Computers and Concrete, *Vol. 26, No. 2 (2020) 107-114*
DOI: https://doi.org/10.12989/cac.2020.26.2.107

Finite element modeling of contact between an elastic layer and two elastic quarter planes

Murat Yaylacı*[1] and Mehmet Avcar[2]

[1]Department of Civil Engineering, Recep Tayyip Erdogan University, 53100, Rize, Turkey
[2]Department of Civil Engineering, Suleyman Demirel University, 32260, Isparta, Turkey

(*Received May 19, 2020, Revised July 6, 2020, Accepted July 8, 2020*)

Abstract. In this study, a two dimensional model of receding contact problem has been analyzed using finite element method (FEM) based software ANSYS and ABAQUS. For this aim finite element modeling of elastic layer and two homogeneous, isotropic and symmetrical elastic quarter planes pressed by means of a rigid circular punch has been presented. Mass forces and friction are neglected in the solution. Since the problem is examined for the plane state, the thickness along the z-axis direction is taken as a unit. In order to check the accuracy of the present models, the obtained results are compared with the available results of the open literature as well as the results of two software are compared using Root Mean Square Error (RMSE) and good agreements are found. Numerical analyses are performed considering different values of the external load, rigid circular radius, quarter planes span length and material properties. The contact lengths and contact stresses of these values are examined, and their results are presented. Consequently, it is concluded that the considered non-dimensional quantities have noteworthy influence on the contact lengths and contact stress distributions, additionally if FEM analysis is used correctly, it can be an efficient alternative method to the analytical solutions that need time.

Keywords: contact mechanics; contact areas; contact stress; FEM

1. Introduction

Practical structural and mechanical components and materials that can keep pace with the changing and developing technology day by day is one of the fundamental issues of today's engineers. For this reason, analytical, numerical and experimental studies on the different behaviors of structural and mechanical components made of different kinds of materials have been the focus of interest of scientists (Gurses *et al*. 2012, Akgoz and Civalek 2015, Civalek and Demir 2016, Haeri and Sarfarazi 2016, Haeri and Marji 2016, Amnieh *et al*. 2018, Avcar 2019, Hajmohammad *et al*. 2019, Arbabi *et al*. 2020, Farokhian and Kolahchi 2020, Hussain *et al*. 2020, Kolahchi *et al*. 2020, Ghamari *et al*. 2020, Taherifar *et al*. 2020). One of the load transferring methods between these structural and mechanical components is contacting them. Stresses and deflections occur between contacting two elastic bodies which may affect the stability of the structural body. Hence, the nature of the stresses arising from the contact between two elastic bodies is of significant importance in engineering field and was first studied by Hertz (1881, 1882). Then, contact problems have found wide application areas in engineering structures of practical importance such as foundations, road and airport superstructures, railways, fuel tanks, grain silos, cylindrical shafts and marbles as well as in biomechanics where human joints, implants or teeth are of consideration. In the last century, Hertzian contact theory has established to be a respected tool for finding the behavior of contacting bodies. Spence (1975) formulated the indentation of an elastic half-space by an axisymmetric punch under a monotonically applied normal force using a mixed boundary value problem under the assumption of Coulomb friction with the coefficient in the region of contact. Keer *et al*. (1972) discussed the problem of frictionless contact of an elastic layer loaded with elastic load and fitted to the elastic half plane, where the contact areas and contact stresses are obtained solving unsymmetrical plane contact problems. The same problem was handled by Ratwani and Erdogan (1973) loading through a frictionless block considering the block profile as rectangular and circular. Cakiroglu and Cakiroglu and Cakiroglu (1991) investigated the continuous and discontinuous contact problem between the elastic layer and the elastic half plane. Pindera and Lane (1993) discussed the contact problem of isotropic, orthotropic, or monoclinic layers. Guler and Erdogan (2004) discussed the contact problem of functionally graded layers supported by an elastic half plane and presented numerical examples. Kahya *et al*. (2007), examined the problem of contact with an anisotropic elastic layer resting on anisotropic elastic half plane and pressed with a rigid punch taking into account a single integral equation where the contact length and contact stress are unknown and solved for some dimensionless parameters. Kuo (2008) presented a two-dimensional contact stress analysis to investigate the effects of multiple inclusions on the contact pressure and subsurface stresses in an elastic half-plane. Rhimi *et al*. (2009, 2011) considered the axisymmetric

*Corresponding author, Ph.D.
E-mail: murat.yaylaci@erdogan.edu.tr

http://www.techno-press.org/?journal=cac&subpage=8
ISSN: 1598-8198 (Print), 1598-818X (Online)

problem of a frictionless receding contact between an elastic functionally graded layer and a homogeneous half-space, when the punch subjected to concentrated normal force and applied distributed load. Liu *et al*. (2010) treated two-dimensional elastic contact problems including normal, tangential, and rolling contacts, using FEM. Zhou and Gao (2013) presented analytical solutions for the problems of an elastic half-space and an elastic half-plane subjected to a distributed normal force. Gandhi *et al.* (2015) examined the effect of material dependency in elastic-plastic contact models by contact analysis of sphere and flat contact model and wheel rail contact model considering the material properties without friction. Öner *et al*. (2015) presented a comparative study of analytical method and FEM for analysis of a continuous contact problem. Vasiliev *et al*. (2017) considered the plane contact problem of the theory of elasticity on the indentation of a non-deformable punch with a flat base into an elastic transversely isotropic half-plane with a transversely isotropic functionally graded coating. Liu *et al*. (2018) considered the smooth receding contact problem between a homogeneous half plane and a composite laminate composed of an in-homogeneously coated elastic layer. Yaylaci *et al*. (2019a) dealt with the numerical analysis of the symmetric contact problem of two bonded layers resting on an elastic half plane compressed with a rigid punch. Yaylaci *et al*. (2019b) examined the contact problem of an elastic layer resting on the rigid foundation.

