Computers and Concrete, *Vol. 25, No. 3 (2020) 245-253*
DOI: https://doi.org/10.12989/cac.2020.25.3.245

Non-local orthotropic elastic shell model for vibration analysis of protein microtubules

Muhammad Taj[1], Afnan Majeed[1], Muzamal Hussain*[3], Muhammad N. Naeem[3], Muhammad Safeer[1,2], Manzoor Ahmad[1], Hidayat Ullah Khan[4] and Abdelouahed Tounsi[5,6]

[1]Department of Mathematics, University of Azad Jammu and Kashmir, Muzaffarabad, 1300, Azad Kashmir, Pakistan
[2]Department of Mathematics, University of Poonch, Rawalwkot 12350, Azad Kashmir, Pakistan
[3]Department of Mathematics, Govt. College University Faisalabad, 38000, Faisalabad, Pakistan
[4]Department of Mathematics, University of Malakand at Chakdara, Dir (Lower), Khyber Pakhtoonkhwa, Pakistan
[5]Materials and Hydrology Laboratory, University of Sidi Bel Abbes, Algeria Faculty of Technology Civil Engineering Department
[6]Department of Civil and Environmental Engineering, King Fahd University of Petroleumand Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia

(*Received February 6, 2020, Revised February 28, 2020, Accepted February 29, 2020*)

Abstract. Vibrational analysis in microtubules is examined based on the nonlocal theory of elasticity. The complete analytical formulas for wave velocity are obtained and the results reveal that the small scale effects can reduce the frequency, especially for large longitudinal wave-vector and large circumferential wave number. It is seen that the small scale effects are more significant for smaller wave length. The methods and results may also support the design and application of nano devices such as micro sound generator etc. The effects of small scale parameters can increase vibrational frequencies of the protein microtubules and cannot be overlooked in the analysis of vibrating phenomena. The results for different modes with nonlocal effect are checked.

Keywords: microtubules; nonlocal theory of elasticity; vibration analysis; small scale effects

1. Introduction

Animals have skeletal system for the support and maintenance of their bodies. Cell is the functional and basic structural unit of the life and therefore this unit should have some skeletal structure for its rigidity and support. All types of cells have skeleton called cytoskeleton. One of the components of cytoskeleton is microtubules (MTs). MTs are like hollow cylinder having outer and inner diameter 30 nanometer (nm) and 20 nanometer (nm) respectively with length up to 100 μm or more (de Pablo *et al.* 2003). MTs are organized by 13 parallel protofilaments (PFs), linked in circular shape (Nogales 2001). These filaments are composed of $\alpha\beta$-tubulin dimmers (Gittes *et al*. 993).

The configuration of MTs can also be illuminated by a pair of integers, N and S. N is PFs number and S is helix start number. The best common type of MTs is assembled in vivo where the PFs run parallel to longitudinal axis (Ishida *et al.* 2007). Sofiyev *et al*. (2006) considered the dynamic stability problem of a cylindrical shell composed of non-homogeneous orthotropic materials with Young's moduli and density varying continuously in the thickness direction under the effect of an axial compressive load varying with a parabolic function of time. At first, the fundamental relations and the modified Donnell type dynamic stability equations of a non-homogeneous orthotropic cylindrical shell are set up. Batou *et al*. (2019) studied the wave propagations in sigmoid functionally graded (S-FG) plates using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. The current higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. MTs are the best rigid filaments of all types of cytoskeleton which are mostly responsible for shape and mechanical inelasticity of the cell and basically involved in large range of functions such as cell division, intercellular transportation and the cell motility (Howard 2001, Boal 2012). Tohidi *et al*. (2017) investigated the dynamic stability of embedded functionally graded (FG)-carbon nanotubes (CNTs)-reinforced micro cylindrical shells. The structure is subjected to harmonic non-uniform temperature distribution and 2D magnetic field. The CNT reinforcement is either uniformly distributed or FG along the thickness direction where the effective properties of nano-composite structure are estimated through Mixture low. Their mechanical characteristics show an essential character in various cellular functions. MTs are tracks for the motor protein together with they change (Fernández and Vico 2011). They help in the movement of chromosomes, during the cell division also form moving cores of flagella and cilia and (Shaw *et al*. 2000). Along PFs, longitudinal bonds are much stronger than lateral bonds between adjacent PFs of MTs (Needleman *et al*. 2005). The elastic moduli along longitudinal and circumferential direction differ remarkably. It is greater in farmer case and lesser in later one (Jánosi *et*

*Corresponding author, Ph.D.
E-mail: muzamal45@gmail.com, muzamalhussain@gcuf.edu.pk

http://www.techno-press.org/?journal=cac&subpage=8
ISSN: 1598-8198 (Print), 1598-818X (Online)

al. 2002). Narwariya *et al*. (2018) presented the vibration and harmonic analysis of orthotropic laminated composite plate. The response of plate is determined using Finite Element Method. The eight noded shell 281 elements are used to analyze the orthotropic plates and results are obtained so that the right choice can be made in applications such as aircrafts, rockets, missiles, etc. to reduce the vibration amplitudes. Hussain *et al*. (2017) demonstrated an overview of Donnell theory for the frequency characteristics of two types of SWCNTs. Fundamental frequencies with different parameters have been investigated with wave propagation approach. Arefi (2018) studied the nonlocal free vibration analysis of a doubly curved piezoelectric nano shell. First order shear deformation theory and nonlocal elasticity theory is employed to derive governing equations of motion based on Hamilton's principle. The doubly curved piezoelectric nano shell is resting on Pasternak's foundation. Hussain and Naeem (2017) examined the frequencies of armchair tubes using Flügge's shell model. The effect of length and thickness-to-radius ratios against fundamental natural frequency with different indices of armchair tube was investigated. Salah *et al*. (2019) examined a simple four-variable integral plate theory is employed for examining the thermal buckling properties of functionally graded material (FGM) sandwich plates. The proposed kinematics considers integral terms which include the effect of transverse shear deformations. Hussain and Naeem (2018a) used Donnell's shell model to calculate the dimensionless frequencies for two types of single-walled carbon nanotubes. The frequency influence was observed with different parameters. Along longitudinal direction, elastic modulus is also greater in magnitude than the shear modulus (de Pablo *et al*. 2003). No doubt, microfilaments of MTs form a helical structure but this helix is not responsible for the mechanical behavior of MTs (Kasas *et al*. 2004). Fatahi-Vajari *et al*. (2019) studied the vibration of single-walled carbon nanotubes based on Galerkin's and homotopy method. This work analyses the nonlinear coupled axial-torsional vibration of single-walled carbon nanotubes (SWCNTs) based on numerical methods. Two-second order partial differential equations that govern the nonlinear coupled axial-torsional vibration for such nanotube are derived. Asghar *et al*. (2019a, b) conducted the vibration of nonlocal effect for double-walled carbon nanotubes using wave propagation approach. Many material parameters are varied for the exact frequencies of many indices of double-walled carbon nanotubes. Shahrma *et al*. (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Benmansour *et al*. (2019) analyzed the dynamic and bending behaviors of isolated protein microtubules. Microtubules (MTs) can be considered as bio-composite structures that are elements of the cytoskeleton in eukaryotic cells and posses considerable roles in cellular activities. They have higher mechanical characteristics such as superior flexibility and stiffness. Particularly, along longitudinal direction, elastic modulus is higher than the shear modulus of the MTs and also elastic modulus along the longitudinal direction is higher than the elastic modulus along with the circumferential directions (Kasas *et al*. 2004, Tuszyński *et al*. 2005). These outcomes provide the basis for the development of a new model named "Orthotropic Elastic Shell Model" for the observation of different mechanical behaviour of MTs (Wang *et al*. 2006a, b). Rezaiee-Pajand *et al*. (2018) investigated the static behavior of non-prismatic sandwich beams composed of functionally graded (FG) materials for the first time. Two types of beams in which the variation of elastic modulus follows a power-law form are studied. The principle of minimum total potential energy is applied along with the Ritz method to derive and solve the governing equations. Sofiyev *et al*. (2009) truncated the free vibration of non-homogeneous truncated conical shells on a Winkler foundation. After formed the fundamental relations and governing equations, the dimensionless frequency parameter of the non-homogeneous isotropic truncated conical shell with or without an elastic foundation are found. Inspired by these ideas, an "Orthotropic shell model" was established to study the propagation of the wave along MTs (Qian *et al.* 2007). A respectable agreement has been made between this model and the available discrete model and the experiments. Ayat *et al*. (2018) resulted the use of optimum content of supplementary cementing materials (SCMs) such as limestone filler (LF) to blend with Portland cement in many environmental and technical advantages, such as increase in physical properties, enhancement of sustainability in concrete industry and reducing CO2 emission are well known. Non-local theory of elasticity is pioneered by Eringen, states that the stress at a point is not simply the function of strain on that point but it also depends on long range interactions of the surrounding areas. Using nonlocal theory of elasticicty, some mathematicians (Gao and An 2010, An and Gao 2010, Gao and Lei 2009) showed that the persistence length of MTs, which is the direct demonstration of the microtubules' stiffness, is reformed by considering the small scale effects into the account. The effects of small scale are also affect the buckling, post-buckling and bending of MTs (Civalek and Demir 2011, Civalek and Demir 2016, Shen 2010). Behera and Kumari (2018) developed an exact solution for free vibration of the Levy-type rectangular laminated plate considering the most efficient Zig-Zag theory (ZIGT) and third order theory (TOT). The plate is subjected to hard simply supported boundary condition (Levy-type) along x axis. Using the equilibrium equations and the plate constitutive relations, a set of 12 m first order differential homogenous equations are obtained. Civalek *et al*. (2010) formulated the equations of motion and bending of Euler-Bernoulli beam using the nonlocal elasticity theory for cantilever microtubules (MTs). The method of differential quadrature (DQ) has been used for numerical modeling. The size effect is taken into consideration using the Eringen's non-local elasticity theory. Jamali *et al*. (2019) devoted to study the post-buckling analysis of functionally graded carbon nanotubes reinforced composite (FG-CNTRC) micro plate with cut out subjected to magnetic field and resting on elastic medium. The basic formulation of plate is based on first order shear deformation theory

