Numerical simulation

The radius of the plate was assumed unity, and κ^2=5/6 was considered in the computations. A large variety of parametric simulations were made for the combinations of the nondimensional parameters presented in Tables 2-4 for simply supported (S), and clamped (C) plates. The material parameters shown in Table 3 were taken from Dumir and Shingal (1986).

4.1 Convergence study and verification of the results

Excellent agreement was obtained in the comparison studies which were made for linear and nonlinear analyses (Tables 5-7). It can be deduced from the convergence studies that N=121, and N=51 are sufficient for admissible accuracy for FDM, and DQM, respectively. The results

Table 1 Boundary conditions

Location	(S)	(C)
$r=a$	$w=u=m_r=0$	$w=u=\psi=0$
$r=0$	$u=\psi=q_r=m_r-m_\theta=0$	

Table 2 Parameter of thickness

Thickness Category	$T1$	$T2$	$T3$	$T4$	$T5$
$c=a/h$	500	100	50	20	10

Table 3 Material properties

Type of Material	ν_θ	β	Γ	Description
$M1$	0.25	1	$1/(2(1+\nu_\theta))$	Isotropic
$M2$	0.25	1	0.1	Transversely Isotropic
$M3$	0.25	3	0.3	Orthotropic

Table 4 Parameters of elastic foundation

K	G	K_3	Foundation Model	Type of Foundation
K>0	G=0	K_3=0	(L) 1-parameter: Winkler	F1
K>0	G>0	K_3=0	(L) 2-parameter: Pasternak	F2
K>0	G=0	$K_3\neq$0	(NL) 2-parameter: (NL) Winkler	F3
K>0	G>0	$K_3\neq$0	(NL) 3-parameter: (NL) Winkler+Pasternak	F4

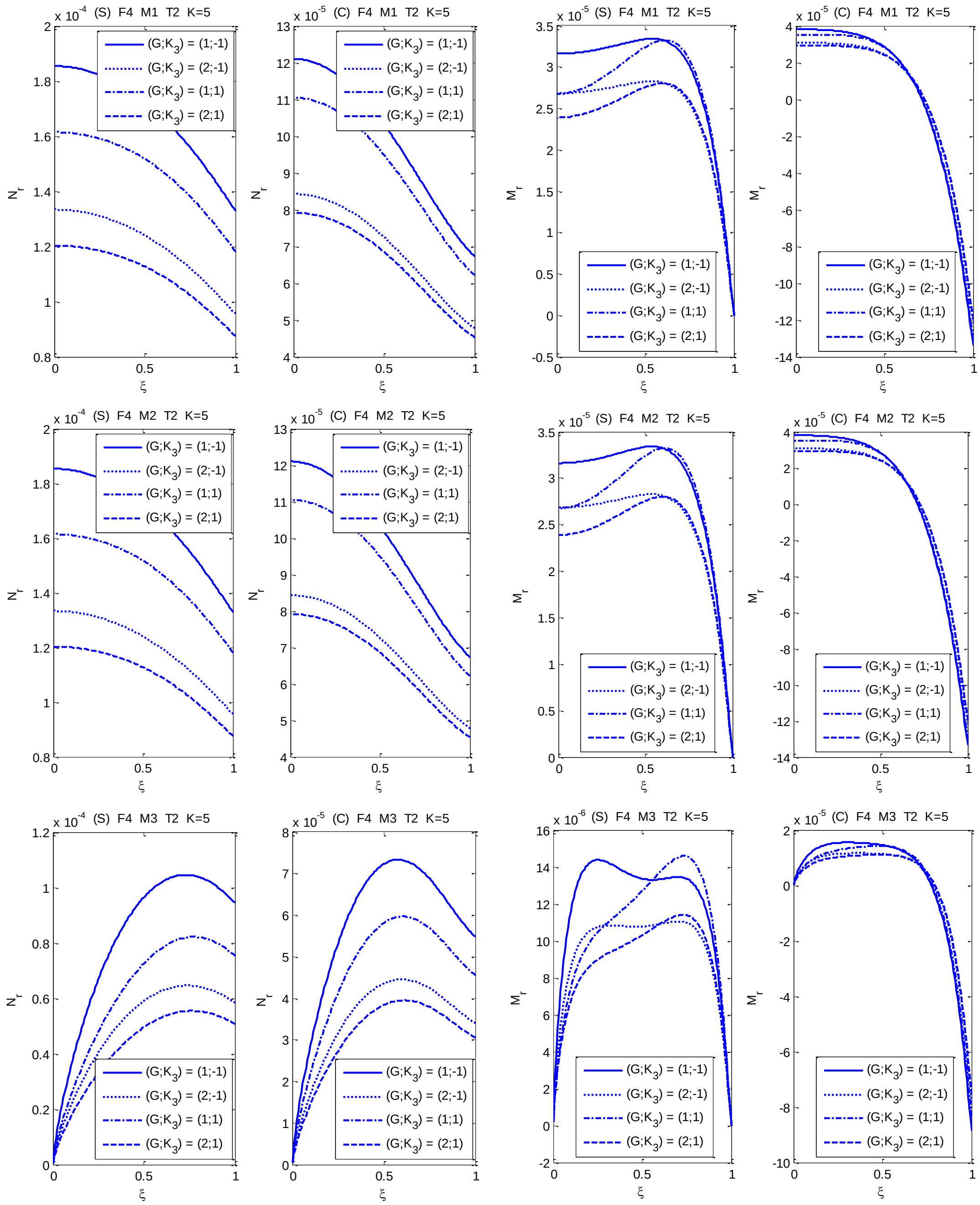

Fig. 1 Influence of the nonlinear elastic foundation on the stress resultants

reveal that compared to FDM, DQM leads to accurate results with less computational effort (Han and Liew 1997).

