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Optimal fiber volume fraction prediction of layered composite using frequency constraints- A hybrid FEM approach

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Abstract. In this research, a hybrid mathematical model is derived using the higher-order polynomial kinematic model in association with soft computing technique for the prediction of best fiber volume fractions and the minimal mass of the layered composite structure. The optimal values are predicted further by taking the frequency parameter as the constraint and the projected values utilized for the computation of the eigenvalue and deflections. The optimal mass of the total layered composite and the corresponding optimal volume fractions are evaluated using the particle swarm optimization by constraining the arbitrary frequency value as mass/volume minimization functions. The degree of accuracy of the optimal model has been proven through the comparison study with published well-known research data. Further, the predicted values of volume fractions are incurred for the evaluation of the eigenvalue and the deflection data of the composite structure. To obtain the structural responses i.e. vibrational frequency and the central deflections the proposed higher-order polynomial FE model adopted. Finally, a series of numerical experimentations are carried out using the optimal fibre volume fraction for the prediction of the optimal frequencies and deflections including associated structural parameter.

Keywords: HSDT; PSO; laminated composite; optimization; fiber volume fraction

1. Introduction

Usage of composites is increasing sharply in the household and industries like aerospace, automobile, biomedical etc. due to their excellent properties and reduction in weight penalty. The gaining popularity need better understanding for the design and analysis. The design variables (fiber volume fraction, thickness, fibre orientation and stacking sequence) including the responses (bending, vibration and buckling) are the important factors for the composite structure. Moreover, variation in one or more design variables may vary the final responses largely due to the change in property and geometry. Hence, to design the final composite structural components need complete understanding for the individual design variables. In this regard, a number of conventional techniques were adopted in the past to complete the task and found to be inefficient because of high solution time. Further, to achieve the computationally sound and predict the required variable a few new techniques are evolved every now and then. In this research, the authors' proposed the optimal fiber volume fractions (design variable) by considering the minimal mass of the composite. Moreover, the structural responses (maximum fundamental frequency and minimum bending deflection) are predicted using the elastic properties evaluated via the optimal volume fractions. The structural model derived using the higher-order shear deformation theory (HSDT) and particle swarm optimization (PSO) for optimal variable prediction. optimization technique adopted for the current reused in this paper is, because of its simplicity in construction and operation. The necessary mathematical relations are derived using rule of mixture and.

The optimization techniques have been adopted for different purposes i.e. to evaluate the optimal frequency data of the composite structure (Bargh and sadr 2012, Sadr and Bargh 2012, Apalak *et al*. 2011, Narita 2003, Le-Anh *et al*. 2015, Topal 2012, Apalak *et al*. 2014) including technique and kinematic theories (finite element method, FEM; first-order shear deformation theory, FSDT; cell-based smoothed discrete shear gap method, Ritz method, finite strip method, classical laminated plate theory). Additionally, a few optimization techniques (genetic algorithm, GA; particle swarm optimization, PSO; artificial bee colony algorithm, ABC; Elitist-Genetic algorithm, layer wise optimization approach, adjusted differential evolution, ADE) for the accurate prediction of the responses.