(FSDT) and the material properties of FG-CNTRCs are presumed. Many mathematicians have been discoursed the vibrations of MTs by considering small scale effects and they showed that these effects have a notable impact on the dynamical properties of the MTs (Heireche *et al.* 2010), (Ansari and Arash, 2013). Recently Hussain and Naeem (2019a, b, c, d, 2020a) and performed the vibration of SWCNTs based on wave propagation approach and Galerkin's method. They investigated many physical parameters for the rotating and non-rotating vibrations of armchair, zigzag and chiral indices. Moreover, the mass density effect of single walled carbon nanotubes with in-plane rigidity has been calculated for zigzag and chiral indices.

Earlier work in literature was limited for the circumferential vibrations of MTs (Wang and Zhang 2008), using orthotropic shell model. In that study, non-local shear deformable Shell model is used to study non-linear vibration analysis of MTs. The numerical results of this work showed that fundamental frequency of MTs are very complex to small scale parameter. This result reveals that small scale parameter decrease the natural frequencies of the MTs but the effect of small scale parameter on non-linear vibration reply is relatively feebler when compared to small scale parameter effects on post reply of the MTs.

2. Materials and methods

Over the past several years vibration of nano and micro structures of various configurations and boundary conditions have been extensively studied (Hussain *et al.* 2018a, Hussain *et al.* 2018b, Hussain *et al.* 2018c, Hussain and Naeem 2018b, Hussain *et al.* 2019a, Hussain and Naeem 2020b, Hussain *et al.* 2019b, Hussain *et al.* 2020a, Sehar *et al.* 2020, Hussain *et al.* 2020b, c, d). Here nonlocal rthotropic Elastic Shell Model is utilized to analyze the vibration of MTs. Nonlocal orthotropic Elastic shell model is developed here to investigate the vibration of protein MTs and the wave propagation approach is applied to find the dispersion relations for free MTs.

2.1 Nonlocal orthotropic shell model for free MTs

In the "non-local theory" of the elasticity, the effects of the small scale parameters are not negligible in the orthotropic modeling of shells, which are elastic in nature. The constitutive relationships of three-dimensional problems take the form given below (Eringen 2002).

$$(1-(e_\circ a)^2\nabla^2)\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{1}$$

Where a is internal length (similar to length of Carbon-Carbon bond etc.), and e_o designated as a material constant depending on the nature of materials; and hence $e_o a$ is the small scale parameter ε_{ij} and σ_{ij} are the strain and stress tensors correspondingly. The C_{ijkl}, represents elastic modulus tensor of local elasticity. The organelle MTs may be taking in consideration as a tube which is circular cylindrical in shape. The depth of bridge is round about 1.1 nm, equivalent depth h is round about 2.7 nm. The effective depth h_o for bending is round about 1.6 nm (Qian *et al.* 2007, Gao and An 2010).

The following relations given by Flugge's shell theory (Wang and Carter 2002, Zou and Foster 1995)

$$\varepsilon_\alpha = \frac{1}{R}\left(\frac{\partial u}{\partial \alpha} - \gamma\frac{\partial^2 w}{\partial^2 \alpha}\right). \tag{2}$$

$$\varepsilon_\beta = \frac{1}{R}\left(\frac{\partial v}{\partial \beta} + w\right) - \frac{\gamma}{R(1+\gamma)}\left(\frac{\partial^2 w}{\partial \beta^2} + w\right) \tag{3}$$

$$\varepsilon_{\alpha\beta} = \frac{\gamma}{R(1+\gamma)}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} + 2\gamma\left(\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial\alpha\partial\beta}\right) + \gamma^2\left(\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial\alpha\partial\beta}\right)\right]. \tag{4}$$

The strain-stress relationship by Hook's law in dimensionless coordinates derived from Eq. (1) is as under (Gao and An 2010).

$$\sigma_\alpha - (e_\circ a)^2\nabla^2\sigma_\alpha = E_1\frac{(\varepsilon_\alpha + \mu_1\varepsilon_\beta)}{(1-\mu_1\mu_2)}\ , \tag{5}$$

$$\sigma_\beta - (e_\circ a)^2\nabla^2\sigma_\beta = E_2\frac{(\mu_2\varepsilon_\alpha + \varepsilon_\beta)}{(1-\mu_1\mu_2)}\ , \tag{6}$$

$$\tau_{\alpha\beta} - (e_\circ a)^2\nabla^2\tau_{\alpha\beta} = G\varepsilon_{\alpha\beta}. \tag{7}$$

where σ_α, σ_β and $\tau_{\alpha\beta}$ are normal and shear stresses, and ε_α , ε_β and $\varepsilon_{\alpha\beta}$ are respective strains; E_1 and E_2 are moduli of elasticity; μ_2 and μ_1 are Poisson's ratios in the directions of α and β correspondingly. G is modulus of rigidity or shear modulus. Also we have $E_1\mu_1 = E_2\mu_2$ and $\nabla^2 = (\partial^2/\partial\alpha^2 + \partial^2/\partial\beta^2)/R^2$ which is the Laplace operator in dimensionless coordinates.

