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Thermomechanical analysis of antisymmetric laminated reinforced composite plates using a new four variable trigonometric refined plate theory

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Abstract. The thermo-mechanical bending behavior of the antisymmetric cross-ply laminates is examined using a new simple four variable trigonometric plate theory. The proposed theory utilizes a novel displacement field which introduces undetermined integral terms and needs only four variables. The validity of the present model is proved by comparison with solutions available in the literature.

Keywords: thermo-mechanical load; laminated plates; analytical modelling

1. Introduction

The composite materials are widely employed in civil, aerospace, automobile and other engineering fields because of their advantage of important stiffness and strength to weight ratio. With the increasing uses of laminated composites in environmental conditions, thermo-mechanical response of such structures has attracted considerable attention. During the operational life, the distribution of temperature diminishes the elastic moduli and degrades the strength of the laminated material.

Shear deformation influences become more considerable in such structures because of the low transverse shear moduli as compared to high in-plane tensile moduli, when subjected to transverse loads. This requires the accurate structural investigation of composite plates. Since the transverse shear deformation is ignored in the classical plate theory (CPT), it cannot be suitable for the analysis of moderately thick or thick plates in which transverse shear deformation impacts are more important. To avoid the problems found in the CPT and accurately introduce the transverse shear influences, many shear deformation models have been proposed. The first order shear deformation theories (FSDTs) accounts for the transverse shear deformation influences, but necessitate a shear correction factor to satisfy the free transverse shear stress conditions on the top and bottom surfaces of the plate (Della Croce and Venini 2004, Ganapathi *et al*. 2006, Zhao and Liew 2009, Zhao *et al*. 2009, Lee *et al*. 2010, Hosseini-Hashemi *et al*. 2010, Hosseini-Hashemi *et al*. 2011, Mantari and Ore, 2015, Mantari and Granados 2015a,b). Although the FSDT gives a sufficiently accurate description of response for thin to moderately thick plates, it is not convenient to use because of the difficulty in evaluation of correct value of the shear correction factor. To avoid the use of shear correction factor, many higher order shear deformation theories (HSDTs) were developed based on the assumption of nonlinear variations of in-plane displacements within the plate thickness. Reddy (1984) has proposed HSDT considering polynomial functions in-terms of thickness coordinate. Soldatos (1992) developed a hyperbolic shear deformation model for homogenous monoclinic plates whereas Thermal flexural investigation of symmetric laminated plates under a single sinusoidal thermal load is presented by Ali *et al*. (1999) by employing displacement based higher-order theory. Rohwer *et al*. (2001) presented higher-order theories for thermal stresses in layered plates. A novel inverse hyperbolic shear deformation model is developed by Grover *et al*. (2013). Karama *et al*. (2003, 2009) developed an exponential function in terms of thickness coordinate for laminated composite beam and plates. Versino *et al*. (2013) have been proposed a refined zigzag theory for the investigation of homogeneous, multilayer composite and sandwich plates. Xiang and Kang (2013) studied bending response of functionally graded plates by employing nth-order shear deformation theory and meshless global collocation method based on the thin plate spline radial basis function. A two-dimensional higher-order deformation theory is proposed by Matsunaga (2009) for the evaluation of displacements and stresses in functionally graded plates subjected to thermal and mechanical loadings.

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Hadji *et al*. (2019) presented an analytical solution for bending and free vibration responses of functionally graded beams with porosities. Ghugal and Kulkarni (2011) presented thermal stresses analysis in cross-ply laminated plates under a sinusoidal thermal load within the thickness of the plate by utilizing refined shear deformation theory. Draiche *et al.* (2014) studied the free vibration of a simply supported laminated composite plate with distributed patch mass using a trigonometric four variable plate theory. Kar *et al*. (2015) examined the nonlinear flexural behavior of laminated composite flat panel under hygro-thermo-mechanical loading. Chattibi *et al.* (2015) investigated the thermomechanical bending response of anti-symmetric cross-ply composite plates using a simple four variable sinusoidal plate theory. Mahapatra *et al*. (2016a) used a micromechanical approach to study the nonlinear flexural response of laminated composite panel under hygro-thermo-mechanical loading. Mahapatra *et al*. (2016b) investigated the geometrically nonlinear flexural behavior of hygro-thermo-elastic laminated composite doubly curved shell panel. Mehar and Panda (2017a) presented a numerical investigation of nonlinear thermomechanical deflection of functionally graded CNT reinforced doubly curved composite shell panel under different mechanical loads. Mahapatra *et al*. (2017) studied the nonlinear thermoelastic deflection of temperature-dependent functionally graded material (FGM) curved shallow shell under nonlinear thermal loading. Mehar and Panda (2017b) analyzed the thermoelastic response of FG-CNT reinforced shear deformable composite plate under various loadings. Mehar and Panda (2017c) discussed the nonlinear static behavior of FG-CNT reinforced composite flat panel under thermomechanical load. Hirwani *et al*. (2018a) examined the thermomechanical deflection and stress responses of delaminated shallow shell structure using higher-order theories. Mehar *et al*. (2018a) employed the finite element method for studying the thermoelastic deflection responses of CNT reinforced sandwich shell structure. Also Mehar *et al*. (2018b) used a finite element approach for investigating the stress, deflection, and frequency analysis of CNT reinforced graded sandwich plate under uniform and linear thermal environment. Mehar and Panda (2018) presented nonlinear finite element solutions of thermoelastic flexural strength and stress values of temperature dependent graded CNT-reinforced sandwich shallow shell structure. Hirwani *et al*. (2018b) analyzed numerical flexural strength of thermally stressed delaminated composite structure under sinusoidal loading. Hirwani and Panda (2019) presented nonlinear finite element solutions of thermoelastic deflection and stress responses of internally damaged curved panel structure. Draiche *et al*. (2019) studied the bending behavior of laminated reinforced composite plates using a simple first-order shear deformation theory. Zarga *et al*. (2019) analyzed the thermomechanical bending response of functionally graded sandwich plates using a simple quasi-3D shear deformation theory. Mehar *et al*. (2019) investigated numerically the buckling response of graded CNT-reinforced composite sandwich shell structure under thermal loading. Mehar and Panda (2019) used a multiscale modeling approach for thermal buckling analysis of nanocomposite curved structure.