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Similarly, the buckling analysis is carried out (Narita and Turvey 2004, Ho-Huu *et al.* 2016, Aymerich and Serra 2008, Jing *et al.* 2015, Adali *et al.* 1995, Topal and Uzman 2008b) to predict critical load capacity of the composite structure via different optimization techniques in association with numerical methods. Further, the stacking sequence optimization (Cho 2013, Hwang *et al.* 2014, Topal and Uzman 2008a, Topal *et al.* 2017, Le Riche and Haftka 1993, Haftka and Walsh 1992, Todoroki and Haftka 1998, Lin and Lee 2004, Nagendra *et al.* 1992) has also been predicted to obtain the best possible ply sequences for the better performances of the layered structure. Ho-Huu *et al.* (2016) used the adaptive elitist differential evolutionary evaluation technique for the optimization of truss structures with discrete design variables. Moreover, the optimal design of the truss structure is reported by Sonmez (2011) using the artificial bee colony optimization technique. Tracking and optimizing the dynamic systems with particle swarm optimization is performed by Eberhart and Shi (2001). Similarly, the dynamic buckling of the sandwich nanocomposite plate structure evaluated using the grey algorithm in association with sinusoidal-visco-piezoelasticity theories by Kolahchi *et al.* (2017). Further, the nonlocal theories adopted to predict the frequency and eigenvalue critical load of the viscoelastic sandwich nano-plates by Kolahchi *et al.* (2017) using the differential cubater (DC), harmonic differential quadrature (HDQ) and differential quadrature (DQ) for the comparison purpose. Vibration and bending deflections of the multilayered plate and shell structures are analyzed using a novel higher-order shear deformation theory (Zine *et al.* 2018). Similarly, the higher and normal-order kinematic models are also adopted by Bousahla *et al.* (2014) to perform the bending deflections and listed the required conclusions.

Liu and Paavola (2016), described a new methodology to find the light weight design of the composite structure using the available optimization techniques by varying the design variables i.e., the fiber volume fractions and fiber orientations along with the frequency constraints. Kave and Ghazaan (2015) adopted two different optimization techniques in association with PSO i.e., Aging Leader and Challengers (ALC-PSO) and harmony ALC-PSO to evaluate the corresponding responses. Similarly, a global numerical approach (adaptive elitist differential evolution in association with cell-based smoothed discrete shear gap method) has proposed by Vo-Duy *et al*. (2017) to design and optimize the lightweight composite plate structure utilizing the frequency constraint.

The survey of literature indicates clearly that the development of the hybrid kind of model i.e., the numerical technique in association with soft computing techniques are already in use for the analysis and subsequent optimal parameter as per the requirement. However, it’s important to indicate that the major research completed using the finite element method (FEM) and soft computing technique majorly adopted the first-order/classical type of deformation kinematics. Additionally, the research relevant to the composite analysis confirms the inevitability of the HSDT displacement field instead of the FSDT. Hence, the current research objective for the optimal mass evaluation of the layered composite structure is proposed a higher-order displacement finite element model in conjunction with PSO considering the modal values as the constraint. In this regard, the objective function has been constructed by considering the minimization of mass with optimal fiber volume fractions (design variable). Further, the derived model i.e. the adequate optimal mass including the fibre volume fraction have been utilized to compute the necessary frequency and deflection parameter. Also, the research outcome includes the minimal mass of the variable layer numbers i.e., four and eight layers of square plate geometry to compute the corresponding structural responses (free vibration frequency and deflection) under the influence of different end boundary conditions.

2. Volume fraction formulation and necessity

The mechanical properties for any composite structure are directly related to their fiber volume fractions and the corresponding mathematical relations adopted as same as the sources (Jones 2014, Gay and Hoa 2007, Mallick 2007, Reddy 2004, Liu 2015)

$$
\begin{aligned}
&E_1 = E_f V_f + E_m\left(1-V_f\right) && E_2 = \frac{E_f E_m}{E_f\left(1-V_f\right)+E_m V_f}\\
&\upsilon_{12} = \upsilon_{21} = \upsilon_f V_f + \upsilon_m\left(1-V_f\right) && G_{12} = \frac{G_f G_m}{G_f\left(1-V_f\right)+G_m V_f}\\
&G_f = \frac{E_f}{2(1+\upsilon_f)} && G_m = \frac{E_m}{2(1+\upsilon_m)}
\end{aligned}
\tag{1}
$$

where, E_f and E_m are the moduli of elasticity of the fibre and matrix, respectively. Additionally, ‘V_f’ fiber fraction, whereas ν_f and ν_m are Poisson’s ratios, respectively. Also, the properties can be obtained the elasticity approach, which are intrinsic in nature. The properties are consisting of the parameter i.e., E_1 and ν_{12} are similar to the mechanics of materials approach, however, E_2 and G_{12} may vary from the predicted values (Vinson and Sierakowski 1987, Kisa 2004, Vinson and Sierakowski 2002, Krawczuk *et al.* 1997, Altenbach *et al.* 2004) and presented in the following new sets of equation