The elements of shell in our coordinates are, N, S, Q which are the stress resultants and M is the moment. The thermal expansion causes pre-stress, which is neglected due to the presence of temperature and considered by means of reference temperature. We arrive at dynamical equilibrium equations.

$$\frac{\partial N_\alpha}{\partial \alpha} + \frac{\partial S_\beta}{\partial \beta} + k = \rho h R\frac{\partial^2 u}{\partial t^2} \tag{8}$$

$$\frac{\partial N_\beta}{\partial \beta} + \frac{\partial S_\alpha}{\partial \alpha} + Q_\beta = \rho h R\frac{\partial^2 v}{\partial t^2}, \tag{9}$$

$$\frac{\partial Q_\sigma}{\partial \alpha} + \frac{\partial Q_\beta}{\partial \beta} - N_\beta = \rho h R\frac{\partial^2 w}{\partial t^2}, \tag{10}$$

$$\frac{\partial M_{\alpha\beta}}{\partial \alpha} + \frac{\partial M_\beta}{\partial \beta} - RQ_\beta = 0, \tag{11}$$

$$\frac{\partial M_{\beta\alpha}}{\partial \beta} + \frac{\partial M_\alpha}{\partial \alpha} - RQ_\alpha = 0. \tag{12}$$

Where ρ is mass density.

The including resultants (N, S, Q) which are derived from overhead set of equations by using stress components.

$$(1-(e_\circ a)^2\nabla^2)\begin{bmatrix} N_\alpha & S_\alpha \\ M_\alpha & M_{\alpha\beta}\end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{bmatrix} \sigma_\alpha & \tau_{\alpha\beta} \\ z\sigma_\alpha & z\tau_{\alpha\beta}\end{bmatrix}\left(1+\frac{z}{R}\right)dz, \tag{13}$$

$$(1-(e_\circ a)^2\nabla^2)\begin{bmatrix} N_\beta & S_\beta \\ M_\beta & M_{\beta\alpha}\end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{bmatrix} \sigma_\beta & \tau_{\beta\alpha} \\ z\sigma_\beta & z\tau_{\beta\alpha}\end{bmatrix}dz, \tag{14}$$

$$(1-(e_\circ a)^2\nabla^2)(Q_\alpha, Q_\beta) = \int_{-\frac{h}{2}}^{\frac{h}{2}}[\tau_{\alpha z}, \tau_{\beta z}]dz. \tag{15}$$

Where h is thickness of the shell. Above equations result in

$$N_\alpha - (e_\circ a)^2\nabla^2 N_\alpha = \frac{K}{R}\left[\frac{\partial u}{\partial \alpha} + \mu_1\left(\frac{\partial v}{\partial \beta} + w\right) - c^2\frac{\partial^2 w}{\partial \alpha^2}\right], \quad (16)$$

$$N_\beta - (e_\circ a)^2\nabla^2 N_\beta = \frac{Kk_1}{R}\left[\frac{\partial v}{\partial \beta} + \mu_2\frac{\partial u}{\partial \alpha} + w + c^2\left(\frac{\partial^2 w}{\partial \beta^2} + w\right)\right] \quad (17)$$

$$S_\alpha - (e_\circ a)^2\nabla^2 S_\alpha = \frac{Kk_2}{R}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} - c^2\left(\frac{\partial^2 w}{\partial \alpha\partial \beta} - \frac{\partial v}{\partial \alpha}\right)\right]. \quad (18)$$

$$S_\beta - (e_\circ a)^2\nabla^2 S_\beta = \frac{Kk_2}{R}\left[\frac{\partial u}{\partial \beta} + \frac{\partial v}{\partial \alpha} + c^2\left(\frac{\partial^2 w}{\partial \alpha\partial \beta} + \frac{\partial v}{\partial \alpha}\right)\right], \quad (19)$$

$$M_\alpha - (e_\circ a)^2\nabla^2 M_\alpha = -Kc^2\left[\frac{\partial u}{\partial \alpha} + \mu_1\frac{\partial v}{\partial \beta} - \left(\frac{\partial^2 w}{\partial \alpha^2} + \mu_1\frac{\partial w}{\partial \beta^2}\right)\right], \quad (20)$$

$$M_\beta - (e_\circ a)^2\nabla^2 M_\beta = Kk_1c^2\left[\frac{\partial^2 w}{\partial \beta^2} + w + \mu_2\frac{\partial^2 w}{\partial \alpha^2}\right], \quad (21)$$

$$M_{\alpha\beta} - (e_\circ a)^2\nabla^2 M_{\alpha\beta} = 2Kk_2c^2\left[\frac{\partial v}{\partial \alpha} - \frac{\partial^2 w}{\partial \alpha\partial \beta}\right], \quad (22)$$

$$M_{\beta\alpha} - (e_\circ a)^2\nabla^2 M_{\beta\alpha} = Kk_2c^2\left[\frac{\partial u}{\partial \beta} - \frac{\partial v}{\partial \alpha} + 2\frac{\partial^2 w}{\partial \alpha\partial \beta}\right], \quad (23)$$

$$Q_\alpha - (e_\circ a)^2\nabla^2 Q_\alpha = \frac{Kc^2}{R}\left[\begin{matrix}\frac{\partial^2 u}{\partial \alpha^2} - k_2\frac{\partial^2 u}{\partial \beta^2} + (k_2+\mu_1)\frac{\partial^2 v}{\partial \alpha\partial \beta} - \frac{\partial^3 w}{\partial \alpha^3} - \\ (2k_2+\mu_1)\frac{w}{\partial \alpha\partial \beta^2}\end{matrix}\right], \quad (24)$$

$$Q_\beta - (e_\circ a)^2\nabla^2 Q_\beta = \frac{Kk_1c^2}{R}\left[2\frac{k_2}{k_1}\frac{\partial^2 v}{\partial \alpha^2} - \frac{\partial^3 w}{\partial \beta^3} - \frac{\partial w}{\partial \beta} - \left(2\frac{\mu_1}{k_1} + \mu_2\right)\frac{\partial^3 w}{\partial \alpha^2\partial \beta}\right]. \quad (25)$$

Where $K = \frac{E_1 h}{(1-\mu_1\mu_1)}$, $k_1 = \frac{E_2}{E_1}$, $k_2 = G(1-\mu_1\mu_1)/E_1$, $c^2 = \frac{h_0^3}{R^2 h}$

$$\left[\frac{\partial^2}{\partial \alpha^2} + k_2(1+c^2)\frac{\partial^2}{\partial \beta^2}\right]u + \left[(\mu_1+k_2)\frac{\partial^2}{\partial \alpha\partial \beta}\right]v + \left[\mu_1\frac{\partial}{\partial \alpha} + c^2\left(k_2\frac{\partial^3}{\partial \alpha\partial \beta^2} - \frac{\partial^3}{\partial \alpha^3}\right)\right]w = \frac{\rho h R^2(1-(e_0 a)^2\nabla^2)}{K}\frac{\partial^2 u}{\partial t^2} \quad (26)$$

$$\left[(\mu_1+k_2)\frac{\partial^2}{\partial \alpha\partial \beta}\right]u + \left[k_2(1+3c^2)\frac{\partial^2}{\partial \alpha^2} + k_1\frac{\partial^2}{\partial \beta^2}\right]v + \left[k_1\frac{\partial}{\partial \beta} - c^2(\mu_1+3k_2)\frac{\partial^3}{\partial \alpha^2\partial \beta}\right]w = \frac{\rho h R^2(1-(e_0 a)^2\nabla^2)}{K}\frac{\partial^2 v}{\partial t^2}. \quad (27)$$

$$\left[\mu_1\frac{\partial}{\partial \alpha} - c^2\left(\frac{\partial^3}{\partial \alpha^3} - k_2\frac{\partial^3}{\partial \alpha\partial \beta^2}\right)\right]u + \left[k_1\frac{\partial}{\partial \beta} - c^2(\mu_1 + 3k_2)\frac{\partial^3}{\partial \alpha^2\partial \beta}\right]v + \left[\left(1+\frac{1}{c^2}\right)k_1 + \frac{\partial^4}{\partial \alpha^4} + k_1\frac{\partial^4}{\partial \beta^4} + 2k_1\frac{\partial^2}{\partial \beta^2} + (2\mu_1+4k_2)\frac{\partial^4}{\partial \alpha^2\partial \beta^2}\right]w = -\frac{\rho h R^2(1-(e_0 a)^2\nabla^2)}{K}\frac{\partial^2 w}{\partial t^2} \quad (28)$$

2.2 Boundry coditions

Three kinds of boundary condition may be assumed while solving such a problems (Ansari and Arash, 2013) . These three conditions are:

- ✓ Clamped-Clamped

$$\alpha = \beta = \gamma = \frac{\partial\gamma}{\partial \alpha} = 0 \quad \text{at} \quad \alpha = 0,\ \alpha = \frac{L}{R} \quad (29)$$

- ✓ Clamped-free

$$\alpha = \beta = \gamma = \frac{\partial\gamma}{\partial \alpha} = 0 \quad \text{at} \quad \alpha = 0,$$

- ✓ Simply supported- Simply supported

$$\beta = \gamma = N_{\alpha\alpha} = 0 \ , \text{ at } \quad \alpha = 0,\ \alpha = \frac{L}{R}. \quad (30)$$

$$N_{\alpha\alpha} = M_{\alpha\alpha} = N_{\alpha\beta} = M_{\alpha\beta} = 0 \quad \text{at} \quad \alpha = \frac{L}{R}. \quad (31)$$

L is the length of microtubules.