In the researches on the contact problems the layers are thought to be resting on the elastic half plane. However, the layers may rest on elastic quarter planes. Limited number of studies performed considering elastic quarter planes. Some of them can be summarized as follows. Erdogan and Ratwani (1974) have solved the problem of discontinuous contact in a layer supported by two quarter planes reducing the problem to a single integral equation where the contact stresses are unknown. Keer *et al*. (1984) solved the problem of the contact between a smooth, rigid indenter and an elastic quarter space using integral transform techniques. Hanson and Keer (1989) used an integral-equation formulation to study the stress analysis and related contact problems for an elastic quarter-plane. Aksoğan *et al*. (1996, 1997) discussed the problem of contact in the case of a symmetrical and unsymmetrical elastic layer which supported by two elastic quarter planes.. Yaylaci and Birinci (2013) considered a receding contact problem for two elastic layers whose elastic constants and heights are different supported by two elastic quarter planes and subjected to a uniformly distributed load is considered according to the theory of elasticity. In addition, this problem was analyzed with based on the FEM using ANSYS software by Yaylaci *et al*. (2014). The numerical and analytical results for the contact pressures, contact areas, and normal are given for various dimensionless quantities. They achieved that the numerical results are verified by comparison with analytical results. A receding contact problem for an elastic layer and functionally graded layer resting on two quarter planes is considered by Adiyaman *et al.* (2015, 2016). The layer is pressed by a stamp and distributed loads. The dimensionless quantities for the contact areas and the contact pressures are calculated. Comez *et al*. (2018) considered the plane receding contact problem for a functionally graded layer resting on two quarter-planes using the theory of linear elasticity.

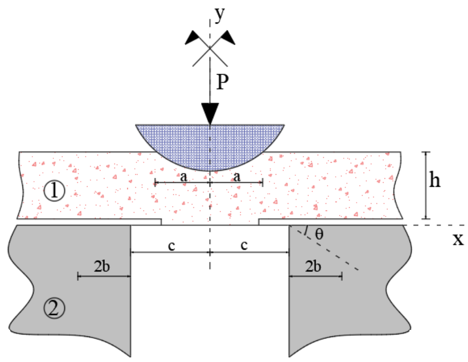

Fig. 1 Geometry of the problem

From the search of open literature, it seen that, finite element modeling of an elastic layer and two homogeneous, isotropic and symmetrical elastic quarter planes and pressed by means of a rigid circular punch has not been presented yet. Hence, an attempt is made to address this problem in this study. For this aim, two-dimensional modeling and simulations are performed using ANSYS and ABAQUS package programs. The accuracy of the present models is validated by comparing the obtained results are with the available results of the open literature. Furthermore, the results of two software are compared using RMSE, also. Numerical analyses are performed considering different values of the external load, radius rigid circular radius, quarter planes span length and material properties. The contact lengths and contact stresses of these values are examined, and their results are presented.

2. The modeling and solution of problem

Fig. 1 shows the geometry of present contact problem composed of a rigidly supported elastic layer and two quarter planes subjected to the concentrated load P applied with circular rigid punch, where, a is the contact distance between the layer and the punch, h is the thickness of the layer, c is the distance between quarter planes, b is the contact distance between the layer and quarter plane. Here, y-axis is the axis of the symmetry, the quarter planes and the layer locate in the range of $-\infty \leq x < \infty$, therefore the calculations are performed in the range of $0 \leq x < \infty$. Besides, the thickness is taken as a unit since the problem is examined for the plane state, as well as the mass forces and friction are neglected. The present contact problem is solved with the use of finite element method-based software called ANSYS (2013) and ABAQUS (2017). The finite element method is an effective means for the solution of the contact problems and it allows considering a real geometry of interacting bodies, complex physical and mechanical properties of materials, mixed boundary conditions (Shrestha *et al*. 2013, Vasudevan and Kothandaraman 2015, Lazzari *et al*. 2017, Zhang *et al*. 2017, Tigdemir *et al*. 2018, Stoner and Polak 2020).

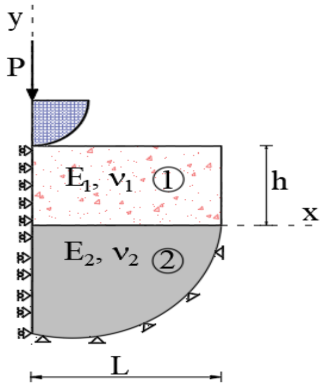

Fig. 2 The geometry for the analysis

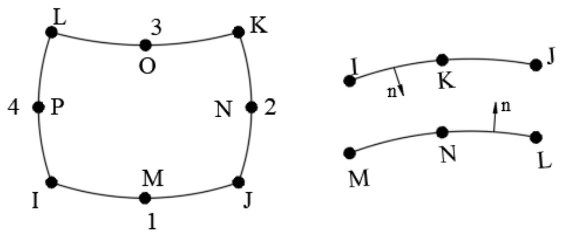

Fig. 3 PLANE183 and TARGE169/CONTA172 contact elements

Fig. 2 shows the geometry for the analyses; here, μ_i, ν_i (i=1, 2) indicate shear modulus and Poisson's ratio of the layer and quarter planes, respectively.

In these analyzes carried out with the help of package programs, many processes such as determination of element types, assignment of material properties of the elements, formation of the geometry of the problem, formation of the mesh structure, assignment of boundary conditions, loading of the problem, solution of the problem and analysis results are performed. Element choice is particularly important for determining the mathematical model to be used in the analysis. The elements are selected according to the type of analysis to be performed, that is, different elements are used for static, thermal, fluid, or electromagnetic analysis. Similarly, whether the model to be analyzed is two or three dimensional is one of the factors in the choice of the element. The type and number of degrees of freedom of the selected element is especially important for the correct analysis. In the analysis, PLANE183 type structural element in ANSYS package library is used. This element is defined by eight joints and each joint has two degrees of freedom and no rotation. Therefore, it can make displacement and deformation in x and y directions. The element has plasticity, withstands large flexes, and has a great deal of deformation, as well as, it performs better than other two-dimensional elements with four connection points in the formation of the network structure of complex geometries. In this study, the surface-to-surface contact model is used to model the contact pair. The surface-to-surface contact pattern also allows for solution if the joints do not overlap. In the problem, a contact pair is formed in the contact area. Contact pairs consist of two element types. These are the TARGET and CONTACT element types. The target surface TARGE169 and the contact surface CONTA172 are used to form the contact pair. The elements TARGE169 and CONTA172 are elements having three nodes and these nodes overlap with the nodes on the surface of the PLANE183 element as can be seen in Fig. 3.