4.2 Numerical examples

Two types of numerical examples were solved for simply supported and clamped plates. First, a comprehensive investigation was made to highlight the influence of various parameters on the central deflection and on the stress resultants for Q_c=20 (Tables 8-12, Figs. 1-5). Very close values of W_{max} regarding to the thickness categories T1 and T2 were obtained for the material groups $M1$ and $M2$ (Tables 8-12). $N_r-\xi$ and $M_r-\xi$ curves for several values of the elastic foundation parameters G and K_3 were demonstrated (Fig. 1). Each material type was considered separately (Fig. 1). $M_r-\xi$ curves for different types of material were shown for different values of the elastic foundation parameters G and K_3 (Fig. 2). $M_r-\xi$ curves of plates on nonlinear Winkler foundation were depicted for different types of material (Fig. 3). Thickness categories $T3$ and $T4$ were considered for the distribution of M_r versus ξ (Fig. 3). The effects of the material properties on M_r and on Q_r were illustrated for $F3$ and $F4$ (Figs. 4-5). Since almost identical results were obtained in the FDM solutions, for brevity, only the DQM solutions for the nondimensional central deflections of (S) and (C) plates subjected to Q_c=20 were presented in Tables 8-12. Next, an inverse problem was solved by imposing a limitation on the central

Table 5 $W_{\max}$ of an isotropic (C) plate for $(K;G;K_3)=(5;2;0)$, and $\nu_\theta=0.30$

(NL) Analysis	Q_c=18	Q_c=24	Q_c=30	Q_c=36	Reference	Thickness
c=100	0.8588	1.0702	1.2521	1.4112	Kutlu and Omurtag (2012)	*T*2
c=100	0.8635	1.0764	1.2597	1.4201	(FDM) N=121	*T*2
c=100	0.8635	1.0763	1.2596	1.4200	(FDM) N=101	*T*2
c=100	0.8634	1.0762	1.2595	1.4198	(FDM) N=81	*T*2
c=100	0.8636	1.0765	1.2599	1.4204	(DQM) N=51	*T*2
c=100	0.8636	1.0765	1.2599	1.4204	(DQM) N=41	*T*2
c=10	0.8763	1.0916	1.2771	1.4395	Kutlu and Omurtag (2012)	*T*5
c=10	0.8788	1.0948	1.2807	1.4435	(FDM) N=121	*T*5
c=10	0.8788	1.0947	1.2806	1.4434	(FDM) N=101	*T*5
c=10	0.8788	1.0946	1.2805	1.4433	(FDM) N=81	*T*5
c=10	0.8789	1.0949	1.2809	1.4437	(DQM) N=51	*T*5
c=10	0.8789	1.0949	1.2809	1.4437	(DQM) N=41	*T*5

Table 6 $\Lambda_{\max}$ of an anisotropic plate ($K=G=K_3=0$, $Q_c=24$)

β	Γ	c	(C)	(S)	Reference: (L) Analysis	(Thickness; Material)
1	0.4	100	0.17586	0.73836	Dumir and Shingal (1986)	(*T*2; *M*1)
1	0.4	100	0.1758	0.7383	(FDM) N=121	(*T*2; *M*1)
1	0.4	100	0.1759	0.7384	(DQM) N=51	(*T*2; *M*1)
1	0.4	10	0.18328	0.74578	Dumir and Shingal (1986)	(*T*5; *M*1)
1	0.4	10	0.1832	0.7457	(FDM) N=121	(*T*5; *M*1)
1	0.4	10	0.1833	0.7458	(DQM) N=51	(*T*5; *M*1)
1	0.1	100	0.17608	0.73858	Dumir and Shingal (1986)	(*T*2; *M*2)
1	0.1	100	0.1760	0.7385	(FDM) N=121	(*T*2; *M*2)
1	0.1	100	0.1761	0.7386	(DQM) N=51	(*T*2; *M*2)
1	0.1	10	0.20578	0.76828	Dumir and Shingal (1986)	(*T*5; *M*2)
1	0.1	10	0.2057	0.7682	(FDM) N=121	(*T*5; *M*2)
1	0.1	10	0.2058	0.7683	(DQM) N=51	(*T*5; *M*2)
3	0.3	100	0.11371	0.34298	Dumir and Shingal (1986)	(*T*2; *M*3)
3	0.3	100	0.1137	0.3430	(FDM) N=121	(*T*2; *M*3)
3	0.3	100	0.1137	0.3430	(DQM) N=51	(*T*2; *M*3)
3	0.3	10	0.12361	0.35288	Dumir and Shingal (1986)	(*T*5; *M*3)
3	0.3	10	0.1236	0.3529	(FDM) N=121	(*T*5; *M*3)
3	0.3	10	0.1236	0.3529	(DQM) N=51	(*T*5; *M*3)

