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Eringen's nonlocal model sandwich with Kelvin's theory for vibration of DWCNT

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Abstract. In this paper, vibration characteristics of chiral double-walled carbon nanotubes entrenched on Kelvin's model. The Eringen's nonlocal elastic equations are being combined with Kelvin's theory to observe small scale response. A nonlocal model has been formulated to explore the frequency spectrum of chiral double-walled CNTs along with diversity of indices and nonlocal parameter. Wave propagation is proposed technique to establish field equations of model subjected to four distinct end supports. The significance of scale effect in relevance of length-to-diameter and thickness- to- radius ratios are discussed and displayed in detail.

Keywords: concrete bridge; concrete structures; fatique; polymer concrete; reinforced concrete buildings; structural analysis/design

1. Introduction

CNTs are exceptionally meagre in structure, so to envision the behaviour through experimental techniques of such nanostructures under various conditions is not an easy task. For that reason, computational simulations have been taken an edge being dynamic tool to inspect the physical and mechanical attributes of CNTs. Owing to remarkable physical and mechanical features of the nanosized structures, carbon nanotubes have been persuasive and contemporary measure in aerospace, microscopic system, actuators, gas exposure, defence, diagnosis devices and several other (Lau and Hui 2002, Zhao 2002, Lieber 2003, Liu and Zang 2004, Kostarelos *et al.* 2009, Sosa *et al.* 2014, Fakhrabadi *et al.* 2015). Since from the last decade carbon nanotubes (CNTs) have become potential subject of scientific research with its vigorous performance in the various fields. CNTs also contribute significantly in material science, medicine and structural engineering (Gittes *et al.* 1993, Nogales 2001, Kasas *et al.* 2004, Regi 2007, Reilly 2007, Gohardani *et al.* 2014, Soldano 2015). Basically CNTs are in shape of cylindrical macromolecules composed of carbon atoms attracted astounding response from scientific community.

Over the last number of years, CNTs have became focus of interest amidst leading scientists from many research areas. At the same time, continuum mechanics has been engaged to examine various features of minuscule and nano-sized suchlike thermo mechanical investigations (Murmu and Adhikari 2010, Rafiee and Moghandam 2014), buckling (Chang *et al.* 2005, Wang *et al.* 2006, Lu *et al.* 2007) and free vibrations (Xu *et al.* 2008, Hu *et al.* 2012, Chang and Lee 2009, Avcar 2019) of CNTs. In recent times, some of the researchers made use of continuum shell model to inquire further advancements in CNTs (Li and Kardomateas 2007, Hu *et al.* 2012, Brischetto 2014). The theory of non-local elasticity happened to be intrinsic factor in continuum mechanics by accommodating the size dependency in nanostructures introduced by Eringen (Eringen 1983, 2002). Kolahchi and Cheraghbak (2017) studied with the nonlocal dynamic buckling analysis of embedded microplates reinforced by single-walled carbon nanotubes (SWCNTs). The material properties of structure are assumed viscoelastic based on Kelvin-Voigt model. Agglomeration effects are considered based on Mori-Tanaka approach. The elastic medium is simulated by orthotropic visco-Pasternak medium.

Kolahchi *et al.* (2017) focussed with general wave propagation in a piezoelectric sandwich plate. The core is consisted of several viscoelastic nanocomposite layers subjected to magnetic field and is integrated with viscoelastic piezoelectric layers subjected to electric field. The piezoelectric layers play the role of actuator and sensor at the top and bottom of the core, respectively. Batou *et al.* (2019) studied the wave propagations in sigmoid functionally graded (S-FG) plates using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. The current higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. Motezaker and Kolahchi (2017a)

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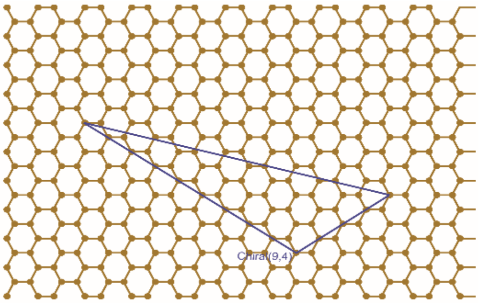
investigated the Seismic response of the concrete column covered by nanofiber reinforced polymer (NFRP) layer. The concrete column. The column is modeled using sinusoidal shear deformation beam theory (SSDT). Mori-Tanaka model is used for obtaining the effective material properties of the NFRP layer considering agglomeration effects. Using the nonlinear strain-displacement relations, stress-strain relations and Hamilton’s principle, the motion equations are derived.

Sharma *et al.* (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Motezaker and Kolahchi (2017b) presented the dynamic analysis of a concrete pipes armed with Silica ($ SiO_2 $) nanoparticles subjected to earthquake load. The structure is modeled with first order shear deformation theory (FSDT) of cylindrical shells. Mori-Tanaka approach is applied for obtaining the equivalent material properties of the structure considering agglomeration effects. Salah *et al.* (2019) examined a simple four-variable integral plate theory for studying the thermal buckling properties of functionally graded material (FGM) sandwich plates. The proposed kinematics considers integral terms which include the effect of transverse shear deformations. Motezaker *et al.* (2020) analysis the vibration, buckling and bending of annular nanoplate integrated with piezoelectric layers at the top and bottom surfaces. The higher order nonlocal theory for size effect and Gurtin-Murdoch theory for surface effects are utilized. The governing equations are derived based on the layer-wise (LW) theory and Hamilton’s principle. The differential cubature method (DCM) as a new numerical procedure is utilized to solve the motion equations for obtaining the frequency, buckling load and deflection. Hussain and Naeem (2017) examined the frequencies of armchair tubes using Flügge’s shell model. The effect of length and thickness-to-radius ratios against fundamental natural frequency with different indices of armchair tube was investigated. Motezaker *et al.* (2020) presented the post-buckling of a cut out plate reinforced through carbon nanotubes (CNTs) resting on an elastic foundation. Material characteristics of CNTs are hypothesized to be altered within thickness orientation which is calculated according to Mori-Tanaka model. For modeling the system mathematically, first order shear deformation theory (FSDT) is applied and using energy procedure, the governing equations can be derived.

