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Free vibration investigation of FG nanoscale plate using nonlocal two variables integral refined plate theory

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Abstract. In this research paper, the free vibrational behavior of the simply supported FG nano-plate is studied using the nonlocal two variables integral refined plate theory. The present model takes into account the small scale effect. The effective's properties of the plate change according to the power law variation (P-FGM). The equations of motion of the system are determined and resolved via Hamilton's principle and Navier procedure, respectively. The validity and efficiency of the current model are confirmed by comparing the results with those given in the literature. At the last section, several numerical results are presented to show the various parameters influencing the vibrational behavior such as the small-scale effect, geometry ratio, material index and aspect ratio.

Keywords: vibrational behaviour; FG nano-plate; small-scale effects; nonlocal theory

1. Introduction

Functionally graded materials attract the attention of many researchers because of their powerful uses and their characteristics (mechanical, chemical, thermal, physical) such as high abrasion resistance (ceramic face), high impact resistance, reactor components and insulating joints. Dimming improves the toughness of the ceramic-face and prevents ceramic-metal detachment. Functionally graded materials (FGM) can be characterized by the gradual variation of the material properties in the thickness (Avcar 2019). Several works have been studied to examine the behavior of the FG nano-plates under different types of loading can be cited as: Ansari and Norouzzadeh (2016) studied the buckling responses of circular, elliptical and asymmetric nanometric FG-plates. Banh-Thien *et al*. (2017) developed a numerical approach for the buckling analysis of the nano-plate by employing isogeometric analysis. The vibrational analysis of the orthotropic monolayer graphene sheets under thermal load is investigated by Ghadiri *et al*. (2017) using GDQM. The effect of the combined thermo-electo-mechanical loads on the buckling and post-buckling behaviors of the piezoelectric nano-plates is examined by Liu *et al*. (2016) using non-local model. Arefi and Zenkour (2017) examined the thermo-electro-magneto-elastic flexural analysis of a three-layer sandwich nano-plate reposed on Pasternak's foundation by employing the non-local and Kirchhoff plate theory.

Askari *et al*. (2017) developed a mathematical model for dynamic analysis of nanoplates using Galerkin's method. Barati and Shahverdi (2017) have modeled a new double-layer FG nano-plate for the vibrational analysis of under Hygro-thermal loads using four variable nonlocal RPT. The effect of the edges boundary conditions on the rigidity of the nano-plate has been studied by Bochkarev (2017). Ebrahimi and Barati (2018) considered the nonlocal and surface effect to examine the dynamic behavior of the flexo-electric nano-plates under thermal load. the nonlinearities due to the thermal force, electrostatic and Casimir attractions is taking into accounts via mathematical model developed by Farrokhabadi and Tavakolian (2017) for examining the vibrational behavior of the clamped and simply supported nano-plate. Karličić *et al*. (2017) used the Eringen's nonlocal theory and the Kirchhoff model for examining the influence of the uniaxial in-plane magnetic field on the "VOMNPS" (Viscoelastic Orthotropic Multi-nano-plate System). Nematollahi *et al*. (2017) presented a novel formulation based on the nonlocal higher order theory for examining the effect of the various thermal conditions on the natural frequencies of the nano-plate. Bensaid *et al*. (2018) presented a dynamic analysis of higher order shear-deformable nanobeams resting on elastic foundation based on nonlocal strain gradient theory.

The thermal vibration characteristics of nano-plate is studied by Satish *et al*. (2017) to show the effect of the surface layer strength, residual stress, nonlocal scale, surface in plane load and mode number on the frequency of the nano-plate. Shahverdi and Barati (2017) analyzed the dynamic behavior of the porous nano-plate by employing a general nonlocal strain-gradient (NSG) elasticity. The model of the porous nano-plate is obtained by modifying

