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Buckling and dynamic behavior of the simply supported CNT-RC beams using an integral-first shear deformation theory

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Abstract. In this work, the buckling and vibrational behavior of the composite beam armed with single-walled carbon nanotubes (SW-CNT) resting on Winkler-Pasternak elastic foundation are investigated. The CNT-RC beam is modeled by a novel integral first order shear deformation theory. The current theory contains three variables and uses the shear correction factors. The equivalent properties of the CNT-RC beam are computed using the mixture rule. The equations of motion are derived and resolved by Applying the Hamilton's principle and Navier solution on the current model. The accuracy of the current model is verified by comparison studies with others models found in the literature. Also, several parametric studies and their discussions are presented.

Keywords: buckling; dynamic behaviour; SW-CNT; Hamilton's principle; navier solution

1. Introduction

The carbon nanotubes (CNTs) has been attracted more attention these last decade, because there excellent properties such as the low density, tensile strength and higher Young modulus (Fiedler *et al*. 2006, Esawi and Farag 2007). This is why they are usable in several sectors as electronics, engineering, physics, chemistry, nanotechnology, and architectural domains (Kolahchi *et al*. 2016a, 2017a, Kolahchi 2017, Golabchi *et al*. 2018, Amnieh *et al*. 2018, Fakhar and Kolahchi 2018, Eltaher *et al*. 2019, Kolahchi *et al*. 2019, Bensattalah *et al*. 2018a, b, Hamidi *et al*. 2018, Eltaher *et al*. 2018, 2019, Selmi 2019). Nowadays, the CNT are considered as the most efficient reinforcement materials for the higher performance structures (Thostenson *et al*. 2001, Gibson *et al* 2007, Rafiee *et al* 2012, 2013). For these benefits, several researchers have studied the behavior of structures reinforced by carbon nanotubes (Wang *et al*. 2011, Zhu *et al*. 2012, Wang and Shen 2012, Jafari Mehrabadi *et al*. 2012, Lei *et al*. 2013a, b, Natarajan *et al*. 2014, Alibeigloo 2014, Wu *et al*. 2016, Mirzaei and Kiani 2015, Eltaher *et al*. 2016, Kiani 2016, Zhang *et al*. 2016a, Selim *et al*. 2016, Zhang *et al*. 2017, Mohamed *et al*. 2019), For examples, the Buckling and dynamic behaviors of CNT-RC beams are investigated by Yas and Samadi (2012).

Wattanasakulpong and Ungbhakorn (2013) studied the various behaviors (static and dynamic) of the CNT-RC beams reposed on elastic foundation. Based on Reissner's model, a unified formulation of FPT (Finite prism techniques) is developed by Wu and Li (2014) to analyze the free vibration of FG CNT-RC plates. Lin and Xiang (2014) utilized the analytical models (FSDBT and TSDBT) and Ritz's method to analyzed vibrational characteristics of the CNT-RC beams. Ansari *et al*. (2015) studied the forced vibrational behavior of the FG CNT-RC plate using the FSDT theory, the Von-Karman type and GDQ (The generalized differential quadrature method). Kamarian *et al*. (2015) have considered the agglomeration effect of CNTs to study the dynamic analysis of FG nano-composite beams. Using the DQ method, Alibeigloo and Liew (2015) investigated on the static and dynamic analysis of the piezoelectric functionally graded CNT-RC beams.

The vibrational behavior of the functionally graded CNT-RC triangular plate subjected to in-plane loads is investigated by Zhang *et al*. (2016b) employing the FSDT element-free method. Kolahchi *et al*. (2016b) examined the dynamic stability of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium based on orthotropic Mindlin plate theory. Using a differential cubature method, Madani *et al*. (2016) studied the vibration behavior of embedded FG-CNT-reinforced piezoelectric cylindrical shells subjected to uniform and non-uniform temperature distributions. Zarei *et al*. (2017) analyzed the vibrational behaviors of temperature-dependent CNT-RC plate with general boundary conditions. Based on FSDT theory, Setoodeh and Shojaee (2017) investigated on critical buckling load of