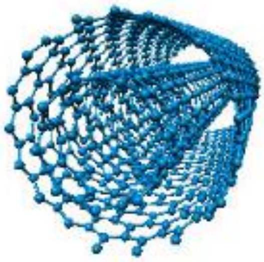
Gafour *et al.* (2020) focused the behavior of non-local shear deformation beam theory for the vibration of functionally graded (FG) nanobeams with porosities that may occur inside the functionally graded materials (FG) during their fabrication, using the nonlocal differential constitutive relations of Eringen. For this purpose, the developed theory accounts for the higher-order variation of transverse shear strain through the depth of the nanobeam. Motezaker and Eyvazian (2020) deals with the buckling and optimization of a nanocomposite beam. The agglomeration of nanoparticles was assumed by Mori-Tanaka model. The harmony search optimization algorithm is adaptively improved using two adjusted processes based on dynamic parameters. The governing equations were derived by Timoshenko beam model by energy method. The optimum conditions of the nanocomposite beam- based proposed AIHS are compared with several existing harmony search algorithms. Hussain and Naeem (2018a) used Donnell’s shell model to calculate the dimensionless frequencies for two types of single-walled carbon nanotubes. The frequency influence was observed with different parameters. Kolahchi and Bidgoli (2016) presented a model for dynamic instability of embedded single-walled carbon nanotubes (SWCNTs). SWCNTs are modeled by the sinusoidal shear deformation beam theory (SSDBT). The modified couple stress theory (MCST) is considered in order to capture the size effects. The surrounding elastic medium is described by a visco-Pasternak foundation model, which accounts for normal, transverse shear, and damping loads. The motion equations are derived based on Hamilton’s principle. Fatahi-Vajari *et al.* (2019) studied the vibration of single-walled carbon nanotubes based on Galerkin’s and homotopy method.This work analyses the nonlinear coupled axial-torsional vibration of single-walled carbon nanotubes (SWCNTs) based on numerical methods. Two-second order partial differential equations that govern the nonlinear coupled axial-torsional vibration for such nanotube are derived. Madani *et al.* (2016) presented vibration analysis of embedded functionally graded (FG)-carbon nanotubes (CNT)-reinforced piezoelectric cylindrical shell subjected to uniform and non-uniform temperature distributions. The structure is subjected to an applied voltage in thickness direction which operates in control of vibration behavior of system. Asghar *et al.* (2019a, b) conducted the vibration of nonlocal effect for double-walled carbon nanotubes using wave propagation approach. Many material parameters are varied for the exact frequencies of many indices of double-walled carbon nanotubes. Boulal *et al.* (2020) investigated the buckling behavior of carbon nanotube-reinforced composite plates supported by Kerr foundation model. In this foundation elastic of Kerr consisting of two spring layers interconnected by a shearing layer. The plates are reinforced by single-walled carbon nanotubes with four types of distributions of uniaxially aligned reinforcement material. The analytical equations are derived and the exact solutions for buckling analyses of such type’s plates are obtained. Hussain *et al.* (2017) demonstrated an overview of Donnell theory for the frequency characteristics of two types of SWCNTs. Fundamental frequencies with different parameters have been investigated with wave propagation approach. The use of nonlocal continuum mechanics evolved small scale effect which conferred the vibrational analysis of CNTs (Erigen 1972, Zang *et al.* 2005, Heireche *et al.* 2008, Ansari *et al.* 2012, Zidour *et al.* 2014, Benguediab *et al.* 2014). Kolahchi *et al.* (2016b) investigated the nonlinear dynamic stability analysis of embedded temperature-dependent viscoelastic plates reinforced by single-walled carbon nanotubes (SWCNTs). The equivalent material properties of nanocomposite are estimated based on the rule of mixture. For the carbon-nanotube reinforced composite (CNTRC) visco-plate, both cases of uniform distribution (UD) and functionally graded (FG) distribution patterns of SWCNT reinforcements are considered. The surrounding elastic medium is modeled by orthotropic temperature-dependent elastomeric medium. The viscoelastic properties of plate are assumed based on Kelvin-Voigt theory. The past research work found on nanotubes carried out by two main methodologies known as continuum mechanics and molecular dynamics (MD). The nonlocal elasticity theory has been extensively utilized for different types of nanostructures such as nano FGM structures (Jung and Han 2013, Kolahchi *et al.* 2015, Nejad *et al.* 2016) and static (Wang and Liew 2007, Pradhan and Reddy 2011, Eltaher *et al.* 2013). Kolahchi *et al.* (2017) studied the dynamic buckling of sandwich nano plate (SNP) subjected to harmonic compressive load based on nonlocal elasticity theory. The material properties of each layer of SNP are supposed to be viscoelastic based on Kelvin-Voigt model. In order to mathematical modeling of SNP, a novel formulation, refined Zigzag theory (RZT) is developed. Furthermore, the surrounding elastic medium is simulated by visco-orthotropic Pasternak foundation model in which damping, normal and transverse shear loads are taken into account. The double-walled CNT is coaxial nano structure in nature comprised of certainly two single-walled CNTs encapsulated one in other. The structure of double-walled CNT makes it rudimentary course of action to estimate the aftermath of inwall coupling hinge on the assorted substantial properties of CNTs. The MD simulation approach is a presentation of molecules of the materials which is distinct solution of Newton's classical equations of motion and has been efficiently exercised to analyze the dynamical properties of single-, double- and multi-walled CNTs (Cornwell and Wille 1997, Liew *et al.* 2005, Hao *et al*. 2008, Hu *et al.* 2008). On comparing the single-walled to double-walled CNTs, it is observed that double-walled CNTs demonstrate strong mechanical ability, thermal combat and effective electronic characteristics. Among armchair, chiral single and double-walled CNTs, limited work is done related to chiral especially double-walled CNTs. The stress aspects of single-walled CNT with respect to chiral dependency of the axial tensile strain was examined (Yoshikazm *et al.* 2005).