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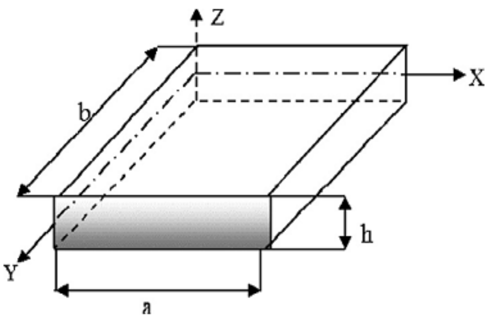

Fig. 1 Geometry of the FG nano-plate

the material properties of the power-law and Mori-Tanaka models. The Dynamic instability and the bifurcation properties of the electrically actuated circular nano-plate is examined by Yang *et al*. (2017) using the Gurtin-Murdoch surface model and the Eringen's nonlocal elasticity. Zhang *et al*. (2018) presented the static analysis of one-dimensional (1D) "quasi crystal piezoelectric hexagonal" nano-plate using the nonlocal theory. The linear free vibration of the micro-/nano-plates has been analyzed by Ziaee (2018) using the classical plate theory, Rayleigh-Ritz method and modified couple stress theory. Ansari *et al*. (2018) applied the FEM (finite element method) and the Eringen's nonlocal elasticity to examine the flexural analysis of the embedded nano-plates. Chen *et al*. (2018) used also the FEM and the nonlocal Kirchhoff plate theory to analyze the dynamic behavior of the nano-plates under temperature loading. Mohseni *et al*. (2018) studied the micro scale vibration of the rectangular thick FG micro-plate using the modified couple stress model and high order theory. Analysis of the waves propagation in the clamped FG porous nanoplate is published by Karami *et al*. (2018) using the FSDT and nonlocal elasticity. Recently, also several research studies of the nano-structures analysis are published such as (Akbas 2018, Faleh *et al*. 2018, Selmi and Bisharat 2018, Bensattalah *et al*. 2018, Karami and Karami 2019, Berghouti *et al*. 2019, Javani *et al*. 2019, Mehar and Panda 2018 and 2019, Hussain and Naeem 2019).

This work presents the analytical studies of the dynamic behavior of the simply supported FG nano-plate using the nonlocal integral refined plate theory (NLIRPT). The present model has a reduced number of equations of motion and not required the shear correction factors. The accuracy of the current model is approved by comparing the results with the analytical model and FEM (finite element method) existing in the literature. The effect of the power law index, the scale effect and the geometry ratio are also discussed in detail.

2. Theoretical formulations

2.1 Nonlocal elasticity of the FG nano-plate

By considering the effect of the small inter-atomic forces presented by (Eringen 2002). The nonlocal stresses can be written as following form (Kolahchi *et al*. 2017, Bensattalah *et al*. 2019)

$$\left(1-\kappa^2\nabla^2\right)\sigma_{ij}^{NL}=\sigma_{ij}^{L} \tag{1}$$

Where "∇^2" is the Laplacian operator and ($i,j=x,y,z$), with $\nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}$

The non-local constitutive equations of the FG nano-plate can be expressed as

$$\left(1-\left(e_0a\right)^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right)\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\ Q_{12}&Q_{22}&0&0&0\\ 0&0&Q_{66}&0&0\\ 0&0&0&Q_{44}&0\\ 0&0&0&0&Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{2}$$

The stiffness coefficients "Q_{ij}" are defined as follows

$$Q_{11}=Q_{22}=\frac{E(z)}{1-\nu^2},Q_{12}=Q_{21}=\frac{\nu E(z)}{1-\nu^2},\quad Q_{44}=Q_{55}=Q_{66}=\frac{E(z)}{2\left(1+\nu\right)} \tag{3}$$

Where "$E(z)$" and "ν" are the Young's modulus and Poisson's ratio, respectively.