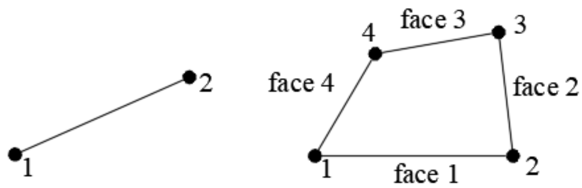

Fig. 4 RAX2 element and CAX4R elements

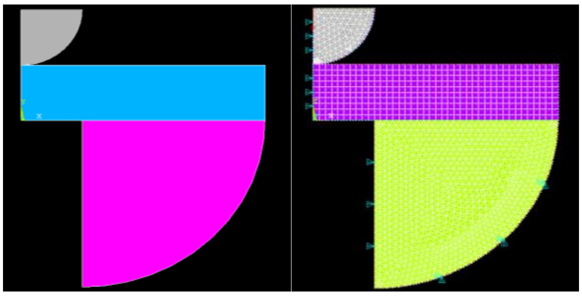

Fig. 5 The finite element mesh in ANSYS

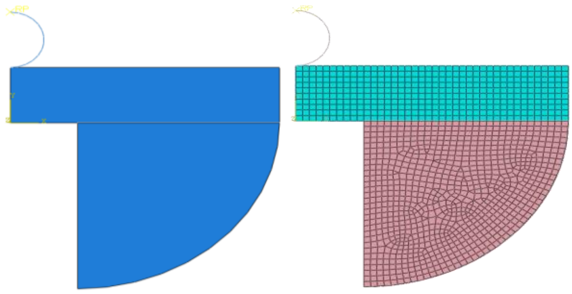

Fig. 6 The finite element mesh in ABAQUS

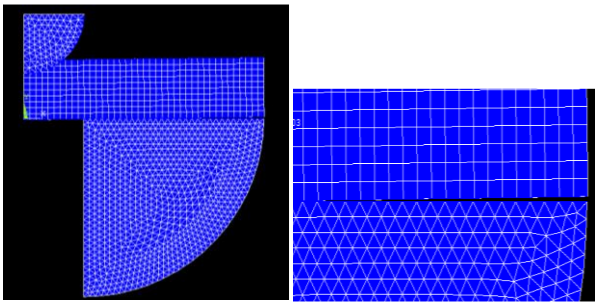

Fig. 7 Deformed shapes of model in ANSYS

In the analyzes performed with ABAQUS program, punch is modeled using 2-node axial symmetrical rigid link element RAX2, the axial symmetrical CAX4R element with four nodes is used while for the other elements. Fig. 4 shows the RAX2 element and CAX4R elements. In contact interaction modeling, the surface-surface contact model is chosen like the ANSYS program.

The finite element models of prior to analysis of the problem created in the ANSYS and ABAQUS software are shown in Figs. 5 and 6. Besides, the deformed shapes that occur after analysis of these models are shown in Figs.7 and 8.

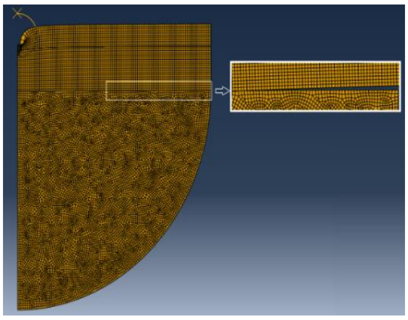

Fig. 8 Deformed shape of model in ABAQUS

3. Numerical studies and discussion

In this section, the contact distances and contact stresses that occur under different external load, material and geometric characteristics are examined using finite element method software called ANSYS and ABAQUS. The results obtained from the numerical analyzes are presented in tables and figures and their findings are discussed. Here, linear, elastic and isotropic materials are used in all parts of the model and concentrated load is taken to be P=100 kN, half length, height, Young's modulus and Poisson's ratio of the layers are taken into account as L=0.5 m, h=0.1 m, E_1=3×10^{10} Pa, ν_1=0.25. The other quantities about material and geometry are taken into consideration as parameters depending on the ratios used in the analyzes.

In all tables and figures the following non-dimensional parameters are used where the values of h and μ_1 parameters kept constant since they affect the other ones.

a/h: contact length between rigid punch and layer
b/h: contact length between quarter planes and layer
c/h: span length of quarter planes
R/h: radius of rigid punch
$\mu_1/(P/h)$: load ratio
$p_1(x)/(P/h)$: Contact stress between rigid punch and layer
$p_2(x)/(P/h)$: Contact stress between quarter planes and layer
κ_1: material property of the layer
κ_2: material property of the quarter planes
μ_1/μ_2: shear modulus ratio

Study 1: Table 1 shows a comparative work for the change in the contact lengths against the non-dimensional load ratio with the analytical results of Cakiroglu and Cakiroglu (1991) to verify the exactness of the present model. Here the following non-dimensional quantities are used: (c/h=0.2, R/h=10, κ_1=κ_2=2, μ_1/μ_2=2). As one can see from Table 1 that the present results are in good agreement with the analytical results of Cakiroglu and Cakiroglu (1991). Thus, it is concluded that the assumed solution method is a good calculation method to examined actual contact problems.

Study 2: Table 2 displays the change in the contact lengths against the radius of rigid punch considering the following non-dimensional parameters: (c/h=0.2, $\mu_1/(P/h)$=250, κ_1=κ_2=2, μ_1/μ_2=2). It is seen that both contact lengths increase with the increase of radius of rigid punch.