Arani and Kolahchi (2016) used a concrete material in construction industry it's been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. For the first time, the nonlinear buckling of straight concrete columns armed with single-walled carbon nanotubes (SWCNTs) resting on foundation is investigated in the present study. The column is modelled with Euler Bernoulli and Timoshenko beam theories. The characteristics of the equivalent composite being determined using mixture rule. The foundation around the column is simulated with spring and shear layer. They applied numerical simulation that worked with a tight binding and first principles density functional theory calculation depicting its authentication. Ghavanloo and Fazelzadeh (2009) studied the vibration frequency spectra of chiral CNT, Flugge shell theory was applied to obtain the isotropic elastic model. Free vibrations of double-walled CNT (Xu *et al.* 2008) showed explicitly the interaction of van der Waals forces being two exclusive beams. The small-scale diameters/aspect ratios were main focus for investigation which revealed the valid use of Donnell shell theory for vibration study (Hashemi *et al.* 2012). Zamanian *et al.* (2017) considered the use of nanotechnology materials and applications in the construction industry. However, the nonlinear buckling of an embedded straight concrete columns reinforced with silicon dioxide (SiO_2) nanoparticles is investigated in the present study. The column is simulated mathematically with Euler-Bernoulli and Timoshenko beam models. Agglomeration effects and the characteristics of the equivalent composite are determined using Mori-Tanaka approach. The foundation around the column is simulated with spring and shear layer.

Double-walled CNTs resonant frequencies were subjected to end layer wise conditions (Rouhi *et al.* 2013). The technique employed to obtain the numerical outcomes for ruling equations was radial point interpolation differential quadrature (RPIDQ) and proposed nonlocal Donnell shell theory which happened to justify the small scale effects. Kolahchi (2017) investigated the bending, buckling and buckling of embedded nano-sandwich plates based on refined zigzag theory (RZT), sinusoidal shear deformation theory (SSDT), first order shear deformation theory (FSDT) and classical plate theory (CPT). In order to present a realistic model, the material properties of system are assumed viscoelastic using Kelvin-Voigt model. Timoshenko beam model framed on nonlocal elasticity theory was utilized (Zidour *et al.* 2014), they performed a study on elastic bending of chiral single-walled CNT considering axial compression. Their work comprehensively covered the chirality of single-walled CNT, its vibrational mode and aspect ratio against the critical buckling load. Bilouei *et al.* (2016) used as concrete the most usable material in construction industry it's been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. For the first time, the nonlinear buckling of straight concrete columns armed with single- walled carbon nanotubes (SWCNTs) resting on foundation is investigated in the present study. The column is modelled with Euler-Bernoulli beam theory. Benguediab *et al.* (2014) inspected mechanical buckling characteristics of a zigzag double-walled CNT incorporated with chirality and small scale effect. Their findings revealed influential reliance of critical buckling load of zigzag CNT by using nonlocal Timoshenko beam model. Kolahchi *et al.* (2016a) concerned with the dynamic stability response of an embedded piezoelectric nanoplate made of polyvinylidene fluoride (PVDF). In order to present a realistic model, the material properties of nanoplate are assumed viscoelastic using Kelvin-Voigt model. The visco-nanoplate is surrounded by viscoelastic medium which is simulated by orthotropic visco-Pasternak foundation. The PVDF visco-nanoplate is subjected to an applied voltage in the thickness direction. Chemi *et al.* (2015) exhibited frequency vibrations of chiral double-walled CNT. The set of governing equations were modelled by nonlocal Euler Bernoulli beam theory. Recently Hussain and Naeem (2019a, b, c, d, 2020a) performed the vibration of SWCNTs



(a)



(b)

Fig. 1 Hexagonally description of chiral DWCNTs on the (a) graphene sheet (b) Rolled DWCNTs

based on wave propagation approach and Galerkin's method. They investigated many physical parameters for the rotating and non-rotating vibrations of armchair, zigzag and chiral indices. Moreover, the mass density effect of single walled carbon nanotubes with in-plane rigidity have been calculated for zigzag and chiral indices.

The foremost intension of this paper to investigate nonlocal vibrations characteristics of chiral double-walled CNT by means of Kelvin's model along with wave propagation technique, which is our intrinsic interest. The suggested method to investigate the solution of fundamental eigen relations, which is a well-known and efficient technique to develop the fundamental frequency equations. It is carefully observed from the literature, no information is seen regarding present established model where aforementioned problem has been considered so it became an incentive to proceed current study. The specific influence of three different end supports based on proposed method such as clamped-clamped, clamped-simply supported, simply supported-simply supported and clamped-free is examined in detail.

Many material researchers calculated the frequency of CNTs using different techniques, for example, Timoshenko beam model (Zidour *et al.* 2014), SiO_2 nanoparticles (Zarei *et al.* 2017, Amnieh *et al.* 2018, Jassas *et al.* 2019), Euler Bernoulli beam theory (Chemi *et al.* 2015), layerwise theory (Hajmohammad *et al.* 2018a, Hajmohammad *et al.* 2019), Flugge shell theory ((Zidour *et al.* 2014), Grey Wolf algorithm (Kolahchi *et al.* 2020), nonlocal Donnell shell theory (Rouhi *et al.* 2013), reinforced polymer layer (Hajmohammad *et al.* 2018b), agglomerated CNTs (agglomerated CNTs), zigzag theory (Kolahchi *et al.* 2017), viscoelastic cylindrical shell (Hosseini and Kolahchi 2018, Hajmohammad *et al* 2018c), and interpolation differential quadrature (Rouhi *et al.* 2013). The aim of current study is to delve for the free vibration characteristics of chiral double-walled CNTs by forming a nonlocal double-walled shell model (DSM). Erigen's nonlocal elasticity equations are acquired by adopting DSM subjected to small scale effect. The investigation is realized by employing the wave propagation approach due to its effective application in studying the structural vibrational analysis related to different parameters and end supports. The domination of numerous end supports alike DSM simply supported (DSM-SS), DSM clamped-supported (DSM-CS), DSM clamped-clamped (DSM-CC) and clamped free (DSM-CF) regarding disparate values of nonlocal parameter are explored numerically and reflected with help of graphs.

2. Formulation of governing nonlocal shell equations

When a graphene sheet is rolled with its hexagonal cells, the structure can be conceptualized as SWCNTs and its circumference and quantum properties depend upon the chirality and diameter described as a pair of (n, m). The indices pair occur during the rolling of tube Fig. 1 shows the schema of the pair indices as (m, n) which occurs on rolling of the tube and this pair of in indices formed as chiral, if $m \neq n$.