Table 7 $\mu_{\max}$ of an isotropic circular plate (Q_c=3, $K=G=K_3=0$, ν_θ=0.30)

(C)	(C)	(S)	(C)	(C)	(L) Analysis
c=100	c=50	c=40	c=20	c=10	Reference
0.01561	0.01563		0.01578	0.01631	Han and Liew (1997)
0.01561	0.01564	0.06388	0.01578	0.01633	Civalek and Ersoy (2009)
0.0156271	0.0156485		0.0157985	0.0163341	Altekin (2018)
0.0156	0.0157	0.0637	0.0158	0.0163	(FDM) N=121
0.0156	0.0157	0.0637	0.0158	0.0163	(DQM) N=51

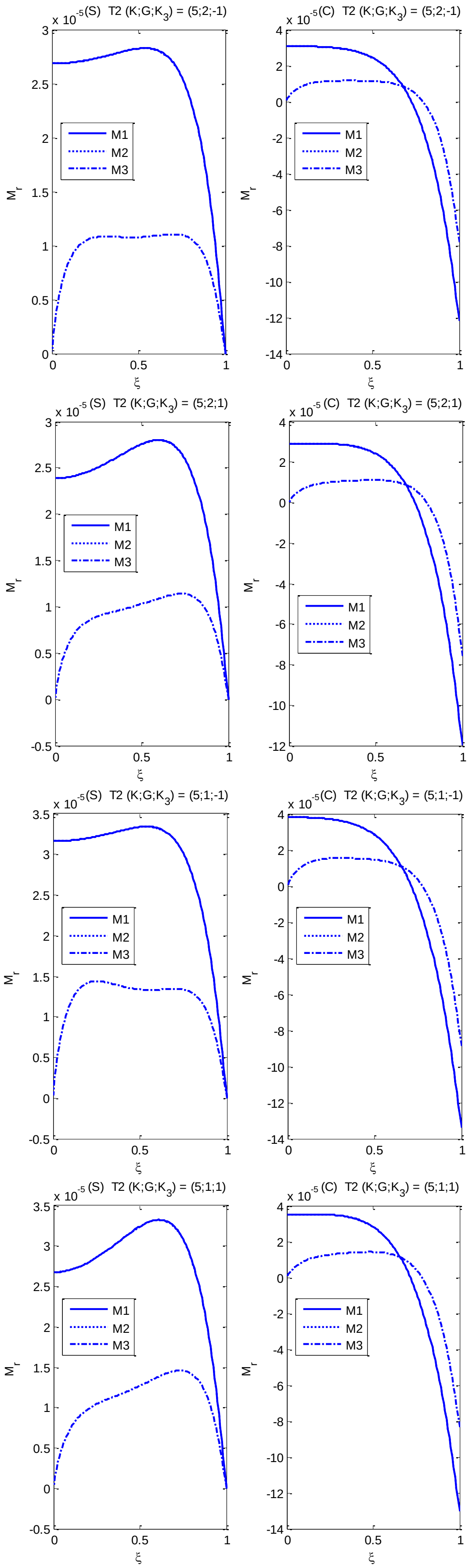

Fig. 2 Influence of the material properties on M_r for plates on nonlinear elastic foundation (*F*4)