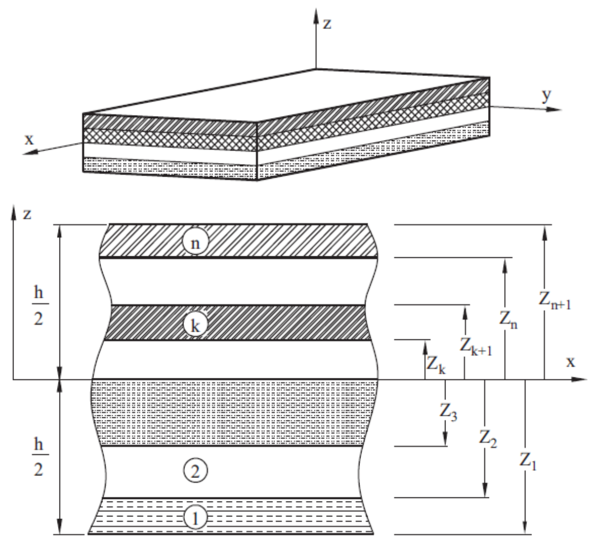

Fig. 1 Coordinate system and layer numbering used for a typical laminated plate

In the present paper, a new simple four variable trigonometric plate theory is proposed for the thermo-mechanical bending response of laminated composite plates. The addition of the integral term in the displacement field leads to a reduction in the number of unknowns and governing equations. Analytical solutions of simply supported antisymmetric cross-ply laminates are determined and the results are compared with the existing solutions. The analysis is relevant to aerospace and nuclear engineering structures experiencing significant heat effects.

2. Mathematical model

The system examined, shown schematically in Fig. 1 is a beam of variable cross section, carrying a so called heavy tip mass M. Its mass moment of inertia with respect to the perpendicular axis at the centroid S is denoted by JS. The publications (Abolghasemi and Jalali 2003, Younesian and Esmailzadeh 2010, Arvin and Bakhtiari-Nejad 2011) are considered also with rotating beams in which nonlinear oscillations are investigated. Analytical and experimental investigations on vibrating frames carrying concentrated masses with characteristics of frames have been studied by using analytical solutions and the finite element method (Cheng *et al*. 2013a, b). ……

Consider a fiber-reinforced rectangular laminated plate of length a, width b and uniform thickness h (see Fig. 1). The plate is composed of n orthotropic layers oriented at angles θ_1; θ_2; . . .; θ_n. The material of each layer is supposed to contain one plane of elastic symmetry parallel to the x-y plane. Perfect bonding between the orthotropic layers and temperature-independent mechanical and thermal characteristics are supposed. Let the plate be subjected to a transverse load $q(x,y)$ and temperature field $T(x,y,z)$.

2.1 Kinematics and strains

In this work, further simplifying assumptions are made to the conventional HSDT so that the number of unknowns is reduced. The displacement field of the conventional HSDT is given by

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\theta_x(x,y,t) \quad (1a)$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+f(z)\theta_y(x,y,t) \quad (1b)$$

$$w(x,y,z,t)=w_0(x,y,t) \quad (1c)$$

where u_0; v_0; w_0, θ_x, θ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ presents shape function representing the variation of the transverse shear strains and stresses across the thickness. In this work a new displacement field with 4 unknowns is proposed

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1 f(z)\int\phi(x,y,t)\,dx \quad (2a)$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+k_1 f(z)\int\phi(x,y,t)\,dy \quad (2b)$$

$$w(x,y,z,t)=w_0(x,y,t) \quad (2c)$$

The constants k_1 and k_2 depends on the geometry and the function $f(z)$ is given by

$$f(z)=\frac{h}{\pi}\sin\left(\frac{\pi}{h}z\right) \quad (3)$$

The strains are related to the displacements given in (2) and can be expressed as

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \quad (4)$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \quad (5a)$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix},\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_1\int\theta\,dy\\ k_2\int\theta\,dx\end{Bmatrix}$$

And

$$g(z)=\frac{df(z)}{dz} \quad (5b)$$

The integrals used in the above relations shall be resolved by a Navier solution and can be expressed by

$$\begin{aligned}&\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y},\\ &\int\theta\,dx=A'\frac{\partial\theta}{\partial x},\int\theta\,dy=B'\frac{\partial\theta}{\partial y}\end{aligned} \quad (6)$$

In which the coefficients A' and B' are determined according to the type of solution considered, in this case via Navier. Thus, A' and B' are defined by

$$A'=-\frac{1}{\alpha^2},B'=-\frac{1}{\beta^2},k_1=\alpha^2,k_2=\beta^2 \quad (7)$$

where α and β are defined in expression (22).

