Computers and Concrete, *Vol. 25, No. 6* (*2020*) *485-495*
DOI: https://doi.org/10.12989/cac.2020.25.6.485

Stability and dynamic analyses of SW-CNT reinforced concrete beam resting on elastic-foundation

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(*Received June 28, 2019, Revised March 24, 2020, Accepted March 31, 2020*)

Abstract. This paper, presents the dynamic and stability analysis of the simply supported single walled Carbon Nanotubes (SWCNT) reinforced concrete beam on elastic-foundation using an integral first-order shear deformation beam theory. The condition of the zero shear-stress on the free surfaces of the beam is ensured by the introduction of the shear correction factors. The SWCNT reinforcement is considered to be uniform and variable according to the *X*, *O* and *V* forms through the thickness of the concrete beam. The effective properties of the reinforced concrete beam are calculated by employing the rule of mixture. The analytical solutions of the buckling and free vibrational behaviors are derived via Hamilton's principle and Navier method. The analytical results of the critical buckling loads and frequency parameters of the SWCNT-RC beam are presented in the form of explicit tables and graphs. Also the diverse parameters influencing the dynamic and stability behaviors of the reinforced concrete beam are discussed in detail.

Keywords: dynamic; stability analysis; SWCNT concrete beam; Hamilton's principle; Navier solution; rule of mixture

1. Introduction

Nowadays, the reinforcement of structures by nanotechnology has become a very attractive research area. The most used is the carbon nanotubes reinforcement, this last has excellent properties such as tensile strength higher and Young modulus lower density (Thostenson *et al*. 2001, Esawi and Farag 2007, Rafiee *et al.* 2013). Among researchers who used CNT (Carbone-Nanotubes) as reinforcement materials, we can cite (Wang and Shen 2011, Natarajan *et al*. 2014, Kiani 2016, Mohammadimehr and Alimirzaei 2016, Tornabene *et al*. 2016, Wu *et al*. 2016, Zhang *et al*. 2017, Lei *et al*. 2018, Zghal *et al*. 2018, Mehar *et al*. 2018, Frikha *et al*. 2018, Daghigh and Daghigh 2018, Chen *et al*. 2019, Qin *et al*. 2019, Selmi 2019). The beams, columns and plates are the structures most concerned by the reinforcements. For this purpose, several research works was carried out to analyze the different behaviors of beams and plates reinforced by carbon nanotubes (Yas and Samadi 2012, Wattanasakulpong and Ungbhakorn 2013, Lin and Xiang 2014, Wu and Li 2014, Alibeigloo and Liew 2015, Kamarian *et al*. 2015, Fantuzzi *et al*. 2017, Setoodeh and Shojaee 2017, Zarei *et al*. 2017 ,Bensattalah *et al*. 2018, Belmahi *et al*. 2018 and 2019). Based on Eshelby-Mori-Tanaka model, Formica *et al*. (2010) proposed a theoretical model to examine the elastodynamic behavior of CNT-RC plate. Ke *et al*. (2010) examined the nonlinear free-vibrational response of FG-CNT RC beams using the Timoshenko's analytical model and nonlinearity geometric of Von Kármán. Arani *et al*. (2011) examined the critical buckling load of SWCNTs laminated-composite plates by employing the both analytical (CPT and TSDT) and numerical (FEM) methods. The dynamic and static responses of SWCNT-RC plate with various boundary conditions have been investigated by Zhu *et al*. (2012) using numerical analysis (FEM) and analytical model (FSDT). The vibration and flexural responses and stability of uniform and FG-SWCNT reinforced composite beam using the FSDT and various HSDTs models have been examined by Wattanasakulpong and Ungbhakorn (2013). Wang and Shen (2011) investigated on nonlinear vibrational analysis of isotropic, UD and FG-SWCNT reinforced-plate seated on foundation under thermal load. Mehrabadi *et al*. (2012) studied the mechanical-stability of UD, FG-X and FG-O SWCNT-RC plate using the FSDT-Mindlin model. The nonlinear low-velocity analysis of single and three layers CNT RC-plate was examined by Wang *et al*. (2014) using the HSDT and Von Kármán-type. The effect of slenderness ratio, agglomeration and curvature of reinforcement on dynamic response of FG CNT-RC beams was investigated by Heshmati *et al*. (2015). A semi analytical-procedure for studying the stability of the reinforced concrete-column subjected to fire was derived by

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ISSN: 1598-8198 (Print), 1598-818X (Online)