$$
\begin{aligned}
&E_2 = E_m\left(\frac{E_f+E_m+\left(E_f-E_m\right)V_f}{E_f+E_m-\left(E_f-E_m\right)V_f}\right)\\
&G_{12} = G_m\left(\frac{G_f+G_m+\left(G_f-G_m\right)V_f}{G_f+G_m-\left(G_f-G_m\right)V_f}\right)\\
&G_{12} = G_{13}
\end{aligned}
\tag{2}
$$

Moreover, this approach also defined Poisson’s ratio ν_{23} and shear modulus G_{23} as follows

$$
\begin{aligned}
&\upsilon_{23} = \upsilon_f V_f + \upsilon_m\left(1-V_f\right)\left(\frac{1+\upsilon_m-\upsilon_{12}E_m/E_1}{1-\upsilon_m{}^2+\upsilon_{12}\upsilon_m E_m/E_1}\right)\\
&G_{23} = \frac{E_2}{2\left(1+\upsilon_{23}\right)}
\end{aligned}
\tag{3}
$$

This can be easily visualized from the Eqs. (1)-(3), the fiber volume fractions are associated with all individual

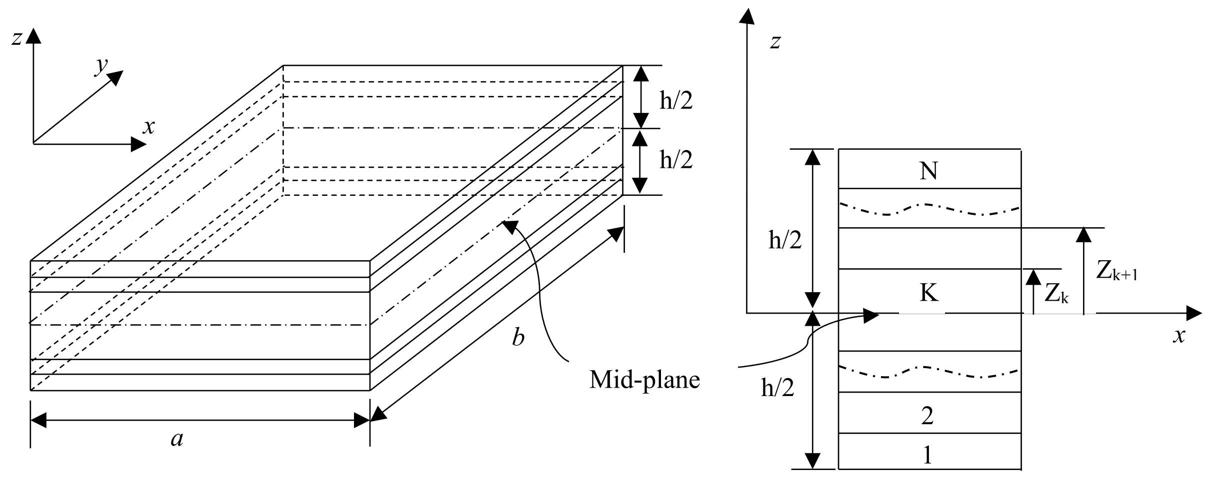

Fig. 1 Representation of laminated composite

properties. Hence, the evaluation of the optimal fibre fractions will be an important data for to obtain the best possible weight of the composite to reduce the weight penalty and improve the final structural performances.