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(FG-CNTRCQ) plates. Feli *et al.* (2017) developed an analytical model based on FSDT and spring-mass model for dynamic analysis of the CNT-RC plate under low-velocity effect. Moradi-Dastjerdi and Payganeh (2017) investigated on the Thermoelastic vibrational behavior of FG CNT-RC cylinders under thermal load using a mesh-free method. Fantuzzi *et al.* (2017) studied the free vibration of the arbitrary shaped FG-CNT armed the sighted plates using the FSDT. Using a finite element approach, Mehar *et al.* (2018a) analyzed Stress, deflection and frequency of CNT reinforced graded sandwich plate under uniform and linear thermal environment. Mehar *et al.* (2019) presented a numerical buckling analysis of graded CNT-reinforced composite sandwich shell structure under thermal loading. Mehar and Panda (2019) developed a multiscale modeling approach for thermal buckling analysis of nanocomposite curved structure. Recently, Motezaker and Eyvazian (2020) studied the post-buckling response of Mindlin cut out-plate reinforced by FG-CNTs. It should be noted that other types of nanocomposites are considered in literature such as (Bilouei *et al.* 2016, Arani and Kolahchi 2016, Motezaker and Kolahchi 2017a, b, Zamanian *et al.* 2017, Kolahchi and Cheraghbak 2017, Hajmohammad *et al.* 2017, 2018a, b, c, Kolahchi *et al.* 2017b, Hosseini and Kolahchi 2018, Jassas *et al.* 2019, Hajmohammad *et al.* 2019, Azmi *et al.* 2019, Kolahchi *et al.* 2020, Arbabi *et al.* 2020). In the last two years, several researchers are also interested by the study of the CNTs such as (Kolahchi and Moniri Bidgoli 2016, Ansari and Kumar 2018, Daghigh and Daghigh 2018, Hassanzadeh-Aghdam *et al.* 2018, Mehar *et al.* 2018b, Hieu and Van Tung 2018, Lei *et al.* 2018, Qin *et al.* 2019, Ahmadi *et al.* 2019, Chen *et al.* 2019). It is reported that most of the works on buckling/vibration are focused on functionally graded or laminated structures (Panda and Katariya 2015, Kar *et al.* 2016, Mehar *et al.* 2016, Katariya and Panda 2016, Ebrahimi and Barati 2017a, Katariya *et al.* 2017a, b and 2018, Belmahi *et al.* 2018, Yüksela and Akbaş 2018, Belkacem *et al.* 2018, Behera and Kumari 2018, Akbas 2019a, b, Forsat *et al.* 2020, Motezaker *et al.* 2020), but little works is devoted to nanocomposites structures (Mehar and Panda 2016, Mehar *et al.* 2017a, b, Mehar and Panda 2018).

In this work, a novel analytical model based on the integral-first shear deformation beam theory is investigated for buckling and dynamic behaviors of the simply supported CNT-RC beams seated on elastic foundation. The present model takes into account the transverse shear effect and introduces the shear correction factors to ensure the nullity of the transverse shear stress in the top and bottom surface. The Hamilton principle and Navier method are also used to derive and solving the equations of motion of the system. To illustrate the accuracy of the present model, the obtained results are compared with those found in the literature. The parametric studies are also presented to show a various parameter influencing the critical buckling loads and the fundamental frequencies of the composite beam armed with single-walled carbon nanotubes (SW-CNT) resting on Winkler-Pasternak elastic foundation.

2. Theoretical formulations

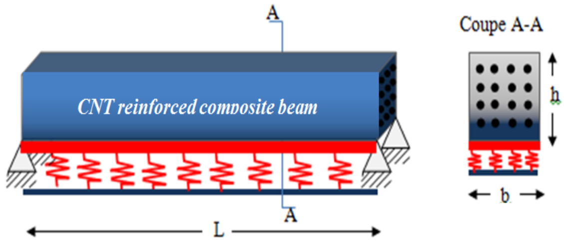

Fig. 1 Geometry of a CNT-RC beam on elastic foundation

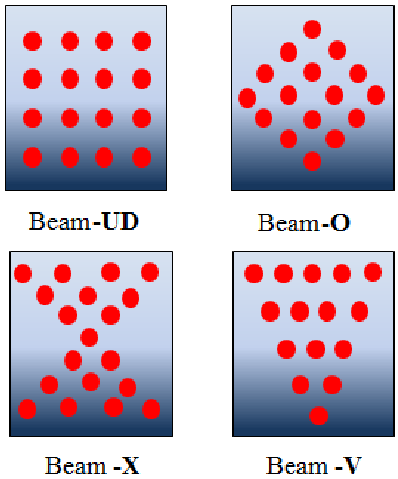

Fig. 2 the various models of the CNT distribution through the thickness of the beam

2.1 Simply supported FG- CNTRC beams

In the present investigation, the beam is supposed made from two material "Isotropic polymer and Single walled CNT". The beam is simply supported and reposed on Winkler-Pasternak elastic foundation (see Fig. 1). Four types of the CNT-RC beams are studied such as UD-beam, FG-O beam, FG-X beam and FG-V beam (as shown in Fig. 2).

For various model of the SWCNT-RC beams, the effective materials properties such as the Young and shear modulus are determined based on the he rule of mixture as (Wattanasakulpong et Ungbhakorn 2013, Ebrahimi and Rostami 2018, Hajlaoui *et al.* 2019, Mohseni and Shakouri 2019)

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E_p \tag{1a}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E_p} \tag{1b}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{G_p} \tag{1c}$$

Where the terms "E_{11}^{cnt}, E_{22}^{cnt} and G_{12}^{cnt}" are the Young's and shear modulus of the SWCNT-RC beam, respectively. "V_{cnt} and V_p" are the volume fraction of the carbon nanotubes (CNT) and polymer with "$V_{cnt}+ V_p=1$". "E_p and G_p" are Young's and shear modulus of the polymer matrix.