Table 1 Comparison of the contact lengths against non dimensional load ratio

$\mu_1/(P/h)$	Source	a/h	b/h
50	Cakiroglu and Cakiroglu (1991)	0.321737	1.007224
	Present (ANSYS)	0.325	1.000
	Present (ABAQUS)	0.320	1.000
100	Cakiroglu and Cakiroglu (1991)	0.223263	0.992050
	Present (ANSYS)	0.225	0.990
	Present (ABAQUS)	0.225	0.990
200	Cakiroglu and Cakiroglu (1991)	0.156233	0.984898
	Present (ANSYS)	0.150	0.985
	Present (ABAQUS)	0.150	0.980
300	Cakiroglu and Cakiroglu (1991)	0.127103	0.982582
	Present (ANSYS)	0.125	0.980
	Present (ABAQUS)	0.130	0.975
600	Cakiroglu and Cakiroglu (1991)	0.089543	0.980302
	Present (ANSYS)	0.090	0.980
	Present (ABAQUS)	0.090	0.970
1000	Cakiroglu and Cakiroglu (1991)	0.069256	0.979398
	Present (ANSYS)	0.070	0.975
	Present (ABAQUS)	0.070	0.970
1500	Cakiroglu and Cakiroglu (1991)	0.056505	0978948
	Present (ANSYS)	0.055	0.970
	Present (ABAQUS)	0.055	0.965

Table 2 The change in the contact lengths against the radius of rigid punch

R/h	Source	a/h	b/h
10	ANSYS	0.140	0.980
	ABAQUS	0.140	0.985
20	ANSYS	0.200	0.990
	ABAQUS	0.200	0.990
50	ANSYS	0.325	1.000
	ABAQUS	0.320	1.000
100	ANSYS	0.475	1.050
	ABAQUS	0.470	1.050
250	ANSYS	0.780	1.150
	ABAQUS	0.780	1.155
500	ANSYS	1.110	1.350
	ABAQUS	1.110	1.355
1000	ANSYS	1.575	1.700
	ABAQUS	1.550	1.705

Study 3: Table 3 presents the change in the contact lengths against the span length of quarter planes considering the following non-dimensional parameters: (R/h=250, $\mu_1/(P/h)$=500, κ_1=κ_2=2, μ_1/μ_2=2). It is found that, as the span length of quarter planes increases, the contact length between the rigid punch and layer increases while the contact length between quarter planes and layer decreases.

Study 4: Table 4 demonstrates the change in the contact lengths against the shear modulus ratio considering the following non-dimensional parameters: (c/h=0.2,

Table 3 The change in the contact lengths against the span length of quarter planes

c/h	Source	a/h	b/h
0.01	ANSYS	0.500	1.265
	ABAQUS	0.500	1.260
0.1	ANSYS	0.510	1.170
	ABAQUS	0.515	1.175
0.2	ANSYS	0.530	1.000
	ABAQUS	0.535	1.055
0.3	ANSYS	0.550	0.950
	ABAQUS	0.555	0.955
0.4	ANSYS	0.570	0.835
	ABAQUS	0.575	0.840
0.6	ANSYS	0.650	0.650
	ABAQUS	0.655	0.655
0.8	ANSYS	0.750	0.500
	ABAQUS	0.750	0.485

Table 4 The change in the contact lengths against the shear modulus ratio

μ_1/μ_2	Source	a/h	b/h
0.1	ANSYS	1.000	2.750
	ABAQUS	1.000	2.500
0.25	ANSYS	0.75	2.000
	ABAQUS	0.70	2.000
0.5	ANSYS	0.650	1.750
	ABAQUS	0.600	1.500
1.0	ANSYS	0.550	1.250
	ABAQUS	0.525	1.250
2.0	ANSYS	0.525	1.000
	ABAQUS	0.500	1.000
4.0	ANSYS	0.500	0.900
	ABAQUS	0.500	0.950
10	ANSYS	0.475	0.850
	ABAQUS	0.450	0.800

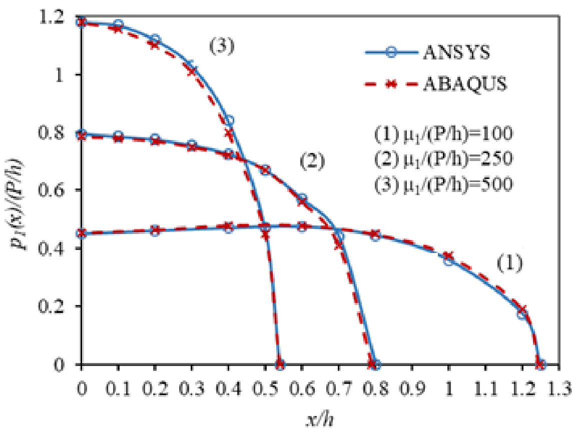

Fig. 9 The change in the contact stresses against the non-dimensional load ratio

$\mu_1/(P/h)$=500, $\kappa_1=\kappa_2$=2, R/h=250). It is observed that both contact lengths decrease with the increase of the shear modulus ratio.

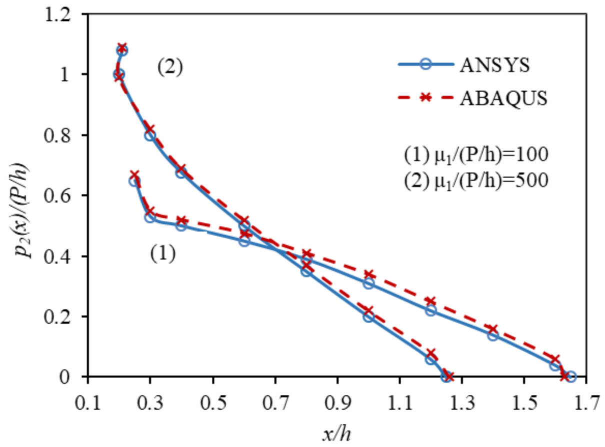

Fig. 10 The change in the contact stresses against the non-dimensional load ratio