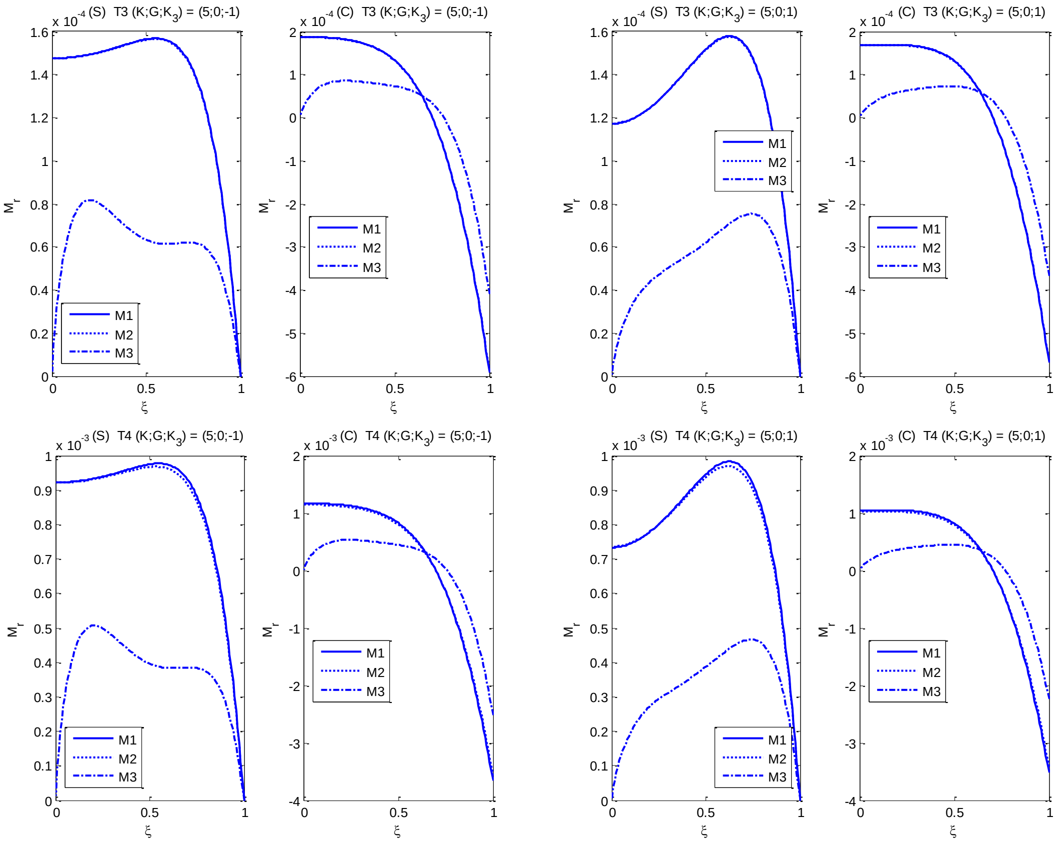

Fig. 3 Influence of the material properties on M_r for plates on nonlinear elastic foundation ($F3$)

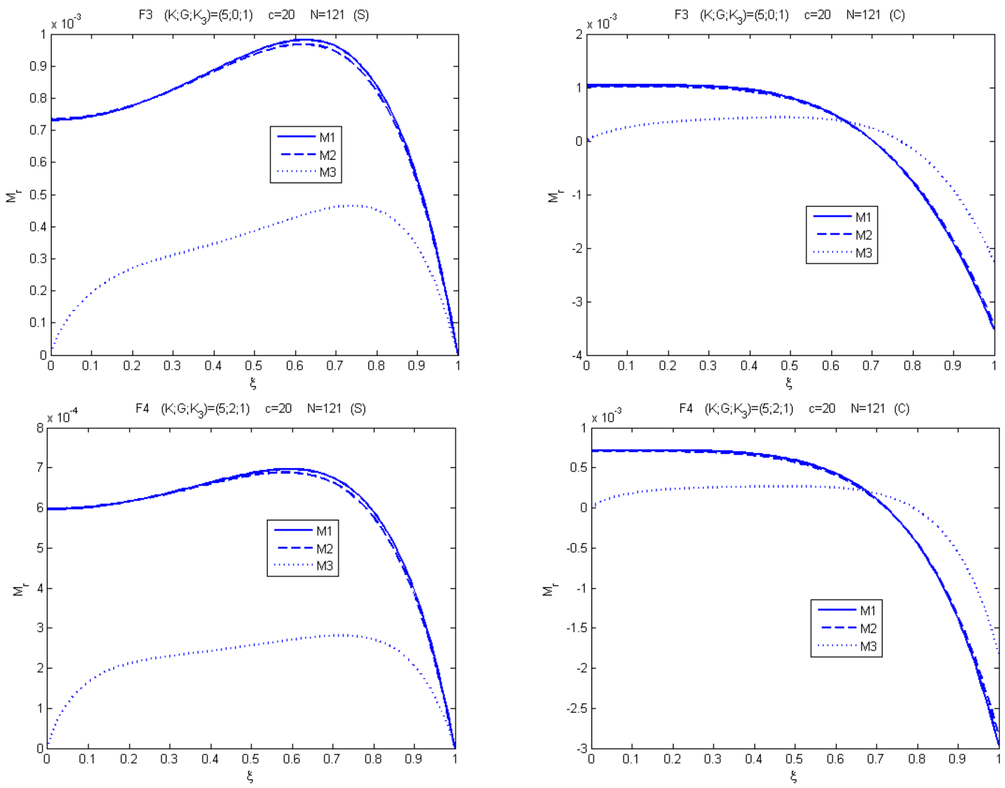

Fig. 4 Influence of the material properties on M_r for plates on nonlinear elastic foundation ($T4$)

deflection. So, the external load Q_c was determined using FDM by setting W_{max}=1 (Tables 13-17). The load carrying capacities of plates each of which deflects the same at the center were compared with one another. The computations