We will apply nonlocal orthotropic elastic shell model to analyze the wave propagation of CNTs. Surrounding medium of CNTs will be modeled by Kelvin model. We will develop nonlocal orthotropic Kelvin-like model by the combination of these models. We will use wave propagation approach to find the wave dispersion relations for CNTs in viscoelastic medium.

2.1 Nonlocal orthotropic Kelvin-like model

Cemal Eringen are pioneers of the nonlocal theory (Kröner 1967, Eringen 1972). For an elastic and homogeneous material the stress strain relationships are given below

$$\sigma_{ij,j} = 0 \tag{1}$$

$$\sigma_{ij}(x) = \int \varphi(|x' - x|, \psi) C_{ijkl}\varepsilon_{kl}(x')dV(x'),\ \forall\, x \in V \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

where, j denotes the derivative with respect to j, σ_{ij} and ε_{kl} are strain tensor and stress tensor respectively, and elastic modulus tensor is denoted by C_{ijkl}, u_i represents the displacements, the attenuation function is $\varphi(|x' - x|, \tau)$, and $|x' - x|$ denotes the usual distance. Also, $\psi = e_0 a/l$, where e_0 is a material constant, internal characteristics length is represented by a and l denotes the external characteristics length.

The differential form of Eq. (2) is used as nonlocal constitutive relation (Eringen 2002)

$$(1 - (e_o a)^2\nabla^2)\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{4}$$

where a is the internal characteristic length.

In this study we have taken $e_0 a$ as a single parameter, known as small scale parameter which represents the effect of size for the nano and micro structures, and ∇^2 is the

Laplace operator. Our coordinate system x, y and z are axial, circumferential and radial coordinates respectively whose dimensionless coordinates are $\alpha = x/R$, $\beta = y/R$ and $\gamma = z/R$.

Along α, β and γ directions, the displacement of middle surface are u, v and w, respectively. The geometrical relations are given by Flugge's shell theory (Flugge 1973, Zou and Foster 1995, Paliwal *et al.* 1995)

$$\varepsilon_\alpha = \frac{1}{R}\left(\frac{\partial u}{\partial \alpha} - \gamma \frac{\partial^2 w}{\partial \alpha^2}\right) \tag{5}$$

$$\varepsilon_\beta = \frac{1}{R}\left(\frac{\partial v}{\partial \beta} + w\right) - \frac{\gamma}{R(1+\gamma)}\left(\frac{\partial^2 w}{\partial \beta^2} + w\right) \tag{6}$$

$$\varepsilon_{\alpha\beta} = \frac{\gamma}{R(1+\gamma)}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} + 2\gamma\left(\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial \alpha \partial \beta}\right) + \gamma^2\left(\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial \alpha \partial \beta}\right)\right] \tag{7}$$

The stress-strain relationships in dimensionless coordinates derived from Eq. (4) is as under (Gao and An 2010)

$$\sigma_\alpha - (e_o a)^2 \nabla^2 \sigma_\alpha = E_1(\varepsilon_\alpha + \mu_1 \varepsilon_\beta)/(1-\mu_1\mu_2) \tag{8}$$

$$\sigma_\beta - (e_o a)^2 \nabla^2 \sigma_\beta = E_2(\varepsilon_\beta + \mu_2 \varepsilon_\alpha)/(1-\mu_1\mu_2) \tag{9}$$

$$\tau_{\alpha\beta} - (e_o a)^2 \nabla^2 \tau_{\alpha\beta} = G\varepsilon_{\alpha\beta} \tag{10}$$

where σ_α, σ_β and $\tau_{\alpha\beta}$ are normal and shear stresses, and ε_α, ε_β and $\varepsilon_{\alpha\beta}$ are respective strains; E_1 and E_2 are moduli of elasticity; Poisson's ratios in the directions of α and β are μ_2 and μ_1 respectively. G is modulus of rigidity or shear modulus. Also we have $E_1\mu_1 = E_2\mu_2$ and $\nabla^2 = (\partial^2/\partial\alpha^2 + \partial^2/\partial\beta^2)/R^2$ which is the Laplace operator in dimensionless coordinates. The element of tube in our coordinates is shown in Fig. 2, where (N, S, Q) are the stress resultants and (M) is the moment. The thermal expansion causes pre-stress, which is neglected because the present temperature is considered as the reference temperature. We arrive at the dynamic equilibrium equations

$$\begin{cases} \dfrac{\partial N_\alpha}{\partial \alpha} + \dfrac{\partial S_\beta}{\partial \beta} + \kappa = \rho h R \dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial N_\beta}{\partial \beta} + \dfrac{\partial S_\alpha}{\partial \alpha} + Q_\beta = \rho h R \dfrac{\partial^2 v}{\partial t^2} \\ \dfrac{\partial Q_\alpha}{\partial \alpha} + \dfrac{\partial Q_\beta}{\partial \beta} + N_\beta + p = \rho h R \dfrac{\partial^2 w}{\partial t^2} \end{cases} \tag{11}$$

$$\begin{cases} \dfrac{\partial M_{\alpha\beta}}{\partial \alpha} + \dfrac{\partial M_\beta}{\partial \beta} - RQ_\beta = 0 \\ \dfrac{\partial M_{\beta\alpha}}{\partial \beta} + \dfrac{\partial M_\alpha}{\partial \alpha} - RQ_\alpha = 0 \end{cases} \tag{12}$$

where ρ is the mass density.