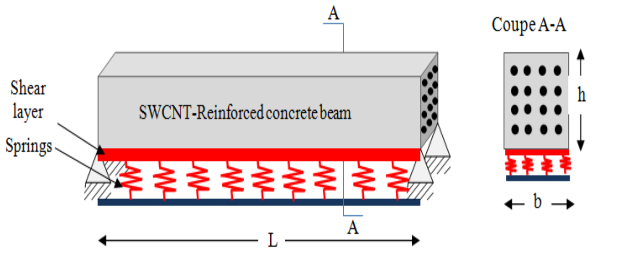

Fig. 1 Geometry of a CNT-RC beam resting on elastic-foundation

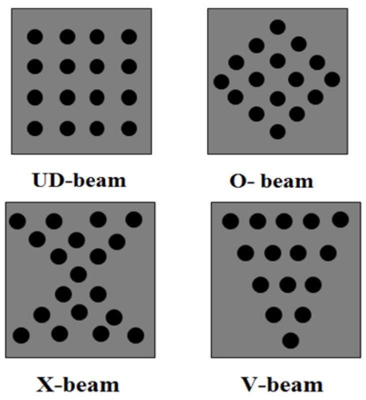

Fig. 2 the various models of the CNT distribution through the thickness of the beam

Bajc *et al*. (2015). Arani and Kolahchi (2016) investigated on mechanical buckling of concrete-columns reinforced with CNT using the Euler-Bernoulli and Timoshenko theories and DQM solution. Using the high order shear deformation theory (HSDT) models, Shen *et al*. (2017) analyzed the vibration response of CNT-RC beam on elastic-foundations with including the initial-deflection caused by thermal-postbuckling. Mercan *et al*. (2018) studied the dynamic behavior of laminated and FGM/CNT composites annular plates using discrete singular convolution method. Recently, Arani *et al*. (2019) investigated on Thermal, electrical and mechanical stability of FG-CNTRC sandwich nano-beams. Also the thermal stability analysis of the FG-CNT reinforced composite beams with temperature-dependent material properties is examined by Bensaid and Kerboua (2019). Based on nonlocal Timoshenko beam theory, Bensattalah *et al*. (2019) studied a free vibrational behavior of chiral SWCNT. Based on refined shear deformation theory, Yazdani and Mohammadimehr (2019) used wave propagation solution for double bonded Cooper-Naghdi micro sandwich cylindrical shells with porous core and CNTRC face sheets. Barati and Shahverdi (2020) analyzed a forced vibration of nanocomposite graphene platelet-reinforced beams. It should be noted that many HSDTs or FSDT can be employed to study the mechanical behavior of such structures and other ones (Xiang and Shi 2011, Abdelmalek *et al*. 2017, Faleh *et al*. 2018, Majeed and Sadiq 2018, Rezaiee-Pajand *et al*. 2018, Sayyad and Ghugal 2018, Barati and Zenkour 2018, Fenjan *et al*. 2019, Mirjavadi *et al*. 2019a, Shahverdi *et al*. 2019, Avcar 2019, Mirjavadi *et al*. 2019, Al-Maliki *et al*. 2019, Ahmed *et al*. 2019, Sahouane *et al*. 2019, Bensattalah *et al*. 2019).

It is remarkable from the literature that the theoretical researches on dynamic and stability of SWCNTs reinforced concrete beam armed are very interesting. The objective of the current paper is to present a novel analytical model for studying the buckling and free vibrational behaviors of CNT reinforced concrete beam and discuss about the nanotechnology effects. Based on recent works (Yas and Samadi 2012, Wattanasakulpong and Ungbhakorn 2013, Arani and Kolahchi 2016, Bousahla *et al*. 2020), the present work aims to apply a new version of Timoshenko's theory (integral Timoshenko's theory) on the mechanical behavior of concrete beams. For this ends, the concrete beam is simulated via the integral first shear deformation beam models. The elastic-foundation is modeled with spring (Winkler parameter) and shear constants (Pasternak parameter). The governing equations are determined by applying the Hamilton's principal and resolved via Navier-solutions. The effects of diverse parameters such as CNT distribution-types, SWCNTs percentage, slenderness ratio and elastic foundation parameters on the buckling and free vibrational responses of reinforced concrete beam are discussed.

2. Mathematical formulations

2.1 CNT reinforced concrete-beams

The simply-supported SWCNT-R concrete beam has length "*L*", thickness "*h*" and width "*b*" as shown in the Fig. 1. The beam is resting on elastic-foundation (Winkler-Pasternak type).