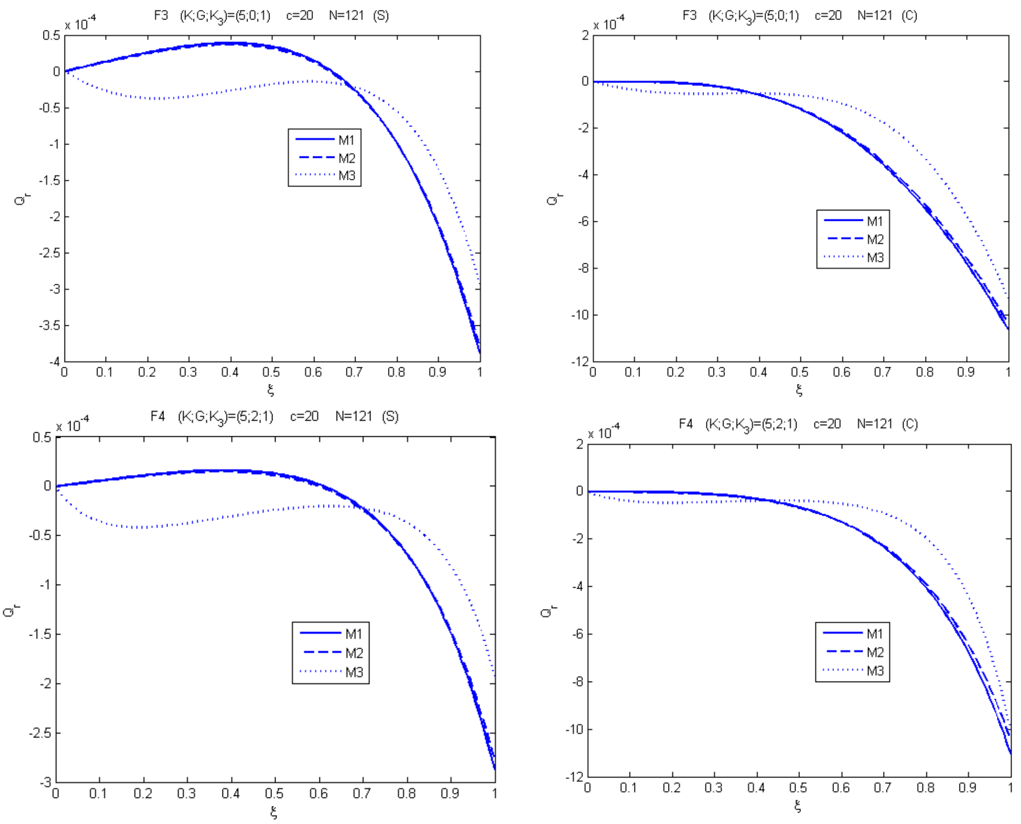

Fig. 5 Influence of the material properties on Q_r for plates on nonlinear elastic foundation ($T4$)

Table 8 $W_{\max}$ of a circular plate on elastic foundation (c=500, Q_c=20, DQM, N=51)

K	G	K_3	$M1$ (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	1.758938	1.758936	2.347098	1.484418	1.484446	2.126179	F1
3	0	0	1.67139	1.671387	2.170597	1.411154	1.411179	1.971608	F1
5	0	0	1.584719	1.584716	1.998733	1.339826	1.339847	1.822807	F1
5	1	0	1.371341	1.371339	1.625727	1.126694	1.126715	1.442601	F2
5	2	0	1.176192	1.176191	1.318589	0.948962	0.948983	1.153477	F2
5	0	-1	1.672038	1.672035	2.358791	1.384764	1.38479	2.049658	F3
5	0	1	1.512682	1.512678	1.788412	1.30047	1.300488	1.673096	F3
5	1	-1	1.429211	1.429209	1.797804	1.153846	1.15387	1.545235	F4
5	2	-1	1.212091	1.21209	1.396678	0.964558	0.964581	1.198308	F4
5	1	1	1.32219	1.322188	1.50894	1.102234	1.102253	1.365477	F4
5	2	1	1.144651	1.14465	1.257898	0.934516	0.934535	1.115924	F4

Table 9 $W_{\max}$ of a circular plate on elastic foundation (c=100, Q_c=20, DQM, N=51)

K	G	K_3	$M1$ (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	1.758917	1.758851	2.347302	1.484642	1.485341	2.126962	F1
3	0	0	1.671367	1.671294	2.170758	1.411353	1.411973	1.972262	F1
5	0	0	1.584694	1.584614	1.998853	1.34	1.340543	1.823341	F1
5	1	0	1.371324	1.37127	1.62582	1.126865	1.127399	1.443099	F2
5	2	0	1.176181	1.176145	1.318665	0.949127	0.94964	1.153941	F2
5	0	-1	1.672017	1.671953	2.35908	1.384973	1.385624	2.050591	F3
5	0	1	1.512652	1.512562	1.788459	1.300617	1.301074	1.673433	F3
5	1	-1	1.429197	1.42915	1.797956	1.154039	1.154642	1.54591	F4
5	2	-1	1.212081	1.212048	1.396775	0.964737	0.965292	1.198852	F4
5	1	1	1.322171	1.322111	1.509001	1.102387	1.102863	1.365866	F4
5	2	1	1.144639	1.1446	1.25796	0.934669	0.935145	1.116328	F4

Table 10 W_{max} of a circular plate on elastic foundation (c=50, Q_c=20, DQM, N=51)

