A mathematical approach for the effect of the rotation on thermal stresses in the piezo-electric homogeneous material

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Abstract. In this work, the analytical solution for the stresses in piezo-thermo-elastic homogeneous, transversely isotropic material under the effect of the rotation has investigated. The thermoelasticity theory has used to study the problem. The material subjected to boundary conditions. Finally, the numerical solution has carried out piezo - thermo-elastic material under the effect of rotation, to illustrate the analytical development. The corresponding simulated results of various physical quantities such as the displacements and the stresses, the temperature and the electrical displacement have presented graphically.

Keywords: rotation; stresses; transversely isotropic; Piezo-thermo-elastic material

1. Introduction

Propagation of waves in piezoelectric plates has been an active research area for several decades. The interaction between the magnetic and thermal fields plays a vital role in geophysics for understanding the effect of Earth's magnetic field on seismic waves. With the development of active material systems, there is a significant interest in the coupling effects between the elastic, magnetic, and temperature for their application in sensing and actuation (Mahmoud 2013, 2016, Mahmoud et al. 2017, Alimirzaei et al. 2019, Karami et al. 2019a, Khorasani et al. 2020). Propagation of waves in piezoelectric plates has been an active research area for several decades because of the application in piezoelectric transducers, resonators, filters, actuators, and other devices such as microelectromechanical systems (MEMS). Several exact solutions of the threedimensional dynamical equations have obtained for widely used materials such as ceramics, various crystal cuts of quartz and materials of other symmetries. Yang (2005) discussed the detailed studies and analysis of piezoelectric vibratory gyroscopes in recent publications. Α comprehensive review of the work on piezoelectricity and related fields has done by Yang and Fang (2002, 2003).

Sharma and Kumar (2000) have studied the propagation of plane harmonic waves in piezo-thermo-elastic materials. Sharma and Othman (2007) investigated the effect of rotation on generalized thermo-viscoelastic Rayleigh-Lamb waves in plates. Sharma and Pal (2004) investigated the propagation of Lamb waves in a transversely isotropic, charge and stress-free piezo-thermo-elastic plate in the context of the conventional coupled theory of piezo-thermoelasticity. They studied the wave characteristics, such as phase velocity and attenuation coefficient of the waves in Cadmium Selenide (CdSe) material. Sharma and Thakur (2006) studied the effect of rotation on Rayleigh-Lamb waves in magneto-thermo-elastic plates.

Recently, Sharma and Walia (2006, 2008a, b) have studied the effect of rotation on Rayleigh waves in piezothermo-elastic half-space, the reflection of piezo-thermoelastic waves from the charge and stress-free boundary of a transversely isotropic half-space. Abd-Alla and Mahmoud (2012, 2013) studied the problem of radial vibrations in the non-homogeneity isotropic cylinder under the influence of initial stress and magnetic field, the influence of rotation and generalized magneto-thermo-elastic on Rayleigh waves in a granular medium under effect of initial stress and gravity field. Mahmoud (2011, 2012) studied analytical solution of wave propagation in non-homogeneous orthotropic rotating elastic media and studied the effect of rotation and magnetic field through the porous medium on peristaltic transport of a Jeffrey fluid in the tube. Ting (2004) studied surface waves in a rotating anisotropic elastic half-space. Yang (2003) studied piezoelectric vibratory gyroscopes. Zhou and Jiang (2001) studied the effects of Coriolis force and centrifugal force on acoustic waves propagating along the surface of a piezoelectric halfspace. Mahmoud (2010) studied wave propagation in cylindrical poro-elastic dry bones. He also discussed the analytical solution for free vibrations of the elastodynamic orthotropic hollow sphere under the influence of rotation (Mahmoud 2014). Other works on wave propagation and vibration problems was also studied in several researches as indicated in Refs (Bourada et al. 2019, Karami et al. 2019b, c, Zaoui et al. 2019, Batou et al. 2019, Karami et al. 2019d). Abd-Alla and Mahmoud (2010) studied the problem magneto-thermo-elastic in rotating nonhomogeneous orthotropic hollow cylindrical under the hyperbolic heat conduction model. It should be noted that