As illustrated in the Fig. 2, four models of reinforcement's distribution of the SW-CNT through the thickness of the concrete beam are considered such as (UD, O, V and X-beam)

The four SW-CNT distributions (UD, O, X and V) are expressed in the mathematical form as (Shen 2009, Zhu *et al*. 2012, Wattanasakulpong et Ungbhakorn 2013, Zghal *et al*. 2018)

- Uniform-CNT distribution

$$C_r = C_r^* \tag{1a}$$

- O- CNTdistribution

$$C_r = 2\left(1 - 2\frac{|z|}{h}\right)C_r^* \tag{1b}$$

- X- CNT distribution

$$C_r = 4\frac{|z|}{h}C_r^* \tag{1c}$$

- V- CNT distribution

$$C_r = \left(1 + 2\frac{|z|}{h}\right)C_r^* \tag{1d}$$

Where C_r^* is the volume fraction of the CNT defined as the following expression

Table 1 Efficiency parameters η_i associated to the volume fractions C_r^*

C_r^*	0.12	0.17	0.28
η_1	1.2833	1.3414	1.3228
η_2	1.0566	1.7101	1.7380
η_3	1.0566	1.7101	1.7380

$$C_r^* = \frac{w_r}{w_r + \left(\frac{\rho_r}{\rho}\right)(1-w_r)} \tag{2}$$

Where index r denotes the reinforcement. C_r^* and w_r is volume and mass fraction of CNT, respectively ρ_r and ρ are the mass density of the reinforcement and concrete, respectively.

The Young modulus E_u and shear modulus G_{12} of the SWCNT-RC beam are computed according to the mixture rule (Wu *et al*. 2016, Shen *et al*. 2017, Zghal *et al*. 2018, Hajlaoui *et al*. 2019)

$$E_{11} = \eta_1 C_r E_{11}^r + (1-C_r) E_c \tag{3a}$$

$$\frac{\eta_2}{E_{22}} = \frac{C_r}{E_{22}^r} + \frac{(1-C_r)}{E_c} \tag{3b}$$

$$\frac{\eta_3}{G_{12}} = \frac{C_r}{G_{12}^r} + \frac{(1-C_r)}{G_c} \tag{3c}$$

Where "C_r" are the volume fraction of SW-CNT with "C_r+C_c=1" (such as C_c is the volume fraction of the concrete), "E_c and G_c" are the effective concrete properties. "E_{11}^r, E_{22}^r and G_{12}^r " are the Young's and shear modulus of the reinforcements (SWCNT), respectively. η_1, η_2 and η_3 are the carbon nanotube efficiency parameteres. E_{11}, E_{22} and G_{12} are the Young modulus and shear modulus of of the SWCNT-RC beam. By employing the same rule of (Yas and Samadi 2012, Hajlaoui *et al*. 2019) the Poison's ratio and the masse density of the SWCNT-R concrete beam are determined

$$\begin{aligned} \rho &= V_r \rho_r + V_c \rho_c \\ v &= V_r v_r + V_c v_c \end{aligned} \tag{4}$$

Where v_r and v_c are Poison's ratio of the SW-CNT and concrete. The mass density ρ_r and ρ_c are corresponding to SW-CNT and concrete, respectively.

The values of the efficiency parameters of the SW-CNT (η_i) are presented in the Table 1 (Han and Elliott 2007)

2.2 Kinematic and constitutive relations

By considering the shear deformation effect on the Euler Bernoulli beam theory, the present First-order shear deformation beam theory can be expressed in analytical integral form as

$$u(x,z,t) = u_0(x,y,t) - z k_1 \int \theta(x,t)\, dx \tag{5a}$$

$$w(x,z,t) = w_0(x,t) \tag{5b}$$

Where the displacements terms (u_0, w_0 and θ) correspond to a mid-plane of the SWCNT-R concrete beam. Using the displacements field of the Eq. (5), the strains (ε_x) and (γ_{xz}) are obtained as

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z k_1\, \theta, \quad \gamma_{xz} = \frac{\partial w_0}{\partial x} - k_1 \int \theta\, dx, \tag{6}$$

According to the type of solution used (in this case Navier method), the undetermined integral ($\int \theta\, dx$) can be expressed as

$$\int \theta\, dx = A' \frac{\partial \theta}{\partial x} \tag{7}$$

With

$$A' = -\frac{1}{\lambda^2} \quad \text{and} \quad k_1 = \lambda^2 \tag{8}$$

Where the term λ is defined in the expression (22).