Table 1 efficiency parameters η_i associated to the volume fractions V_{cnt}^*

V_{cnt}^*	0.12	0.17	0.28
η_1	1.2833	1.3414	1.3228
η_2	1.0566	1.7101	1.7380
η_3	1.0566	1.7101	1.7380

Table 2 The volume fraction formulations of the CNT through the thickness

CNTs Distributions	V_{cnt}
UD	$V_{cnt} = V_{cnt}^*$
FG-O-beam	$V_{cnt} = 2\left(1-2\frac{\lvert z\rvert}{h}\right)V_{cnt}^*$
FG-X-beam	$V_{cnt} = 4\frac{\lvert z\rvert}{h}V_{cnt}^*$
FG-V-beam	$V_{cnt} = \left(1+2\frac{\lvert z\rvert}{h}\right)V_{cnt}^*$

The terms η_i (i=1,2,3) are the efficiency parameters of the CNT. They are used to take into account the scale-dependent of SWCNT. Based on the mixture law and MD simulation, the efficiency parameters of the CNT are determined (Han and Elliott 2007). The values of the η_i (i=1, 2, 3) of the CNT are abstracted in the Table 1.

The mass density and the poison's ratio of the SWCNT-RC beams are defined by applying the same rule as **(**Yas and Samadi 2012, Mohseni and Shakouri 2019, Hajlaoui *et al*. 2019**)**

$$\begin{aligned} \rho &= V_{cnt}\rho^{cnt} + V_p\rho^p \\ \nu &= V_{cnt}\nu^{cnt} + V_p\nu^p \end{aligned} \tag{2}$$

Where ρ^{cnt}, ρ^p and V^{cnt}, V^p are the masse density and poison's ratio of the CNT and the polymer matrix, respectively.

The mathematical formulations of the CNT distributions through the cross section (see Fig. 2) are presented in the Table 2.

Where the volume fraction of the CNT V_{cnt}^* is obtained from the following expression

$$V_{cnt}^* = \frac{w_{cnt}}{w_{cnt} + \left(\frac{\rho^{cnt}}{\rho}\right)(1-w_{cnt})} \tag{3}$$

From the Table 2, it is remarkable that the UD-beam gives the uniform distributions of the reinforcement. But the *O*, *V* and *X* are the *FG* beams which the material properties varies gradually through the thickness *h*.

2.2 Kinematic and constitutive relations

Based on the Timoshenko's hypothesis and supposing that the transverse rotation $\phi_x = \int \theta(x,t)dx$. The displacement field of the present model can be expressed in the undetermined integral form as

$$u(x,z,t) = u_0(x,y,t) - zk_1\int \theta(x,t)\,dx \tag{4a}$$

$$w(x,z,t) = w_0(x,t) \tag{4b}$$

Where (u_0, w_0 and θ) are the three displacements terms at mid-plane of the CNT-RC beam.

The non-zero strains ε_x and γ_{xz} associated to the displacements $u(x,z,t)$ and $w(x,z,t)$ can be obtained as follow

$$\varepsilon_x = \frac{\partial u_0}{\partial x} - z\,k_1\,\theta\,, \quad \gamma_{xz} = \frac{\partial w_0}{\partial x} - k_1\int \theta\,dx\,, \tag{5}$$

The undetermined integral $\int \theta\,dx$ used in the above section is resolved via the employed solutions type. In the present case the $\int \theta\,dx$ can be expressed in the derivative form as

$$\int \theta\,dx = A'\frac{\partial \theta}{\partial x} \tag{6}$$

Where the coefficient A' is adopted according to Navier method, the coefficients A' and k_1 are obtained as

$$A' = -\frac{1}{\lambda^2} \quad , k_1 = \lambda^2 \tag{7}$$

Where λ is defined in the expression (23).

The stress-strain relations can be expressed based on the Hooke's relation as

$$\sigma_x = Q_{11}(\varepsilon_x) \text{ and } \tau_x = k_s Q_{55}\gamma_x \tag{8a}$$

$$Q_{11}(z) = \frac{E_{11}(z)}{1-\nu^2} \text{ and } Q_{55}(z) = G_{12}(z) \tag{8b}$$

Where k_s represent the shear correction factor and it takes three values (k_s=1,2/3 and 5/6).

2.3 Equations of motion

The three equations of motion are determined using the Hamilton's principle (Talha and Singh 2010, Li *et al*. 2010, Shahrjerdi *et al*. 2011, Yas and Samadi 2012, Ebrahimi and Rostami 2018, Mohseni and Shakouri 2019, Avcar 2019). The principle can be expressed in the analytical form as

$$\int_{t_1}^{t_2} (\delta U - \delta K + \delta V)dt = 0 \tag{9}$$

Where δU, δK and δV are the variations of the virtual strain energy, virtual kinetic energy and the work done by external load, respectively.