3. General solutions of vibrations

In case of non- axisymmetric vibration, the solutions of Eqs (26-28) with boundary conditions (29-31) are given by

$$u(\alpha,\beta,t) = U\sin k\alpha.\cos n\beta\ e^{i\omega t},$$
$$v(\alpha,\beta,t) = V\cos k\alpha.\sin n\beta\ e^{i\omega t},$$
$$w(\alpha,\beta,t) = W\cos k\alpha.\cos n\beta\ e^{i\omega t}. \quad (32)$$

Where U, V and W are waves amplitudes of the vibration with the direction of x, y and z correspondingly. Wave vecto $k = \frac{\pi m R}{L}$ is dimensionless in the longitudinal direction m is the half wave number in the longitudinal direction and ω is the frequency of vibration Putting Eq. (32) in Eqs. (26)-(28) will give us the following three homogeneous equations.

$$\left[(1+c^2)k_2n^2 + k^2 - \frac{\eta h\rho\omega^2R^2}{K}\right]U + nk(k_2+\mu_1)$$
$$V + K[c^2(k^2 - k_2n^2) + \mu_1]W = 0,$$
$$nk\binom{k_2+}{\mu_1}U + \left[(1+3c^2)k_2k^2 + k_1n^2 - \frac{\eta h\rho\omega^2R^2}{K}\right]V$$
$$+n[k_1 + c^2k^2(3k_2+\mu_1)]W = 0,$$
$$k[c^2(k^2 - k_2n^2) + \mu_1]U + n[k_1 + c^2k^2(3k_2+\mu_1)]$$
$$V + [(1+c^2)k_1 + c^2[k^4 + k_1n^2(n^2-2) + 2k^2n^2$$
$$(3k_2+\mu_1)] - \frac{\eta h\rho\omega^2R^2}{K}\Big]w = 0. \quad (33)$$

Where $\eta = 1 + (n^2 + k^2)\left(\frac{e_0 a}{R}\right)^2$

Writing the above set of equation (33) in the matrix form as

$$\left[H^{(1)}(k,\omega)\right]_{3\times 3}\begin{bmatrix}U\\V\\W\end{bmatrix} = [0 \quad 0 \quad 0]^T, \quad (34)$$

Nonzero solution of above system exist if

$$Det\left[H^{(1)}(k,\omega)\right]_{3\times 3} = 0. \quad (35)$$

Eq. (35) is written as

$$\Omega^3 + \Omega^2 e + \Omega f + g = 0 \quad (36)$$

Where

$$\Omega = \frac{\eta h\rho\omega^2R^2}{K},\ e = -(a_{11} + a_{22} + a_{33}),$$
$$f = (a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33}) - (a_{12}^2 + a_{23}^2 + a_{13}^2),$$
$$g = (a_{11}a_{23}^2 + a_{22}a_{13}^2 + a_{33}a_{23}^2) + 2a_{11}a_{12}a_{13} - a_{11}a_{22}a_{33}\ .$$

And in the above equations

$$a_{11} = k^2 + (1+c^2)k_2n^2,\quad a_{12} = -kn(k_2+\mu_1),$$
$$a_{13} = k[(k^2 + k_2n^2)c^2 + \mu_1]$$
$$a_{21} = 0,\ a_{22} = k^2(1+3c^2) + n^2k_1,$$
$$a_{23} = nk_1 + nc^2k^2(\mu_1 + 3k_2),$$

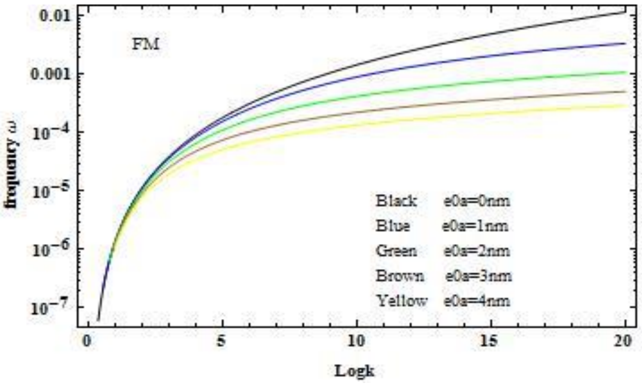

Fig. 1 Effect of non-local parameter (e_0a) on the vibrational frequency for non-axisymmetric vibrations (n=1) for first mode

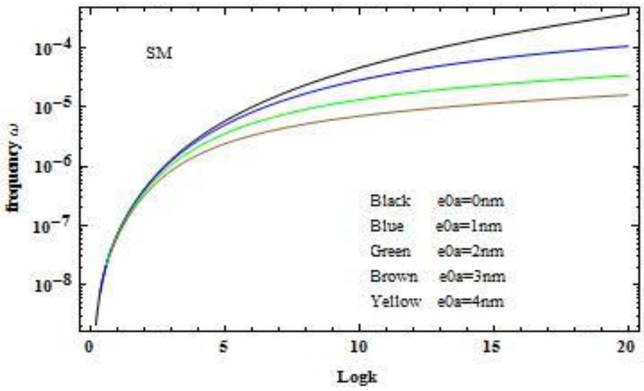

Fig. 2 Effect on non-local parameter (e_0a) on the vibrational frequency for non-axisymmetric vibrations (n=1) for second mode of frequency

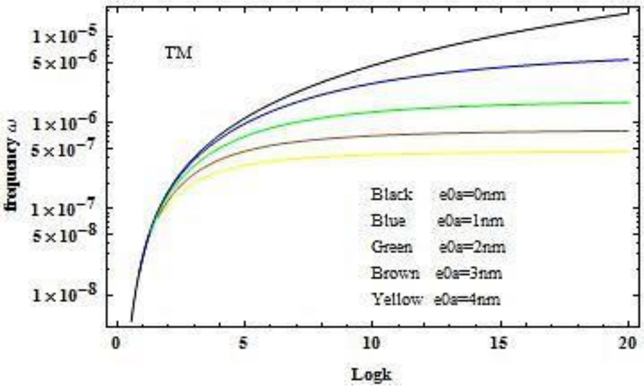

Fig. 3 Effect on non-local parameter (e_0a) on the vibrational frequency for non-axisymmetric vibrations (n=1) for the third mode of frequency

$$a_{31} = 0, \quad a_{32} = 0, \quad a_{33} = (1+c^2)k_{1,} + c^2[k^4 + n^2k_1(n^2-2) + 2k^2n^2(2k_2+\mu_1)].$$

From Eq. (36), after long calculations and heavy simplifications, we get the analytic formulas for wave velocity

$$\omega_1^2 = \left(\sqrt[3]{T+\sqrt{D}} + \sqrt[3]{T-\sqrt{D}} - e/3\right) K/(\,h\eta\rho R^{\,2}), \quad (37)$$

$$\omega_2^2 = \left(\lambda_1\sqrt[3]{T+\sqrt{D}} + \lambda_2\sqrt[3]{T-\sqrt{D}} - e/3\right) K/(\,h\eta\rho R^2), \, (38)$$

$$\omega_3^2 = \left(\lambda_2\sqrt[3]{T+\sqrt{D}} + \lambda_1\sqrt[3]{T-\sqrt{D}} - e/3\right) K/(\,h\eta\rho R^2). \; (39)$$

Where $D = T^2 + \left(\frac{f}{3} - \frac{e^2}{9}\right)^3$, $T = -\frac{e^3}{27} + \frac{ef}{6} - \frac{g}{2}$, $\lambda_1 = -\frac{1+\sqrt{3}}{2}$, $\lambda_2 = -\frac{1-i\sqrt{3}}{2}$ and $i = \sqrt{-1}$ is imaginary unit.