2.1 Mass of the composite

Now the mass and fiber volume fraction of the composite are related as (Vo-Duy *et al.* 2017)

$$m\left(V_f, h\right) = \sum_{k=1}^{N} \rho^{(k)} . V_f^{(k)} \Omega_k . h^k \tag{4}$$

Subjected to $f\left(V_f, h\right) \geq f_1$

$$0 < V_f^{(k)} < V_f^{\max}$$

Where $\rho^{(k)}$ is the mass density of the kth layer and is given by (Vo-Duy *et al.* 2017, Jones 2014, Gay and Hoa 2007, Mallick 2007, Reddy 2004, Liu 2015)

$$\rho^{(k)} = \rho_f V_f^{(k)} + \rho_m \left(1 - V_f^{(k)}\right) \tag{5}$$

where V_f and h are the design variable vectors of fiber volume fractions $V_f^{(k)}$ and thickness h^k of layers, respectively; N is the number of layers; Ω_k is the area of the kth layer (Fig. 1)

To handle constraints for the optimization problem Eq. (4), we use the penalty function as presented in (Kaveh and Ghazaan 2015). Therefore, the optimization problem is redefined as follows

$$M_{penalty}\left(V_f, h\right) = (1 + \in_1 .\upsilon)^{\in_2} \times m\left(V_f, h\right) \tag{6}$$

Where $\upsilon = \max\{0, (f_1 - f(V_f, h)\}$, $\in_1$ and $\in_2$ are the constants which show the exploration and the exploitation rates of the search space. In this paper, the value of $\in_1$ is set to be 1, and $\in_2$ starts from 20 and then linearly increases to 40 (Ho-Huu *et al.* 2016).

2.2 Frequency analysis

The plate model has been established with the assumption that the plate is composed of the finite number of orthotropic layers of uniform thickness as depicted in Fig. 1. The dimensions of the plate are, i.e., length a, width b in x and y-direction, respectively, whereas h defined as the thickness of the component in z direction.

2.2.1 Displacement field

In this study, a higher-order shear deformable model has been utilized for the modeling of the laminated composite plate. The displacement field is considered as same as the source (Patle *et al.* 2018) (the displacement variation throughout the thickness is assumed to be constant thus giving zero transverse normal strain)

$$\left.\begin{aligned} \bar{u}(x,y,z) &= u(x,y) + z\theta_1(x,y) + z^2\lambda_1(x,y) + z^3\psi_1(x,y) \\ \bar{v}(x,y,z) &= v(x,y) + z\theta_2(x,y) + z^2\lambda_2(x,y) + z^3\psi_2(x,y) \\ \bar{w}(x,y,z) &= w(x,y) \end{aligned}\right\} \tag{7}$$

Here, the displacement of any arbitrary point in the plate geometry is defined as $\bar{u}$, $\bar{v}$ and $\bar{w}$ along x, y and z direction. Additionally, the above-mentioned equation associated with u, v and w are the mid-plane displacement values and the rotations i.e., θ_1 and θ_2 are the rotation of normal to the mid-plane about y and x direction, respectively. Further, the equation also consist of some more terms θ_3, λ_1, λ_2, ψ_1 and ψ_2 are known to be the higher-order terms encompassed from Taylor's series expansion for the necessary parabolic distribution of shear stress through the entire thickness of the plate.

2.2.2 Constitutive relation

For any k^{th} layer laminae, the stress-strain relationship (Jones 2014) is expressed mathematically as below by considering an arbitrary angle 'Θ' of the fiber orientation as

$$\left\{\sigma_{ij}\right\} = \left[\bar{Q}_{ij}\right]\left\{\varepsilon_{ij}\right\} \tag{8}$$

where, $\{\sigma_{ij}\}$, $[\bar{Q}_{ij}]$ and $\{\varepsilon_{ij}\}$ are the stress matrix, the elastic property matrix and the strain matrix respectively. Further, the elastic property matrix and strain matrix can be explored as

$$\left[\bar{Q}_{ij}\right]=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & \bar{Q}_{26}\\ 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0\\ 0 & 0 & \bar{Q}_{54} & \bar{Q}_{55} & 0\\ \bar{Q}_{16} & \bar{Q}_{26} & 0 & 0 & \bar{Q}_{66}\end{bmatrix} \tag{9}$$

$$\{\varepsilon_{ij}\}=\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{Bmatrix}=\left(\frac{\partial\bar{u}}{\partial x}\quad \frac{\partial\bar{v}}{\partial y}\quad \frac{\partial\bar{v}}{\partial z}+\frac{\partial\bar{w}}{\partial y}\quad \frac{\partial\bar{u}}{\partial z}+\frac{\partial\bar{w}}{\partial x}\quad \frac{\partial\bar{u}}{\partial y}+\frac{\partial\bar{v}}{\partial x}\right) \tag{10}$$