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several works on thermoelastic problems was investigated using different methods as is shown in (Lal *et al.* 2017, Shinde *et al.* 2018, Mehar and Panda 2018a, b, 2020, Semmah *et al.* 2019, Belbachir *et al.* 2019, Abualnour *et al.* 2019, Hirwani and Panda 2019, Tounsi *et al.* 2020, Boussoula *et al.* 2020).

In this paper, stress components, the displacement components, stress components, and the electric potential, the temperature in transversely isotropic, piezo-thermoelastic homogeneous material under the effect of the rotation has investigated. The corresponding numerical results of various physical quantities as displacements and the stresses, the temperature and the electrical displacement, have presented graphically.

2. Formulation of the problem

We consider homogeneous, transversely isotropic (6 mm class), generalized piezothermo-elastic material. The material at a uniform temperature T_0 is used in the undisturbed state. The origin of the coordinate system (x, y, z) on the middle surface of the material is used in this work. One has chosen the XY-plane so that it coincides with the middle surface and z-axis normal to it along with the thickness. The x-axis is chosen in the wave propagation direction to ensure that all the particles on a line parallel to the y-axis are equally spaced. Accordingly, not all field quantities depend on yz-coordinates. Material surfaces are represented by $x = \pm d$ which are governed by isothermal, electrically shorted (closed circuit), stress-free and thermally insulated boundary conditions. Let u(x,t) =(u, 0, 0) represents the displacement vector, $\phi(x, t)$ represents the electric potential, $\psi(x,t)$ represents the magnetic potential and T(x,t) refers to temperature change in the material. They considered this case in the non-existence of heat sources, charge density, and body forces in dimensionless form linear generalized theories of piezo-thermo-elasticity.

Now, One assumes an anisotropic piezo-thermo-elastic homogeneous medium for temperature variation T(x, t), The basic governing field equations of displacement vector hexagonal type, $\overline{U}(x, t) = (u, 0, 0)$, electric potential $\phi(x, t)$ take the following form

$$\frac{\partial}{\partial x}\tau_{xx} + \rho \left(\overline{\Omega} \times \overline{\Omega} \times \overline{U}\right)_r = \rho \frac{\partial^2}{\partial t^2} u, \qquad (1a)$$

$$\frac{\partial}{\partial x}D_x = 0, \tag{1b}$$

$$\frac{\partial}{\partial x}B_x = 0, \qquad (1c)$$

$$K_{11}\frac{\partial^2}{\partial x^2}T - \rho C_e \left(\frac{\partial}{\partial t} + t_0 \frac{\partial^2}{\partial t^2}\right)T$$

= $\beta_1 T_0 \left(1 + t_0 \delta_{1k} \frac{\partial}{\partial t}\right) \left(\frac{\partial^2}{\partial x \partial t}u\right).$ (1d)

where $\overline{\Omega} = (0,0,\Omega)$, $(\overline{\Omega} \times \overline{\Omega} \times \overline{U})_r$ is a component of the centripetal acceleration in the radial direction (\overline{r}), due to

the time-varying motion only. The hexagonal crystal symmetry electric displacement along with the constitutive relations take the following form

$$\tau_{xx} = c_{11}\varepsilon_{xx} - \beta_1 \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t} \right) T, \qquad (2a)$$

$$\tau_{yy} = c_{12}\varepsilon_{xx} - \beta_1 \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t} \right) T, \qquad (2b)$$

$$\tau_{zz} = c_{13}\varepsilon_{xx} - \beta_3 \left(1 + t_1 \delta_{2k} \frac{\partial}{\partial t} \right) T, \qquad (2c)$$

$$D_x = \varsigma_{11} E_x + m_{11} H_x,$$
 (2d)

$$B_x = m_{11}E_x + \mu_{11}H_x, (2e)$$

where, $\beta_1 = (c_{11} + c_{12})\gamma_1 + c_{13}\gamma_3$, $\beta_3 = 2c_{13}\gamma_1$