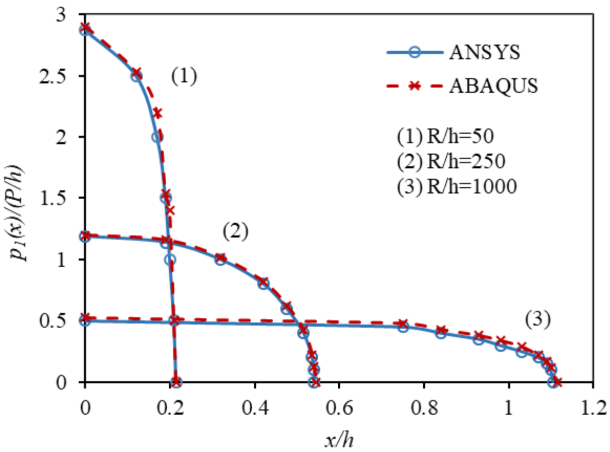

Fig. 11 The change in the contact stresses against the radius of rigid punch

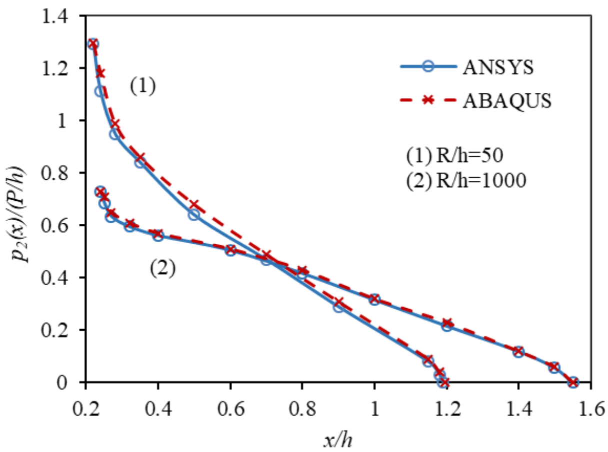

Fig. 12 The change in the contact stresses against the radius of rigid punch

Study 5: Figs. 9-10 show the change in the contact stresses against the load ratio considering the following non-dimensional parameters: (c/h=0.2, R/h=250, $\kappa_1=\kappa_2$=2, μ_1/μ_2=2). It is concluded that as the non-dimensional load ratio increase, both contact stresses increase.

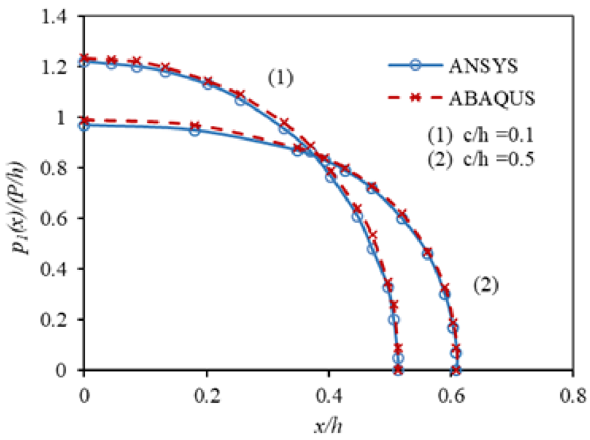

Fig. 13 The change in the contact stresses against the span length of quarter planes

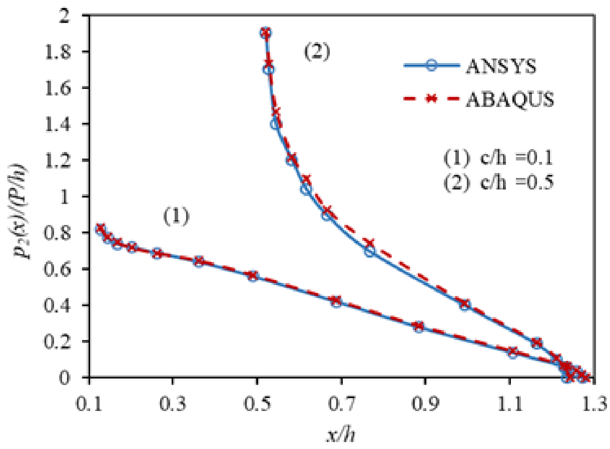

Fig. 14 The change in the contact stresses against the span length of quarter planes

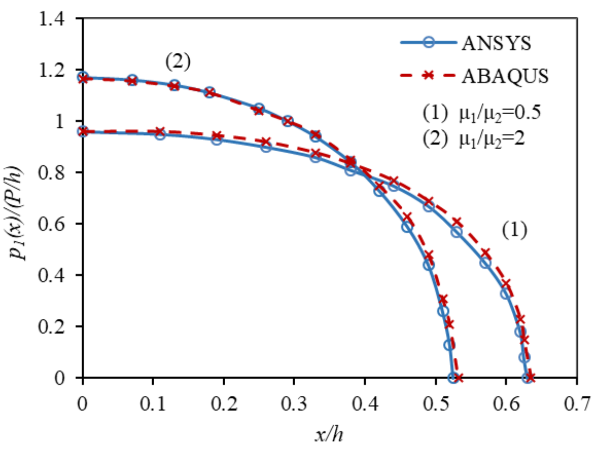

Fig. 15 The change in the contact stresses against the shear modulus ratio

Study 6: Figs. 11-12 show the change in the contact stresses against the radius of rigid punch considering the following non-dimensional parameters: (c/h=0.2, $\mu_1/(P/h)$=500, $\kappa_1=\kappa_2$=2, μ_1/μ_2=2). It is seen that both contact stresses decrease with the increase of radius of rigid punch.

Study 7: Figs. 13-14 show the change in the contact stresses against the span length of quarter planes considering the following non-dimensional parameters: (R/h=250, $\mu_1/(P/h)$=500, $\kappa_1=\kappa_2$=2, μ_1/μ_2=2). It is found that as the span length of quarter planes increases, the contact stresses between the rigid punch and layer decrease while the contact stresses between the quarter planes and layer increase.