Based on the Hooke's relation, the constitutive relations (stress-strain) can be given as

$$\sigma_x = Q_{11}(\varepsilon_x) \quad \text{and} \quad \tau_x = k_s Q_{55} \gamma_x \tag{9a}$$

$$Q_{11}(z) = \frac{E_{11}(z)}{1-v^2} \quad \text{and} \quad Q_{55}(z) = G_{12}(z) \tag{9b}$$

Where the factor (k_s) is the shear correction coefficient.

2.3 Equations of motion

In this investigation the equations of motion of the SWCNT-RC beam are derived via Hamilton's principle. For the beam reposed on foundation and subjected to external load, the principle takes the following form (Ebrahimi and Barati 2017a, b, 2018a, Hadji *et al*. 2019)

$$\int_{t_1}^{t_2} (\delta U - \delta K + \delta V) dt = 0 \tag{10}$$

With

$$\begin{aligned} \delta U &= \int_0^L \int_A \left(\sigma_x \delta\, \varepsilon_x + \tau_{xz} \delta\, \gamma_{xz} \right) dA dx \\ &= \int_0^L \left(N \frac{\delta \partial u_0}{\partial x} - M \delta k_1 \theta - Q k_1 A' \frac{\delta \partial \theta}{\partial x} + Q \frac{\delta \partial w_0}{\partial x} \right) dx \end{aligned} \tag{11a}$$

$$\begin{aligned} \delta K &= \iint \rho (\dot u \delta \dot u + \dot w \delta \dot w) dx dz \\ &= \iint \rho \left[\left(\dot u_0 - z K_1 A' \frac{\partial \dot\theta}{\partial x} \right) \delta \left(\dot u_0 - z K_1 A' \frac{\partial \dot\theta}{\partial x} \right) + \dot w_0 \delta \dot w_0 \right] dx dz \end{aligned} \tag{11b}$$

$$\delta V = \iint \left[f_e\ \delta w + N_x^0 \left(\frac{dw}{dx} \frac{d\delta w}{dx} \right) \right] dx dz \tag{11c}$$

Where the expressions δU, δK represents the variations of the virtual strain energy and virtual kinetic energy associated to the above displacement field, respectively. δV is the variation of the work done by external load (axial load and reaction of elastic-foundation).

The resultants stresses N, M and Q appears in the above equation are defined as

$$(N,M)=\int_{-h/2}^{h/2}(1,z)\sigma_x dz \text{ and } Q=\int_{-h/2}^{h/2}G\tau_{xz}dz \quad (12)$$

The foundation reaction f_e applied on the SWCNT-R concrete beam is as

$$f_e=K_w-K_s\frac{\partial^2 w}{\partial x^2} \quad (13)$$

The Winkler and Pasternak constants K_w and K_s are given as

$$K_w=\frac{\beta_w A_{110}}{L^2} \quad (14)$$

$$K_s=\beta_s A_{110} \quad (15)$$

Where the terms β_w and β_s corresponding to spring and shear layer. The extension stiffness of the concrete only is expressed by

$$A_{110}=\int_{-h/2}^{h/2}\frac{E_c}{1-\nu_c^{\,2}}dz \quad (16)$$

By substituting the Eq. (11) in the Hamilton's principle (Eq. (10)). Integrating by part and arranging the equations according to displacements terms (δu_0, δw_0 and δu). We obtain the following equations of motion

$$\delta u_0:\frac{\partial N}{\partial x}=-I_0\ddot{u}+k_1A'I_1\frac{\partial\dot{\theta}}{\partial x} \quad (17a)$$

$$\delta w_0:\frac{\partial Q}{\partial x}-f_e+N_x^0\left(\frac{d^2w_0}{dx^2}\right)=-I_0\ddot{w}_0 \quad (17b)$$

$$\delta\theta:-k_1M+k_1A'\frac{\partial Q}{\partial x}=I_2(k_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2} \quad (17c)$$

Using the equation (12), the resultants stresses N, M and Q can be expressed as function of displacement terms as

$$N=A_{11}\frac{\partial u_0}{\partial x}-B_{11}k_1\theta \quad (18a)$$

$$M=B_{11}\frac{\partial u_0}{\partial x}-D_{11}k_1\theta \quad (18b)$$

$$Q=A_{55}\left(\frac{\partial w_0}{\partial x}-k_1A'\frac{\partial\theta}{\partial x}\right) \quad (18c)$$