In the expression of wave speed the small scale parameter e_0a occurs only in η so η can visualize the small scale effects that are affected by long range relation. $\eta = 1$ Refers to the classical case because $\eta = 1$ for $e_0a = 0$. If we take $e_0a = 0$, the equations will degrade to the local theory (Qian *et al*. 2007). For nonlocal cases η takes the values greater than 0.

4. Results and discussion

In this section we will explain the vibrational frequency designed against wave vector graphically. We will elaborate the circumferential half wave number and small scale effect on the wave speed. In order to analyze the effects of small scale, present study is independent of time. The material parameters of microtubules can be obtained by theoretical and experimental methods and few of them are précised (Wang and Gao 2016). In our discussions, we have $E_1 = 1\times10^9$ Pa, $E_2 = 1\times10^6$ Pa, $\mu_2 = 0.3$, $G = 1\times10^6$ Pa. e_0a be influenced by many features. In order to check the effects of the e_0a (Small scale parameter), we have taken the range of e_0a between 0 to 4 nm. Fig. 1, shows that for the first mode of frequency, there is a slight difference between classical and nonlocal results, if we give the values to the wave vector (k) from 0 to 3.5. But there exist a significant difference between the both classical and non-local results, if we give the values to the wave vector (k) from 3.6 to 20, the non local parameter (e_0a) influences on the maximum values of second mode (ω_1). At $e_0a = 0nm$, $e_0a = 1nm$, $e_0a = 2nm$, $e_0a = 3nm$ and $e_0a = 4nm$, ω_2 has maixium value 1×10^{-1}, 4×10^{-2}, 10^{-2}, 5×10^{-4}, and 2×10^{-4} at the value of wave vector $k = 20$ respectively. Fig. 2, also shows that for the second mode of frequency, there is negligiale dufference between the claassical and non local results when the wave vector (k) obtain the values between 0 and 4. But there exist an outstading difference between clasical and non local results when the values of wave vector between 4.2 and 20. The non local parameter (e_0a) influences on the maximum values the of second mode (ω_2). At $e_0a = 0nm$, $e_0a = 1nm$, $e_0a = 2nm$, $e_0a = 3nm$ and $e_0a = 4nm$, ω_2 has maixium value $10^{-3}, 10^{-4}, 5\times10^{-5}, 2\times10^{-5}$, and 9×10^{-6} at the value of wave vector $k = 20$ respectively. Fig. 3, shows that for the third mode of the frequency, there is a slight difference between classical and non-local result when the wave vectors (k) obtains the values between 0 to 3. But there is huge difference between the classical and nonlocal results when the wave vector (k) obtains values between 3.1 to 20. The non local parameter (e_0a) influences on the maximum values of second mode (ω_2). At $e_0a = 0nm$, $e_0a = 1nm$, $e_0a = 2nm$, $e_0a = 3nm$ and $e_0a = 4nm$, ω_2 has maixium value

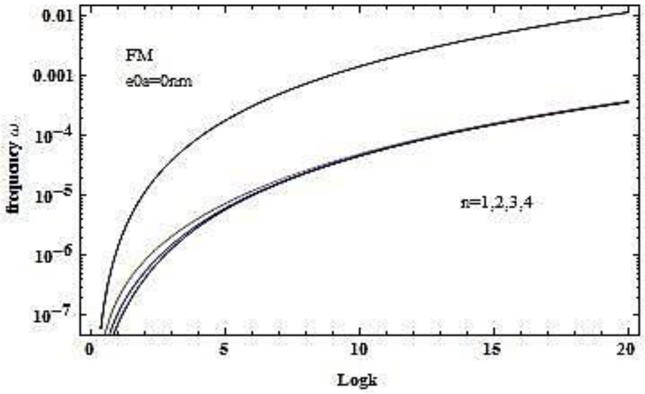

Fig. 4 Effects of half circumferential wave number n on vibrational frequency for nonlocal case $e_0a = 0nm$ of ω_1

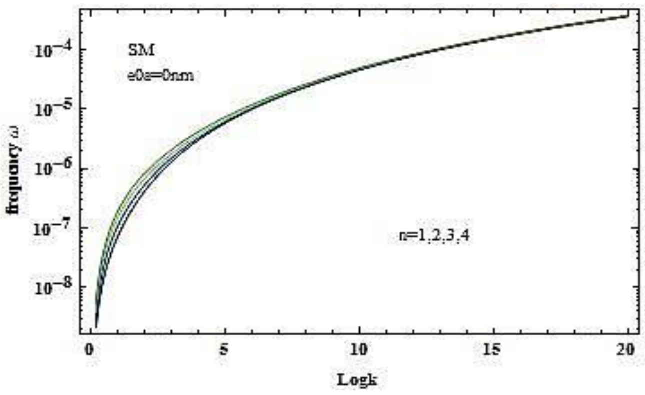

Fig. 5 Effects of half circumferential wave number n on vibrational frequency for nonlocal case $e_0a = 0nm$ of ω_2

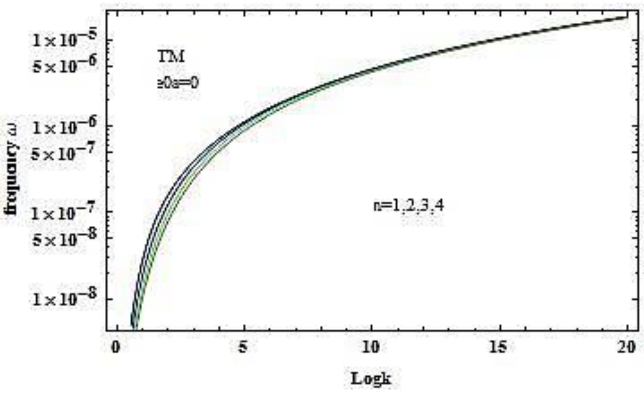

Fig. 6 Effects of half circumferential wave number n on vibrational frequency for nonlocal case $e_0a = 0nm$ of (ω_3) (Third mode)

3×10^{-5}, 5×10^{-6}, 1×10^{-6}, 5×10^{-7} and 3×10^{-7} at the $k = 20$ respectively. Fig. 4, shows effects different values of n (half circumferential wave number) on the vibrational frequency for nonlocal case $e_0a = 0nm$ of the first mode. effects of different values of n (half circumferential wave number) on the vibrational frequency for nonlocal case $e_0a = 0nm$ of the second mode as shown in Fig, 5. Fig. 6, depicts the effect of different values of n (half circumferential wave number) on the vibrational frequency for nonlocal case $e_0a = 0nm$ of the third mode. The effects of different values of n (half circumferential