2.2 Constitutive and governing equations

The stress-strain relationships, accounting for thermal and transverse shear deformation influences, in the plate coordinates for the kth layer can be defined as

$$\begin{aligned}&\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}^{(k)}=\begin{bmatrix}\bar{Q}_{11}&\bar{Q}_{12}&\bar{Q}_{16}\\ \bar{Q}_{12}&\bar{Q}_{22}&\bar{Q}_{26}\\ \bar{Q}_{16}&\bar{Q}_{26}&\bar{Q}_{66}\end{bmatrix}^{(k)}\begin{Bmatrix}\varepsilon_x-\alpha_x T\\ \varepsilon_y-\alpha_y T\\ \gamma_{xy}-\alpha_{xy}T\end{Bmatrix}^{(k)}\\ &and\begin{Bmatrix}\tau_{yz}\\ \tau_{zx}\end{Bmatrix}^{(k)}=\begin{bmatrix}\bar{Q}_{44}&\bar{Q}_{45}\\ \bar{Q}_{45}&\bar{Q}_{55}\end{bmatrix}^{(k)}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{zx}\end{Bmatrix}^{(k)}\end{aligned} \quad (8)$$

where $T=T(x,y,z)$ is the temperature distribution; and (α_x, α_y, α_{xy}) are the thermal expansion coefficients in the plate coordinates, and are related to the coefficients (α_L, α_T,0) in the material principal directions. $\bar{Q}_{ij}$ are the transformed elastic coefficients are the transformed material constants given as (Bogdanovich and Pastore 1996)

$$\begin{aligned}\bar{Q}_{11}&=Q_{11}\cos^2\theta+2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta+Q_{22}\sin^4\theta\\ \bar{Q}_{12}&=(Q_{11}+Q_{22}-4Q_{66})\sin^2\theta\cos^2\theta+Q_{12}(\sin^4\theta+\cos^4\theta)\\ \bar{Q}_{22}&=Q_{11}\sin^2\theta+2(Q_{12}+2Q_{66})\sin^2\theta\cos^2\theta+Q_{22}\cos^4\theta\\ \bar{Q}_{16}&=(Q_{11}-Q_{12}-2Q_{66})\sin\theta\cos^3\theta+(Q_{12}-Q_{22}+2Q_{66})\sin^3\theta\cos\theta\\ \bar{Q}_{26}&=(Q_{11}-Q_{12}-2Q_{66})\sin^3\theta\cos\theta+(Q_{12}-Q_{22}+2Q_{66})\sin\theta\cos^3\theta\\ \bar{Q}_{66}&=(Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta\cos^2\theta+Q_{66}(\sin^4\theta+\cos^4\theta)\\ \bar{Q}_{44}&=Q_{44}\cos^2\theta+Q_{55}\sin^2\theta\\ \bar{Q}_{45}&=(Q_{55}-Q_{44})\cos\theta\sin\theta\\ \bar{Q}_{55}&=Q_{55}\cos^2\theta+Q_{44}\sin^2\theta\end{aligned} \quad (9)$$

where Q_{ij} are the (plane stress-reduced) material stiffness of the lamina

$$\begin{aligned}&Q_{11}=\frac{E_{xx}}{1-\nu_{xy}\nu_{yx}},Q_{12}=\frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}},Q_{22}=\frac{E_y}{1-\nu_{xy}\nu_{yx}},\\ &Q_{66}=G_{xy},Q_{44}=G_{yz},Q_{55}=G_{xz}\end{aligned} \quad (10)$$

in which E_x and E_y are Young's moduli in the x and y material principal directions, respectively; ν_{xy} and ν_{yz} are Poisson's ratios; and G_{xy}; G_{yz} and G_{xz} are shear moduli in the x-y, y-z and x-z surfaces, respectively.

The stress and moment resultants of a laminated composite plate made up of n layers of orthotropic laminate can be determined by integrating (8) over the thickness, and are expressed as

$$\begin{Bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \\ \begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} \\ \begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} \end{Bmatrix} = \begin{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} & \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} & \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} \\ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} & \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} & \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} \\ \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} & \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} & \begin{bmatrix} H_{11}^s & H_{12}^s & H_{16}^s \\ H_{12}^s & H_{22}^s & H_{26}^s \\ H_{16}^s & H_{26}^s & H_{66}^s \end{bmatrix} \end{bmatrix} \begin{Bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \\ \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} \\ \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \end{Bmatrix} - \begin{Bmatrix} \begin{Bmatrix} N_1^T \\ N_2^T \\ N_6^T \end{Bmatrix} \\ \begin{Bmatrix} M_1^{bT} \\ M_2^{bT} \\ M_6^{bT} \end{Bmatrix} \\ \begin{Bmatrix} M_1^{sT} \\ M_2^{sT} \\ M_6^{sT} \end{Bmatrix} \end{Bmatrix} \tag{11a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{11b}$$

and stiffness components are given as

$$(A_{ij}, B_{ij}, B_{ij}^s, D_{ij}, D_{ij}^s, H_{ij}^s) = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} (1, z, f(z), z^2, z\, f(z), f(z)^2)\, dz\ , (i, j = 1, 2, 6), \tag{12a}$$

$$A_{ij}^s = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} \,(g(z))^2\, dz,\ (i, j = 4, 5) \tag{12b}$$

Note that, z_k denotes the distance from the mid-surface to the lower surface of the kth layer. The stress and moment resultants, N_x^T , N_y^T; . . . etc., due to thermal loading are expressed by

$$\begin{Bmatrix} N_1^T, M_1^{bT}, M_1^{sT} \\ N_2^T, M_2^{bT}, M_2^{sT} \\ N_6^T, M_6^{bT}, M_6^{sT} \end{Bmatrix} = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} (1, z, f(z)) T dz \tag{13}$$

