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Predicting the shear strength of reinforced concrete beams using Artificial Neural Networks

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Abstract. In this research study, the artificial neural networks approach is used to estimate the ultimate shear capacity of reinforced concrete beams with transverse reinforcement. More specifically, surrogate approaches, such as artificial neural network models, have been examined for predicting the shear capacity of concrete beams, based on experimental test results available in the pertinent literature. The comparison of the predicted values with the corresponding experimental ones, as well as with available formulas from previous research studies or code provisions highlight the ability of artificial neural networks to evaluate the shear capacity of reinforced concrete beams in a trustworthy and effective manner. Furthermore, for the first time, the (quantitative) values of weights for the proposed neural network model, are provided, so that the proposed model can be readily implemented in a spreadsheet and accessible to everyone interested in the procedure of simulation.

Keywords: artificial neural networks; heuristic algorithm; reinforced concrete beams; stirrups; soft computing; shear strength

1. Introduction

Determination of shear capacity is a critical step in structural element design of reinforced concrete structures. Numerous mechanistic models have been proposed researchers standardisation committees to evaluate the shear capacity of reinforced concrete beams. Typically these models are empirical and account for various parameters, not always the same and often with quite different results. This has led to researchers to turn towards non-deterministic techniques for wish an in-depth review and critical literature examination can be found in the works of Flood and Kartam (1994), Adeli (2001), Asteris and Plevris (2013, 2017) and Asteris *et al.* (2016a, b), Sarir *et al*. 2019.

Amongst the non-deterministic methods, the method of Artificial Neural Networks (ANN) appears to be the most attractive and reliable. ANNs have materialized as an innovative simulation technique, with wide spectrum of applications in a variety of technological disciplines. Over the last two decades, there has been extensive use of ANNs in predicting the behavior and evaluating the mechanical properties of structural materials and in particular of concrete (Waszczyszyn and Ziemiański 2001, Asteris and Kolovos 2019, Asteris *et al.* 2017). The pertinent literature includes studies on the application of ANNs in the determination of compressive strength and elasticity modulus of concrete (Dias and Pooliyadda 2001, Lee 2003, Topçu and Saridemir 2008, Trtnik *et al.* 2009), for which various other methods of artificial intelligence, such as the fuzzy logic and genetic algorithms were also (Baykasoğlu *et al.* 2004, Akkurt *et al*. 2004, Özcan *et al*. 2009, Asteris *et al.* 2019a). ANNs have also been used for the determination of the shear strength of reinforced concrete structural elements (Sanad and Saka 2001, Mansour *et al*. 2004, Seleemah 2005, 2012, El-Chabib *et al.* 2006, Amani and Moeini 2012, Mohammadhassani *et al*. 2014, 2015, Kotsovou *et al.* 2017, Keskin 2017, Kaveh *et al*. 2018, Sarveghadi *et al*. 2019, Yaseen *et al*. 2018, Yavuz 2016, 2019).

This paper examines the adoption of Artificial Neural Networks for the estimation of shear strength of reinforced concrete. In particular, a heuristic algorithm is proposed to determine the optimal Artificial Neural Network architecture for estimating shear resistance of reinforced concrete members in terms of mean square error. For the training of the network, a research database is used, which includes the shear resistance of reinforced concrete beams specimens with various dimensions, materials and geometric properties. The backward propagation method is examined in the procedure of neural network design and development, while the well accepted Levenberg-Marquardt algorithm (Lourakis 2005) is used as the training algorithm. For the training, nine parameters concerning the mechanical and geometric characteristics of the beams are used as input parameters, while the experimental test shear capacity is used as the output parameter. Comparisons are made with

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other models to determine accuracy and efficiency.

In addition to the architecture of the proposed optimum neural system, a supplementary materials section is included which provides a simple design/education tool which can assist both in teaching, as well as the interpretation of the behavior of reinforced concrete beams under shear loading. The decision to include all the necessary information for anyone to be able to test the proposed model, in addition to the reliability it ensures, also provides the means for other researchers, students or field practitioners to further test the reliability of the proposed model.