2.2 Geometry and concept of the FG nano-plate

In the current work the FG nano-plate have the length "a", width "b", and the thickness "h" in the coordinate system (x,y,z) that is placed in the middle of the nano-plate and the coordinate parameters are limited as

$$0\le x\le a\ ;\ 0\le y\le b\ \ and\ \ -h/2\le z\le +h/2 \tag{4}$$

The material properties of the FG nano-plate are considered to vary according to power law function in the thickness direction. The effectives properties ($E(z)$ and mass density $\rho(z)$) can be written as

$$E(z)=E_m+\left(E_c-E_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p,\quad \rho(z)=\rho_m+\left(\rho_c-\rho_m\right)\left(\frac{z}{h}+\frac{1}{2}\right)^p \tag{5}$$

2.3 Kinematics and deformations

Considering the some simplification on the classical higher order shear deformation plate theory in which to reduce the number of unknown displacement the kinematic of the present model can be given as

$$u(x,y,z,t)=-z\frac{\partial w_0}{\partial x}+k_1\ f(z)\int\theta(x,y,t)\,dx$$

$$v(x,y,z,t) = -z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy$$
$$w(x,y,z,t) = w_0(x,y,t) \tag{6}$$

Where the warping function $f(z)$ is used in the current work can be written as

$$f(z) = z\left(\frac{5}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right) \tag{7}$$

Based on the present displacement field of Eq. (6), the non-zero strain can be obtained as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} \eta_x^s \\ \eta_y^s \\ \eta_{xy}^s \end{Bmatrix}, \varepsilon_z = 0,$$
$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{xy}^s \\ \gamma_{xy}^s \end{Bmatrix} \tag{8a}$$

With

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \begin{Bmatrix} \eta_x^s \\ \eta_y^s \\ \eta_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 A'\theta \\ k_2 B'\theta \\ k_1 A'\dfrac{\partial}{\partial y}\int \theta dx + k_2 B'\dfrac{\partial}{\partial x}\int \theta dy \end{Bmatrix},$$
$$\begin{Bmatrix} \gamma_{xz}^s \\ \gamma_{yz}^s \end{Bmatrix} = \begin{Bmatrix} k_1 A'\int \theta dx \\ k_2 B'\int \theta dy \end{Bmatrix}, g(z) = df(z)/dz \tag{8b}$$

By using the Navier type method, the integrals considered in the above equations can be treated via the following expressed

$$\frac{\partial}{\partial y}\int \theta\, dx = A'\frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x}\int \theta\, dy = B'\frac{\partial^2 \theta}{\partial x \partial y},$$
$$\int \theta\, dx = A'\frac{\partial \theta}{\partial x}, \quad \int \theta\, dy = B'\frac{\partial \theta}{\partial y} \tag{9a}$$

where coefficients A', B', k_1 and k_2 are given by

$$A' = -\frac{1}{\mu^2}, \quad B' = -\frac{1}{\lambda^2}, \quad k_1 = \lambda^2, \quad k_2 = \mu^2 \tag{9b}$$

with λ and μ are defined in Eq. (17).

2.4 Equations of motion of the FG nano-plate

In the current research the equations of motion are obtained via Hamilton's principle, the principle of the present kinematic can be written as follow

$$\int_{h/2}^{-h/2}\int_S \left[\sigma_X \delta\varepsilon_X + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right] dSdz$$
$$- \int_{h/2}^{-h/2}\int_S \rho\left[\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w\right] dSdzdt = 0 \tag{10}$$

By replacing Eqs. (6) and (8) into Eq. (10). The Hamilton principle of the present model can be rewritten as function of resulting constraints (M, S and Q) as

$$\int_s \begin{bmatrix} M_x \delta k_x^b + M_y \delta k_y^b + M_{xy}\delta k_{xy}^b + S_x \delta k_x^s \\ +S_y \delta k_y^s + S_{xy}\delta k_{xy}^s + Q_{yz}\delta\gamma_{yz}^s + Q_{xz}\delta\gamma_{xz}^s \end{bmatrix} ds$$
$$-\int_{-h/2}^{h/2}\int_s \rho \begin{bmatrix} (-z\ddot{k}_x^b + f\ddot{\eta}_{xx}^s)\delta(-zk_x^b + f\eta_x^s) + \\ (-z\ddot{k}_y^b + f\ddot{\eta}_y^s)\delta(-zk_y^b + f\eta_y^s) + \ddot{w}_0\delta w_0 \end{bmatrix} dSdz = 0 \tag{11}$$