The considered temperature variation T (x, y, z) within the thickness are supposed to be

$$T(x, y, z) = T_1(x, y) + \frac{z}{h} T_2(x, y) + \frac{f(z)}{h} T_3(x, y), \tag{14}$$

The governing equations of equilibrium can be obtained by employing the principle of virtual displacements. The governing equations associated with the present theory are

$$\begin{aligned} &\delta\, u_0: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\delta\, v_0: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\delta\, w_0: \quad \frac{\partial^2 M_x^b}{\partial x^2} + \frac{\partial^2 M_y^b}{\partial y^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + q = 0 \\ &\delta\, \theta: \ \text{-}k_1\, A' \frac{\partial^2 M_x^s}{\partial x^2} \text{-}k_2\, B' \frac{\partial^2 M_y^s}{\partial y^2} - (k_1\, A' + k_2 B') \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1\, A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = 0 \end{aligned} \tag{15}$$

Eq. (15) can be presented in terms of displacements (u_0, v_0, w_0 and θ) by substituting for the stress resultants from Eq. (11). For homogeneous laminates, the governing Eq. (15) become

$$\begin{aligned} &A_{11} d_{11} u_0 + 2 A_{16}\, d_{12} u_0 + A_{66}\, d_{22} u_0 + (A_{12} + A_{66}) d_{12} v_0 + A_{26}\, d_{22} v_0 + A_{16}\, d_{11} v_0 \\ &-(B_{11} d_{111} w_0 + 3 B_{16} d_{112} w_0 + (B_{12} + 2B_{66}) d_{122} w_0 + B_{26} d_{222} w_0) \\ &+(k_1 A' + k_2 B') B_{66}^s\, d_{122}\theta + (k_1 B_{11}^s + k_2 B_{12}^s)\, d_1\theta + (k_1 A' + k_2 B') B_{16}^s\, d_{112}\theta \\ &-(k_1 B_{16}^s + k_2 B_{26}^s) d_2\theta = F_1, \end{aligned} \tag{16a}$$

$$\begin{aligned} &A_{11} d_{16} u_0 + (A_{12} + A_{66}) d_{12} u_0 + A_{26}\, d_{22} u_0 + A_{66}\, d_{22} v_0 + 2A_{26}\, d_{12} v_0 + A_{22}\, d_{22} v_0 \\ &-(B_{16} d_{111} w_0 + 3 B_{26} d_{122} w_0 + (B_{12} + 2B_{66}) d_{112} w_0 + B_{22} d_{222} w_0) \\ &+(k_1 A' + k_2 B') B_{66}^s\, d_{112}\theta + (k_2 B_{22}^s + k_1 B_{12}^s)\, d_2\theta + (k_1 A' + k_2 B') B_{26}^s\, d_{122}\theta \\ &+(k_1 B_{16}^s + k_2 B_{26}^s) d_1\theta = F_2, \end{aligned} \tag{16b}$$

$$\begin{aligned} &(B_{11} d_{111} u_0 + 3 B_{16} d_{112} u_0 + (B_{12} + 2B_{66}) d_{122} u_0 + B_{26} d_{222} u_0) \\ &+(B_{16} d_{111} v_0 + 3 B_{26} d_{122} v_0 + (B_{12} + 2B_{66}) d_{112} v_0 + B_{22} d_{222} v_0) \\ &-D_{11} d_{1111} w_0 - 2(D_{12} + 2 D_{66})\, d_{1122} w_0 - D_{22}\, d_{2222} w_0 \\ &-4 D_{16}\, d_{1112} w_0 - 4 D_{26}\, d_{1222} w_0 \\ &+(k_1 D_{11}^s + k_2 D_{12}^s)\, d_{11}\theta + 2(k_1 A' + k_2 B') D_{66}^s\, d_{1122}\theta + \\ &(k_1 D_{12}^s + k_2 D_{22}^s)\, d_{22}\theta + 2(k_1 D_{16}^s + k_2 D_{26}^s)\, d_{12}\theta \\ &+(k_1 A' + k_2 B') D_{16}^s\, d_{1112}\theta + (k_1 A' + k_2 B') D_{26}^s\, d_{1222}\theta + q = F_3 \end{aligned} \tag{16c}$$

$$\begin{aligned} &-((k_1 A' + k_2 B') B_{66}^s\, d_{122} u_0 + (k_1 B_{11}^s + k_2 B_{12}^s)\, d_1 u_0 + (k_1 A' + k_2 B') B_{16}^s\, d_{112} u_0 \\ &-(k_1 B_{16}^s + k_2 B_{26}^s) d_2 u_0) \\ &-((k_1 A' + k_2 B') B_{66}^s\, d_{112} v_0 + (k_2 B_{22}^s + k_1 B_{12}^s)\, d_2 v_0 + (k_1 A' + k_2 B') B_{26}^s\, d_{122} v_0 \\ &+(k_1 B_{16}^s + k_2 B_{26}^s) d_1 v_0) \\ &+(k_1 D_{11}^s + k_2 D_{12}^s)\, d_{11} w_0 + 2(k_1 A' + k_2 B') D_{66}^s\, d_{1122} w_0 + (k_1 D_{12}^s + k_2 D_{22}^s)\, d_{22} w_0 \\ &+2(k_1 D_{16}^s + k_2 D_{26}^s)\, d_{12} w_0 \\ &+(k_1 A' + k_2 B') D_{16}^s\, d_{1112} w_0 + (k_1 A' + k_2 B') D_{26}^s\, d_{1222} w_0 \\ &-H_{11}^s k_1\theta - H_{22}^s\, k_2\theta - 2 H_{12}^s k_1 k_2\theta - (k_1 A' + k_2 B')^2\, H_{66}^s\, d_{1122}\theta \\ &-2(k_1 A' + k_2 B')(H_{16}^s + H_{26}^s) d_{12}\theta \\ &+A_{44}^s (k_2 B')^2\, d_{22}\theta + A_{55}^s\, (k_1 A')^2\, d_{11}\theta + 2 A_{45}^s\, k_1 k_2\, A' B' d_{12}\, \theta = F_4 \end{aligned} \tag{16d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$\begin{aligned} &d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j},\ d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\ d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \\ &d_i = \frac{\partial}{\partial x_i} \qquad (i, j, l, m = 1, 2). \end{aligned} \tag{17}$$