2. Research significance

Despite the abundance of research works, both experimental and theoretical, conducted since the middle of the previous century (Clark 1951) up to today, the determination of the shear stress value still remains an open issue of great interest in structural engineering. The need for further research is indicated by the fact that the majority of available models, use different parameter and lead to different results with a high degree of variation. This may be due to the many (more than ten) parameters which influence the shear capacity estimation. Even though five values for each parameter can be considered as a satisfactory set to work with, that would demand the results of 9765625 (5^{10}) experiments to comprehend and interpret the ten-dimensional space underlying the specific problem. An experimental endeavor of such magnitude is unfeasible; hence, other approaches that can give results from fewer results are necessary.

Non-deterministic techniques, such as soft computing techniques, can contribute towards the solution of this problem. To this end, a soft computing mathematical model based on Artificial Neural Networks (ANNs) is proposed herein, aiming to predict shear strength of reinforced concrete beams with or without stirrups.

3. Shear capacity of reinforced concrete beams

This section presents and discusses the experimental test results and main empirical expressions proposed for the determination of shear resistance. Typical geometrical parameters for a beam test are shown in Fig. 1.

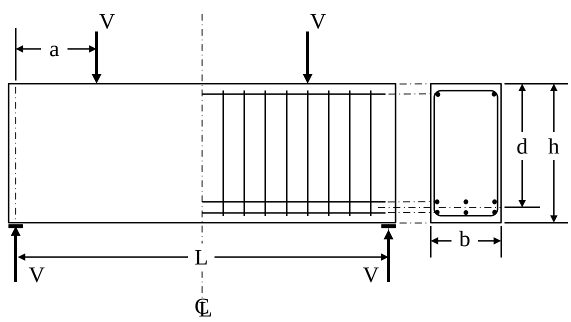

Fig. 1 Reinforced concrete beam under shear force

3.1 Experimental tests results

Since the early 1950s, a number of comprehensive experimental works have been published on the shear capacity of reinforced concrete beams with or without shear reinforcement (stirrups). The widely utilised works by Clark (1951), Placas and Regan (1971) and Fukuhara and Kokusho (1982) are worth mentioning and these are included in the baseline used in the present study as well as the experimental results of another six studies, those of Xie *et al.* (1994), Yoon *et al.* (1996), Angelakos *et al.* (2001), Zararis *et al.* (2009), Ismail (2009), Londhe (2011), which are presented in detail in the next section. These works were selected to better cover all the ranges of values of the parameters involved in the problem under consideration.

Fig. 2 shows the values of shear strength for the 300 experimental results (datasets) examined versus six of the most important parameters that affect the shear resistance of reinforced concrete beams. These are the shear ratio (*a/d*), compressive strength of concrete (*fc*), percentage of lateral/shear reinforcement, percentage of longitudinal reinforcement (*ρl*), (*ρw*), and the yield strength of shear and longitudinal reinforcement.

In fact, these three experimental databases are included in the baseline used in the present study along with the experimental results of the studies by Xie *et al.* (1994), Yoon *et al.* (1996), Angelakos *et al.* (2001), Zararis *et al.* (2009), Ismail (2009), Londhe (2011). Diagrams of Fig. 2 show the non-linear and complex behavior of the aforementioned parameters which may explain the inability of deterministic methods to formulate an analytical relationship to evaluate the value of shear capacity of reinforced concrete beams.

3.2 Shear strength according to building codes

In addition to transverse reinforcement, other mechanisms as consider to contribute to the shear capacity of a concrete member including: shear transfer in the compressive region, dowel action of longitudinal reinforcement and aggregate interlock. The total shear resistance V_n of a concrete member without axial force is normally given as the summation of two contributions

$$V_n = V_c + V_s \tag{1}$$

where V_c is the concrete shear contribution and V_s is the transverse reinforcement contribution.

Different design codes propose different expressions for both of these contributions, but vary much more in the determination of the concrete shear contribution.