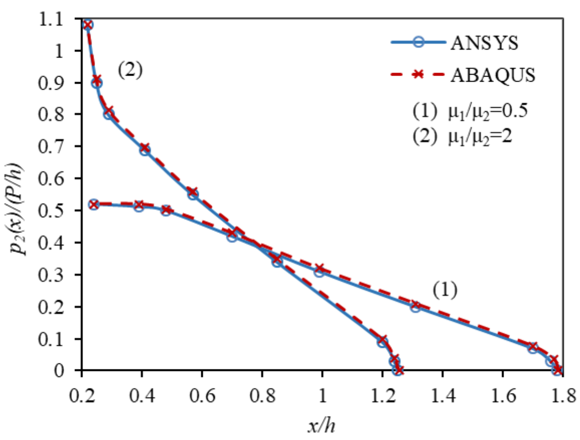

Fig. 16 The change in the contact stresses against the shear modulus ratio

Table 5 RMSE for dimensionless contact pressures

Tables	Parameters	$p_1(x)/(P/h)$	$p_2(x)/(P/h)$
Figs. 9-10	$\mu_1/(P/h)$=100	8.125×10^{-5}	4.83×10^{-4}
	$\mu_1/(P/h)$=500	1.25×10^{-6}	2.69×10^{-5}
Figs. 11-12	R/h=50	2.9057×10^{-2}	9.52×10^{-4}
	R/h=1000	4.22×10^{-4}	1.19×10^{-4}
Figs. 13-14	c/h=0.1	8.92×10^{-4}	6.15×10^{-5}
	c/h=0.5	2.83×10^{-4}	1.088×10^{-3}
Figs. 15-16	μ_1/μ_2=0.5	1.048×10^{-3}	5.43×10^{-5}
	μ_1/μ_2=2	9.2×10^{-4}	8.11×10^{-5}

Study 8: Figs. 15-16 show the change in the contact stresses against the shear modulus ratio considering the following non-dimensional parameters: (c/h=0.2, $\mu_1/(P/h)$=500, $\kappa_1=\kappa_2$=2, R/h=250). It is found that as the shear modulus ratio increases, both contact stresses decrease.

Study 9: Table 5 shows the comparisons of results of ANSYS and ABAQUS software for non-dimensional contact stresses given in Figs. 9-16 using Root Mean Square Error (RMSE) as

$$\mathrm{RMSE} = \left[\frac{1}{N}\sum_{i=1}^{N}\left(Y_i - X_i\right)^2\right]^{1/2}$$

where *Yi, Xi*, and *N* are the values gotten from ABAQUS, ANSYS, and the number of samples, respectively.

It is found that non-dimensional contact stresses obtained from ANSYS and ABAQUS software agree well.

4. Conclusions

In this paper, the contact analysis between an elastic layer and two homogeneous, isotropic and symmetrical elastic quarter planes pressed by means of a rigid circular punch is presented. For this aim, several analyzes are carried out with the help of ANSYS and ABAQUS package programs, many processes such as determination of element types, assignment of material properties of the elements, formation of the geometry of the problem, formation of the mesh structure, assignment of boundary conditions, loading of the problem, solution of the problem and analysis results are performed.

Briefly, the following results are obtained:

- Contact lengths increase with the increase of radius of rigid punch
- The contact length between the rigid punch and layer increases while the contact length between quarter planes and layer decreases with the increase of the span length of quarter planes
- Contact lengths decrease with the increase of the shear modulus ratio
- Contact stresses increase with the increase of non-dimensional load ratio
- Contact stresses decrease with the increase of radius of rigid punch
- The contact stresses between the rigid punch and layer decrease while the contact stresses between the quarter planes and layer increase with the increase of span length of quarter planes
- Contact stresses decrease with the increase of shear modulus ratio

Finally, it is concluded that the considered non-dimensional quantities have noteworthy influence on the normal and shear stress distributions as well as if FEM analysis is used correctly, it can be an efficient alternative numerical method to the analytical solutions need time.

References

ABAQUS (2017), ABAQUS/Standard: User's Manual, Dassault Systèmes Simulia, Johnston, RI.

Adıyaman, G., Birinci, A., Oner, E. and Yaylacı, M. (2016), "A receding contact problem between a functionally graded layer and two homogeneous quarter plane", *Acta Mech.*, **227**(6), 1753-1766. https://doi.org/10.1007/s00707-016-1580-y.

Adıyaman, G., Yaylacı, M. and Birinci, A. (2015), "Analytical and finite element solution of a receding contact problem", *Struct Eng. Mech.*, **54**(1), 69-85. https://doi.org/10.12989/sem.2015.54.1.069.

Akgoz, B. and Civalek, O. (2015), "A microstructure-dependent sinusoidal plate model based on the strain gradient elasticity theory", *Acta Mech.*, **226**, 2277-2294. https://doi.org/10.1007/s00707-015-1308-4.

Aksoğan, O., Akavcı, S. and Becker, A.A. (1996), "A comparative study of the contact problem of an elastic layer supported by two elastic quarter planes", *J. Fac. Eng. Arch. Çukurova Univ.*, **11**, 25-31.

Aksoğan, O., Akavcı, S. and Becker, A.A. (1997), "The solution of the nonsymmetrical contact problem of an elastic layer supported by two elastic quarter planes using three different methods", *J. Fac. Eng. Arch. Çukurova Univ.*, **12**, 1-14.

Amnieh, H.B., Zamzam, M.S. and Kolahchi, R. (2018), "Dynamic analysis of non-homogeneous concrete blocks mixed by SiO_2 nanoparticles subjected to blast load experimentally and theoretically", *Constr. Build. Mater.*, **174**, 633-644. https://doi.org/10.1016/j.conbuildmat.2018.04.140.

ANSYS (2013), Swanson Analysis Systems Inc., Houston, PA, USA.