and $\{F\} = \{F_1, F_2, F_3, F_4\}^t$ is a generalized force vector given by

$$\begin{aligned} &F_1 = -\frac{\partial N_x^T}{\partial x} - \frac{\partial N_{xy}^T}{\partial y},\ F_2 = -\frac{\partial N_{xy}^T}{\partial x} - \frac{\partial N_y^T}{\partial y}, \\ &F_3 = -\frac{\partial^2 M_x^{bT}}{\partial x^2} - \frac{\partial^2 M_y^{bT}}{\partial y^2} - 2\frac{\partial^2 M_{xy}^{bT}}{\partial x \partial y} - q, \\ &F_4 = \ k_1\, A' \frac{\partial^2 M_x^{sT}}{\partial x^2} + k_2\, B' \frac{\partial^2 M_y^{sT}}{\partial y^2} + (k_1\, A' + k_2 B') \frac{\partial^2 M_{xy}^{sT}}{\partial x \partial y} \\ &-k_1\, A' \frac{\partial S_{xz}^T}{\partial x} - k_2 B' \frac{\partial S_{yz}^T}{\partial y} \end{aligned} \tag{18}$$

3. Analytical solutions for anti-symmetric cross-ply laminates

The Navier procedure is utilized to deduce the analytical solutions of the partial differential equations in Eq. (16) for simply supported rectangular plates. For anti-symmetric cross-ply laminates, the following plate stiffnesses are identically zero

$$
\begin{aligned}
&A_{16} = A_{26} = D_{16} = D_{26} = D_{16}^s = D_{26}^s = H_{16}^s = H_{26}^s = 0 \\
&B_{12} = B_{16} = B_{26} = B_{66} = B_{12}^s = B_{16}^s = B_{26}^s = B_{66}^s = 0 \\
&B_{22} = -B_{11};\ B_{22}^s = -B_{11}^s
\end{aligned}
\tag{19}
$$

To solve this problem, Navier supposed that the transverse mechanical and temperature loads q, T_1, T_2 and T_3 in the form of a double trigonometric series as

$$
\begin{Bmatrix} q \\ T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} q_0 \\ t_1 \\ t_2 \\ t_3 \end{Bmatrix} \sin(\alpha\, x)\sin(\beta\, y) \tag{20}
$$

where q_0, t_1, t_2 and t_3 are constants.

Based on the Navier method, the following expansions of displacements are adopted to automatically respect the simply supported boundary conditions of plate

$$
\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{21}
$$

where U_{mn}, V_{mn} and X_{mn} are coefficients, and α and β are expressed as

$$
\alpha = m\pi / a \ \text{and}\ \beta = n\pi / b \tag{22}
$$

Substituting Eqs. (21) and (20) into Eq. (16), the Navier solution of anti-symmetric cross-ply plates can be deduced from equations

$$
\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} \tag{23}
$$

where

$$
\begin{aligned}
&S_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right), S_{12} = -\alpha\beta\left(A_{12} + A_{66}\right), \\
&S_{13} = \alpha\left(B_{11}\alpha^2\right), S_{14} = \alpha\left(k_1 B_{11}^s\right) \\
&S_{22} = -\left(A_{66}\alpha^2 + A_{22}\beta^2\right), S_{23} = \beta\left(B_{22}\beta^2\right), \\
&S_{24} = \beta\left(k_2 B_{22}^s\right) \\
&S_{33} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right) \\
&S_{34} = -\left(k_1 D_{11}^s\alpha^2 + k_2 D_{12}^s\beta^2\right) + 2(k_1 A' + k_2 B')\, D_{66}^s\alpha^2\beta^2 \\
&-\left(k_2 D_{22}^s\beta^2 + k_1 D_{12}^s\alpha^2\right) \\
&S_{44} = -k_1\left(k_1 H_{11}^s + k_2 H_{12}^s\right) - (k_1 A' + k_2 B')\left((k_1 A' + k_2 B') H_{66}^s\alpha^2\beta^2\right) \\
&-k_2\left(k_1 H_{12}^s + k_2 H_{22}^s\right) - \left(k_1 A'\right)^2 A_{55}^s\alpha^2 - \left(k_2 B'\right)^2 A_{44}^s\beta^2
\end{aligned}
\tag{24}
$$