2.2.3 Finite element formulation

FEM provides the accurate numerical solutions for any critical engineering problems with least possible errors. Hence, in the current analysis the geometry of the structural component is modeled using the FEM with the help of an available isoparametric quadrilateral Lagrangian element and nine degrees of freedom per node. Now, the new displacement vector at any point on the mid-surface for the proposed model will be written in the following form

$$\{\delta\}=\left[u\ v\ w\ \theta_1\ \theta_2\ \lambda_1\ \lambda_2\ \psi_1\ \psi_2\right]^T=\sum_{i=1}^{9}\left[N_i\right]\{\delta_i\} \tag{11}$$

$[N_i]$ represents the interpolation function and the displacement field vector for the i^{th} node as $\{\delta_i\}$The nodal points generally signify the physical characteristics and the shape functions of the current element (Cook *et al.* 2003). Now, the mid-plane strain vector can be written as

$$\{\varepsilon\}_i=\left[B_i\right]\{\delta_i\} \tag{12}$$

where, $[B_i]$ represents the strain displacement relation matrix in accordance with the type of model.

2.2.4 Energy calculation

The strain energy (S) of the laminate can be expressed as

$$S=\frac{1}{2}\iint\left[\int_{-h/2}^{+h/2}\{\varepsilon\}^T\{\sigma\}\,dz\right]dxdy \tag{13}$$

Similarly, the kinetic energy (V) of the free vibrated composite panel can be expressed as

$$V=\frac{1}{2}\iint\left\{\sum_{k=1}^{N}\int_{x_{3k-1}}^{x_{3k}}\rho^k\left\{\dot{d}\right\}^T\left\{\dot{d}\right\}dz\right\}dxdy \tag{14}$$

where, expression ρ and $\{\dot{d}\}$ denotes the mass density and the velocity vector, respectively.

The elemental stiffness matrix ($[K]$) and the mass matrix ($[M]$) can be expressed as following

$$[K]=\int_A\left(\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}[B_L]^T[D][B_L]dz\right)dA$$

$$[M]=\int_A\left(\sum_{k=1}^{n}\int_{x_{k-1}}^{x_k}[N]^T[N]\rho dz\right)dA \tag{15}$$

The work done due to externally applied transverse mechanical loading is expressed in the following lines

$$W=\int_A\{\delta\}^T\{F\}\,dA \tag{16}$$

Now, the governing equation is obtained by reducing the order of the total energy using to Hamilton's principle and denoted as in (Cook *et al.* 2003)

$$\delta\int_{t_1}^{t_2}(V-S)dt=0 \tag{17}$$

Substituting Eqs. (13) and (14) into Eq. (17), the final form of the equation will be conceded as

$$[M]\left\{\ddot{\delta}\right\}+[K]\{\delta\}=0 \tag{18}$$

where, $\ddot{\delta}$ is the acceleration, δ is the displacement.

The natural frequency of the system is computed from the eigenvalue solution of the proposed equation and summarized as

$$\left([K]-\omega^2[M]\right)\Delta=0 \tag{19}$$

where ω and $\Delta$are the natural frequency and the corresponding eigen vector, respectively.

3. Bending analysis

The bending test is performed to find the optimum bending deflection by taking the optimal volume fiber fraction values that are obtained in optimal mass calculations. The governing equation of the bending analysis of the layered composite plate is obtained via variational principle and can be expressed as

$$\partial\Pi=\partial(V-W)=0 \tag{20}$$

Now, substituting the values of energy and work-done part in Eq. (20) and presented above equation, it can be re written as

$$[K]\{\delta\}=\{F\} \tag{21}$$

Eqs. (19) and (21) are solved by using the given sets of boundary conditions shown in Table 1 to avoid rigid body movements and to reduce the number of unknowns.