Arbabi, A., Kolahchi, R. and Bidgoli, M.R. (2020), "Experimental study for ZnO nanofibers effect on the smart and mechanical properties of concrete", *Smart Struct. Syst.*, **25**(1), 97-104. https://doi.org/10.12989/sss.2020.25.1.097.

Avcar, M. (2019), "Free vibration of imperfect sigmoid and power law functionally graded beams", *Steel Compos. Struct.*, **30**(6), 603-615. https://doi.org/10.12989/scs.2019.30.6.603.

Cakiroglu, A.O. and Cakiroglu, F.L. (1991), "Continuous and discontinuous contact problems for strips on an elastic semi-infinite plane", *Int. J. Eng. Sci.*, **29**(1), 99-111. https://doi.org/10.1016/0020-7225(91)90080-M.

Civalek, O. and Demir, C. (2016) "A simple mathematical model of microtubules surrounded by an elastic matrix by nonlocal finite element method", *Appl. Math. Comput.*, **289**, 335-352. https://doi.org/10.1016/j.amc.2016.05.034.

Çömez, İ., Kahya, V. and Erdöl, R. (2018), "Plane receding contact problem for a functionally graded layer supported by two quarter-planes". *Arch. Mech.*, **70**(6), 485-504. https://doi.org/10.24423/aom.2846.

Erdoğan, F. and Ratwani, M. (1974), "The contact problem for an elastic layer supported by two elastic quarter planes", *ASME J. Appl. Mech.*, **41**(3), 673-677. https://doi.org/10.1115/1.3423369.

Farokhian, A. and Kolahchi, R. (2020), "Frequency and instability responses in nanocomposite plate assuming different distribution of CNTs", *Struct. Eng. Mech.*, **73**(5), 555-563. https://doi.org/10.12989/sem.2020.73.5.555.

Gandhi, V.C., Kumaravelan, R., Ramesh, S. and Sriram, K. (2015), "Analysis of material dependency in an elastic-plastic contact models using contact mechanics approach", *Struct Eng. Mech.*, **53**(5), 1051-1066. https://doi.org/10.12989/sem.2015.53.5.1051.

Ghamari, A., Kurdi, J., Shemirani, A.B. and Haeri, H. (2020), "Experimental investigating the properties of fiber reinforced concrete by combining different fibers", *Comput. Concrete*, **25**(6), 509-516. https://doi.org/10.12989/cac.2020.25.6.509.

Guler, M.A. and Erdoğan, F. (2004), "Contact mechanics of graded coatings", *Int. J. Solid. Struct.*, **41**(14), 3865-3889. https://doi.org/10.1016/j.ijsolstr.2004.02.025.

Gurses, M., Akgoz, B. and Civalek, O. (2012), "Mathematical modeling of vibration problem of nano-sized annular sector plates using the nonlocal continuum theory via eight-node discrete singular convolution transformation", *Appl. Math. Comput.*, **219**, 3226-3240. https://doi.org/10.1016/j.amc.2012.09.062.

Haeri, H. and Marji, M.F. (2016), "Simulating the crack propagation and cracks coalescence underneath TBM disc cutters", *Arab. J. Geosci.*, **9**(2), 124. https://doi.org/10.1007/s12517-015-2137-4.

Haeri, H. and Sarfarazi, V. (2016), "The deformable multilaminate for predicting the elasto-plastic behavior of rocks", *Comput. Concrete*, **18**(2), 201-214. https://doi.org/10.12989/cac.2016.18.2.201.

Hajmohammad, M.H., Kolahchi, R., Zarei, M.S. and Nouri, A.H. (2019), "Dynamic response of auxetic honeycomb plates integrated with agglomerated CNT-reinforced face sheets subjected to blast load based on visco-sinusoidal theory", *Int. J. Mech. Sci.*, **153**, 391-401. https://doi.org/10.1016/j.ijmecsci.2019.02.008.

Hanson, M.T. and Keer, L.M. (1989), "Stress analysis and contact problems for an elastic quarter-plane", *Q. J. Mech. Appl. Math.*, **42**(3), 364-383. https://doi.org/10.1093/qjmam/42.3.364.

Hertz, H. (1881), "On the contact of elastic solids", J. Reine Angew. Math., 92, 156-171. Translated and reprinted in English in Hertz's Miscellaneous Papers, Macmillan & Co., London, 1896, Ch. 5.

Hertz, H. (1882), "On hardness", Verh. Ver. Beförderung Gewerbe

Fleisses 61, 410. Translated and reprinted in English in Hertz's Miscellaneous Papers, Macmillan &Co, London, 1896, Ch. 6.

Hussain, M., Naeem, M.N., Khan, M.S. and Tounsi, A. (2020), "Computer-aided approach for modelling of FG cylindrical shell sandwich with ring supports", *Comput. Concrete*, **25**(5), 411-425. https://doi.org/10.12989/cac.2020.25.5.411.

Kahya, V., Ozsahin, T.S., Birinci, A. and Erdöl, R. (2007), "A receding contact problem for an anisotropic elastic medium consisting of a layer and a half plane", *Int. J. Solid. Struct.*, **44**(17), 5695-5710. https://doi.org/10.1016/j.ijsolstr.2007.01.020.

Keer, L.M., Dundurs, J. and Tsai, K.C. (1972), "Problem involving a receding contact problem between a layer and half space", *ASME J. Appl. Mech.*, **39**(4), 1115-1120. https://doi.org/10.1115/1.3422839.

Keer, L.M., Lee, J.C. and Mura, T. (1984), "A contact problem for the elastic quarter space", *Int. J. Solid. Struct.*, **20**(5), 513-524. https://doi.org/10.1016/0020-7683(84)90016-7.

Kolahchi, R., Zhu, S.P., Keshtegar, B. and Trung, N.T. (2020), "Dynamic buckling optimization of laminated aircraft conical shells with hybrid nanocomposite martial", *Aerosp. Sci. Technol.*, **98**, 105656. https://doi.org/10.1016/j.ast.2019.105656.