Where p denotes the exerted pressure on i tube through van der Waals (*vdW*) interaction forces. The proposed vdW model accounts the effects of interlayer interactions between the tubes of double-walled CNT.

$$p = w_i \sum_{j=1}^{2} c_{ij} - \sum_{j=1}^{2} c_{ij} w_j \quad (i = 1,2) \tag{13}$$

c_{ij} is *vdW* coefficient, depicting the pressure increment contributing from ith to jth tube.

$$c_{ij} = \left[\frac{1001\pi\varepsilon\sigma^{12}}{3a^4} E_{ij}{}^{13} - \frac{1120\pi\varepsilon\sigma^6}{9a^4} E_{ij}{}^{7}\right] R_j \tag{14}$$

Here C-C bond length is given by $a = 1.42\dot{A}$, depth of potential by ε, σ as parameter concluded by equilibrium distance, R_j as radius of jth tube and $E_{ij}{}^m$ be as elliptic integral which is given as

$$E_{ij}{}^m = (R_j + R_i)^{-m} \int_0^{\pi/2} \frac{d\theta}{(1 - K_{ij}\cos^2\theta)^{m/2}} \tag{15}$$

being m as integer and coefficient K_{ij} is defined by

$$K_{ij} = \frac{4R_j R_i}{(R_j + R_i)^2} \tag{16}$$

The resultants (N, S, Q) are derived from above set of integral equations using the stress components.

$$(1 - (e_0 a)^2 \nabla^2) \begin{bmatrix} N_\alpha, S_\alpha, \\ M_\alpha, M_{\alpha\beta} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_\alpha, \tau_{\alpha\beta}, \\ z\,\sigma_\alpha, z\,\tau_{\alpha\beta} \end{bmatrix} \left(1 + \frac{z}{R}\right) dz \tag{17}$$

$$(1 - (e_0 a)^2 \nabla^2) \begin{bmatrix} N_\beta, S_\beta, \\ M_\beta, M_{\beta\alpha} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_\beta, \tau_{\beta\alpha}, \\ z\,\sigma_\beta, z\,\tau_{\beta\alpha} \end{bmatrix} dz \tag{18}$$

$$(1 - (e_0 a)^2 \nabla^2)(Q_\alpha, Q_\beta) = \int_{-\frac{h}{2}}^{\frac{h}{2}} [\tau_{\alpha z}, \tau_{\beta z}]\, dz \tag{19}$$

where h is thickness of the shell. Above equations result in

$$N_\alpha - (e_o a)^2 \nabla^2 N_\alpha = \frac{K}{R}\left[\frac{\partial u}{\partial \alpha} + \mu_1\left(\frac{\partial v}{\partial \beta} + w\right) - c^2 \frac{\partial^2 w}{\partial \alpha^2}\right] \tag{20}$$

$$N_\beta - (e_o a)^2 \nabla^2 N_\beta = \frac{Kk_1}{R}\left[\frac{\partial v}{\partial \beta} + \mu_2 \frac{\partial u}{\partial \alpha} + w + c^2\left(\frac{\partial^2 w}{\partial \beta^2} + w\right)\right] \tag{21}$$

$$S_\alpha - (e_o a)^2 \nabla^2 S_\alpha = \frac{Kk_2}{R}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} - c^2\left(\frac{\partial^2 w}{\partial \alpha \partial \beta} - \frac{\partial v}{\partial \alpha}\right)\right] \tag{22}$$

$$S_\beta - (e_o a)^2 \nabla^2 S_\beta = \frac{Kk_2}{R}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} + c^2\left(\frac{\partial^2 w}{\partial \alpha \partial \beta} + \frac{\partial v}{\partial \alpha}\right)\right] \tag{23}$$

$$M_\alpha - (e_o a)^2 \nabla^2 M_\alpha = -Kc^2\left[\frac{\partial u}{\partial \alpha} + \mu_1 \frac{\partial v}{\partial \beta} - \left(\frac{\partial^2 w}{\partial \alpha^2} + \mu_1 \frac{\partial^2 w}{\partial \beta^2}\right)\right] \tag{24}$$

$$M_\beta - (e_o a)^2 \nabla^2 M_\beta = Kk_1 c^2\left(\frac{\partial^2 w}{\partial \beta^2} + w + \mu_2 \frac{\partial^2 w}{\partial \alpha^2}\right) \tag{25}$$

$$M_{\alpha\beta} - (e_o a)^2 \nabla^2 M_{\alpha\beta} = 2Kk_2 c^2\left(\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial \alpha \partial \beta}\right) \tag{26}$$

$$M_{\beta\alpha} - (e_o a)^2 \nabla^2 M_{\beta\alpha} = Kk_2 c^2\left(\frac{\partial u}{\partial \beta} - \frac{\partial v}{\partial \alpha} + 2\frac{\partial^2 w}{\partial \alpha \partial \beta}\right) \tag{27}$$

$$Q_\alpha - (e_o a)^2 \nabla^2 Q_\alpha = \frac{Kc^2}{R}\begin{bmatrix} \dfrac{\partial^2 u}{\partial \alpha^2} - k_2 \dfrac{\partial^2 u}{\partial \beta^2} + (k_2 + \mu_1)\dfrac{\partial^2 v}{\partial \alpha \partial \beta} - \\ \dfrac{\partial^3 w}{\partial \alpha^3} - (2k_2 + \mu_1)\dfrac{\partial^3 w}{\partial \alpha \partial \beta^2} \end{bmatrix} \tag{28}$$

$$Q_\beta - (e_o a)^2 \nabla^2 Q_\beta = \frac{Kk_1 c^2}{R}\begin{bmatrix} 2\dfrac{k_2}{k_1}\dfrac{\partial^2 v}{\partial \alpha^2} - \dfrac{\partial^3 w}{\partial \beta^3} - \\ \dfrac{\partial w}{\partial \beta} - \left(2\dfrac{k_2}{k_1} + \mu_2\right)\dfrac{\partial^3 w}{\partial \alpha^2 \partial \beta} \end{bmatrix} \tag{29}$$

where $K = E_1 h/(1-\mu_1\mu_2)$, $k_1 = E_2/E_1$, $k_2 = G(1-\mu_1\mu_2)/E_1$,

$c^2 = h_o^3/(12R^2 h)$.

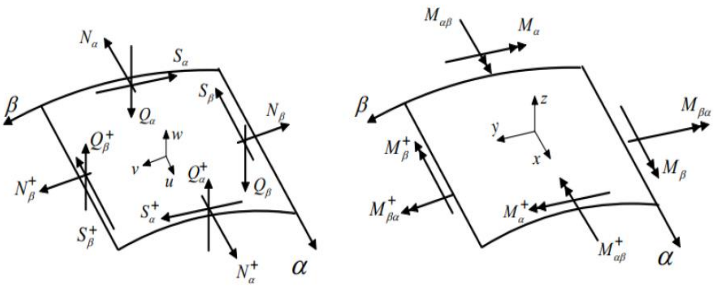


Fig. 2 Resolution of components of stress and moments of the middle surface of CNTs

Using Kelvin model and Eqs. (11) and (12), we get Kelvin-like nonlocal orthotropic elastic shell model.