with

$$(M_x, M_y, M_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) z dz$$
$$(S_x, S_y, S_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy})\, f dz \tag{12}$$
$$(Q_{yz}, Q_{yz}) = \int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) g dz$$

substituting the Eq. (2) into Eq. (12) and integrating across the thickness, resulting constraints (M, S and Q) are given as follows

$$\begin{Bmatrix} M \\ S \end{Bmatrix} - (e_0 a)^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix} M \\ S \end{Bmatrix} = \begin{bmatrix} B_{ij} & D_{ij} & D_{ij}^f \\ B_{ij}^f & D_{ij}^f & F_{ij}^f \end{bmatrix}\begin{Bmatrix} \kappa \\ \eta \end{Bmatrix} \tag{13a}$$

$$Q - (e_0 a)^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)Q = \gamma A_{ij}^f \tag{13b}$$

Where B_{ij}, D_{ij} etc., are the rigidity of the FG nano-plate, it can be defined as

$$\left\{D_{ij}, D_{ij}^s, F_{ij}^s\right\} = \int_{-h/2}^{h/2}\left\{z^2, zf(z), f(z)^2\right\}Q_{ij}dz, (i,j = 1,2,6)$$
$$\left\{A_{ij}^s\right\} = \int_{-h/2}^{h/2}\left\{g^2\right\}Q_{ij}dz,, (i,j = 4,5) \tag{14}$$

By integrating by parts and separating the displacement coefficients (δw_b and δw_s) of Eq. (11). The equations of motion of the current model can be obtained as

$$\delta w_0 : \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\, \partial y} + \frac{\partial^2 M_y}{\partial y^2} = I_0 \frac{\partial^2 w_0}{\partial t^2}$$
$$-I_2\left(\frac{\partial^4 w_0}{\partial x^2 \partial t^2} + \frac{\partial^4 w_0}{\partial y^2 \partial t^2}\right) + J_2\left(k_1 A'\frac{\partial^4 \theta}{\partial x^2 \partial t^2} + k_2 B'\frac{\partial^4 \theta}{\partial y^2 \partial t^2}\right) \tag{15a}$$

$$\delta\theta : -k_1 S_x - k_2 S_y - (k_1 A' + k_2 B')\frac{\partial^2 S_{xy}}{\partial x \partial y} +$$
$$k_1 A'\frac{\partial Q_{xz}}{\partial x} + k_2 B'\frac{\partial Q_{yz}}{\partial y} = +J_2\left(k_1 A'\frac{\partial^4 w_0}{\partial x^2 \partial t^2} + k_2 B'\frac{\partial^4 w_0}{\partial y^2 \partial t^2}\right)$$
$$-K_2\left((k_1 A')^2\frac{\partial^4 \theta}{\partial x^2 \partial t^2} + (k_2 B')^2\frac{\partial^4 \theta}{\partial y^2 \partial t^2}\right) \tag{15b}$$

3. Analytical solutions

In the present work, the Navier procedure is employed to satisfy automatically the boundary conditions of the simply supported FG nano-plate. The Navier solution can be expressed by a double series Fourier of as

Table 1 Comparison of frequency frequency "$\varpi = \omega h\sqrt{\rho_c/G_c}$" for FG square nano-plate (η=1, δ=0.1 and p=5)