Where c_{ij} is an isothermal elastic tensor, τ_{ij} is the stress vector, ε_{ij} is strain tensor, ς_{ij} is a piezoelectric parameter, m_{ii} are dielectric moduli, t_1 is thermal relaxation time, β_1 , β_3 are the isothermal thermo-elastic parameters, γ_1 , γ_3 is the coefficient of linear thermal expansion and thermal conductivities along and perpendicular to the axis of symmetry, respectively. The relation between the electric field vector E_i and the electric potential ϕ , and Similarly, the magnetic field H_i is related to the magnetic potential ψ as: is given by

$$E_x = -\frac{\partial}{\partial x}\varphi, \qquad \qquad H_x = -\frac{\partial\psi}{\partial x}, \qquad (3)$$

Using Eqs. (1)-(2) moreover, Eq. (3), One gets

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$$c_{11}\frac{\partial^2}{\partial x^2}u - \beta_1 \left(\frac{\partial}{\partial x}T + \delta_{2k}t_1\frac{\partial^2}{\partial x\partial t}T\right) + \rho \left(\overline{\Omega} \times \overline{\Omega} \times \overline{U}\right)_r = \rho \frac{\partial^2}{\partial t^2}u,$$
(4a)

$${}_{11}\frac{\partial^2}{\partial x^2}\varphi + m_{11}\frac{\partial^2}{\partial x^2}\psi = 0, \qquad (4b)$$

$$a_{11}\frac{\partial^2}{\partial x^2}\varphi + \mu_{11}\frac{\partial^2}{\partial x^2}\psi = 0, \qquad (4c)$$

$$\frac{K_{11}}{\rho C_e} \frac{\partial^2}{\partial x^2} T - \left(\frac{\partial}{\partial t} + t_0 \frac{\partial^2}{\partial t^2}\right) T$$

$$= \frac{\beta_1 T_0}{\rho C_e} \left(1 + \delta_{1k} t_0 \frac{\partial}{\partial t}\right) \left(\frac{\partial^2}{\partial x \, \partial t} u\right).$$
(4d)

In order to simplify, one will implement the following dimensionless variables.

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\beta_1 T_0}, \quad \bar{\beta} = \frac{\beta_1}{c_{11}}, \quad K = \frac{K_{11}}{\rho C_e},$$

$$\bar{\rho} = \frac{\rho}{c_{11}}, \quad \epsilon = \frac{\beta_1 T_0}{\rho C_e}, \quad \epsilon = \frac{T_0 \beta_1^2}{\rho C_e c_{11}},$$

$$\beta_1 = (c_{11} + c_{12})\gamma_1 + c_{13}\gamma_3, \quad \beta_3 = 2c_{13}\gamma_1,$$

$$m_1 = \frac{m_{11}}{\varsigma_{11}}, \quad \mu_1 = \frac{\mu_{11}}{m_{11}}, \quad D_i' = \frac{D_i}{\beta_1 T_0},$$
(5)

Eqs. (11)-(14) in the non-dimensional forms (after suppressing the primes) reduce to

$$\frac{\partial^2}{\partial x^2} u - \bar{\beta} \left(\frac{\partial}{\partial x} + \delta_{2k} t_1 \frac{\partial^2}{\partial x \partial t} \right) T + \rho \left(\overleftarrow{\Omega} \times \overleftarrow{\Omega} \times \overleftarrow{U} \right)_r$$