The obtained model is as follows

$$\left[\frac{\partial^2}{\partial\alpha^2}+k_2(1+c^2)\frac{\partial^2}{\partial\beta^2}\right]u+\left[(\mu_1+k_2)\frac{\partial^2}{\partial\alpha\partial\beta}\right]v+\left[6+\frac{\partial}{\partial\alpha}+c^2\left(k_2\frac{\partial^3}{\partial\alpha\partial\beta^2}-\frac{\partial^3}{\partial\alpha^3}\right)\right]w=\frac{\rho hR^2[1-(e_0a)^2\nabla^2]}{K}\frac{\partial^2u}{\partial t^2} \tag{30}$$

$$\left[(\mu_1+k_2)\frac{\partial^2}{\partial\alpha\partial\beta}\right]u+\left[k_2(1+3c^2)\frac{\partial^2}{\partial\alpha^2}+k_1\frac{\partial^2}{\partial\beta^2}\right]v+\left[k_1\frac{\partial}{\partial\beta}-c^2(\mu_1+3k_2)\frac{\partial^3}{\partial\alpha^2\partial\beta}\right]w=\frac{\rho hR^2[1-(e_0a)^2\nabla^2]}{K}\frac{\partial^2v}{\partial t^2} \tag{31}$$

$$\left[\mu_1\frac{\partial}{\partial\alpha}-c^2\left(\frac{\partial^3}{\partial\alpha^3}-k_2\frac{\partial^3}{\partial\alpha\partial\beta^2}\right)\right]u+\left[k_1\frac{\partial}{\partial\beta}-c^2(\mu_1+3k_2)\frac{\partial^3}{\partial\alpha^2\partial\beta}\right]v+\left[\left(1+\frac{1}{c^2}\right)k_1+\frac{\partial^4}{\partial\alpha^4}+k_1\frac{\partial^4}{\partial\beta^4}+2k_1\frac{\partial^2}{\partial\beta^2}+(2\mu_1+4k_2)\frac{\partial^4}{\partial\alpha^2\partial\beta^2}\right]c^2w+\frac{R^2}{K}(1-(e_0a)^2\nabla^2)\left[Ew+\eta\frac{\partial w}{\partial t}\right]\left[W_i\sum_{j=1}^{2}C_{ij}-\sum_{j=1}^{2}C_{ij}W_j\right]=-\frac{\rho hR^2[1-(e_0a)^2\nabla^2]}{K}\frac{\partial^2w}{\partial t^2} \tag{32}$$

where $K=\frac{E_1h}{1-\mu_1\mu_2}$, medium has stiffness E, and the viscosity of the medium is η and the nonlocal parameter is $\Im=(e_oa)^2$. Two kinds of boundary conditions may be assumed while solving such problems. These three conditions are:

Clamped-clamped

$$\alpha=\beta=\gamma=\frac{\partial\gamma}{\partial\alpha}=0,\ \text{at}\ \alpha=0,\alpha=L/R \tag{33}$$

Clamped-free

$$\begin{cases}\alpha=\beta=\gamma=\frac{\partial\gamma}{\partial\alpha}=0 & \text{at}\ \alpha=0\\ N_{\alpha\alpha}=M_{\alpha\alpha}=N_{\alpha\beta}=M_{\alpha\beta}=0 & \text{at}\ \alpha=L/R\end{cases} \tag{34}$$

where L is the length of CNTs.

Using any combination of above three conditions we come close to nonlocal Flugge's shell model. Above system of equations is the nonlocal orthotropic Kelvin-like shell model for CNTs. To understand the waves propagating in CNTs, we need to derive the dispersion relations.

2.2 Application of wave propagation approach

Over the past several years vibration of nanostructures of various configurations and boundary conditions have been extensively studied (Hussain *et al.* 2018a, Hussain *et al*. 2018b, Hussain *et al*. 2018c, Hussain and Naeem 2018b; Hussain *et al.* 2019a, Hussain *et al.* 2019b, Hussain *et al.* 2020a, Hussain and Naeem 2020b, Sehar *et al.* 2020, Hussain *et al.* 2020b, c, d, Taj *et al.* 2020a, Taj *et al.* 2020a, b, c). Here, we will discuss wave solutions for single-walled carbon nanotubes.

The solutions of system of Eqs. (30)-(32) for axisymmetric waves is given by Wang and Gao (2016)

$$\begin{cases}u(\alpha,t)=U\mathrm{e}^{ik\left(\alpha-\frac{\nu t}{R}\right)}\\ v(\alpha,t)=V\mathrm{e}^{ik\left(\alpha-\frac{\nu t}{R}\right)}\\ w(\alpha,t)=W\mathrm{e}^{ik\left(\alpha-\frac{\nu t}{R}\right)}\end{cases} \tag{35}$$

where U, V and W are the amplitudes of waves along the direction of x, y and z respectively, the dimensionless wave vector in the longitudinal direction is $k=\frac{\pi mR}{L}$, in longitudinal direction m is the half axial wave number and ν is the wave phase velocity.