The variation of strain energy δU associated to the present model (Eq. (4)) is given as

$$\delta U = \int_0^L \int_A \left(\sigma_x \delta\,\varepsilon_x + \tau_{xz}\delta\,\gamma_{xz}\right) dA dx \tag{10}$$

$$= \int_0^L \left(-M\delta k_1\theta - Qk_1A'\frac{\delta\partial\theta}{\partial x} + Q\frac{\delta\partial w_0}{\partial x} \right) dx \tag{11}$$

Where *N, M* and *Q* the resultants stresses are defined as

$$(N,M)=\int_{-h/2}^{h/2}(1,z)\sigma_x dz \quad \text{and} \quad Q=\int_{-h/2}^{h/2}G\tau_{xz}dz \tag{12}$$

The kinetic energy variation δK can be expressed as

$$\delta K=\iint \rho(\dot{u}\delta\dot{u}+\dot{w}\delta\dot{w})dxdz$$
$$=\iint \rho\left[\begin{array}{l}\left(\dot{u}_0-zK_1A'\dfrac{\partial\dot{\theta}}{\partial x}\right)\delta\left(\dot{u}_0-zK_1A'\dfrac{\partial\dot{\theta}}{\partial x}\right)\\ +\dot{w}_0\delta\dot{w}_0\end{array}\right]dxdz \tag{13}$$

The virtual work done by external load δV can be given as

$$\delta V=\iint\left[f_e\ \delta w+N_x^0\left(\frac{dw}{dx}\frac{d\delta w}{dx}\right)\right]dxdz \tag{14}$$

Where f_e and N_x^0 are the reaction force of the foundation and axial force. The foundation reaction can be given as

$$f_e=K_w-K_s\frac{\partial^2 w}{\partial x^2} \tag{15}$$

Where K_w and K_s are the constants of elastic foundations Winkler and Pasternak, respectively. With

$$K_w=\frac{\beta_w A_{110}}{L^2}\ \ and\ \ K_s=\beta_s A_{110} \tag{16}$$

Where β_w and β_s are the corresponding of spring and shear layer constant factors. A_{110} is the extension stiffness of the A_{11} of the polymer only and can be defined as

$$A_{110}=\int_{-h/2}^{h/2}\frac{E_p}{1-{\nu_p}^2}dz \tag{17}$$

Using the Eqs. (9), (11), (13), (14) and (15), integrating by part and separating the terms of displacements. The three equations of motion are obtained as

$$\delta u_0:\frac{\partial N}{\partial x}=-I_0\ddot{u}+k_1A'I_1\frac{\partial\dot{\theta}}{\partial x} \tag{18a}$$

$$\delta w_0:\frac{\partial Q}{\partial x}-f_e+N_x^0\left(\frac{d^2w_0}{dx^2}\right)=-I_0\ddot{w}_0 \tag{18b}$$

$$\delta\theta:-k_1M+k_1A'\frac{\partial Q}{\partial x}=I_2(k_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2} \tag{18c}$$

Where A_{11}, B_{11} and D_{11} are the Membrane flexural and coupling rigidity, defined by

$$(A_{11},B_{11},D_{11})=\int_{-h/2}^{h/2}Q_{11}(1,z,z^2)dz \tag{19}$$

Where I_i is the inertia of the beam, defined by

$$(I_0,I_1,I_2)=\int_{-h/2}^{h/2}\rho(1,z,z^2)dz \tag{20}$$

3. Analytical solutions

The CNT-RC beam is considered simply supported in the edges. Using the Navier method, the analytical solutions of dynamic and stability of the CNT-RC beam are derived. The Navier solution can be presented in the trigonometric form as (Yas and Samadi 2012)

$$\begin{Bmatrix}u_0\\ w_0\\ \theta\end{Bmatrix}=\sum_{m=1}^{\infty}\begin{Bmatrix}U_{mn}\cos(\lambda x)e^{iwt}\\ w_{mn}\sin(\lambda x)e^{iwt}\\ \theta_{mn}\cos(\lambda x)e^{iwt}\end{Bmatrix} \tag{21}$$

With

$$\lambda=m\pi/L \tag{22}$$

Replacing the above equation (Eq. (21)) in the Eq. (18) as function of displacement. The solution of the dynamic and stability analysis of the CNT-RC beam can be obtained in the matrix form as

$$\left(\begin{bmatrix}S_{11}&S_{12}&S_{13}\\ S_{12}&S_{22}+\overline{N}&S_{23}\\ S_{13}&S_{23}&S_{33}\end{bmatrix}-\omega^2\begin{bmatrix}m_{11}&m_{12}&m_{13}\\ m_{12}&m_{22}&m_{23}\\ m_{13}&m_{23}&m_{33}\end{bmatrix}\right)\begin{Bmatrix}U_{mn}\\ W_{mn}\\ \theta_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \tag{23}$$

Where $[K]$ is the stiffness matrix defined by the following elements

$$\begin{gathered}S_{11}=-A_{11}\lambda^2,\ S_{12}=0,\ S_{13}=B_{11}k_1\lambda\\ S_{22}=-A_{55}\lambda^2-\beta_w-\beta_s\lambda^2,\ S_{33}=-A_{55}\lambda^2-D_{11}\lambda^4\end{gathered} \tag{24}$$

And $[M]$ is the mass matrix defined by the following elements

$$\begin{gathered}m_{11}=-I_0\ ,\ m_{12}=m_{21}=0,\ m_{13}=m_{31}=-I_1\lambda,\\ m_{22}=-I_0\ ,\ m_{23}=m_{32}=0,\ m_{33}=-I_2\lambda\end{gathered} \tag{25}$$

4. Results and discussions

In this work, the stability and dynamic behaviors of the FG CNT-RC beam reposed on the Winkler-Pasternak foundations are presented using a novel integral-first shear deformation beam theory. The beams are composed by Poly methacrylate “PMMA” as matrix and the armchair (10, 10) single walled Carbone nanotubes (SW-CNTs) as reinforcement. The properties of these components are given as