Method	Model	Dimensionless Frequency		
		κ=0	κ=1	κ=2
(Zargaripoor *et al*. 2018)	FEM	0.0444	0.0405	0.0376
(Natarajan *et al*. 2012)	FEM	0.0441	0.0403	0.0374
Chikh (2019)	RPT	0.0442	0.0404	0.0374
Present	2D	0.0442	0.0404	0.0374

$$\begin{Bmatrix} w_0 \\ \theta \end{Bmatrix} = \begin{Bmatrix} W_0 \sin(\lambda\ x)\sin(\mu\ y) \\ \theta_0 \sin(\lambda\ x)\sin(\mu\ y) \end{Bmatrix} e^{i\omega t} \tag{16}$$

where "ω" is the frequency of the FG nanoplate, $\sqrt{i} = -1$ the imaginary unit, noting that

$$\lambda = m\pi / a\ ,\quad \mu = n\pi / b \tag{17}$$

Substituting Eqs. (13) and (16) into equations of motion of (Eq. (14)). We obtain the following matrix system

$$\left(\left[K\right] - \omega^2\left[M\right]\right)\left\{\Delta\right\} = \left\{0\right\}, \tag{18}$$

Where

$$\left\{\Delta\right\} = \begin{Bmatrix} W_0 \\ \theta_0 \end{Bmatrix},\ \left[K\right] = \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix},\ \left[M\right] = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} \tag{19}$$

The elements k_{ij}=k_{ji}; m_{ij}=m_{ji} of the Matrix [K] and [M] are

$$\begin{aligned} k_{11} &= D_{11}\lambda^4 + D_{22}\mu^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 \\ k_{12} &= -D^s_{22}\mu^4 - D^s_{11}\lambda^4 - 2(D^s_{12} + 2D^s_{66})\lambda^2\mu^2 \\ k_{22} &= -F^s_{11}\lambda^4 - F^s_{22}\mu^4 - 2(F^s_{12} + 2F^s_{66})\lambda^2\mu^2 - A^s_{44}\mu^2 - A^s_{55}\lambda^2 \\ \psi &= 1 + \kappa(\lambda^2 + \mu^2) \end{aligned} \tag{20}$$

$$\begin{aligned} \left\{D_{ij}, D^s_{ij}, F^s_{ij}\right\} &= \int_{-h/2}^{h/2} \left\{z^2, zf(z), f(z)^2\right\} Q_{ij} dz, (i,j = 1,2,6) \\ \left\{A^s_{ij}\right\} &= \int_{-h/2}^{h/2} \left\{g^2\right\} Q_{ij} dz,, (i,j = 4,5) \end{aligned} \tag{21}$$

4. Numerical results and discussion

4.1 Comparison and validation

In this section, the validity and accuracy of the current model are verified by comparing the results with the analytical model (Aghababaei and Reddy 2009) and chikh (2019) and finite element method of (Natarajan *et al*. 2012) and (Zargaripoor *et al*. 2018).

The material properties of the SUS 304 (Metal) and Si3N4 (Ceramic) used in the present study are as follows:

SUS 304 (*Metal*): $E_m = 201.04\,GPa,\ \ \rho_m = 8166\,kg/m^3,\ \ \nu_m = 0.3$

$\mathbf{Si_3N_4}$ (*Ceramic*): $E_c = 348.46\,GPa,\ \ \rho_c = 2370\,kg/m^3,\ \ \nu_c = 0.3$

The dimensionless frequency parameter and other parameters employed in the following results are

$$\eta = \frac{a}{b},\ \delta = \frac{h}{a},\ \varpi = \omega h\sqrt{\frac{\rho_c}{G_c}},\ \ Fr = \frac{\varpi_{NL}}{\varpi_L} \tag{22}$$

Table 2 Comparison of dimensionless frequency parameter "$\varpi = \omega h\sqrt{\rho/G}$" or a simply supported nano-plate

η	δ	κ	Dimensionless Frequency			
			Present	Chikh (2019)	Zargaripoor *et al*. (2018)	Aghababaei and Reddy (2009)
1	10	0	0.0931	0.0931	0.0930	0.0935
		1	0.0851	0.0851	0.0850	0.0854
		2	0.0789	0.0789	0.0788	0.0791
	20	0	0.0239	0.0239	0.0239	0.0239
		1	0.0218	0.0218	0.0218	0.0218
		2	0.0202	0.0202	0.0202	0.0202
2	10	0	0.0589	0.0589	0.0589	0.0591
		1	0.0556	0.0556	0.0556	0.0557
		2	0.0528	0.0528	0.0527	0.0529
	20	0	0.0150	0.0150	0.0150	0.0150
		1	0.0141	0.0141	0.0141	0.0141
		2	0.0134	0.0134	0.0134	0.0134