K	G	K_3	$M1$ (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	1.758851	1.758586	2.347935	1.485341	1.488104	2.129392	F1
3	0	0	1.671294	1.671004	2.17126	1.411973	1.414423	1.974294	F1
5	0	0	1.584614	1.584297	1.999227	1.340543	1.34269	1.824999	F1
5	1	0	1.37127	1.371056	1.626109	1.127399	1.129508	1.444641	F2
5	2	0	1.176145	1.176002	1.318901	0.94964	0.951666	1.155375	F2
5	0	-1	1.671953	1.671696	2.359983	1.385624	1.388197	2.05349	F3
5	0	1	1.512562	1.5122	1.788606	1.301074	1.302881	1.674478	F3
5	1	-1	1.42915	1.428967	1.798429	1.154642	1.157027	1.548002	F4
5	2	-1	1.212048	1.211921	1.397076	0.965292	0.967487	1.200535	F4
5	1	1	1.322111	1.321874	1.509191	1.102863	1.104742	1.367071	F4
5	2	1	1.1446	1.144445	1.258152	0.935145	0.937023	1.117579	F4

Table 11 W_{max} of a circular plate on elastic foundation (c=20, Q_c=20, DQM, N=51)

K	G	K_3	$M1$ (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	1.758389	1.756793	2.352278	1.490142	1.50602	2.145644	F1
3	0	0	1.670787	1.669035	2.174707	1.416231	1.430327	1.987904	F1
5	0	0	1.584061	1.582143	2.001799	1.344274	1.356631	1.836118	F1
5	1	0	1.370896	1.369607	1.628073	1.131063	1.143186	1.454898	F2
5	2	0	1.175895	1.17504	1.320492	0.953157	0.964735	1.164819	F2
5	0	-1	1.671504	1.669952	2.36619	1.390098	1.405008	2.073065	F3
5	0	1	1.511931	1.509748	1.789617	1.304212	1.31454	1.681441	F3
5	1	-1	1.42883	1.427728	1.801655	1.158786	1.172549	1.561974	F4
5	2	-1	1.211826	1.211063	1.399109	0.969103	0.98168	1.211642	F4
5	1	1	1.321697	1.320267	1.510482	1.106126	1.116885	1.375062	F4
5	2	1	1.14433	1.143401	1.259447	0.938405	0.949111	1.1258	F4

Table 12 W_{max} of a circular plate on elastic foundation (c=10, Q_c=20, DQM, N=51)

K	G	K_3	$M1$ (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	1.756793	1.75119	2.366607	1.50602	1.553447	2.194414	F1
3	0	0	1.669035	1.662849	2.186152	1.430327	1.472627	2.028997	F1
5	0	0	1.582143	1.575342	2.010391	1.356631	1.393793	1.869797	F1
5	1	0	1.369607	1.365118	1.634386	1.143186	1.180042	1.48525	F2
5	2	0	1.17504	1.172127	1.325425	0.964735	0.999714	1.191936	F2
5	0	-1	1.669952	1.664463	2.386837	1.405008	1.450816	2.13388	F3
5	0	1	1.509748	1.501994	1.793007	1.31454	1.344913	1.702011	F3
5	1	-1	1.427728	1.423915	1.812084	1.172549	1.214914	1.603918	F4
5	2	-1	1.211063	1.208489	1.405451	0.98168	1.019982	1.243793	F4
5	1	1	1.320267	1.315276	1.514606	1.116885	1.149217	1.398465	F4
5	2	1	1.143401	1.140224	1.263438	0.949111	0.981223	1.149259	F4

Table 13 Q_c of a circular plate on elastic foundation (c=500, W_{max}=1, FDM, N=121)

K	G	K_3	$M1$ (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	4.838758	4.838764	3.027352	9.500943	9.500573	4.931898	F1
3	0	0	6.292719	6.292736	4.584295	10.861654	10.861299	6.398603	F1
5	0	0	7.773777	7.773805	6.188862	12.240253	12.239915	7.901281	F1
5	1	0	11.783449	11.783476	10.206818	16.847697	16.847241	12.46591	F2
5	2	0	15.790993	15.791019	14.232508	21.412313	21.411703	16.921331	F2
5	0	-1	7.265743	7.265762	5.579202	11.796744	11.796394	7.362928	F3
5	0	1	8.295696	8.295733	6.826872	12.69115	12.690825	8.457739	F3
5	1	-1	11.270367	11.270388	9.627442	16.38532	16.384853	11.922563	F4
5	2	-1	15.275221	15.275243	13.667142	20.937346	20.936727	16.378232	F4
5	1	1	12.30704	12.307073	10.802971	17.316411	17.315965	13.021511	F4
5	2	1	16.315188	16.31522	14.809686	21.892788	21.892187	17.473656	F4

Table 14 Q_c of a circular plate on elastic foundation (c=100, W_{max}=1, FDM, N=121)