4. Particle swarm optimization

Table 1 End constraint of structural component

End conditions	Longitudinal i.e., x=0, a	Transverse i.e., y=0, b
Simply-support (S):	$v=w=\theta_2=\lambda_2=\psi_2=0$	$u=w=\theta_1=\lambda_1=\psi_1=0$
Clamped (C):	$u=v=w=\theta_1=\theta_2=\lambda_1=\lambda_2=\psi_1=\psi_2=0$	
Free (F):	$u\neq v\neq w\neq\theta_1\neq\theta_2\neq\lambda_1\neq\lambda_2\neq\psi_1\neq\psi_2\neq 0$	

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy in 1995, inspired by social behavior of bird flocking or fish schooling. PSO learned from the scenario and used it to solve the optimization problems. In PSO, each single solution is a "bird" in the search space. We call it "particle". All of particles have fitness values which are evaluated by the fitness function to be optimized, and have velocities which direct the flying of the particles. The particles fly through the problem space by following the current optimum particles.

PSO is initialized with a group of random particles (solutions) and then searches for optima by updating generations. In every iteration, each particle is updated by following two "best" values. The first one is the best solution (fitness) it has achieved so far. (The fitness value is also stored.) This value is called pbest. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far by any particle in the population. This best value is a global best and called gbest. When a particle takes part of the population as its topological neighbors, the best value is a local best and is called lbest.

After finding the two best values, the particle updates its velocity and positions with following equations (a) and (b).

v[] = v[] + c1 * rand() * (pbest[] - present[]) + c2 * rand() * (gbest[] - present[]) (22)

present[] = persent[] + v[] (23)

v[] is the particle velocity, persent[] is the current particle (solution). pbest[] and gbest[] are defined as stated before. rand () is a random number between (0,1). c1, c2 are learning factors. usually c1=c2=2.

The pseudo code of the procedure is as follows

```
For each particle
    Initialize particle
END
Do
    For each particle
        Calculate fitness value
        If the fitness value is better than the best fitness value (pBest) in history
            set current value as the new pBest
    End
    Choose the particle with the best fitness value of all the particles as the gBest
    For each particle
        Calculate particle velocity according equation (a)
        Update particle position according equation (b)
    End
While maximum iterations or the minimum error criteria is attained.
```

5. Numerical examples

Initially, a square plate of geometrical dimension i.e., 2×2×0.04 m has been utilized for the analysis comparison purpose and the input data are as same as the source (Liu and Paavola 2016, Vo-Duy *et al.* 2017). The square plate with eight numbers of layer i.e., N=8 including the matrix material (QY9511) properties i.e., E_m=4.2 GPa, v_m=0.3, ρ_m=1240 Kg/m^3 and the fiber (T800H) properties are (E_f=294 GPa, v_f=0.2, ρ_f=1810 Kg/m^3) incurred for the current computational purpose. Further, the example includes the fiber orientations of symmetric type of laminate i.e., $[0°/90°/45°/-45°]_s$. The present results and the reference data are provided in Table 2, which indicates the necessary agreement of the proposed higher-order model considering the frequency constraint.

Table 2 Optimal results of the both the methods

Design constraint	Methodology	Optimal mass (Kg)
f_1=10 Hz	Present	211.949
	Liu and Paavola (2016)	211
	Vo-Duy *et al*. (2017)	209.477
f_1=5 Hz	Present	199.5844
	Liu and Paavola (2016)	201
	Vo-Duy *et al*. (2017)	200.219

Similarly, the optimal mass of the composite has been obtained using the current model with the help of input data i.e., glass/epoxy composite with properties of fiber: E_f=71 GPa, v_f=0.22, ρ_f=2450 Kg/m^3 and matrix: E_m=3.5 GPa, v_m=0.33, ρ_m=1540 Kg/m^3. The optimal mass including the individual layer fibre fractions are presented in the tabular form (Table 3) for the different support conditions considering the frequency as 10 Hz. Moreover, the results are computed for different end boundary conditions (CCCC, SSSS, SCSC, CFCF, CFFF) and different thicknesses values of the plate structure.