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$$=\bar{\rho}\left(\frac{\partial^2}{\partial t^2}\right)u,\tag{6a}$$

$$\frac{\partial^2}{\partial x^2}\varphi + m_1 \frac{\partial^2}{\partial x^2}\psi = 0, \tag{6b}$$

$$\frac{\partial^2}{\partial x^2}\varphi + \mu_1 \frac{\partial^2}{\partial x^2}\psi = 0, \tag{6c}$$

$$K\frac{\partial^2}{\partial x^2}T - \left(\frac{\partial}{\partial t} + t_0\frac{\partial^2}{\partial t^2}\right)T$$

= $\left(1 + \delta_{1k}t_0\frac{\partial}{\partial t}\right)\left(\in \frac{\partial^2}{\partial x \partial t}u\right).$ (6d)

Where ρ is density, C_e is the specific heat at constant strain, t_0, t_1 , are thermal relaxation times. The superimposed dots indicate time differentiation, and comma notation has been utilized for spatial derivatives. The symbol $\delta_{jk}, j = 1, 2$, is the Kronecker's delta in which k =1 corresponds to Lord and Shulman and k = 2 refers to Green and Lindsay theory of thermo-elasticity. The theories mentioned above are abbreviated as GL and SL, respectively. The prime was repressed for simplification. $\varphi, \psi D_i$ and B_i , are respectively, the electric potential, magnetic potential, electrostatic displacement, and magnetic induction, ε_{jk} and μ_{jk} are, respectively, the dielectric and magnetic permeability coefficients, e_{kj}, d_{kj} and m_{jk} are, respectively, the piezoelectric, piezo-magnetic and magneto-electric material coefficients.

3. The solution to the problem

We assume the solution of the form

$$\mathbf{u} = \bar{\mathbf{u}}(\mathbf{x})\mathbf{e}^{\mathbf{i}\omega\mathbf{t}},\qquad(7a)$$

$$b = \bar{\psi}(\mathbf{x}) \mathbf{e}^{\mathrm{i}\omega \mathrm{t}},\tag{7b}$$

$$\varphi = \bar{\varphi}(\mathbf{x}) \mathbf{e}^{\mathbf{i}\omega \mathbf{t}},\tag{7c}$$

$$T = \bar{T}(x)e^{i\omega t}$$
(7d)

Where ω is the angular frequency, and prime was repressed for simplification $\bar{u}(x)$, $\bar{\varphi}(x)$, $\bar{\psi}(x)$ Moreover, $\bar{T}(x)$ are the displacement component u, electric potential, magnetic potential, and temperature change respectively. Using solutions (7) into Eqs. (6) to get a system of four coupled equations of components $(\bar{u}(x), \bar{\varphi}(x), \bar{\psi}(x), \bar{T}(x))^T$

$$(\nabla_1^2 - \rho \omega^2)\overline{u} - \beta_1 (1 + i\omega t_1) \frac{d\overline{r}}{dx} = 0, \qquad (8a)$$

$$\nabla_1^2 \bar{\varphi} + m_1 \nabla_1^2 \bar{\psi} = 0, \tag{8b}$$

$$\nabla_1^2 \bar{\varphi} + \mu_1 \nabla_1^2 \bar{\psi} = 0, \qquad (8c)$$

$$\varepsilon_1(i\omega - t_0\delta_{1k}c^2)\frac{d}{dx}\bar{u} - \left(\nabla_1^2 - (i\omega - t_0\omega^2)\right)\bar{T} = 0, \ (8d)$$