Substituting Eq. (35) in system of Eqs. (30)-(32) and simplifying, in matrix form, we get the following system

$$[M^{(1)}(k,\nu)]_{3\times3}\begin{bmatrix}U\\V\\W\end{bmatrix}=[0\quad0\quad0]^T$$

For the nontrivial solution of above equation, we have

$$Det\left[M^{(1)}(k,\nu)\right]=0 \tag{33}$$

4. Results and discussions

On the basis of established DSM by practicing wave propagation approach, the dominance of end conditions of double-walled CNTs is presented. Sundry studies can be seen for authentic application of present technique to conclude governing equation system of CNTs and to examine the fundamental frequency of double-walled CNTs (Wang *et al.* 2006, Xu *et al* 2008, Rouhi *et al.* 2011, Ansari *et al.* 2013). The procedure proposed in the previous section is here applied to study the size-dependent vibration behavior double-walled CNTs. Wave propagation approach is also applied to form the presented model, whereby the size-dependent effect is considered by means of the application of the Eringen's nonlocal differential model. Thus, the vibration phenomena of the nanostructure are solved mathematically via the suggested approach for different boundary conditions. The parametric study presented in this work analyzes the sensitivity of the size-

Table 1 Comparison of natural frequencies with Rakrak *et al.* (2016)

	Chiral	
(m, n)	Rakrak *et al.* 2016	Present
(12, 6)	0.94964	0.94964
(14, 6)	0.82448	0.82448
(16, 8)	0.95618	0.95618
(18, 9)	0.96508	0.96508
(20, 12)	0.97029	0.97029
(24, 11)	0.97648	0.97648

Table 2 Comparison of natural frequencies with experimental result of Raman Spectroscopy (RRS)

	f (THz)	
	Chiral	
(m, n)	(16, 7)	(18, 6)
RRS (Jorio *et al.* 2001)	4.617	4.317
Present	4.617	4.317
Difference %	4.85	4.04

dependent vibration response of double-walled CNTs mechanical parameters (i.e., the nonlocal parameter), as well as to some geometrical parameters, (namely, the length, radius and height. The preliminary focus of the investigation was on the precision of the proposed technique with existing model, whose results are summarized in Table 1, a very good match was observed, which confirms the accuracy of the proposed formulation for similar problems. The influence of aspect ratio and chirality are discussed in the Table 1. For comparison, various chiral indices against distinct length-to-diameter ratios determined by the non-local Euler Bernoulli beam model are recorded in Table 1. For the current work the non- dimensional frequency parameters are also calculated to evaluate convergence rate of end supports of both chiral CNTs. The results generated by present model solved with wave propagation approach seen to have accordance with those experimental outputs of Jorio *et al.* (2001) obtained by the deformation theory are specified in Table 2. The results obtained here specifically deal with the small scale effect versus length and thickness to radius of both tubes. For the purpose of numerical computations estimates of Young's modulus and Poisson's ratio are E=1TPA, ρ=2.3 g/cm^3 remain unchanged as used (Rouhi *et al.* 2011). Furthermore, inner tube radius R_1=8.5 nm and thickness to radius ratio are observed along with calibrated values of nonlocal parameter. Carbon nanotubes structure exhibit in forms as i) armchair, ii) zigzag, iii) chiral, for the ongoing investigation of vibration spectra of chiral CNT based on DSM are demonstrated together with end supports clamped-clamped (DSM-CC), clamped-simply supported (DSM-CS), simply supported-simply supported (DSM-SS) and clamped-free (DSM- CF). The curves in Fig. 3(a) and 3(b) are frequencies with nonlocal parameter e_0a=0.45 for chiral double-walled CNT with indices (15, 2) versus aspect ratio. The other parameters remain same for these calculations.

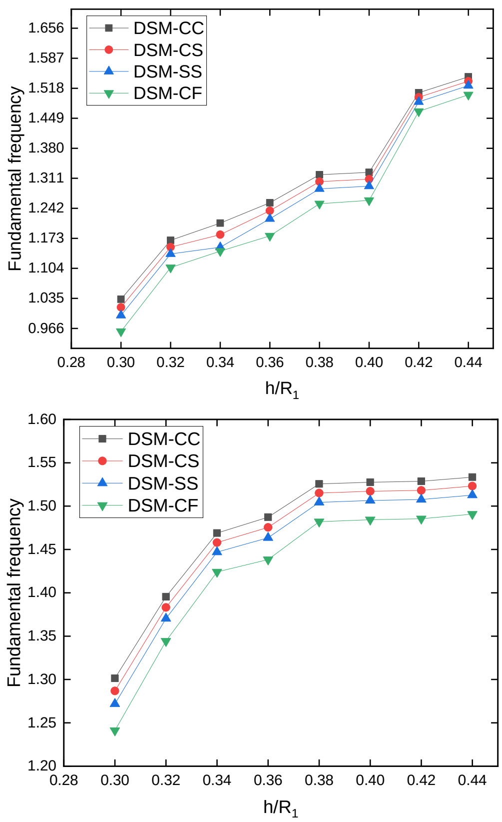


Fig. 3(a) Fundamental frequency for chiral double walled CNT (15, 2) against h/R_1 with e_0a=0.45

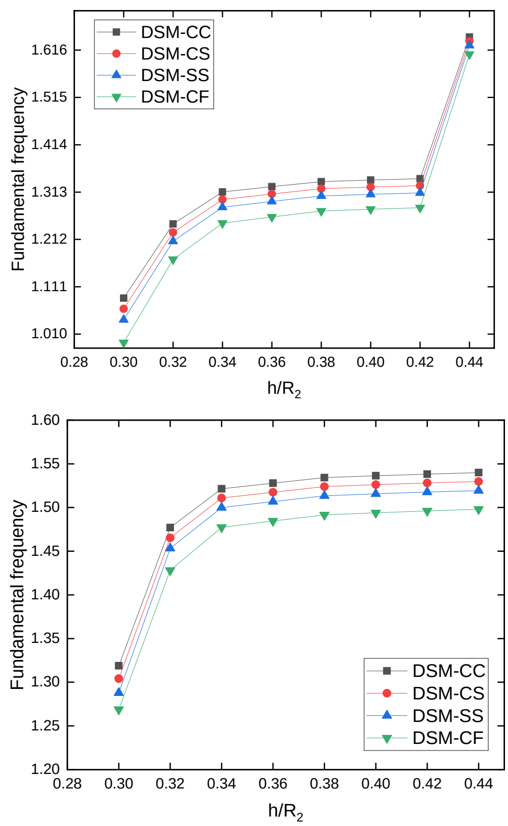


Fig. 3(b) Fundamental frequency for chiral double walled CNT (15, 2) against h/R_2 with e_0a=0.45