Table 3 Optimal mass of total composite and individual layer (f_1=10 Hz)

Boundary conditions	Optimal mass (Kg)	Thickness of the plate (mm)	Optimal fiber volume fraction of layers (%)			
CCCC	0.8328	6	0.2683	0.7716	0.1952	0.2985
SSSS	1.3846	10	0.1768	0.3793	0.7932	0.1585
SCSC	0.9301	8	0.6221	0.1841	0.2116	0.3231
CFCF	0.8354	7	0.4481	0.1823	0.5603	0.1869
CFFF	4.0568	30	0.6734	0.2043	0.4762	0.1564

5.1 New results

Based on the comprehensive testing of the derived model, it is now extended to compute the optimal parameter for the square plate component with geometrical dimension i.e., 0.3×0.3×0.004 (0.002) m. Moreover, the values of the natural frequencies and the deflection parameter evaluated for the optimal data including different layers i.e., N=8 and N=4, respectively. The material properties of Glass/epoxy as same as the reference (Vo-Duy *et al.* 2017) adopted for to evaluate the optimum fiber volume fractions of the composite.

5.1.1 Vibration and bending analysis of a composite plate

5.1.1.1 Case 1

A square plate of 0.3×0.3×0.002 m and N=4 is considered in this case. Using mass and fiber volume

Table 4 Predicted optimal mass and optimal fiber volume fractions of four layered composite using PSO

Optimal mass (in Kg)	Optimal fiber volume fraction of layers			
0.1743	0.3568	0.178	0.178	0.3568

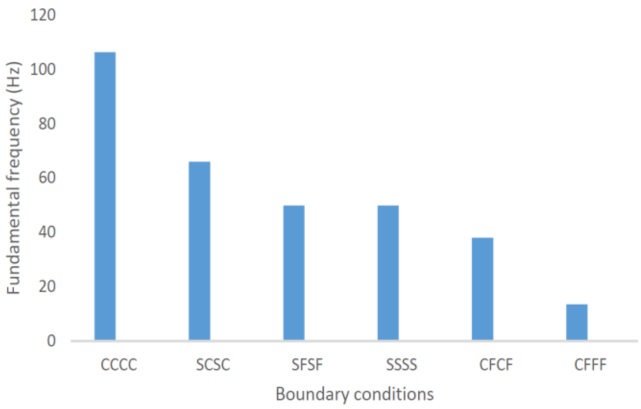

Fig. 2 Variation of fundamental frequency for boundary conditions of a four layered composite structure

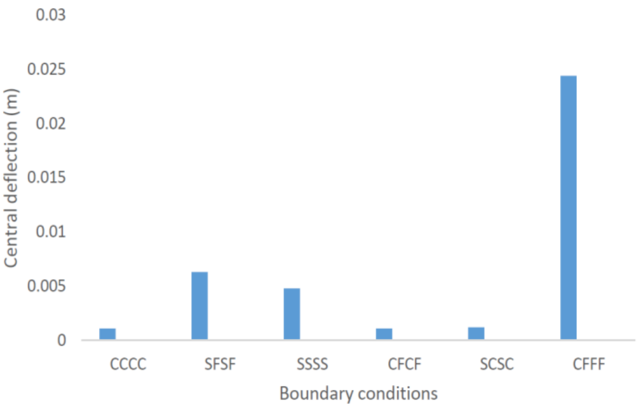

Fig. 3 Variation of central deflection for boundary conditions of a four layered composite structure

fraction relation as discussed above, optimal values of fiber volume fractions (Table 4) are found and then by using these optimal fiber volume fractions, maximum fundamental frequency and minimum bending deflection are found under the effect of different boundary conditions (CCCC, SSSS, SCSC, CFCF, CFFF). The variation of fundamental frequency and minimum bending deflection (for load=1000 N) with boundary condition are plotted as shown in Figs. 2-3 respectively. From graph it is clear that both fundamental frequency and central deflection are more for clamped condition (CCCC) and minimum for cantilever condition (CFFF).