Table 3 Comparison of dimensionless frequency "$\varpi = \omega h\sqrt{\rho_c/G_c}$" for simply supported square FG nano-plate (η=1, δ=0.1)

p	κ		Dimensionless Frequency			
			Mode1 (1,1)	Mode2 (1,2)	Mode3 (2,1)	Mode4 (2,2)
0	0	Present	0.0931	0.2226	0.2226	0.342
		Chikh (2019)	0.0931	0.2226	0.2226	0.342
		Zargaripoor *et al*. (2018)	0.0930	0.2225	0.2225	0.3407
	1	Present	0.0851	0.1822	0.1822	0.2558
		Chikh (2019)	0.0851	0.1822	0.1822	0.2558
		Zargaripoor *et al*. (2018)	0.0850	0.1820	0.1820	0.2547
	2	Present	0.0789	0.1579	0.1579	0.2130
		Chikh (2019)	0.0789	0.1579	0.1579	0.2130
		Zargaripoor *et al*. (2018)	0.0788	0.1578	0.1578	0.2122
1	0	Present	0.0548	0.1309	0.1309	0.2011
		Chikh (2019)	0.0548	0.1309	0.1309	0.2011
		Zargaripoor *et al*. (2018)	0.0552	0.1310	0.1310	0.2008
	1	Present	0.0501	0.1071	0.1071	0.1504
		Chikh (2019)	0.0501	0.1071	0.1071	0.1504
		Zargaripoor *et al*. (2018)	0.0504	0.1072	0.1072	0.1501
	2	Present	0.0464	0.0929	0.0929	0.1253
		Chikh (2019)	0.0464	0.0929	0.0929	0.1253
		Zargaripoor *et al*. (2018)	0.0467	0.0930	0.0930	0.1250
5	0	Present	0.0442	0.1052	0.1052	0.1613
		Chikh (2019)	0.0442	0.1052	0.1052	0.1613
		Zargaripoor *et al*. (2018)	0.0444	0.1052	0.1052	0.1608
	1	Present	0.0404	0.0861	0.0861	0.1206
		Chikh (2019)	0.0404	0.0861	0.0861	0.1206
		Zargaripoor *et al*. (2018)	0.0405	0.0861	0.0861	0.1202
	2	Present	0.037	0.0746	0.0746	0.1005
		Chikh (2019)	0.037	0.0746	0.0746	0.1005
		Zargaripoor *et al*. (2018)	0.0376	0.0747	0.0747	0.1002

In which "ϖ_L" is the dimensionless frequency corresponding to the non-local parameter "κ=0".

The Table 1 presents the non-dimensional frequency

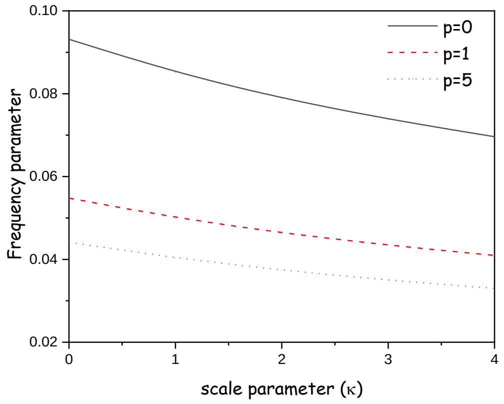

Fig. 2 Scale parameter effect on nondimensional frequency of simply supported square FG nano-plate for different values of material index (h/a=0.1)