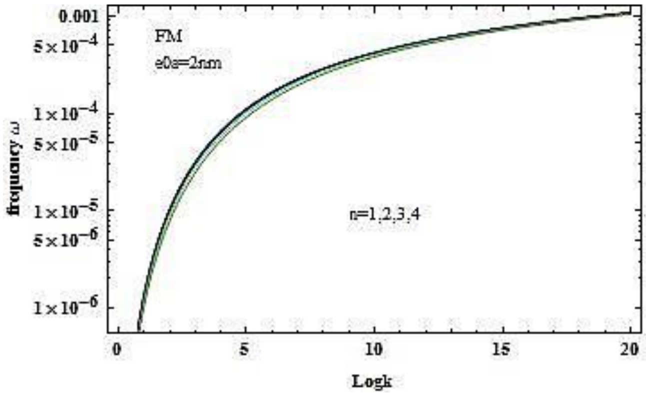

Fig. 7 Effects of half circumferential wave number n on vibrational frequency for non-local case $e_0a = 2nm$ of the first mode

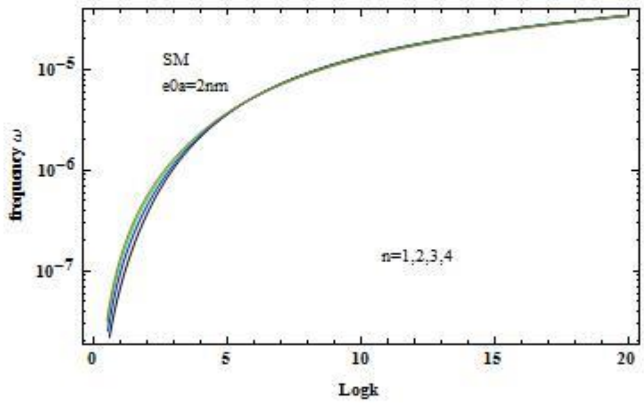

Fig. 8 Effects of half circumferential wave number n on vibrational frequency for nonlocal case $e_0a = 2nm$ of the (ω_2) (Second mode)

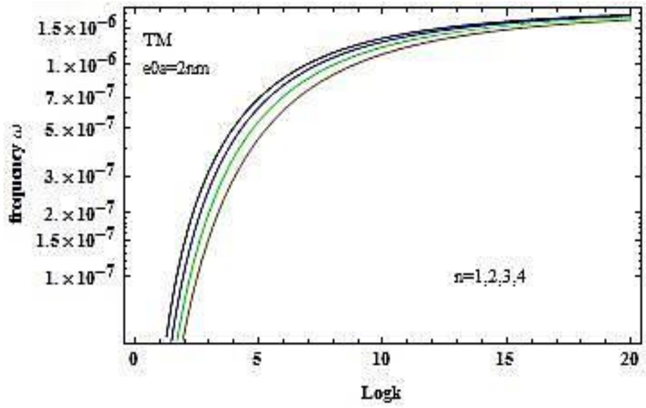

Fig. 9 Effects of half circumferential wave number n on vibrational frequency for nonlocal case $e_0a = 2nm$ of the ω_3 (Third mode)

wave number) on the vibrational frequency for nonlocal case $e_0a = 2nm$ of the first mode as shown in Fig 7. Fig. 8, show the effects of different values of n (half circumferential wave number) on the vibrational frequency for nonlocal case $e_0a = 2nm$ of the second mode. Fig. 9, show the effects of the different values of n (half circumferential wave number) on the vibrational frequency for nonlocal case $e_0a = 2nm$ of the third mode.

5. Conclusion

MTs are modeled as non-local orthotropic elastic shell model and "wave propagation approach" is used to solve the obtained model. We observed that due to non-local effects the frequency of vibration is increased with the increase in wave vector. The effects of small scale parameters can increase vibrational frequencies of the protein microtubules, and cannot be overlooked in analysis of vibrating phenomena. We checked that the non-local parameter (e_0a) affected the first mode (ω_1) more than second mode (ω_2) and third mode (ω_3) of vibration. In the future, we may try to check the effects of surrounding medium and thermal stress on the vibration of MTs taking into account its non-linearity

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

References

An, L. and Gao, Y. (2010), "Mechanics behavior of microtubules based on nonlocal anisotropic shell theory", *IOP Conference Series: Materials Science and Engineering*, **10**(1), 012181. doi:10.1088/1757-899X/10/1/012181.

Asghar, S., Hussain, M. and Naeem, M. (2019), "Non-local effect on the vibration analysis of double walled carbon nanotubes based on Donnell shell theory", *J. Physica E: Low Dimens. Syst. Nanostruct.*, **116**, 113726. https://doi.org/10.1016/j.physe.2019.113726.

Arefi, M. (2018), "Nonlocal free vibration analysis of a doubly curved piezoelectric nano shell", *Steel Compos. Struct.*, **27**(4), 479-493. https://doi.org/10.12989/scs.2018.27.4.479.

Asghar, S., Naeem, M.N., Hussain, M. and Tounsi, A. (2020), "Nonlocal vibration of DWCNTs based on Flügge shell model using wave propagation approach", *Steel Compos. Struct.*, **34**(4), 599-613. https://doi.org/10.12989/scs.2020.34.4.599.

Ansari, R. and Arash, B. (2013), "Nonlocal Flügge shell model for vibrations of double-walled carbon nanotubes with different boundary conditions", *J. Appl. Mech.*, **80**(2), 021006. https://doi.org/10.1115/1.4007432.

Ayat, H., Kellouche, Y., Ghrici, M. and Boukhatem, B. (2018), "Compressive strength prediction of limestone filler concrete using artificial neural networks", *Adv. Comput. Des.*, **3**(3), 289-302. https://doi.org/10.12989/acd.2018.3.3.289.

Batou, B., Nebab, M., Bennai, R., Atmane, H.A., Tounsi, A. and Bouremana, M. (2019), "Wave dispersion properties in imperfect sigmoid plates using various HSDTs", *Steel Compos. Struct.*, **33**(5), 699-716. https://doi.org/10.12989/scs.2019.33.5.699.

Behera, S. and Kumari, P. (2018), "Free vibration of Levy-type rectangular laminated plates using efficient zig-zag theory", *Adv. Comput. Des.*, **3**(3), 213-232. https://doi.org/10.12989/acd.2018.3.3.213.

Boal, D. (2012), *Mechanics of the Cell*, Cambridge University Press, New York, Cambridge, UK.

Bilouei, B.S., Kolahchi, R. and Bidgoli, M.R. (2016), "Buckling of concrete columns retrofitted with Nano-Fiber Reinforced Polymer (NFRP)", *Comput. Concrete*, **18**(5), 1053-1063. https://doi.org/10.12989/cac.2016.18.5.1053.

Benmansour, D.L., Kaci, A., Bousahla, A.A., Heireche, H., Tounsi, A., Alwabli, A.S., ... and Mahmoud, S.R. (2019), "The nano scale bending and dynamic properties of isolated protein microtubules based on modified strain gradient theory", *Adv. Nano Res.*, **7**(6), 443. https://doi.org/10.12989/anr.2019.7.6.443.

Civalek, Ö., Demir, Ç. and Akgöz, B. (2010), "Free vibration and bending analyses of cantilever microtubules based on nonlocal continuum model", *Math. Comput. Appl.*, **15**(2), 289-298. https://doi.org/10.3390/mca15020289.

Civalek, Ö. (2006), "An efficient method for free vibration analysis of rotating truncated conical shells", *Int. J. Press. Ves. Pip.*, **83**(1), 1-12. https://doi.org/10.1016/j.ijpvp.2005.10.005.

Civalek, Ö. and Demir, C. (2016), "A simple mathematical model of microtubules surrounded by an elastic matrix by nonlocal finite element method", *Appl. Math. Comput.*, **289**, 335-352. https://doi.org/10.1016/j.amc.2016.05.034. 0 096-30.

Civalek, Ö. and Demir, Ç. (2011), "Bending analysis of microtubules using nonlocal Euler-Bernoulli beam theory", *Appl. Math. Model.*, **35**(5), 2053-2067. https://doi.org/10.1016/j.apm.2010.11.004.

de Pablo, P.J., Schaap, I.A., MacKintosh, F.C. and Schmidt, C.F. (2003), "Deformation and collapse of microtubules on the nanometer scale", *Phys. Rev. Lett.*, **91**(9), 098101. https://doi.org/10.1103/PhysRevLett.91.098101.