5.1.1.2 Case 2

Now, the same analysis has been continued for eight layered composite square structure with 0.3×0.3×0.004 m dimensions instead of four layers and the results of the optimal mass including the optimal fiber volume fractions (Table 5) reported. Figs. 4-5 are showing the values of the maximum fundamental frequencies and the minimum bending deflections (for load=1000 N), respectively.

Table 5 Predicted optimal mass and optimal fiber volume fraction of eight layered composite using PSO

Optimal mass (in Kg)	Optimal fiber volume fraction of layers							
0.6985	0.4716	0.8509	0.2589	0.2899	0.2899	0.2589	0.8509	0.4716

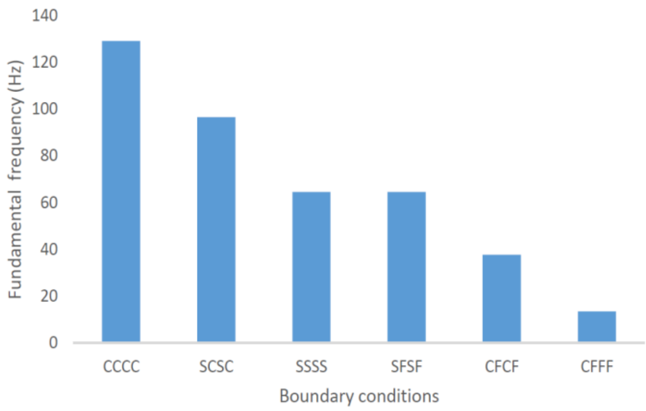

Fig. 4 Variation of fundamental frequency for boundary conditions of a eight layered composite structure

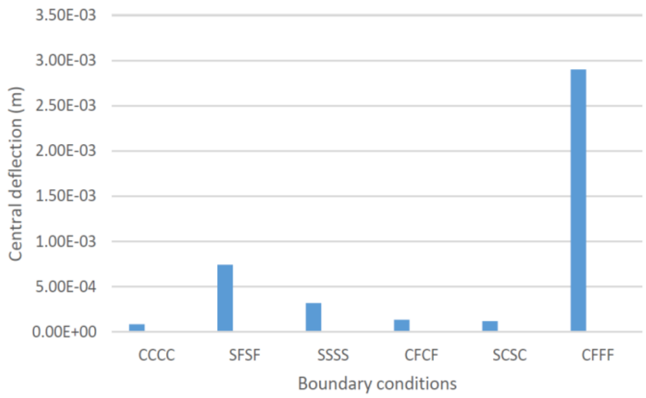

Fig. 5 Variation of central deflection for boundary conditions of a eight layered composite structure

6. Conclusions

A hybrid model (which combines FEM with Soft computing techniques) with an objective is to minimize the mass of the composite structures and to get the corresponding optimal fiber volume fractions by including the frequency constraint developed in this study. In order to analyze the composite structures for frequency and bending, a higher-order FE model in association with PSO technique is adopted. Initially, the accuracy of the proposed model is checked by comparing the results with published data. Further, the model has been extended to evaluate frequency and deflection parameters using the corresponding predicted optimal values. Also, the analysis includes the number of layers since the structural stiffness largely depends on the thickness parameter. The new results obtained for the different boundary conditions considering the condition of the high frequency and low deflection for any structure as same as the real service life. In general, the results follow the expected line i.e., the frequency is increasing while the number of constraint increases whereas a reverse line observed for the deflection parameter. Additionally, the optimal data are predicted for two cases of layer numbers (four and eight layer) including the necessary structural parameter. From the results it can be concluded that the current model is good enough to utilized for the layered composite structure for the optimal prediction with adequate accuracy.

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