- ***Poly methacrylate “PMMA”***

  $E_p=2.5GPa,\ \rho_p=1190\ kg/m^3,\ \nu^p=0.3$

- ***The armchair (10, 10) SWCNTs***

  $E_{11}^{cnt}=600\,GPa,\ E_{22}^{cnt}=10\,GPa,$

  $\rho^{cnt}=1400\ \ kg/m^3,$

  $\nu^{cnt}=0.19,G_{12}^{cnt}=17.2\,GPa$

For convenience purposes, the obtained results are presented in the nondimensional form as

- ***Critical buckling load***

Table 3 Dimensionless critical buckling load of the CNT-RC beams with "L=15h"

(β_w, β_s)	V_{cnt}^{*}	Theory	UD	V	O	FG-X
(0.00, 0.00)	0.12	Wattanasakulpong and Ungbhakorn (2013)	0.1032	-	0.0604	0.1367
		present (K_s=1)	0.1032	0,0969	0.0604	0.1368
		Yas and Samadi (2012)	0,0985	0,0925	0,0588	0,1288
		present (K_s=5/6)	0,0983	0,0927	0,0587	0,1284
	0.17	Yas and Samadi (2012)	0,1505	0,1407	0,0877	0,1999
		present (K_s=5/6)	0,1500	0,1408	0,0875	0,1989
		present (K_s=1)	0,1566	0,1466	0,0897	0,2106
	0.28	Yas and Samadi (2012)	0,2209	0,2093	0,1338	0,2896
		present (K_s=5/6)	0,2193	0,2075	0,1331	0,2867
		present (K_s=1)	0,2315	0,2183	0,1373	0,307
(0.1, 0.00)	0.12	Yas and Samadi (2012)	0,1087	0,1026	0,0689	0,139
		present (K_s=5/6)	0,1085	0,1028	0,0688	0,1385
		present (K_s=1)	0,1133	0,1071	0,0705	0,1469
	0.17	Yas and Samadi (2012)	0,1606	0,1508	0,0978	0,2101
		present (K_s=5/6)	0,1602	0,15099	0,0976	0,2091
		present (K_s =1)	0,1667	0,1568	0,0998	0,22068
	0.28	Yas and Samadi (2012)	0,231	0,2194	0,1439	0,2998
		present (K_s=5/6)	0,2294	0,2176	0,1432	0,2968
		present (K_s=1)	0,2416	0,2285	0,1475	0,3171
(0.1, 0.02)	0.12	Wattanasakulpong and Ungbhakorn (2013)	0.1333	-	0.0905	0.1668
		present (K_s=1)	0.1333	0,1271	0.0905	0.1669
		Yas and Samadi (2012)	0,1287	0,1226	0,0889	0,159
		present (K_s=5/6)	0,1285	0,1228	0,0888	0,1585
	0.17	Yas and Samadi (2012)	0,1807	0,1709	0,1178	0,2301
		present (K_s=5/6)	0,1801	0,171	0,1176	0,2291
		present (K_s=1)	0,1867	0,1768	0,1198	0,2407
	0.28	Yas and Samadi (2012)	0,251	0,2394	0,1639	0,3198
		present (K_s=5/6)	0,2494	0,2376	0,1633	0,3168
		present (K_s=1)	0,2617	0,2485	0,1675	0,3371

$$\overline{N} = \frac{N_x^0}{A_{110}} \tag{26}$$

➢ ***Fundamental frequency parameter***

$$\overline{\omega} = \omega L \sqrt{\frac{I_{00}}{A_{110}}} \tag{27}$$

Where I_{00} is the extension inertia of the I_0 of the polymer only.

4.1 Stability analysis of the CNT-RC beams

In this first part, we present the buckling analysis of the simply supported CNT-RC beam under mechanical axial load reposed on the Winkler-Pasternak elastic foundation.

The Table 3 presents the comparison of the nondimensional critical buckling load "$\overline{N}$" versus the elastic foundation parameters "β_w, β_s", the nanotube volume fraction "V_{cnt}^{*}" and reinforcement distribution of the simply supported CNT-RC beams (*UD*, *V*, *O* and *X*). The obtained results computed using the current model are compared with those given by first shear deformation beam theory developed by Wattanasakulpong and Ungbhakom (2013) with shear correction factor K_s=1 and those obtained by FSDT proposed by Yas and Samadi (2012) using the correction factor K_s=5/6. From the presented table, a good concordance is confirmed between the current results and those found in the literature. It can be also seen that the increase of the nanotube volume fraction "V_{cnt}^{*}" leads to an increase of the critical buckling load "$\overline{N}$". The largest values of the "$\overline{N}$" are obtained for FG-X CNT-RC beams.

The effect of the Winkler parameter "β_w" on the adimensional critical buckling load "$\overline{N}$" of the various CNT-RC beams is illustrated in the Fig. 3. From the plotted graphs, it can be noted the adimensional critical buckling load "$\overline{N}$" is in direct correlation relation with the Winkler parameter "β_w". The lowest values of the adimensional critical buckling load "$\overline{N}$" are obtained for The FG-O distribution.