The components of the generalized force vector $\{F\}=\{F_1, F_2, F_3, F_4\}^t$ are given by

$$
\begin{aligned}
&F_1 = \alpha\left(A_1^T t_1 + \frac{B_1^T}{h} t_2 + \frac{B_1^{Ts}}{h} t_3\right) \\
&F_2 = \beta\left(A_2^T t_1 + \frac{B_2^T}{h} t_2 + \frac{B_2^{Ts}}{h} t_3\right) \\
&F_3 = -\left(\begin{aligned}&\left(B_1^T\alpha^2 + B_2^T\beta^2\right)t_1 + \frac{\left(D_1^T\alpha^2 + D_2^T\beta^2\right)}{h} t_2 \\ &+ \frac{\left(D_1^{Ts}\alpha^2 + D_2^{Ts}\beta^2\right)}{h} t_3\end{aligned}\right) - q_0 \\
&F_4 = \left(\alpha^2 A' k_1 B_1^{Ts} + \beta^2 B' k_2 B_2^{Ts}\right) t_1 + \frac{\left(\alpha^2 A' k_1 D_1^{Ts} + \beta^2 B' k_2 D_2^{Ts}\right)}{h} t_2 \\
&+ \frac{\left(\alpha^2 A' k_1 H_1^{Ts} + \beta^2 B' k_2 H_2^{Ts}\right)}{\text{h}} t_3
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
&\left(A_i^T, B_i^T, D_i^T, B_i^{Ts}, D_i^{Ts}, H_i^{Ts}\right) = \\
&\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left(\alpha_x\overline{Q}_{1i}^{(k)} + \alpha_y\overline{Q}_{2i}^{(k)}\right)\left(1, z, z^2, f(z), z\, f(z), f(z)?\right)dz \qquad (i,j) = (1,2)
\end{aligned}
\tag{26}
$$

4. Numerical results and discussion

In this section, various numerical results are presented for verifying the exactitude and efficiency of the present model in predicting the thermo-mechanical behavior of simply-supported anti-symmetric cross-ply composite plates. The exact closed-form solution of Reddy and Hsu (1980) for simply supported rectangular plates under sinusoidal thermal and mechanical loading is employed to demonstrate the validity of the present method. Calculations were carried out for the fundamental mode (i.e., m=n=1). All of the lamina are considered to be of the same thickness and made of the same orthotropic material. In all problems, the lamina characteristics are supposed to be

$$
E_x = 25\times10^6\ \text{psi}, E_x = 10^6\ \text{psi}, G_{xy} = G_{xz} = 0.5\times10^6\ \text{psi}
$$

$$
G_{yz} = 0.2\times10^6\ \text{psi}, \nu_{xy} = 0.25
$$

Note that, values of ($\alpha_x(\equiv\alpha_1)$) and $\alpha_y(\equiv\alpha_2)$ are given during the discussion of material results. We will supposed in all of the considered cases (unless otherwise stated) that a/h=10, a/b=1, t_1=1, and α_2/α_1=3.

Table 1 contains results of non-dimensional center transverse displacements $\bar{w} = 10wh/(\alpha_1\bar{T}_2 a^2)$ of two-layer cross-ply (0°/90°) plates engendered by thermal loading. After a detailed comparison investigation, it can be said that the present plate model provides very closed results to the values computed by the conventional sinusoidal plate theory (SSDT) obtained by Zenkour (2004), Chatibi *et al* (2015).

In Table 2, the following non-dimensional transverse displacement $\tilde{w}$ of two-layer cross-ply (0°/90°) plates under to combined loading is employed (see Reddy and Hsu 1980)

Table 1 Nondimensional center deflections $\bar{w}=10wh/(\alpha_1\bar{T}_2 a^2)$ of cross-ply square plates (0°/90°) subjected to thermal loading ($\bar{T}_3=0$)

a/h	Exact [(a)]	Chattibi *et al.* (2015)	HSDT [(b)]	SSDT [(b)]	Present
100	1.6765	1.6766	1.6766	1.6766	1.6766
50	1.6765	1.6767	1.6767	1.6767	1.6767
25	1.6765	1.6771	1.6770	1.6771	1.6771
20	1.6765	1.6774	1.6773	1.6774	1.6774
12.5	1.6765	1.6789	1.6786	1.6789	1.6789
10	1.6765	1.6802	1.6798	1.6802	1.6802
6.25	1.6765	1.6858	1.6848	1.6858	1.6858
5	1.6765	1.6910	1.6894	1.6910	1.6910

[(a)] Reddy and Hsu (1980)
[(b)] Zenkour (2004)

Table 2 Nondimensional center deflections $\tilde{w}$ of cross-ply square plates (0°/90°) subjected to combined loading (q_0=100, $\bar{T}_2$=100, $\bar{T}_3$=0, α_1=10^{-6})

a/h	Exact [(a)]	Chattibi *et al.* (2015)	HSDT [(b)]	SSDT [(b)]	Present
100	2.4451	2.4481	2.4481	2.4481	2.4481
50	2.4597	2.4585	2.4586	2.4584	2.4585
25	2.5083	2.4999	2.5006	2.4996	2.4999
20	2.5443	2.5309	2.5321	2.5304	2.5309
12.5	2.7001	2.6650	2.6679	2.6636	2.6650
10	2.8438	2.7885	2.7927	2.7859	2.7885
6.25	3.4666	3.3186	3.3273	3.3090	3.3186
5	4.0415	3.8013	3.8120	3.7821	3.8013

[(a)] Reddy and Hsu (1980)
[(b)] Zenkour (2004)

$$\tilde{w}=w\left[\frac{q_0 a^4}{h^3\zeta}+\frac{\alpha_1\bar{T}_2 a^2}{10h}\right]^{-1},$$

$$with\ \ \zeta=\frac{1}{12}\pi^4\left[4G_{xy}+\frac{E_x+(1+\nu_{xy})E_y}{1-\nu_{xy}\nu_{yx}}\right]^{-1}$$

The computed results are given in Table 2 and are compared to those calculated via various plate theories (Zenkour 2004, Chattibi *et al.* 2015) and the solution of Reddy and Hsu (1980). It can be observed that the present model with only four unknowns agree extremely well with those determined in (Zenkour 2004, Chattibi *et al.* 2015).