K	G	K_3	$M1$ (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	4.838808	4.838964	3.02698	9.497985	9.488763	4.928758	F1
3	0	0	6.29285	6.293258	4.584036	10.858815	10.849967	6.395616	F1
5	0	0	7.773997	7.774686	6.188748	12.237552	12.229133	7.898507	F1
5	1	0	11.783667	11.78435	10.206245	16.844048	16.832684	12.460876	F2
5	2	0	15.791203	15.79186	14.231594	21.407439	21.392279	16.91353	F2
5	0	-1	7.265896	7.266375	5.578974	11.793949	11.785239	7.36	F3
5	0	1	8.29599	8.296907	6.826895	12.688551	12.680451	8.455156	F3
5	1	-1	11.270536	11.271064	9.626839	16.381588	16.36997	11.917468	F4
5	2	-1	15.275394	15.275931	13.66623	20.932404	20.91703	16.370417	F4
5	1	1	12.307311	12.308157	10.802436	17.312849	17.30176	13.016556	F4
5	2	1	16.315439	16.316221	14.808776	21.887987	21.873054	17.46588	F4

Table 15 Q_c of a circular plate on elastic foundation (c=50, W_{max}=1, FDM, N=121)

K	G	K_3	M1 (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	4.838964	4.839584	3.025817	9.488763	9.452222	4.918989	F1
3	0	0	6.293258	6.294883	4.583227	10.849967	10.814921	6.386326	F1
5	0	0	7.774686	7.777429	6.188391	12.229133	12.195806	7.889887	F1
5	1	0	11.78435	11.787063	10.20446	16.832684	16.787846	12.445273	F2
5	2	0	15.79186	15.794469	14.228755	21.392279	21.332679	16.889433	F2
5	0	-1	7.266375	7.268283	5.57826	11.785239	11.750748	7.350897	F3
5	0	1	8.296907	8.300562	6.826966	12.680451	12.648399	8.447129	F3
5	1	-1	11.271064	11.273162	9.624963	16.36997	16.324113	11.901672	F4
5	2	-1	15.275931	15.278066	13.663395	20.91703	20.856575	16.346276	F4
5	1	1	12.308157	12.311525	10.800773	17.30176	17.258018	13.001201	F4
5	2	1	16.316221	16.319328	14.805951	21.873054	21.814366	17.441864	F4

Table 16 Q_c of a circular plate on elastic foundation (c=20, W_{max}=1, FDM, N=121)

K	G	K_3	$M1$ (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	4.840047	4.843782	3.017748	9.425175	9.211037	4.852456	F1
3	0	0	6.296096	6.305999	4.577602	10.788996	10.58425	6.323166	F1
5	0	0	7.779478	7.796231	6.185892	12.171173	11.977192	7.83141	F1
5	1	0	11.789082	11.805393	10.192255	16.754849	16.49911	12.3412	F2
5	2	0	15.796409	15.811996	14.209618	21.289038	20.956822	16.731825	F2
5	0	-1	7.269706	7.281335	5.573298	11.725242	11.524012	7.289057	F3
5	0	1	8.303292	8.325633	6.827425	12.624721	12.438677	8.392794	F3
5	1	-1	11.274722	11.287294	9.612121	16.290351	16.028272	11.796215	F4
5	2	-1	15.279651	15.292361	13.644262	20.812291	20.474705	16.18826	F4
5	1	1	12.314032	12.334318	10.789408	17.225843	16.976918	12.898891	F4
5	2	1	16.321639	16.34024	14.786926	21.771411	21.444911	17.284903	F4

Table 17 Q_c of a circular plate on elastic foundation (c=10, W_{max}=1, FDM, N=121)

K	G	K_3	M1 (S)	$M2$ (S)	$M3$ (S)	$M1$ (C)	$M2$ (C)	$M3$ (C)	(NL) Analysis
1	0	0	4.843782	4.856791	2.989838	9.211037	8.516323	4.638467	F1
3	0	0	6.305999	6.342136	4.558058	10.58425	9.926589	6.121341	F1
5	0	0	7.796231	7.857913	6.176983	11.977192	11.361585	7.646176	F1
5	1	0	11.805393	11.86216	10.152326	16.49911	15.727418	12.029436	F2
5	2	0	15.811996	15.864819	14.150084	20.956822	20.008577	16.288405	F2
5	0	-1	7.281335	7.323869	5.556015	11.524012	10.880215	7.091997	F3
5	0	1	8.325633	8.40813	6.828542	12.438677	11.85395	8.22206	F3
5	1	-1	11.287294	11.330523	9.569979	16.028272	15.232633	11.479275	F4
5	2	-1	15.292361	15.334889	13.584492	20.474705	19.506305	15.74261	F4
5	1	1	12.334318	12.405467	10.752359	16.976918	16.230893	12.593528	F4
5	2	1	16.34024	16.403851	14.727995	21.444911	20.518011	16.844459	F4

showed that the material classifications *M*1 and *M*2 had similar effects on Q_c regarding to the thickness categories *T*1 and *T*2 (Tables 13-17).

5. Conclusions

A parametric study on the axisymmetric bending of circular plates was presented in the current work. The problem was formulated on the basis of FSDT, and Von Karman type geometric nonlinearity was used. Cylindrically orthotropic plates interacting with a nonlinear three-parameter elastic foundation were investigated. Numerical simulations were performed to discuss the influences of the material properties, the boundary conditions, and the foundation parameters on the deflection, on the stress resultants, and on the external load. The accuracy of the results was verified via comparison studies.