Fig. 4 illustrate the effect of the shear parameter of Pasternak "β_s" on the adimensional critical buckling load "$\overline{N}$" of the simply supported UD, FG-V, FG-O and FG-X beams with (V_{cnt}^{*}=0.17; L/h=10; β_s=0.4). It is remarkable that the adimensional critical buckling load "$\overline{N}$" increase linearly as function of Pasternak parameter "β_s" and that

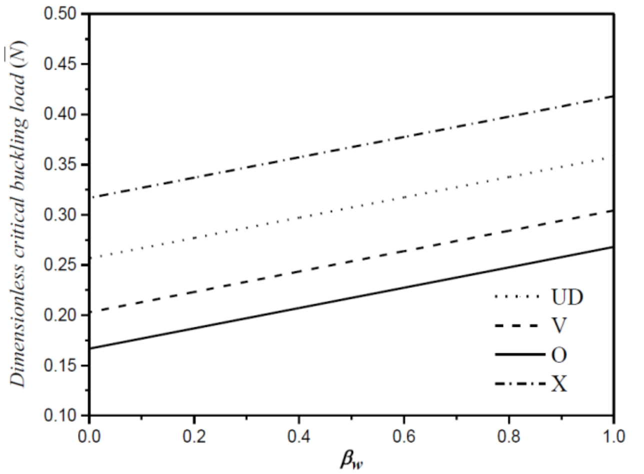

Fig. 3 Effect of Winkler modulus parameter on the critical buckling loads of CNT-RC beams (V_{cnt}^{*}=0.14; L/h=10; β_s=0)

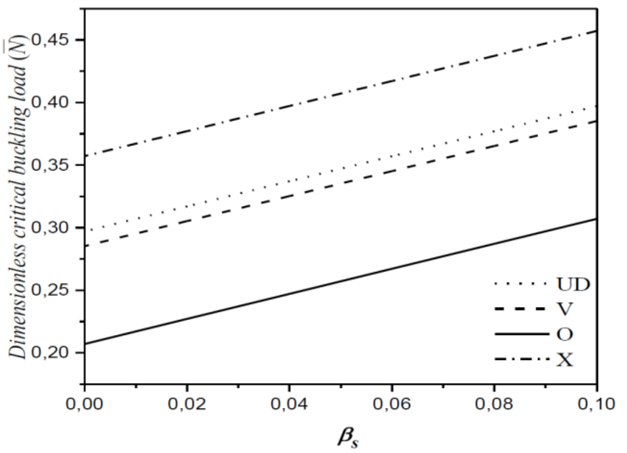

Fig. 4 Effect of Pasternak shear modulus parameter the critical buckling loads of CNT-RC beams (V_{cnt}^{*}=0.17; L/h=10; β_s=0.4)

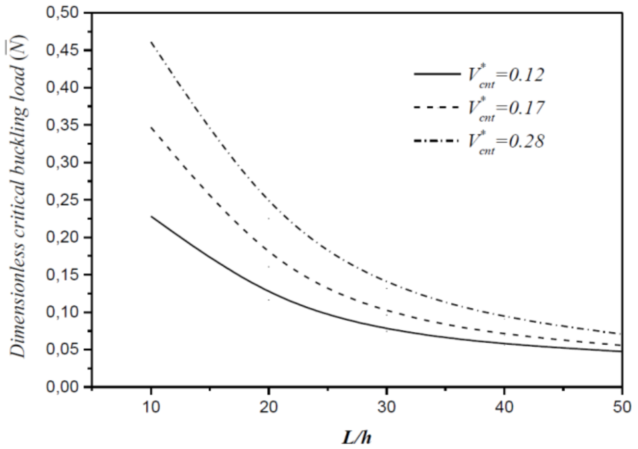

Fig. 5 Dimensionless critical buckling loads of *FG-X* Beam on elastic foundation with various thickness ratios and CNT volume fraction V_{cnt}^{*} (β_w=0.1; β_s=0.02)

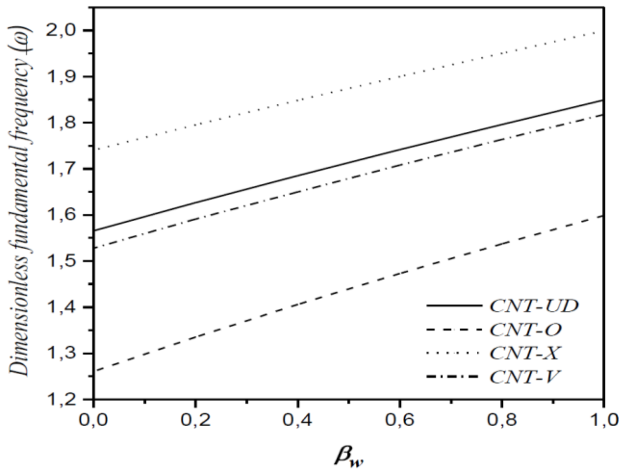

Fig. 6 Effect of Winkler modulus parameter on the fundamental frequencies of CNT-RC beams with (L/h=10, β_s=0, V_{cnt}^{*}=0.17)

because of the beam that becomes rigid.