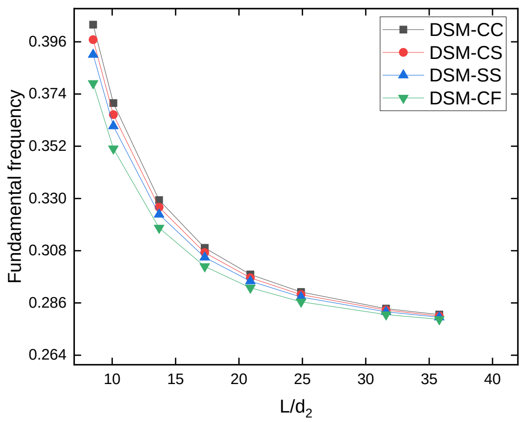


Fig. 4(a) Fundamental frequency for chiral double- walled CNT (11, 2) against L/d_2 with e_0a=0.45

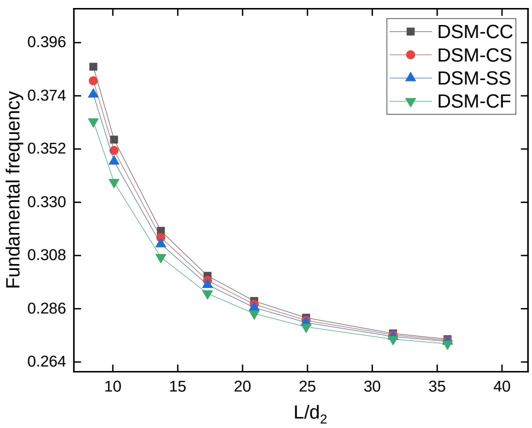


Fig. 4(b) Fundamental frequency for chiral double- walled CNT (11, 2) against L/d_2 with e_0a=0.90

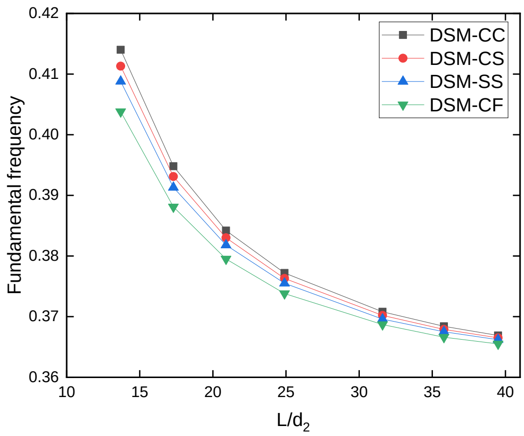


Fig. 5(a) Fundamental frequency for chiral double- walled CNT (9, 4) against L/d_2 with e_0a= 0.45

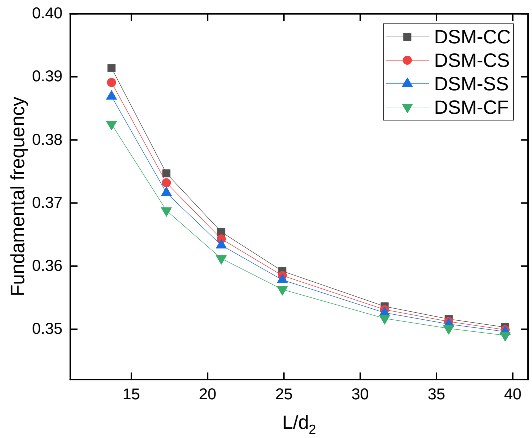


Fig. 5(b) Fundamental frequency for chiral double- walled CNT (9, 4) against L/d_2 with e_0a=0.90

The frequencies are displaying increasing pattern with an increase in indices of chiral double-walled CNT, as (11, 2) exhibited less values of frequencies, whereas (15, 2) have shown the higher frequencies. The boundary condition (DSM-CF) is at a minor difference from the other three conditions, on the other hand (DSM-CC), (DSM-CS) and (DSM-SS) possess the less difference. So as indices increases for chiral double-walled CNT the frequencies also increases. The same behaviour is noticed for both aspect ratios as for aspect ratio h/R_2 frequencies are followed by h/R_1.

The Fig. 4(a) and 4(b) are frequency curves of chiral double-walled CNT with indices (11,2), on the other hand Fig. 5(a) and 5(b) show the frequency graphs of (9, 4) chiral double-walled CNT. In Fig. 4(a) and 4(b) the calculated natural frequencies are shown against e_0a=0.45 and aspect ratio L/d_2 however, for Figs. 4(b) and 5(b) versus e_0a=0.90. The aspect ratio L/d_2 ranges from 13.7 to 39.5 for these four graphs subjected to clamped-clamped (DSM-CC), clamped supported (DSM-CS), simply supported -simply supported (DSM-SS) and clamped-free (DSM-CF). The natural frequency via aspect ratio L/d_2 descends as length-to-diameter expands subjected to all end supports. The difference between the conditions is obvious in beginning and vanishes as length of tube continues to extend. This behaviour and trend of curves reflects the fact that small scale effect becomes negligible for longer tubes and also as scale rises the frequency declines for all end supports. The frequency for chiral (9, 4) attains increasing frequency pattern over the chiral (8, 3) as the previous chiral doubled-walled CNTs have been observed.

5. Conclusions

In present study, influence of four end supports against length-to-diameter ratio and thickness-to-radius ratio with varying nonlocal parameter are discussed and shown graphically. The free vibration analysis of chiral double-walled CNTs is presented based on nonlocal Kelvin's model by exercising wave propagation approach. Different indices are considered against aspect ratio to show the diversity of vibration characteristic of chiral. According to the results, it is found that nonlocal effect is more prominent in increasing length of tubes for chiral double-walled CNTs. The fundamental frequency curves show an increasing pattern as indices of chiral increases. Also it is examined that if thickness- to -radius ratio expands, the frequency tend to increase too within specified range. In addition as length- to- diameter ratio increases the difference between end supports become negligible and meet all curves at end for respective four end conditions. The fundamental frequency curves displayed in article show the dependence of vibrations attributes on the chiral double -walled CNT and nonlocal parameter. The aspect ratio thickness-to-radius are compared with both radii, it is observed that h/R_1 show the lowest frequency as compare to h/R_2. The nonlocal parameter value with an increase show the reduced frequency for chiral. The adoption of wave propagation method sustain the fact that nonlocal effect is insignificant for longer carbon nanotubes and can be engage effectively for further work on triple-walled CNT.

Declaration of conflicting interests

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