The non-dimensional transverse displacement calculated by employing using various theories for two-, four-, six- and ten-layer anti-symmetric cross-ply square laminates are reported in Table 3. The results clearly show that the present new simple four variable plate theory and the conventional sinusoidal theory (SSDT) provide identical results.

The variation of non- dimensionalized vertical displacement $\bar{w}$ versus the ratio *a/h* for anti-symmetric two- and four-layer cross-ply square plates is shown in Figs. 2 and 3, respectively. An interesting result deduced from Figs. 2 and 3 is that the vertical displacement $\bar{w}$ is independent of the side to-thickness ratio for the case of the CPT. On the other hand, with the consideration of the shear deformation effect, all responses of the present theory, HSDT, SSDT, and FSDT become dependent on the side to-thickness ratio. It is known that the dependency of the responses on the side to-thickness ratio for the present theory, HSDT, SSDT, and FSDT is uniquely due to the effect of shear deformation. The obtained results are compared with those generated by HSDT, SSDT, and FSDT as is shown in Figs. 2 and 3. In addition, it is seen that the vertical displacement $\bar{w}$ decreases with increasing the side to-thickness ratio for two- layer plates, whereas for four-layer plates ones the increase in vertical displacement due to the same theories is shown.

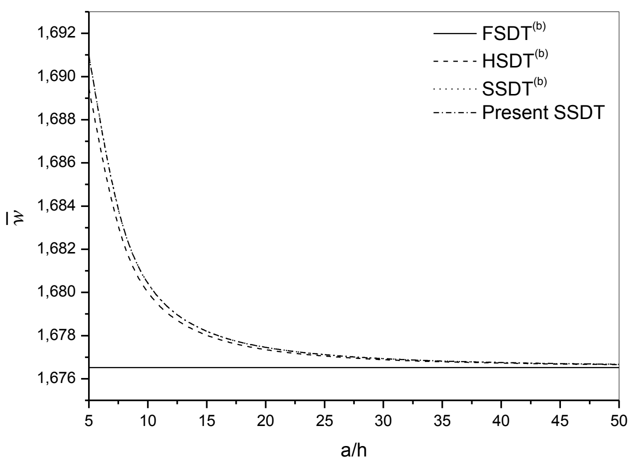

Fig. 2 Effect of thickness on the dimensionless deflection $\bar{w}$ of a two-layer, anti-symmetric cross-ply (0/90) square plate (t_3=0)

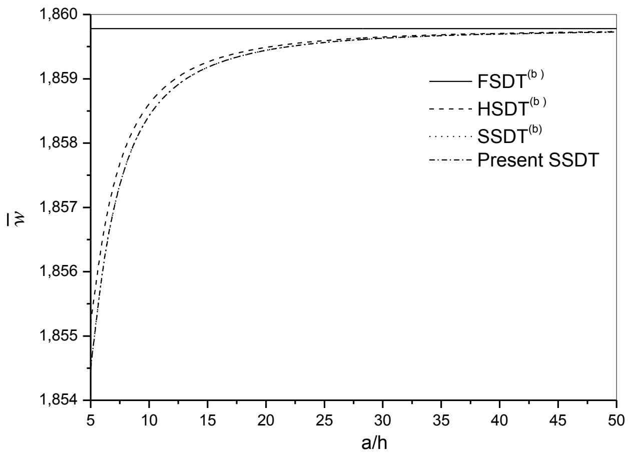

Fig. 3 Effect of thickness on the dimensionless deflection $\bar{w}$ of a four-layer, anti-symmetric cross-ply $(0/90)^2$ square plate (t_3=0)

The effect of the ratio of thermal expansion coefficients (α_2/α_1) on the bending response of anti-symmetric four-layer cross-ply square plate is demonstrated in Fig. 4. It can be seen that the vertical displacement is linearly proportional to the α_2/α_1 ratio.

Fig. 5 demonstrates the effects of the aspect ratio (*a/b*) on the non- dimensionalized vertical displacement $\bar{w}$ of anti-symmetric four-layer cross-ply square plate subjected to linear temperature distribution and/or mechanical loading. It is found that the aspect ratio effect is more pronounced on the thermal bending deflection $\bar{w}$ (q=0) of

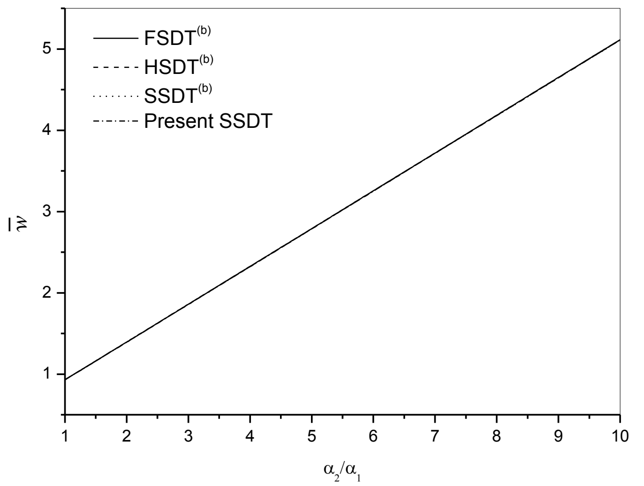

Fig. 4 Effect of the ratio of thermal expansion coefficients α_2/α_1 on the dimensionless deflection $\bar{w}$ of a four-layer, anti-symmetric cross-ply $(0/90)^2$ square plate (t_3=0)