Fig. 5 shows the variation of the adimensional critical buckling load "$\overline{N}$" of the FG-X beam versus the slenderness ratio "L/h" and CNT volume fraction "V_{cnt}^{*}" with (β_w=0.1; β_s=0.02). We can see through the results that the adimensional critical buckling load "$\overline{N}$" is in inverse relation with the slenderness ratio "L/h". We can also conclude that the increase in the CNT volume fraction "V_{cnt}^{*}" leads to increase the critical buckling load "$\overline{N}$".

4.2 Dynamic analysis of the CNT-RC beams

In this section present the vibrational behavior of the UD, FG-V, FG-O and FG-X simply supported CNT-RC beams on elastic foundation (Winkler-Pasternak type).

Table 4 presents the first three adimensional frequencies "$\overline{\omega}$" versus the elastic foundation parameters for various beams models (UD, FG-V, FG-O and FG-X). Three types of the foundation are considered. The first one the CNT-RC beams is reposed on only Pasternak elastic foundation (β_w, β_s)=(0.1, 0.02), the second type, the CNT-RC beam is reposed on Winkler elastic foundation (β_w, β_s)=(0.1, 0.00) and the last type, the CNT-RC beam is without elastic foundation (β_w, β_s)=(0.0, 0.00).

From the obtained results, we can notice that the presence of a Winkler-Pasternak foundation type leads to an increase in the first three adimensional fundamental frequencies "$\overline{\omega}$" and this is confirmed for different types of CNT-distribution (*UD*, *V*, *O* and *X*). It can also be observed that the adimensional fundamental frequency "$\overline{\omega}$" increase with increasing of the CNT volume fraction "V_{cnt}^{*}". It can be noted that the lowest values of the adimensional frequencies "$\overline{\omega}$" are obtained for FG-O distribution unlike the FG-X distribution which gives the largest values of the nondimensional frequency "$\overline{\omega}$".

The variation of the fundamental frequencies parameters "$\overline{\omega}$" as function of the Winkler and Winkler-Pasternak parameters "β_w, β_s" are presented in the Figs. 6 and 7, respectively. The computed results are for various CNT-RC beams with (V_{cnt}^{*}=0.17, L/h=10). It can be seen from the results that the adimensional fundamental frequencies "$\overline{\omega}$" increase almost linearly with increasing of the elastic

Table 4 First three dimensionless natural frequency $\bar{\omega} = \omega L\sqrt{I_{10}/A_{110}}$ of simply supported CNT-RC beams with (L/h=15)

V_{cnt}^{*}	model	Theory	(β_w=0, β_s=0)			(β_w=0.1, β_s=0)			(β_w=0.1, β_s=0.02)		
			ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
0.12	*UD*	Yas and Samadi (2012)	0.9753	2.8728	4.8704	1.0241	2.8898	4.8804	1.1144	3.0203	5.0552
		present (K_s=5/6)	0.9740	2.8640	4.8530	1.0229	2.8810	4.8630	1.1132	3.0127	5.0380
	V	Yas and Samadi (2012)	0.9453	2.6424	4.6675	0.9957	2.6607	4.6780	1.0883	2.8013	4.8596
		present (K_s=5/6)	0.9400	2.6300	4.6200	0.9950	2.6571	4.6380	1.0880	2.7982	4.8215
	O	Yas and Samadi (2012)	0.7527	2.4562	4.4320	0.8150	2.4760	4.4430	0.9258	2.6268	4.6338
		present (K_s=5/6)	0.7521	2.4510	4.4180	0.8144	2.4700	4.4200	0.9250	2.6219	4.6200
	X	Yas and Samadi (2012)	1.1150	3.0814	5.0695	1.1581	3.0972	5.0791	1.2386	3.2194	5.2474
		present (K_s=5/6)	1.1130	3.0710	5.0490	1.1561	3.0860	5.0587	1.2360	3.2090	5.2270
0.17	*UD*	Yas and Samadi (2012)	1.1999	3.6276	6.2363	1.2396	3.6409	6.2441	1.3145	3.7444	6.3804
		present (K_s=5/6)	1.1977	3.6127	6.2030	1.2300	3.6261	6.2108	1.3120	3.7300	6.3478
	V	Yas and Samadi (2012)	1.1609	3.3084	5.9498	1.2019	3.3229	5.9579	1.2790	3.4354	6.1002
		present (K_s=5/6)	1.1604	3.3000	5.8840	1.2014	3.3150	5.8900	1.2786	3.4280	6.0300
	O	Yas and Samadi (2012)	0.9155	3.0577	5.6139	0.9670	3.0734	5.6225	1.0612	3.1951	5.7731
		present (K_s=5/6)	0.9145	3.0480	5.5884	0.9660	3.0641	5.5970	1.0600	3.1860	5.7480
	X	Yas and Samadi (2012)	1.3830	3.9293	6.5447	1.4176	3.9416	6.5521	1.4836	4.0375	6.6822
		present (K_s=5/6)	1.3790	3.9000	6.5042	1.4142	3.9210	6.5117	1.4800	4.0180	6.6420
0.28	UD	Yas and Samadi (2012)	1.4401	4.1362	6.9245	1.4728	4.1477	6.9314	1.5352	4.2372	7.0523
		present (K_s=5/6)	1.4340	4.1049	6.8500	1.4676	4.1160	6.8660	1.5300	4.2060	6.9800
	V	Yas and Samadi (2012)	1.4027	3.8639	6.7618	1.4362	3.8762	6.7688	1.5002	3.9714	6.8923
		present	1.3950	3.8404	6.6600	1.4290	3.8520	6.8370	1.4930	3.9400	6.9500
	O	Yas and Samadi (2012)	1.1202	3.6056	6.4434	1.1619	3.6187	6.4508	1.2400	3.7208	6.5802
		present (K_s=5/6)	1.1170	3.5800	6.3800	1.1500	3.5900	6.3900	1.2370	3.6900	6.5250
	X	Yas and Samadi (2012)	1.6493	4.4752	7.3068	1.6779	4.4858	7.3133	1.7330	4.5688	7.4280
		present (K_s=5/6)	1.6400	4.4330	7.2250	1.6690	4.4400	7.2300	1.7240	4.5200	7.3480