In the current study, four design codes are examined for the estimation of the shear capacity of reinforced concrete beams: the American ACI-318-14 (2015), the Canadian CSA-A23.3-04 (2004), the New Zealand NZS-3101 (2006) and the European Eurocode 2 (EN 1992-1 2004) as presented below:

3.2.1 Eurocode 2 - EN 1992-1 (2004)

According to the Eurocode (EC2) concrete shear contribution V_c of members without shear reinforcement and without axial force is given by the following expression

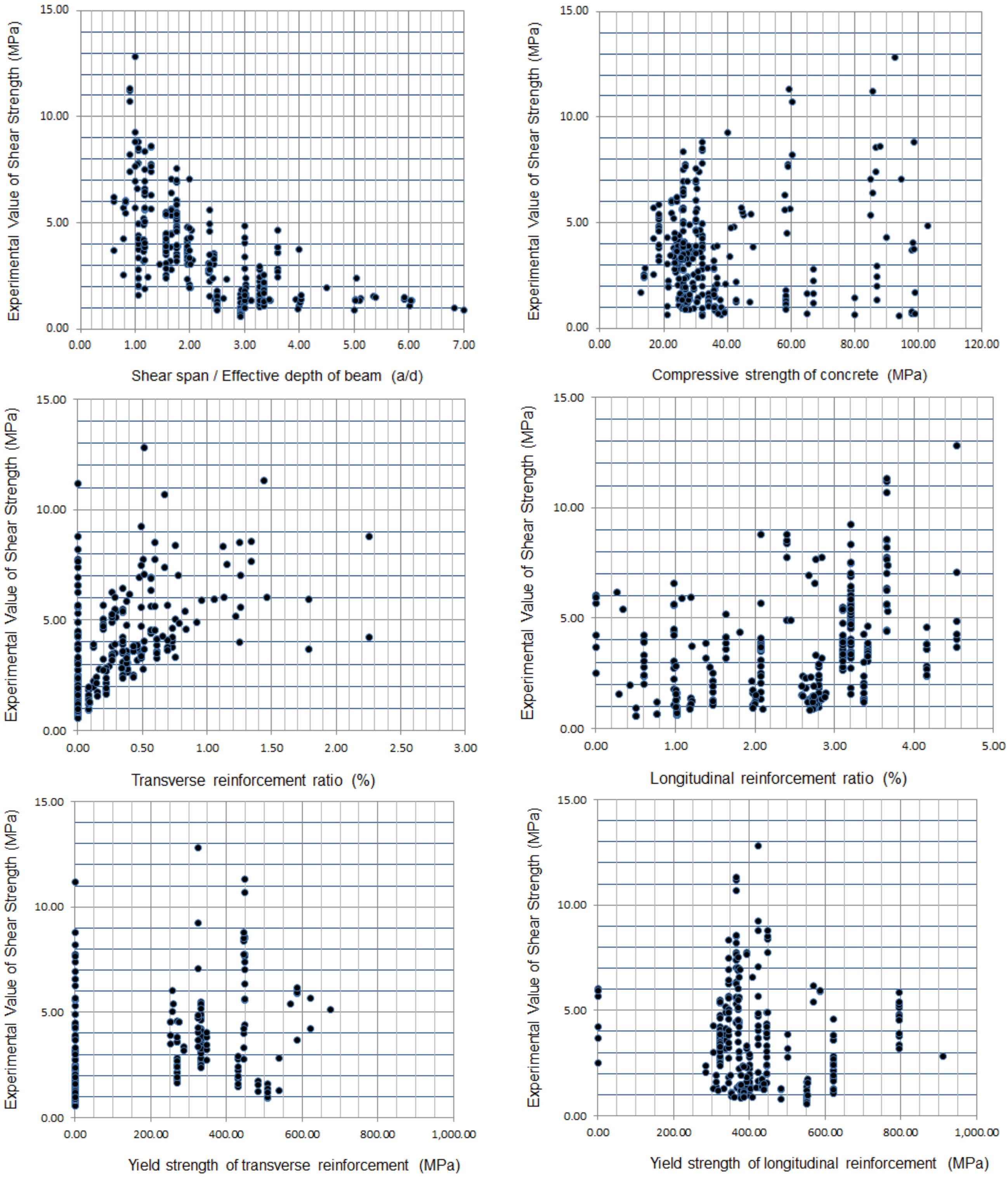

Fig. 2 Experimental values of shear strength of reinforced concrete beams

$$V_c = \left[C_{R,c}\, k\, (100\, \rho_l f_c)^{1/3}\right] b_w d \geq 0.035\, k^{3/2} \sqrt{f_c}\, b_w d \quad (2)$$