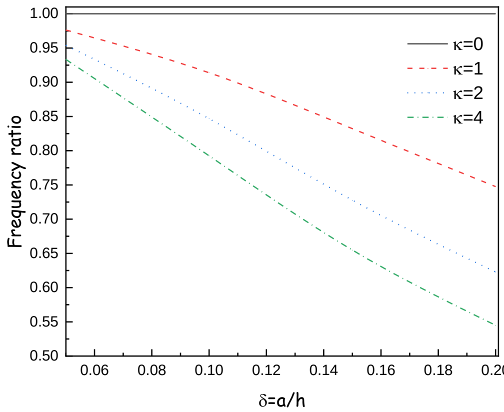

Fig. 3 Effect of the geometry ratio on frequency ratio of simply supported square FG nano-plate for different value of nonlocal parameters (p=5)

"ϖ" of the simply supported FG nano-plate with (η=1, δ=0.1 and p=5). The current results obtained via refined plate theory are compared with those given by Natarajan *et al*. (2012), Zargaripoor *et al*. (2018) using the FEM (finite element method) and Chikh (2019) using analytical model (RPT). It can be observed from the table that a very good agreement is confirmed between the present results and those of the finite element method (FEM).

The variation of the dimensionless frequency "$\varpi = \omega h\sqrt{\rho/G}$" of the FG nano-plate versus the nonlocal parameter "κ", geometry ratio "δ" and dimension ratio "η" is presented in the Table 2. The obtained results are compared with those given by the Third shear deformation theory developed by Aghababaei and Reddy (2009), the finite element method proposed by Zargaripoor *et al*. (2018) and refined plate theory published by Chikh (2019). From the tabulated results, it is remarkable that the actual results are almost identical with those given in the literature. It can be also seen that the dimensionless frequency "$\varpi = \omega h\sqrt{\rho/G}$" is in inverse relation with nonlocal parameter "κ", geometry ratio "δ" and dimension ratio "η".

Table 3 Show the effect of the vibrational mode the power index "n" and the non-local parameter "κ" on the dimensionless natural frequency "$\varpi = \omega h\sqrt{\rho_c/G_c}$" of the square FG nano-plate. From the table, it is confirmed again that the actual results obtained by the two unknowns integral model are very close to those obtained by RPT theory given by Chikh (2019) and Zargaripoor *et al*. (2018) using the FEM. The lower values of the natural frequency "ϖ" are obtained by the vibrational mode (1,1).

4.2 Parametric studies

This section focuses on the study of different parameters influencing the non-dimensional frequency "ϖ" of the simply supported FG nano-plates.

Fig. 2 illustrates the effect of the nonlocal parameter "κ" on the dimensionless frequency "ϖ" of simply supported square FG nano-plate for different values of the power law

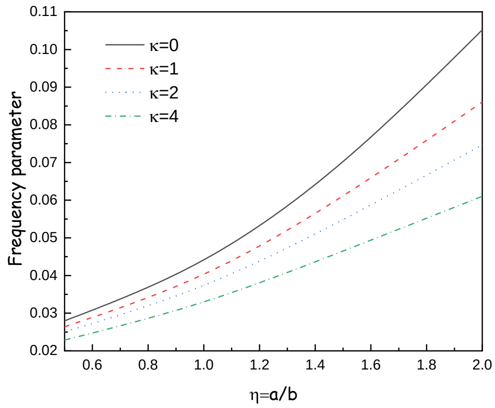

Fig. 4 Effect of dimension ratio on frequency parameter of FG nano-plate for different scale parameters (p=5, δ=0.1)

index "p" with (h/a=0.1). From the plotted graphs, we can see that the non-local parameter "κ" affects inversely on the non-dimensional frequency "ϖ". It can be also concluded that the ceramic "p=0" gives the biggest values of the frequency "ϖ" because his high Young modulus "E_c".

The effects of the geometry ratio on the frequency ratio "Fr" of the simply supported square FG nanoplate with (p=5) is presented in the Fig. 3. From the curves, it is remarkable that the frequency ratio "Fr" is not influenced by the thickness to length ratio "δ=0.1" in the case "κ=0" because the both frequency are equal $\varpi_{NL} = \varpi_L$. In the case "$\kappa \neq 0$" the non-local dimensionless frequency "ϖ_{NL}" decreased relative to the local dimensionless frequency "ϖ_L".