The non-trivial solutions of Eq. (8) are represented by

$$\overline{u}(x) = \frac{1}{4\lambda} \left((D_1 e^{-A_1 x} - D_2 e^{A_1 x}) + (D_3 e^{-A_2 x} - D_4 e^{A_2 x}) \right)$$
^(9a)

$$\bar{T}(x) = D_1 e^{-A_3 x} + D_2 e^{A_3 x} + D_3 e^{-A_4 x} + D_4 e^{A_4 x}, \quad (9b)$$

$$\bar{\varphi}(x) = D_5 x + D_6, \tag{9c}$$

$$\bar{\psi}(x) = D_7 x + D_8. \tag{9d}$$

Moreover, substituting Eqs. (9) into Eqs (8), One obtain

$$u(x,t) = \frac{1}{4\lambda} ((D_1 e^{-A_2 x} - D_2 e^{A_2 x}) + (D_3 e^{-A_4 x} - D_4 e^{A_4 x}))e^{i\omega t},$$
^(10a)

$$T(x,t) = (D_1 e^{-A_3 x} + D_2 e^{A_3 x} + D_3 e^{-A_4 x} + D_4 e^{A_4 x}) e^{i\omega t},$$
(10b)

$$\varphi(x,t) = (D_5 x + D_6)e^{i\omega t}, \qquad (10c)$$

$$\psi(x,t) = (D_7 x + D_8)e^{i\omega t}.$$
 (10d)

Moreover, substituting Eqs. (10) along Eqs. (1) One obtains the stresses

$$\tau_{xx} = \frac{-c_{11}}{4\lambda} \begin{pmatrix} A_1(D_1e^{-A_1x} + D_2e^{A_1x}) + \\ A_3(D_3e^{-A_3x} + D_4e^{A_3x}) \end{pmatrix} e^{i\omega t} \\ -(\beta_1 + i\omega t_1) \begin{pmatrix} D_1e^{-A_3x} + D_2e^{A_3x} \\ +D_3e^{-A_4x} + D_4e^{A_4x} \end{pmatrix} e^{i\omega t},$$
(11a)

$$\tau_{yy} = \frac{-c_{12}}{4\lambda} \begin{pmatrix} A_1(D_1 e^{-A_1 x} + D_2 e^{A_1 x}) \\ +A_3(D_3 e^{-A_3 x} + D_4 e^{A_3 x}) \end{pmatrix} e^{i\omega t} \\ -(\beta_1 + i\omega t_1) \begin{pmatrix} D_1 e^{-A_3 x} + D_2 e^{A_3 x} \\ +D_2 e^{-A_4 x} + D_4 e^{A_4 x} \end{pmatrix} e^{i\omega t},$$
(11b)

$$\tau_{zz} = \frac{-c_{13}}{4\lambda} \begin{pmatrix} A_1(D_1e^{-A_1x} + D_2e^{A_1x}) \\ +A_3(D_3e^{-A_3x} + D_4e^{A_3x}) \end{pmatrix} e^{i\omega t} \\ -(\beta_3 + i\omega t_1) \begin{pmatrix} D_1e^{-A_3x} + D_2e^{A_3x} \\ +D_3e^{-A_4x} + D_4e^{A_4x} \end{pmatrix} e^{i\omega t},$$
(11c)

$$D_x = -(\varsigma_{11}D_5 + m_{11}D_7)e^{i\omega t}, \qquad (11d)$$

$$B_x = -(m_{11}D_5 + \mu_{11}D_7)e^{i\omega t}, \qquad (11e)$$

Where E_X , D_X , H_X , B_X are the electric field, electrical displacement, magnetic field, and magnetic induction respectively, A_i , i = 1,2,3,4. and are given as follows:

$$\begin{split} \lambda_{1} &= -\beta_{1}t_{1}^{2}c^{6}\varepsilon^{2}t_{0}^{6}\delta_{1k}^{2} + 2it_{0}\beta_{1}\delta_{1k}\varepsilon t_{1}(\beta_{1}(t_{1}+t_{0}\delta_{1k})\varepsilon \\ &+ t_{0}c_{11}\delta_{2k} + \rho)c^{5} + (\beta_{1}^{2}(t_{1}^{2}+\delta_{1k}^{2}t_{0}^{2}+4t_{0}\delta_{1k}t_{1})\varepsilon^{2} \\ &+ 2(t_{0}^{2}\delta_{1k}\delta_{2k}c_{11} + (t_{1}(\delta_{1k}+\delta_{2k})c_{11} + \rho\delta_{1k})t_{0} + t_{1}\rho)\beta_{1}\varepsilon \\ &+ (t_{0}c_{11}\delta_{2k} - \rho)^{2}c^{4} + (-2i\beta_{1}^{2}(t_{1}+t_{0}\delta_{1k})\varepsilon^{2} \\ &+ 2i\beta_{1}\left(\left((-\delta_{2k}-\delta_{1k})c_{11}\right)t_{0} - \rho - t_{1}c_{11}\right)\varepsilon - (t_{0}c_{11}\delta_{2k} - \rho)c_{11})c^{3} + (-\beta_{1}^{2}\varepsilon^{2} - 2\beta_{1}(c_{11})\varepsilon - c_{11}^{2})c^{2} \\ \lambda_{2} &= i\beta_{1}t_{1}c^{3}\varepsilon t_{0}\delta_{1k} + (\beta_{1}(t_{1}+t_{0}\delta_{1k})\varepsilon \\ &+ t_{0}c_{11}\delta_{2k} + \rho)c^{2} - i(\beta_{1}\varepsilon + c_{11})c, \\ \lambda_{3} &= i\beta_{1}t_{1}c^{3}\varepsilon t_{0}\delta_{1k} + (\beta_{1}(t_{1}+t_{0}\delta_{1k})\varepsilon \\ &+ t_{0}c_{11}\delta_{2k} - \rho)c^{2} - i(\beta_{1}\varepsilon + c_{11})c, \\ \lambda_{3} &= \frac{\varepsilon\varepsilon\rho(i + t_{0}c\delta_{1k})(c^{2})c_{11}}{\sqrt{2}}, \\ \lambda_{1} &= B_{2}\sqrt{-2B_{1}}, \quad A_{2} &= \frac{1}{\sqrt{2}}\frac{\sqrt{-B_{1}}}{c_{11}}, \\ A_{3} &= B_{4}\sqrt{-B_{3}}, \quad A_{4} &= \frac{1}{\sqrt{2}}\frac{\sqrt{-B_{1}}}{c_{11}}, \\ B_{1} &= \left(\sqrt{\lambda_{1}} + \lambda_{1}\right)c_{11}, \quad B_{2} &= -\sqrt{\lambda_{1}} + \lambda_{3}, \\ B_{3} &= \left(-\sqrt{\lambda_{1}} + \lambda_{1}\right)c_{11}, \quad B_{4} &= \sqrt{\lambda_{1}} + \lambda_{3}, \end{split}$$



Fig. 1 Dispersion curves for the displacement u, versus x with t=0.8, in the cases of the rotation $\Omega = 1.5$ and non rotation $\Omega = 0$



Fig. 3 Dispersion curves for the temperature *T*, versus *x* with *t*=0.8, in the cases of the rotation $\Omega = 1.5$ and non rotation $\Omega = 0$

4. Boundary conditions

The material proposed to be electrically shorted and thermally insulated/isothermal. Consequently, the following boundary conditions have to be satisfied with the material, at x = 0, and the mechanical, electrical, magnetic and thermal boundary condition are

where $p_0 e^{i\omega t}$ is the periodic loading

5. Numerical results and discussion

With the view of illustrating theoretical results obtained in the proceeding sections, we give the physical data for material (Sharma and Walia 2008b)

$$\begin{split} \rho &= 5504 Kgm^{-3}, \quad \varsigma_{13} = -0.160 Cm^{-2}, \\ f_1^* &= 10^8, T_0 = 298 K, \\ \beta_1 &= 0.621 \times 10^6 N k^{-1} K^{-1}, \\ \beta_3 &= 0.551 \times 10^6 N K^{-1} m^{-2}, \\ m_{11} &= 8.26 \times 10^{-11} C^2 N^{-1} m^{-2}, \\ K_{11} &= 9W m^{-1} K^{-1}, C_e = 260 J K g^{-1} K^{-1}, \\ c_{11} &= 7.41 \times 10^{10} N m^{-2}, \qquad c_{12} = 4.52 \times 10^{10} N m^{-2}, \\ c_{11} &= 3.93 \times 10^{10} N m^{-2}, \end{split}$$