Where the terms of rigidity A_{11}, B_{11} and D_{11} are defined as

$$(A_{11},B_{11},D_{11})=\int_{-h/2}^{h/2}Q_{11}(1,z,z^2)dz \text{ and } A_{55}=\int_{-h/2}^{h/2}Q_{55}dz \quad (19)$$

and the inertia I_0, I_1 and I_2 are given as

$$(I_0,I_1,I_2)=\int_{-h/2}^{h/2}\rho(1,z,z^2)dz \quad (20)$$

3. Analytical solutions

The Navier's method is employed here to satisfy the boundary conditions of the simply supported SWCNT-RC beam. The displacements terms take the following form (Yas and Samadi 2012, Wattanasakulpong and Ungbhakorn 2013, Safa *et al*. 2019, Zouatnia and Hadji 2019)

$$\begin{Bmatrix}u_0\\w_0\\\theta\end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix}U_{mn}\cos(\lambda x)e^{i\omega t}\\w_{mn}\sin(\lambda x)e^{i\omega t}\\\theta_{mn}\cos(\lambda x)e^{i\omega t}\end{Bmatrix} \quad (21a)$$

With

$$\lambda=m\pi/L \quad (21b)$$

To obtain the analytical solutions of the frequency and the buckling load of the SWCNT-RC beam, just replace the Navier method (Eq. (21)) in the Eq. (17). The analytical solution takes the following matrix form

$$\left(\begin{bmatrix}S_{11}&S_{12}&S_{13}\\S_{12}&S_{22}+\overline{N}&S_{23}\\S_{13}&S_{23}&S_{33}\end{bmatrix}-\omega^2\begin{bmatrix}m_{11}&m_{12}&m_{13}\\m_{12}&m_{22}&m_{23}\\m_{13}&m_{23}&m_{33}\end{bmatrix}\right)\begin{Bmatrix}U_{mn}\\W_{mn}\\\theta_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \quad (22)$$

With

$$S_{11}=-A_{11}\lambda^2,\quad S_{12}=0,\quad S_{13}=B_{11}k_1\lambda$$
$$S_{22}=-A_{55}\lambda^2-\beta_w-\beta_s\lambda^2,\quad S_{23}=A_{55}\lambda^2, \quad (23)$$
$$S_{33}=-A_{55}\lambda^2-D_{11}\lambda^4$$

$$m_{11}=-I_0,\ m_{12}=m_{21}=0,\ m_{13}=m_{31}=-I_1\lambda,$$
$$m_{22}=-I_0,\ m_{23}=m_{32}=0,\ m_{33}=-I_2\lambda \quad (24)$$

The solutions of the natural frequency are calculated by putting det([S]−ω^2[M]=0) and $\overline{N}=0$ and to obtain the critical buckling load just we put det([S]=0) and (ω=0).

4. Results and discussions

In this investigation, the used concrete has the Young's modulus E_c=20 GPa and Poison's ratio ν_c=0.27.

The all obtained results are computed by adimensional form as (Yas and Samadi 2012, Wattanasakulpong and Ungbhakorn 2013)

√ ***Fundamental frequency parameter***

$$\overline{\omega}=\omega L\sqrt{\frac{I_{00}}{A_{110}}} \quad (25)$$

Where I_{00} is the extension inertia of the I_0 of the concrete.

√ ***Critical buckling load parameter***

$$\overline{N}=\frac{N_x^0}{A_{110}} \quad (26)$$

Table 2 First three dimensionless natural frequencies $\bar{\omega}$ of simply supported CNT reinforced concrete beam with (L/h=15, k_s=5/6)

C_r^*	Distribution	model	(β_w=0, β_s=0)			(β_w=0.1, β_s=0)			(β_w=0.1, β_s=0.02)		
			ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0.12	*UD*	present	0.4436	1.6388	3.3092	0.5498	1.6705	3.3249	0.7145	1.9022	3.5929
	V	present	0.3933	1.4740	3.0277	0.5100	1.5091	3.0448	0.6843	1.7615	3.3338
	O	present	0.3451	1.3112	2.7387	0.4739	1.3505	2.7576	0.6579	1.6279	3.0738
	X	present	0.5201	1.8740	3.6826	0.6132	1.9018	3.6968	0.7644	2.1085	3.9404
0.17	*UD*	present	0.5322	1.9880	4.0660	0.6256	2.0148	4.0791	0.7778	2.2153	4.3054
	V	present	0.4628	1.7532	3.6488	0.5677	1.7834	3.6634	0.7318	2.0065	3.9122
	O	present	0.4046	1.5504	3.2753	0.5214	1.5846	3.2916	0.6966	1.8323	3.5666
	X	present	0.6301	2.3001	4.5831	0.7107	2.3234	4.5948	0.8478	2.4995	4.7975
0.28	*UD*	present	0.6723	2.4553	4.8950	0.7526	2.4784	4.9067	0.8901	2.6535	5.1078
	V	present	0.5758	2.1485	4.3899	0.6678	2.1747	4.4027	0.8196	2.3717	4.6245
	O	present	0.5032	1.9054	3.9633	0.6063	1.9350	3.9775	0.7704	2.1541	4.2215
	X	present	0.7981	2.8288	5.4682	0.8669	2.8489	5.4786	0.9887	3.0028	5.6601