The numerical simulations reveal that

• The solutions regarding to thin plates such as T1 and *T*2 are very close to each other for the material categories *M*1 and *M*2.

• Maximum deflection of a clamped plate increases in the order *M*1-*M*2-*M*3 (i.e., *M*1 minimizes $W_{\max}$, and *M*3 maximizes $W_{\max}$).

• Maximum deflection of a simply supported plate increases in the order *M*2-*M*1-*M*3 (i.e., *M*2 minimizes $W_{\max}$, and *M*3 maximizes $W_{\max}$).

• For (*C*) plates *M*3 minimizes Q_c, and *M*1 maximizes Q_c.

• For (*S*) plates *M*3 minimizes Q_c, and *M*2 maximizes Q_c.

• The Pasternak parameter has a remarkable effect on Q_c especially for *M*3.

• The material properties have remarkable effect on N_r which is minimized by *M*3. The dispersion of N_r along the radial coordinate looks like each other for *M*1 and *M*2, but the distribution of N_r versus ξ is totally different for the material group *M*3. These statements also hold for M_r.

• For (*S*) plates M_r does not reach its maximum value at the center.

• The influence of the foundation parameters on M_r is significant. The effect of K_3 should not be ignored.

• M1 and M2 have almost similar influence on M_r and on Q_r. The magnitudes of Q_r and $\mathrm{M_r}$ are minimized for M3 at the support.

• With increasing thickness, the distribution of M_r versus ξ tends to differ for the material categories *M*1 and *M*2.

Compared to FDM, the rate of convergence for DQM is faster with smaller number of grid points. Since the memory storage requirement of DQM is less than that of FDM, it can be concluded that DQM requires less computational effort. The execution time of a code depends on the algorithm. In the current study, very similar algorithms were used for FDM and DQM, and the FDM code ran faster than the DQM code.

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CC

Nomenclature

a, g, h	radius of the plate, Pasternak parameter, thickness of the plate
r, q	radial coordinate, uniform pressure
k, k_3	linear and nonlinear Winkler parameters
w, u	deflection, horizontal displacement
m_r, m_θ	bending moments
n_r, n_θ	normal forces
q_r, s_s	shear force, step size in FDM
N, D, G_{rz}	number of sampling points, bending rigidity, shear modulus
E_r, E_θ	Young's moduli
ψ, κ^2	rotation, shear correction factor
ν_r, ν_θ	Poisson ratios

Abbreviation List

CPT, FSDT	classical plate theory, first order shear deformation theory
FDM, DQM	finite difference method, differential quadrature method
FEM, FVM, GQ	finite element method, finite volume method, Gaussian quadrature
MWR, PDE	methods of weighted residuals, partial differential equation
(L), (NL), C-G-L	linear, nonlinear, Chebyshev-Gauss-Lobatto
(C), (S)	clamped, simply supported

Appendix A: An overview to FDM and DQM

With the advance of computer technology, the role of numerical simulations in science and engineering has been growing (Wang 2015). “Various numerical methods such as FEM, FDM, FVM, and MWR have been used as powerful tools for solving PDEs. However, none of the aforementioned numerical methods is versatile and none of them is capable of solving all problems efficiently. Each method has its own merits and limitations. Therefore, along with the ever-growing advancement of faster computers, the research into the development of new efficient methods is an ongoing activity” (Bert and Malik 1996, Wang 2015).

DQM is essentially a generalization of GQ which is used for numerical integration of functions (Zong and Zhang 2009). GQ approximates a finite integral as a weighted sum of the integrand values at selected points whereas DQM approximates the derivatives of a smooth function at a given discrete point as a weighted linear sum of the function values at all discrete points (Striz *et al*. 1988, Zong and Zhang 2009). “This is in contrast to the standard FDM in which a solution value at a point is a function of values at adjacent points only. Even if FDM is of a high enough order to cover all points, there is still a fundamental difference in the fact that DQM is a polynomial fitting while the higher order FDM is a Taylor series expansion. Besides, the weighting coefficients in DQM are independent of the boundary conditions, and thus, they need to be determined only once” (Striz *et al*. 1988).

Like FDM, DQM transforms the differential equations into a set of analogous algebraic equations in terms of the unknown function values at the grid points (Civalek and Çatal 2003). “Owing to the higher order polynomial approximation, DQM usually requires fewer gid points in comparison to FDM. However, DQM leads to nonsymmetric and nonbanded system matrices, and it appears to be very sensitive to the choice of the grid points” (Lamacchia *et al*. 2014). The accuracy of the results can be increased by shifting the mesh points from a uniformly distributed grid towards a Gauss-Chebyshev-Lobatto grid (Lamacchia *et al*. 2014) in which the points are concentrated close to the boundaries (Hejripour and Saidi 2011).

From computational aspects, the memory storage requirements are often considerable for the numerical methods (Civalek and Çatal 2003). Due to its attractive features such as rapid convergence, high accuracy with a considerably smaller number of grid points, and computational efficiency, the DQM is a well-known method worldwide (Shu 2000, Wang 2015).