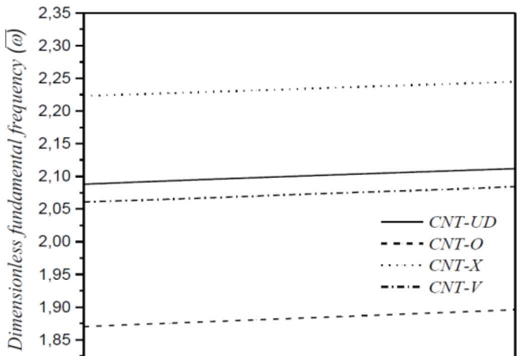

Fig. 7 Effect of Pasternak shear modulus parameter the fundamental frequencies of CNT-RC Beams with (V_{cnt}^{*}=0.17, L/h=10, β_w=0.2)

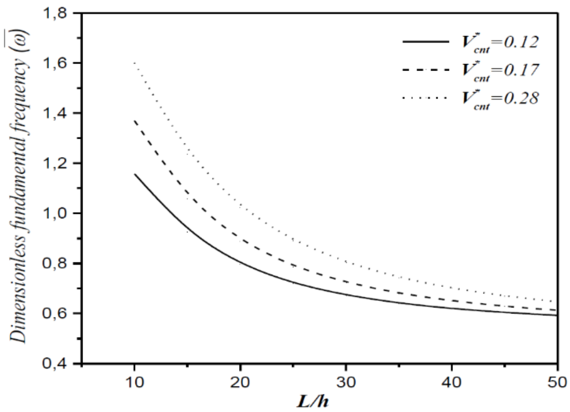

Fig. 8 Dimensionless fundamental frequencies of FG-O Beam on elastic foundation with various thickness ratios (β_w=0.1, β_s=0.02)

foundation parameters (Winkler-Pasternak). The fundamental frequencies "$\bar{\omega}$" obtained for FG-O beam are smaller compared to other CNT-distributions (*UD*, *V* and *X*).

Fig. 8 presents the effect of the slenderness ratio "*L/h*" and the CNT volume fraction "V_{cnt}^{*}" on the values of the fundamental frequencies "$\bar{\omega}$" of the FG-O. From the plotted curves, it can be seen that the increase in the CNT volume fraction "V_{cnt}^{*}" leads to an increase in the fundamental frequency "$\bar{\omega}$". We can also conclude that the fundamental frequency "$\bar{\omega}$" is inverse relation with slenderness ratio "*L/h*".

5. Conclusions

In this investigation, Based on integral first order shear deformation beam theory, the buckling and dynamic behaviors of the nano-composite beam reinforced by Single walled (SW) carbon nanotubes (CNT) resting on elastic foundation (Winkler-Pasternak type) have been studied. The current model contains only three unknowns' variables and uses the shear correction factors. The effectives properties of the CNT-RC nano-composite beam have been estimated though the mixture rule. The Hamilton's principle has been applied to derive the equations of motion and this last are resolved by analytical solutions of Navier. The accuracy of the present model has been confirmed by comparing the obtained results with those found in the literature. Finally, we can conclude that the CNT volume fraction, elastic foundation parameter and CNT distribution have significant effects on critical buckling load and natural frequencies of CNT-RC beams. An improvement of the present formulation will be considered in the future work to consider other type of structures and materials (Sharma *et al*. 2009, Ebrahimi and Barati 2017b, Lal *et al*. 2017, Daouadji 2017, Dihaj *et al*. 2018, Faleh *et al*. 2018, Ayat *et al*. 2018, Panjehpour *et al*. 2018, Pascon 2018, Li *et al*. 2018, Narwariya *et al*. 2018, Rezaiee-Pajand *et al* 2018,Rajabi and Mohammadimehr 2019, Fadoun 2019, Al-Osta 2019, Mehar *et al*. 2018c, Abdou *et al*. 2019, Barati *et al*. 2019, Belmahi *et al*. 2019, Bensattalah *et al*. 2019, Katariya *et al*. 2019, Othman *et al*. 2019, Yüksela and Akbaş 2019, Eltaher *et al*. 2020).

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