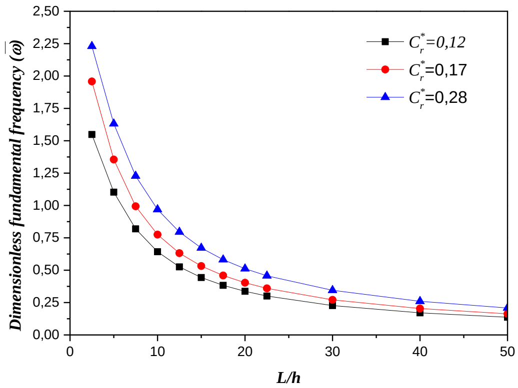

Fig. 3 Dimensionless fundamental frequency ($\bar{\omega}$) of UD-CNT reinforced concrete beam without elastic foundation versus the percentage (C_r^*)

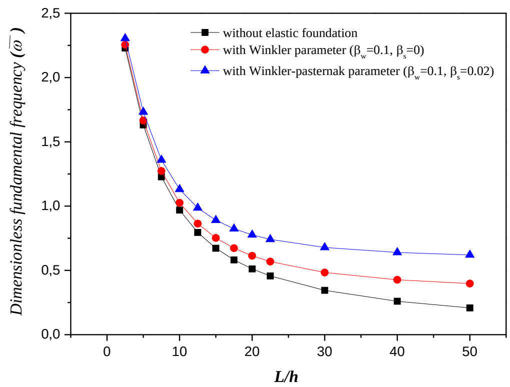

Fig. 4 Dimensionless fundamental frequency (C_r^*) of UD-CNT reinforced concrete beam versus the elastic-foundation parameters with (C_r^*=0.28)

4.1 Dynamic analysis

This part presents the free vibrational analysis of simply supported concrete beams reinforced with single walled carbon nanotubes (SW-CNT).

Tables 2 shows the variation of the first three dimensionless frequencies ($\bar{\omega}$) of concrete beams reinforced with SW-CNTs with and without elastic-foundation versus the volume fraction of the SW-CNT (C_r^*) and this for the different distributions of this latter (SW-CNT). From the obtained results, it can be seen that the values of the frequency parameter ($\bar{\omega}$) increase with the increase of the values of the elastic-foundation parameter (β_w, β_s) and this is due to the fact that the presence of the elastic-foundation makes the beam more rigid. It can be also noted that the increase in the values of reinforcement percentage (C_r^*) leads to an increase in the dimensionless frequency ($\bar{\omega}$). We can also conclude that the largest values of the frequency ($\bar{\omega}$) are obtained by a distribution of CNTs through the thickness of the "X-form ".

The effect of the percentage of the reinforcement (C_r^*) and slenderness ratio (L/h) on dimensionless fundamental frequency ($\bar{\omega}$) of the concrete beam without elastic-foundation (β_w=β_s=0) is presented in Fig. 3. From the plotted curves, it is clear that the dimensionless frequency parameter ($\bar{\omega}$) is in inverse relation with slenderness ratio (L/h) because the SWCNT-RC beam becomes flexible. It can be also observed that the increasing of the percentage of the SWCNT C_r^* () has an important role to increase the frequency ($\bar{\omega}$).

The effect of the elastic-foundation parameters (β_w, β_s) on dimensionless fundamental frequency ($\bar{\omega}$) of the UD-reinforced concrete beam with SWCNT percentage (C_r^*=0.28) is illustrated in the Fig. 4. From the obtained graphs, it can be noted that the influence of the elastic foundation parameters (β_w, β_s) is important in the case of in the case of slender beams. It is confirmed again that the Winkler-Pasternak foundation become the reinforced concrete beam more rigid.

4.2 Stability analysis

In this second section, the buckling analysis of simply-supported concrete beams reinforced with CNTs is presented.