Eringen, A.C. (2002), *Nonlocal Continuum Field Theories*, Springer Science and Business Media.

Esmaeili, J. and Andalibia, K. (2019), "Development of 3D Meso-Scale finite element model to study the mechanical behavior of steel microfiber-reinforced polymer concrete", *Comput. Concrete*, **24**(5), 413-422. https://doi.org/10.12989/cac.2019.24.5.413.

Fatahi-Vajari, A., Azimzadeh, Z. and Hussain, M. (2019), "Nonlinear coupled axial-torsional vibration of single-walled carbon nanotubes using Galerkin and Homotopy perturbation method", *Micro Nano Lett.*, **14**(14), 1366-1371. https://doi.org/10.1049/mnl.2019.0203.

Fernández, J.D. and Vico, F.J. (2011)., "Automating the search of molecular motor templates by evolutionary methods", *Biosyst.*, **106**(2-3), 82-93. https://doi.org/10.1016/j.biosystems.2011.07.002.

Gao, Y. and An, L. (2010)., "A nonlocal elastic anisotropic shell model for microtubule buckling behaviors in cytoplasm", *Physica E: Low Dimens. Syst. Nanostruct.*, **42**(9), 2406-2415. https://doi.org/10.1016/j.physe.2010.05.022.

Gao, Y. and Lei, F.M. (2009, "Small scale effects on the mechanical behaviors of protein microtubules based on the nonlocal elasticity theory", *Biochem. Biophys. Res. Commun.*, **387**(3), 467-471. https://doi.org/10.1016/j.bbrc.2009.07.042.

Golabchi, H., Kolahchi, R. and Bidgoli, M.R. (2018), "Vibration and instability analysis of pipes reinforced by SiO_2 nanoparticles considering agglomeration effects", *Comput. Concrete*, **21**(4), 431-440. https://doi.org/10.12989/cac.2018.21.4.431.

Gittes, F., Mickey, B., Nettleton, J. and Howard, J. (1993), "Flexural rigidity of microtubules and actin filaments measured from thermal fluctuations in shape", *J. Cell Biol.*, **120**(4), 923-934. https://doi.org/10.1083/jcb.120.4.923.

Hussain, M., Naeem, M.N. and Tounsi, A. (2020d), "Response of orthotropic Kelvin modeling for single-walled carbon nanotubes: Frequency analysis", *Adv. Nano Res.* (in Press)

Hussain, M., Naeem, M.N. and Tounsi, A. (2020a), "Simulating vibration of single-walled carbon nanotube based on Relagh-Ritz Method".

Hussain, M. and Naeem, M.N. (2020a), "Mass density effect on vibration of zigzag and chiral SWCNTs", *J. Sandw. Struct. Mater.*, 1099636220906257. https://doi.org/10.1177/1099636220906257.

Hussain, M. and Naeem, M.N. (2020b), "Vibration characteristics of zigzag FGM single-walled carbon nanotubes based on Ritz method with ring-stiffeners", *Ind. J. Phys.*, **140**, 85-98. https://doi.org/10.1016/j.tws.2019.03.018.

Hussain, M., Naeem, M.N. and Tounsi, A, (2020c), "Numerical Study for nonlocal vibration of orthotropic SWCNTs based on Kelvin's model", *Adv. Concrete Constr.*, **9**(3). (in Press)

Hussain, M., Naeem, M.N. and Tounsi, A. (2020b), "On mixing the Rayleigh-Ritz formulation with Hankel's function for vibration of fluid-filled Fluid-filled cylindrical shell", *Adv. Comput. Des.* (in Press)

Hussain, M. and Naeem, M. (2019d), "Rotating response on the vibrations of functionally graded zigzag and chiral single walled carbon nanotubes", *Appl. Math. Model.*, **75**, 506-520. https://doi.org/10.1016/j.apm.2019.05.039.

Hussain, M. and Naeem, M. (2018a), "Vibration of single-walled carbon nanotubes based on Donnell shell theory using wave propagation approach", Chapter 5, Intechopen, Novel Nanomaterials - Synthesis and Applications, ISBN 978-953-51-5896-7. https://doi.org/10.5772 /intechopen.73503.

Hussain, M. and Naeem, M. (2019a), "Vibration characteristics of single-walled carbon nanotubes based on non-local elasticity theory using wave propagation approach (WPA) including chirality", *Perspective of Carbon Nanotubes*, IntechOpen. https://doi.org/10.5772/intechopen.85948.

Hussain, M. and Naeem, M.N. (2018b), "Effect of various edge conditions on free vibration characteristics of rectangular plates", Chapter 3, Intechopen, Advance Testing and Engineering, ISBN 978-953-51-6706-8, Intechopen.

Hussain, M. and Naeem, M.N. (2019b), "Effects of ring supports on vibration of armchair and zigzag FGM rotating carbon nanotubes using Galerkin's method", *Compos.: Part B. Eng.*, **163**, 548-561. https://doi.org/10.1016/j.compositesb.2018.12.144.

Hussain, M. and Naeem, M.N. (2019c), "Vibration characteristics of zigzag and chiral FGM rotating carbon nanotubes sandwich with ring supports", *J. Mech. Eng. Sci., Part C*, **233**(16), 5763-5780. https://doi.org/10.1177/0954406219855095.

Hussain, M., Naeem, M., Shahzad, A. and He, M. (2018a), "Vibration characteristics of fluid-filled functionally graded cylindrical material with ring supports", Chapter 14, Intechopen, Computational Fluid Dynamics, ISBN 978-953-51-5706-9. https://doi.org/10.5772 /intechopen.72172.

Hussain, M., Naeem, M.N. and Isvandzibaei, M. (2018c), "Effect of Winkler and Pasternak elastic foundation on the vibration of rotating functionally graded material cylindrical shell", *Proc. Inst. Mech. Eng., Part C: J. Mech. Eng. Sci.*, **232**(24), 4564-4577. https://doi.org/10.1177/0954406217753459.

Hussain, M., Naeem, M.N. and Taj, M. (2019b), "Effect of length and thickness variations on the vibration of SWCNTs based on Flügge's shell model", *Micro Nano Lett.*, **15**(1), 1-6. https://doi.org/10.1049/mnl.2019.0309.

Hussain, M., Naeem, M.N., Shahzad, A., He, M. and Habib, S. (2018b), "Vibrations of rotating cylindrical shells with FGM using wave propagation approach", *IMechE Part C: J Mech. Eng. Sci.*, **232**(23), 4342-4356. https://doi.org/10.1177/0954406218802320.

Hussain, M., Naeem, M.N., Tounsi, A. and Taj, M. (2019a), "Nonlocal effect on the vibration of armchair and zigzag SWCNTs with bending rigidity",. *Adv. Nano Res.*, **7**(6), 431-442. https://doi.org/10.12989/anr.2019.7.6.431.

Hussain, M. and Naeem., M.N. (2017), "Vibration analysis of single-walled carbon nanotubes using wave propagation approach", *Mech. Sci.*, **8**(1), 155-164. https://doi.org/10.5194/ms-8-155-2017.

Hussain, M., Naeem., M.N., Shahzad, A. and He, M. (2017), "Vibrational behavior of single-walled carbon nanotubes based on cylindrical shell model using wave propagation approach", *AIP Adv.*, **7**(4), 045114. https://doi.org/10.1063/1.4979112.

Heireche, H., Tounsi, A., Benhassaini, H., Benzair, A., Bendahmane, M., Missouri, M. and Mokadem, S. (2010). "Nonlocal elasticity effect on vibration characteristics of protein microtubules"., *Physica E: Low Dimens. Syst. Nanostruct.*, **42**(9), 2375-2379. https://doi.org/10.18720/MPM.4022018_19.