Kuo, C.H. (2008), "Contact stress analysis of an elastic half-plane containing multiple inclusions", *Int. J. Solid. Struct*., **45**(16), 4562-4573. https://doi.org/10.1016/j.ijsolstr.2008.03.032.

Lazzari, P.M., Lazzari, B.M. and Pacheco, A.R. (2017), "Structural analysis of a prestressed segmented girder using contact elements in ANSYS", *Comput. Concrete*, **20**(3), 319-327. https://doi.org/10.12989/cac.2017.20.3.319.

Liu, C.H., Cheng, I., Tsai, A.C., Wang, L.J. and Hsu, J.Y. (2010), "Using multiple point constraints in finite element analysis of two-dimensional contact problems", *Struct. Eng. Mech.*, **36**(1), 95-110. https://doi.org/10.12989/sem.2010.36.1.095.

Liu, Z., Yan, J. and Mi, C. (2018), "On the receding contact between a two-layer inhomogeneous laminate and a half-plane", *Struct. Eng. Mech.*, **66**(3), 329-341. https://doi.org/10.12989/sem.2018.66.3.329.

Oner, E., Yaylacı, M. and Birinci, A. (2015), "Analytical solution of a contact problem and comparison with the results from FEM", *Struct. Eng. Mech.*, **54**(4), 607-622. https://doi.org/10.12989/sem.2015.54.4.607.

Pindera, M.J. and Lane, M.S. (1993), "Frictionless contact of layered half-planes analysis Part I: Analysis", *J. Appl. Mech.*, **60**(3), 633-639. https://doi.org/ 10.1115/1.2900851.

Ratwani, M. and Erdoğan, F. (1973), "On the plane contact problem for a frictionless elastic layer", *Int.J. Solid. Struct.*, **9**(8), 921-936. https://doi.org/10.1016/0020-7683(73)90021-8.

Rhimi, M., El-Borgi, S. and Lajnef, N. (2011), "Double receding contact axisymmetric problem between a functionally graded layer and a homogeneous substrate", *Mech. Mater.*, **43**(12), 787-798. https://doi.org/ 10.1016/j.mechmat.2011.08.013.

Rhimi, M., El-Borgi, S., Saïd, B.W. and Jemaa, B.F. (2009), "A receding contact axisymmetric problem between a functionally graded layer and a homogeneous substrate", *Int. J. Solid. Struct.*, **46**(20), 3633-3642. https://doi.org/10.1016/j.ijsolstr.2009.06.008.

Shrestha, R., Smith, S.T. and Samali, B. (2013), "Finite element modelling of FRP-strengthened RC beam-column connections with ANSYS", *Comput. Concrete*, **11**(1), 1-20. https://doi.org/10.12989/cac.2013.11.1.001.

Spence, D.A. (1975), "The Hertz contact problem with finite friction", *J. Elasticit.*, **5**(3-4), 297-319. https://doi.org/10.1007/BF00126993.

Stoner, J.G. and Polak, M.A. (2020), "Finite element modelling of GFRP reinforced concrete beams", *Comput. Concrete*, **25**(4), 369-382. https://doi.org/10.12989/cac.2020.25.4.369.

Taherifar, R., Mahmoudi, M., Nasr Esfahani, M.H., Khuzani, N.A., Esfahani, S.N. and Chinaei, F. (2019), "Buckling analysis of concrete plates reinforced by piezoelectric nanoparticles", *Comput. Concrete*, **23**(4), 295-301. https://doi.org/10.12989/cac.2019.23.4.295.

Tigdemir, M., Jafarzadyeganeh, M., Bayrak, M.Ç. and Avcar, M. (2018), "Numerical modelling of wheel on the snow", *Int. J. Eng. Appl. Sci.*, **10**(2), 64-72. http://dx.doi.org/10.24107/ijeas.437861.

Vasiliev, A.S., Volkov, S.S., Aizikovich, S.M. and Mitrin, B.I. (2017), "Plane contact problem on indentation of a flat punch into a transversely-isotropic half-plane with functionally graded transversely-isotropic coating", *Z. Angew. Math. Phys*., **68**(1), 4. https://doi.org/10.1007/s00033-016-0746-8.

Vasudevan, G. and Kothandaraman, S. (2015), "RC beams retrofitted using external bars with additional anchorages-a finite element study", *Comput. Concrete*, **16**(3), 415-428. https://doi.org/10.12989/cac.2015.16.3.415.

Yaylaci, M. and Birinci, A. (2013), "The receding contact problem of two elastic layers supported by two elastic quarter planes", *Struct. Eng. Mech.*, **48**(2), 241-255. https://doi.org/10.12989/sem.2013.48.2.241.

Yaylaci, M., Bayrak, M.Ç. and Avcar, M. (2019b), "Finite element modeling of receding contact problem", *Int. J. Eng. Appl. Sci.*, **11**(4), 468-475. https://doi.org/10.24107/ijeas.646718.

Yaylacı, M., Oner, E. and Birinci, A. (2014), "Comparison between analytical and ANSYS calculations for a receding contact problem", *ASCE J. Eng. Mech.*, **140**(9), 1-10. https://doi.org/ 10.1061/(ASCE)EM.1943-7889.0000781.

Yaylaci, M., Terzi, C. and Avcar, M. (2019a) "Numerical analysis of the receding contact problem of two bonded layers resting on an elastic half plane", *Struct. Eng. Mech.*, **72**(6), 775. https://doi.org/10.12989/sem.2019.72.6.775.

Zhang, Y., Zhao, K., Li, Y., Gu, J., Ye, Z. and Ma, J. (2018), "Study on the local damage of SFRC with different fraction under contact blast loading", *Comput. Concrete*, **22**(1), 63-70. https://doi.org/10.12989/cac.2018.22.1.063.

Zhou, S. and Gao, X.L. (2013), "Solutions of half-space and half-plane contact problems based on surface elasticity", *Z. Angew. Math. Phys.*, **64**(1), 145-166. https://doi.org/10.1007/s00033-012-0205-0.

HK