Table 3 presents the dimensionless values of the critical buckling load ($\bar{N}$) of (UD, V, O, and X) reinforced concrete-beams with and without elastic-foundation versus the volume fraction of the CNT (C_r^*). From the obtained results, it can be seen that the values of the critical buckling load

Table 3 Critical buckling load ($\bar{N}$) of CNT reinforced concrete beam with L/h=15

(β_w, β_s)	C_r^*	Theory	UD	V	O	X
(0.00, 0.00)	0.12	present (k_s=1)	0,0191	0,0149	0,0115	0,0263
		present (k_s=2/3)	0,0186	0.01470	0,0113	0,0255
		present (k_s=5/6)	0,0189	0,0149	0,0114	0,0259
	0.17	present (k_s=1)	0,0267	0,0202	0,0154	0,0376
		present (k_s=2/3)	0,0262	0,0199	0,0152	0,0366
		present (k_s=5/6)	0,0265	0,0200	0,0153	0,0372
	0.28	present (k_s=1)	0,0404	0,0296	0,0226	0,0572
		present (k_s=2/3)	0,0394	0,029	0,0222	0,0551
		present (k_s=5/6)	0,0400	0,0294	0,0224	0,0564
(0.1, 0.00)	0.12	present (k_s=1)	0,0292	0,0251	0,0216	0,0364
		present (k_s=2/3)	0,0288	0,0248	0,0215	0,0356
		present (k_s=5/6)	0,0290	0,0250	0,0216	0,0361
	0.17	present (k_s=1)	0,0369	0,0303	0,0255	0,0477
		present (k_s=2/3)	0,0364	0,0300	0,0254	0,0467
		present (k_s=5/6)	0,0367	0,0302	0,0255	0,0473
	0.28	present (k_s=1)	0,0506	0,0397	0,0327	0,0673
		present (k_s=2/3)	0,0495	0,0392	0,0324	0,0653
		present (k_s=5/6)	0,0501	0,0395	0,0326	0,0665
(0.1, 0.02)	0.12	present (k_s=1)	0,0492	0,0451	0,0416	0,0564
		present (k_s=2/3)	0,0488	0,0448	0,0415	0,0556
		present (k_s=5/6)	0,049	0,0449	0,0416	0,0561
	0.17	present (k_s=1)	0,0569	0,0503	0,0455	0,0677
		present (k_s=2/3)	0,0564	0,0500	0,0454	0.0667
		present (k_s=5/6)	0,0567	0,0502	0,0455	0,0673
	0.28	present (k_s=1)	0,0706	0,0597	0,0527	0,0873
		present (k_s=2/3)	0,0695	0,0591	0,0524	0,0853
		present (k_s=5/6)	0,0701	0,0595	0,0525	0,0865

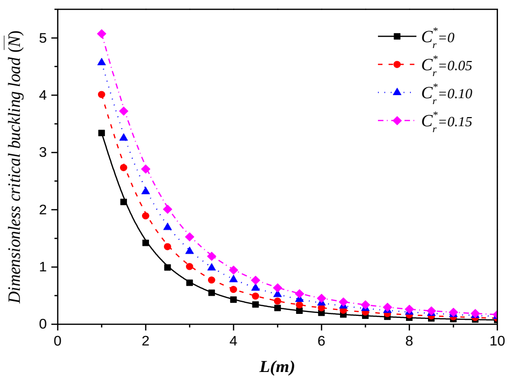

Fig. 5 The effect of SWCNT percentage on critical buckling load ($\bar{N}$) of UD-CNT reinforced concrete beam with (h=0.3 m)

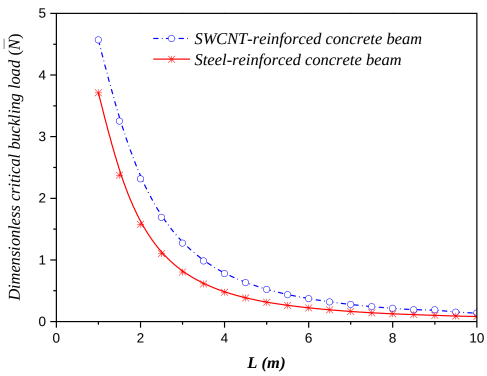

Fig. 6 Comparison of dimensionless critical buckling loads ($\bar{N}$) of steel and SWCNT reinforced concrete beam with (C_r^*=0.10 and h=0.3 m)

($\bar{N}$) increases with the increasing in the values of the elastic-foundation parameter (β_w, β_s) and this is due to the presence of the elastic foundation increase the beam rigidity. It can also be noted that the critical buckling load ($\bar{N}$) is in direct correlation relation with SWCNT percentage (C_r^*). We can also conclude that the most flexible beam is obtained by a distribution of carbon nanotubes of “O” shape.