Fig. 4 shows the variation of non-dimensional frequency "ϖ" of the FG nano-plate versus the scale effect and aspect ratio. From the figure, we can observe that the dimensionless frequency "ϖ" is in direct correlation relation with both aspect ratio "η" and non-local parameter "κ". The largest values of the frequency "ϖ" are obtained by aspect ratio "η=2" because the nano-plate becomes flexible.

The effect of the vibrational mode and nonlocal parameter "κ" on the values of the frequency ratio "Fr" of

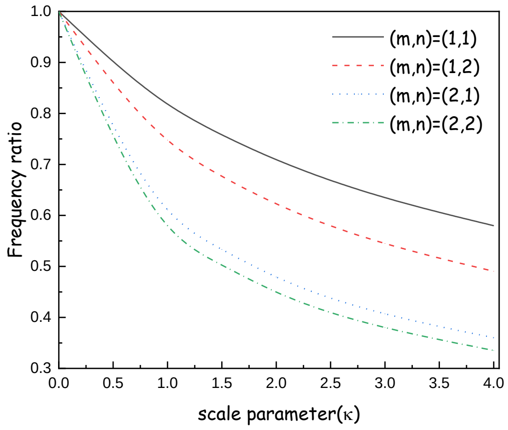

Fig. 5 Effect of the scale parameter on frequency ratio of rectangular FG nano-plate for different mode numbers p=5, h/a=0.1, η=2

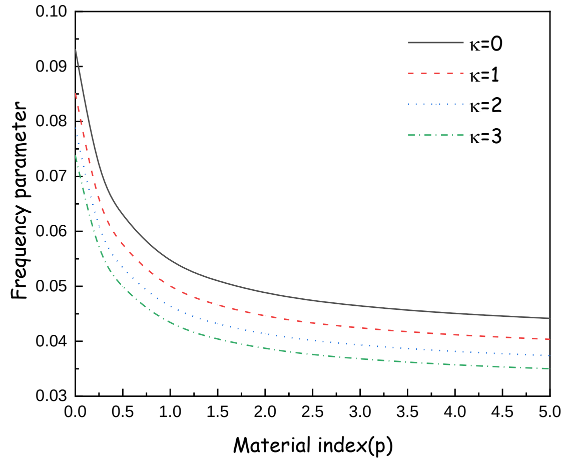

Fig. 6 Effect of the material index on the frequency parameter of simply supported square nano-plate (δ=0.1)

the rectangular FG nano-plate is illustrated in the Fig. 5. From the drawn graphs. It can be seen that the frequency ratio "*Fr*" is in inverse relation with value of the small scale effect "κ". It is remarkable also that the vibrational mode (2,2) give the lowest values of the frequency ratio "*Fr*".

Fig. 6 illustrate the effect of the material index "p" on the non-dimensional frequency "ϖ" of FG nanoplate with (δ=0.1, η=1) for the various values of the nonlocal parameter "κ". from the plotted graphs, we can confirm once again that the non-dimensional frequency "ϖ" decrease with increasing of the power index "p" and nonlocal effect "κ".

5. Conclusions

In this research work, the free vibrational analysis of the simply supported FG nano-plate is studied using a four variable nonlocal refined plate theory. The nano-plate is modeled according to the power law function model which the properties change through the thickness. The four equations of motion determined by Hamilton's principle have been solved via Navier procedure. This model has examined the various parameters influencing the vibrational frequency such as (the small scale effect, aspect ratio, geometry ratio, power index and vibrational mode). Finally, the comparisons made with FEM we can conclude that the current model is precise and efficient to solve the problem of the vibration of the simply supported FG nano-plate. An improvement of the present formulation will be considered in the future work to consider other type of materials (Daouadji 2017, Fadoun 2019, Rajabi and Mohammadimehr 2019, Salah *et al*. 2019, Al-Osta 2019, Batou *et al*. 2019).

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