Howard, J. (2001), *Mechanics of Motor Proteins and the Cytoskeleton*, 384.

Ishida, T., Thitamadee, S. and Hashimoto, T. (2007), " Twisted growth and organization of cortical microtubules", *J. Plant Res.*, **120**(1), 61-70. https://doi.org/10.1007/s10265-006-0039-y.

Jamali, M., Shojaee, T., Mohammadi, B. and Kolahchi, R. (2019), "Cut out effect on nonlinear post-buckling behavior of FG-CNTRC micro plate subjected to magnetic field via FSDT", *Adv. Nano Res.*, **7**(6), 405-417. https://doi.org/10.12989/anr.2019.7.6.405.

Jánosi, I.M., Chrétien, D. and Flyvbjerg, H. (2002)., "Structural microtubule cap: Stability, catastrophe, rescue, andthird state", *Biophys. J.*, **83**(3), 1317-1330. https://doi.org/10.1016/S0006-3495(02)73902-7.

Kasas, S., Cibert, C., Kis, A., De Rios, P.L., Riederer, B., Forro, L.,... Catsicas, S. (2004), "Oscillation modes of microtubules", *Biol. Cell*, **96**(9), 697-700. https://doi.org/10.1016/j.biolcel.2004.09.002.

Loghman, A., Arani, A.G. and Barzoki, A.A.M. (2017), "Nonlinear stability of non-axisymmetric functionally graded reinforced nano composite microplates", *Comput. Concrete*, **19**(6), 677-687. https://doi.org/10.12989/cac.2017.19.6.677.

Mousavi, M., Mohammadimehr, M. and Rostami, R. (2019), "Analytical solution for buckling analysis of micro sandwich hollow circular plate", *Comput. Concrete*, **24**(3), 185-192. https://doi.org/10.12989/cac.2019.24.3.185.

Narwariya, M., Choudhury, A. and Sharma, A.K. (2018), "Harmonic analysis of moderately thick symmetric cross-ply laminated composite plate using FEM", *Adv. Comput. Des.*, **3**(2), 113-132. https://doi.org/10.12989/acd.2018.3.2.113.

Needleman, D.J., Ojeda-Lopez, M.A., Raviv, U., Ewert, K., Miller, H.P., Wilson, L. and Safinya, C.R. (2005), "Radial compression of microtubules and the mechanism of action of taxol and associated proteins", *Biophys. J.*, **89**(5), 3410-3423. https://doi.org/10.1529/biophysj.104.057679.

Nogales, E. (2001), "Structural insights into microtubule function", *Ann. Rev. Biophys. Biomolecul. Struct.*, **30**(1), 397-420. https://doi.org/10.1146/annurev.biophys.30.1.397.

Qian, X., Zhang, J. and Ru, C. (2007), "Wave propagation in orthotropic microtubules", *J. Appl. Phys.*, **101**(8), 084702. https://doi.org/10.1063/1.2717573.

Rezaiee-Pajand, M., Masoodi, A.R. and Mokhtari, M. (2018)., "Static analysis of functionally graded non-prismatic sandwich beams", *Adv. Comput. Des.*, **3**(2), 165-190. https://doi.org/10.12989/acd.2018.3.2.165.

Salah, F., Boucham, B., Bourada, F., Benzair, A., Bousahla, A.A. and Tounsi, A. (2019), "Investigation of thermal buckling properties of ceramic-metal FGM sandwich plates using 2D integral plate model", *Steel Compos. Struct.*, **32**(5), 595-610. https://doi.org/10.12989/scs.2019.33.6.805.

Sehar. A., Hussain M., Naeem M.N and Tounsi. A. (2020), "Prediction and assessment of nolocal natural frequencies DWCNTs: Vibration analysis", *Comput. Concrete*, **25**(2), 133-144. https://doi.org/10.12989/cac.2020.25.2.133.

Sharma, P., Singh, R. and Hussain, M. (2019), "On modal analysis of axially functionally graded material beam under hygrothermal effect", *Proc. Inst. Mech. Eng., Part C: J. Mech. Eng. Sci.*, 095440621988823
4.

https://doi.org/10.1177/0954406219888234.
Sofiyev, A.H., Avcar, M., Ozyigit, P. and Adigozel, S. (2009), "The free vibration of non-homogeneous truncated conical shells on a Winkler foundation", *Int. J. Eng. Appl. Sci.*, **1**(1), 34-41.
Sofiyev, A.H., Yücel, K., Avcar, M. and Zerin, Z. (2006), "The dynamic stability of orthotropic cylindrical shells with non-homogenous material properties under axial compressive load varying as a parabolic function of time", *J. Reinf. Plast. Compos.*, **25**(18), 1877-1886. https://doi.org/10.1177/0731684406069914.
Tohidi, H., Hosseini-Hashemi, S.H., Maghsoudpour, A. and Etemadi, S. (2017), "Dynamic stability of FG-CNT-reinforced viscoelastic micro cylindrical shells resting on nonhomogeneous orthotropic viscoelastic medium subjected to harmonic temperature distribution and 2D magnetic field", *Wind Struct.*, **25**(2), 131-156. https://doi.org/10.12989/was.2017.25.2.131.
Shaw, M.K., Compton, H.L., Roos, D.S. and Tilney, L.G. (2000), "Microtubules, but not actin filaments, drive daughter cell budding and cell division in Toxoplasma gondii", *J. Cell Sci*., **113**(7), 1241-1254. https://doi.org/10.1242/jcs.021089.
Shen, H.S. (2010).,"Buckling and postbuckling of radially loaded microtubules by nonlocal shear deformable shell model", *J. Theo. Biol.*, **264**(2), 386-394. https://doi.org/10.1016/j.jtbi.2010.02.014.
Tuszyński, J., Luchko, T., Portet, S. and Dixon, J. (2005), "Anisotropic elastic properties of microtubules", The European *Phys. J. E*, **17**(1), 29-35. https://doi.org/10.1140/epje/i2004-10102-5.
Wang, C., Ru, C. and Mioduchowski, A. (2006a), "Orthotropic elastic shell model for buckling of microtubules", *Phys. Rev. E*, **74**(5), 052901. https://doi.org/10.1103/physreve.74.052901.
Wang, C., Ru, C. and Mioduchowski, A. (2006b), "Vibration of microtubules as orthotropic elastic shells", *Physica E: Low Dimens. Syst. Nanostruct.*, **35**(1), 48-56. https://doi.org/10.1155/2014/591532.
Wang, C. and Zhang, L. (2008), "Circumferential vibration of microtubules with long axial wavelength", *J. Biomech.*, **41**(9), 1892-1896. https://doi.org/10.1016/j.jbiomech.2008.03.029.
Wang, J. and Gao, Y. (2016)., "Nonlocal orthotropic shell model applied on wave propagation in microtubules", *Appl. Math. Model.*, **40**(11-12), 5731-5744. doi: 10.1016/j.apm.2016.01.013.
Wang, Y.A. and Carter, E.A. (2002). "Orbital-free kinetic-energy density functional theory", *Theoretical Methods in Condensed Phase Chemistry*, 117-184, Springer.
Yazdani, R. and Mohammadimehr, M. (2019). "Double bonded Cooper-Naghdi micro sandwich cylindrical shells with porous core and CNTRC face sheets: Wave propagation solution", *Comput. Concrete*, **24**(6), 499-511. https://doi.org/10.12989/cac.2019.24.6.499.
Zou, R. and Foster, C. (1995) "Simple solution for buckling of orthotropic circular cylindrical shells", *Thin Wall. Struct*., **22**(3), 143-158. https://doi.org/10.1155/2008/685805.
Zamani, A., Kolahchi, R. and Bidgoli, M.R. (2017), "Seismic response of smart nanocomposite cylindrical shell conveying fluid flow using HDQ-Newmark methods", *Comput. Concrete*, **20**(6), 671-682. https://doi.org/10.12989/cac.2017.20.6.671.

CC