The Fig. 5 shows the effect of the SWCNT percentage (C_r^*) on the critical buckling load ($\bar{N}$) of the UD-CNT concrete beam. From the plotted curve, we can notice that the critical buckling load ($\bar{N}$) is in direct correlation relationship with the percentage of the SWCNT volume fraction (C_r^*), therefore we can conclude that the increase in the SWCNT percentage (C_r^*) makes the beam more rigid.

The effect of reinforcement type on the critical buckling load ($\bar{N}$) of simply-supported UD-CNT concrete beams is presented in the Fig. 6. Two types of reinforcement are used, namely steel and single-walled carbon nanotubes (SW-CNT). From the results, it can be noted that the critical buckling load ($\bar{N}$) of the SW-CNT (single-walled Carbon-Nanotubes) reinforced concrete beam is greater than ($\bar{N}$) of

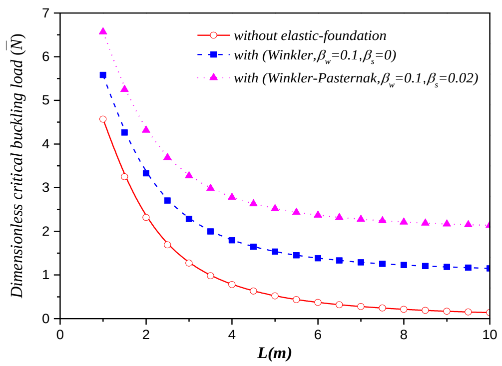

Fig. 7 the effect of elastic foundation parameter on critical buckling load of UD-CNT reinforced concrete beam with (C_r^*=0.10 and h=0.3 m)

steel reinforced concrete beams.

The effect of the elastic-foundation parameters (β_w, β_s) on the critical buckling load ($\bar{N}$) of SWCNT-RC beams with (C_r^*=0.10) is presented in the Fig. 7. From the plotted curves, it can be noted that the critical buckling load ($\bar{N}$) decreases with increasing of length (L) because the beam becomes slender. It can be noted that the presence of the elastic-foundation (β_w, β_s) leads to an increase in the values of the critical buckling load ($\bar{N}$). We can conclude that the beam on Winkler-Pasternak foundation type (spring and shear layer, β_w=0.1, β_s=0.02) is more rigid than a beam on only Winkler elastic foundation (spring only) and the beam without elastic foundation (β_w=0, β_s=0) is the least rigid.

5. Conclusions

In this paper, the dynamic and stability analysis of reinforced concrete beam resting on Winkler-Pasternak elastic-foundation is investigated by employing a novel first shear deformation beam theory. The analytical solutions for buckling and free vibration have been determined via Hamilton's principle and Navier procedure. Several parametric studies have been performed to illustrate the influences of the reinforcement percentage, slenderness ratio, elastic-foundation parameters and SWCNT distributions on non-dimensional frequency parameter and critical buckling loads of simply-supported SWCNT-RC beams. An improvement of the developed formulation will be considered in the future work to consider other type of structures and materials (Sharma *et al*. 2009, Civalek and Ozturk 2010, Akgoz and Civalek 2011, Sedighi *et al*. 2015, Avcar 2016a, b, Sedighi and Bozorgmehri 2016, Ouakad *et al*. 2017, Daouadji 2017, Lal *et al*. 2017, Mirjavadi *et al*. 2018a, b, Ebrahimi and Barati 2018b, Avcar 2018, Barati 2018, Narwariya *et al*. 2018, Hamidi *et al*. 2018, Othman and Mahdy 2018, Behera and Kumari 2018, Ayat *et al*. 2018, Dihaj *et al*. 2018, Ebrahimi *et al*. 2019, Abdou *et al*. 2019, Bakhshi and Taheri-Behrooz 2019, Fládr *et al*. 2019, Fenjan *et al*. 2019, Othman *et al*. 2019, Mandi *et al*. 2019, Oucif *et al*. 2019, Barati and Zenkour 2019, Arhamnamazi *et al*. 2019, Mirjavadi *et al*. 2019b, c, Al-Osta 2019, Selmi 2019, Bakhshi and Taheri-Behrooz 2019, Timesli 2020, Akbaş 2019a, b, Rajabi and Mohammadimehr 2019, Hamed *et al*. 2019, 2020, Forsat *et al*.2020, Faleh *et al*. 2020, Mirjavadi *et al*. 2020, Ghannadpour and Mehrparvar 2020, Shokrieh and Kondori 2020, Ghadimi 2020, Khater *et al*. 2020).

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