Fig. 2 Dispersion curves for the normal stress τ_{xx} , versus x with t=0.8 in the cases of the rotation $\Omega = 1.5$ and non rotation $\Omega = 0$



Fig. 4 Dispersion curves for the electrical displacement D_x , versus x with t=0.8, in the cases of the rotation $\Omega = 1.5$ and non rotation

The thermal relaxation time $t_0 = 0.8$ and t_1 is selected as multiple of t_0 . The thermomechanical coupling factor, specific loss factor, and relative frequency shift numerically analyzed. The computed results in respected dispersion curves.

Fig. 1 shows a comparison between the displacement component u. The computations are carried out for in the cases of the rotation $\Omega = 1.5$ and non-rotation $\Omega = 0$, the time t = 0.8 on the surface plane.

Fig. 2, shows the comparison between the normal stresses components τ_{xx} and the computations carried out in the cases of the rotation $\Omega = 1.5$ and non-rotation $\Omega = 0$, the time t = 0.8.

Fig. 3 shows the comparison between the temperature *T*; the computations are carried out for the time t = 0.8 and in the cases of the rotation $\Omega = 1.5$ and non-rotation $\Omega = 0$, on the surface plane.

Fig. 4 shows the comparison between the electric displacement D_x the computations are carried out in the cases of the rotation $\Omega = 1.5$ and non-rotation $\Omega = 0$, and the time t = 0.8, on the surface medium. According to the above numerical results, one can observe that All the physical quantities agree with the boundary conditions.

Finally, one can observe that the analytical solutions under the effect of the rotation based upon normal mode analysis for transversely isotropic, piezoelectric thermoelastic homogeneous medium have been developed. The significant effect of the rotation and magnetic field, thermal time relaxation has observed in all the various physical quantities of the material since all the profiles of considered functions are quite distinguishable.

6. Conclusion

The mechanical stresses under the effect of the rotation based upon normal mode analysis in a homogeneous, transversely isotropic, piezo-thermo-elastic material have studied. The thermoelasticity theory has used to investigate the problem. It was then subjecting the conditions, electrical and thermally insulated thermally. Results in the forms of graphs and so each variable such as the temperature, electrical displacement, the stresses and displacements presented graphically. This work can be extended in the future work for other type of materials (Singh and Panda 2013, 2015, 2017, Singh et al. 2016a,b, 2019, Arani and Kolahchi 2016, Dutta et al. 2017, Daouadji 2017, Katariya et al. 2017, Ayat et al. 2018, Panjehpour et al. 2018, Behera and Kumari 2018, Malikan 2018, 2019, Rezaiee-Pajand et al 2018, Othman and Fekry 2018, Narwariya et al. 2018, Fenjan et al. 2018, 2019, Ahmed et al. 2019, Hussain and Naeem 2019, Mehar et al. 2019, Chaabane et al. 2019, Selmi 2019, Yazdani and Mohammadimehr 2019, Al-Maliki et al. 2019, Mirjavadi et al. 2019, Draoui et al. 2019, Bakhshi and Taheri-Behrooz 2019, Hussain et al. 2019, 2020a, b, Berghouti et al. 2019, Sahla et al. 2019, Tounsi et al. 2019, Boukhlif et al. 2019, Salah et al. 2019, Bensattalah et al. 2019, Karami et al. 2019e, Ahmed et al. 2019, Avcar 2019, Faleh et al. 2018, 2020, Bakhti et al. 2020, Kaddari et al. 2020, Asghar et al. 2020a, b, Khosravi et al. 2020, Taj et al. 2020a, b).

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