Table 3 The non-dimensional transverse displacement calculated by employing using various theories for two-, four-, six- and ten-layer anti-symmetric cross-ply square laminates

No. of layers	Theory	a/h					
		2	4	10	20	50	100
$(0/90)_1$	CLPT	1.6765	1.6765	1.6765	1.6765	1.6765	1.6765
	FSDT	1.6765	1.6765	1.6765	1.6765	1.6765	1.6765
	SSDT	1.7590	1.6989	1.6802	1.6774	1.6767	1.6766
	Present	1.7590	1.6989	1.6802	1.6774	1.6767	1.6766
$(0/90)_2$	CLPT	1.8598	1.8598	1.8598	1.8598	1.8598	1.8598
	FSDT	1.8598	1.8598	1.8598	1.8598	1.8598	1.8598
	SSDT	1.8296	1.8516	1.8584	1.8594	1.8597	1.8598
	Present 2	1.8296	1.8516	1.8584	1.8594	1.8597	1.8598
$(0/90)_3$	CLPT	1.8745	1.8745	1.8745	1.8745	1.8745	1.8745
	FSDT	1.8745	1.8745	1.8745	1.8745	1.8745	1.8745
	SSDT	1.8602	1.8706	1.8739	1.8744	1.8745	1.8745
	Present 2	1.8602	1.8706	1.8739	1.8744	1.8745	1.8745
$(0/90)_5$	CLPT	1.8811	1.8811	1.8811	1.8811	1.8811	1.8811
	FSDT	1.8811	1.8811	1.8811	1.8811	1.8811	1.8811
	SSDT	1.8759	1.8797	1.8809	1.881	1.8811	1.8811
	Present 2	1.8759	1.8797	1.8809	1.881	1.8811	1.8811

a plate under non-uniform temperature distribution.

The effect of the modulus ratio (E_1/E_2) on the bending response of anti-symmetric four-layer cross-ply square plate is shown in Fig. 6. It can be deduced that the bending response of the composite plate depends strongly on the material anisotropy of the layer.

4. Conclusions

A simple four variable sinusoidal plate theory has been successfully developed for the thermo-mechanical of simply supported laminated plates. The theory accounts for the shear deformation effects without requiring a shear correction factor. By dividing the transverse displacement into bending, shear and stretching components, the number of unknowns and governing equations of the present theory is reduced to four and is therefore less than alternate theories. The accuracy and efficiency of the present theory has been demonstrated for thermo-mechanical bending behavior of antisymmetric cross-ply laminates.

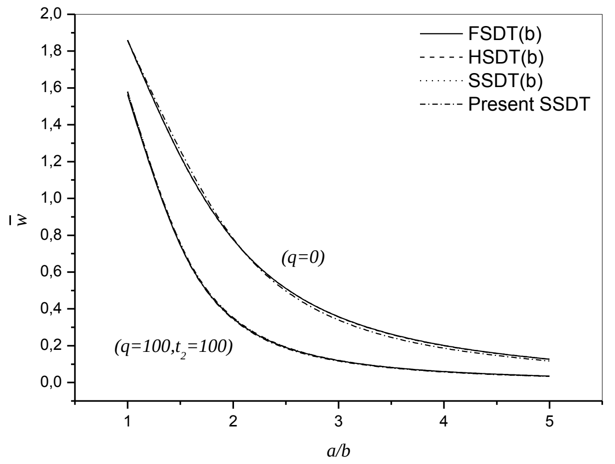

Fig. 5 Effect of aspect ratio on the dimensionless combined deflection $\bar{w}$ of a four-layer, anti-symmetric cross-ply (0/90)2 plate (t_3=0)

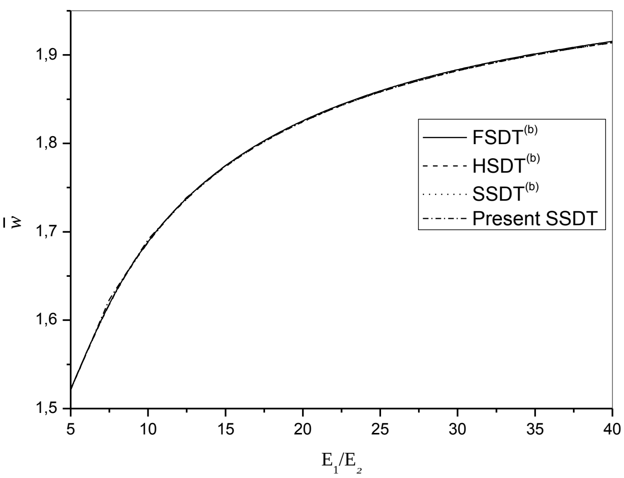

Fig. 6 The effect of material anisotropy E_1/E_2 on the dimensionless deflection $\bar{w}$ of a four-layer, anti-symmetric cross-ply $(0/90)^2$ square plate (t_3=0)

An improvement of the present formulation will be considered in the future work to consider other type of materials (Benferhat *et al*. 2016, Kar *et al*. 2017, Eltaher *et al*. 2018, Selmi and Bisharat 2018, Natanzi *et al*. 2018, Panjehpour *et al*. 2018, Shahadat *et al*. 2018, Faleh *et al*. 2018, Karami *et al*. 2018, Chemi *et al*. 2018, Bensattalah *et al*. 2018 and 2019, Hussain and Naeem 2019, Avcar 2019, Alimirzaei *et al*. 2019, Addou *et al*. 2019, Rajabi and Mohammadimehr 2019, Karami *et al*. 2019, Safa *et al*. 2019, Hadji and Zouatnia 2019, Fadoun 2019).

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