where f_c (MPa) is the concrete compressive strength, b_w (mm) is the smallest width of the cross-section in the tensile area, d (mm) is the effective depth of a cross-section, $C_{R,c} = 0.18$, $\mathrm{k} = 1 + \sqrt{\frac{200}{\mathrm{d}}} \leq 2.0$ (d in mm), $\rho_\mathrm{l} = \frac{\mathrm{A_{sl}}}{\mathrm{b_w d}} \leq 0.02$ is the longitudinal reinforcement ratio and A_{sl} is the area of the longitudinal reinforcement.

The transverse reinforcement contribution V_s is expressed as

$$V_s = \frac{A_{sw}}{s}\, z\, f_{yw} \cot\theta \quad (3)$$

Where A_{sw} is the area of the shear reinforcement, s is the spacing of the stirrups, z is the inner lever arm with an approximate value z=0.9d for members without axial force, f_{yw} (MPa) is the yield stress of the shear reinforcement, θ is the angle between the concrete compression strut and the beam axis perpendicular to the shear force, 1≤cotθ≤2.5, $\rho_w = \frac{A_{sw}}{b_w\, s}$ the transverse reinforcement ratio.

The maximum value of the shear strength is equal to

$$V_{max} = a_{cw} b_w\, z\, v_1 \frac{f_c}{(cot\,\theta + tan\,\theta)} \tag{4}$$

where $a_{cw} = 1.0$ and $v_1 = v = 0.6\left[1 - \frac{f_c}{250}\right]$.

3.2.2 ACI building code - ACI 318-14(2015)

According to ACI 318-14 building code, the concrete shear contribution, V_c, for members without axial force is given by

$$V_c = 0.17\lambda\sqrt{f_c} b_w d \quad \text{with} \quad \sqrt{f_c} \le 8.3\, MPa \tag{5}$$

where λ=1.0 for normal weight concrete. The transverse reinforcement contribution V_s is expressed as

$$V_s = \frac{A_{sw}}{s}\, d\, f_{yw} \le 0.66\sqrt{f_c} b_w d \tag{6}$$

3.2.3 Canadian Standard Code-CSA-A23.3-04

The Canadian Building Code (CSA-A23.3-04) in the simplified method proposes the following expressions for the calculation of the concrete shear contribution, V_c, of elements without axial load. The expression of the shear strength depends on the height of the cross-section and the area of the transverse reinforcement.

For $h \le 250$ mm

$$\mathrm{V_c} = 0.21\sqrt{\mathrm{f_c}}\mathrm{b_w d} \tag{7}$$

If $A_{sw} \ge \frac{0.06\sqrt{f_c} b_w s}{f_{yw}}$

$$V_c = 0.18\sqrt{f_c} b_w d \tag{8}$$

If $A_{sw} < \frac{0.06\sqrt{f_c} b_w s}{f_{yw}}$ and $h > 250$ mm

$$V_c = \left(\frac{230}{1000 + d_v}\right)\sqrt{f_c} b_w d \tag{9}$$

where $\sqrt{f_c} \le 8.0\sqrt{\mathrm{MPa}}$ and dv = max{0.9d , 0.77h}

The transverse reinforcement contribution V_s is given by

$$V_s = \frac{A_{sw}}{s} d_v f_{yw}\, cot\,\theta \le 0.66\sqrt{f_c} b_w d \tag{10}$$

In the simplified method, the Canadian building code proposes that the angle between the concrete compression strut and the beam axis perpendicular to the shear force should be taken equal to 42° for beams with height $h \le 250$ mm or else to be taken equal to 35°.

3.2.4 New Zealand Standards - NZS 3101

The New Zealand code is valid for concrete with compressive strength that does not exceed 100 MPa. The nominal shear stress $v_n = V_n/b_w d$ should be equal to or less than the smaller of $0.2 f_c$ or 8 MPa.

According to that code, the concrete stress contribution is equal to

$$V_c = k_d(0.07 + 10\rho_l)\sqrt{f_c} b_w d \quad \begin{matrix} \ge 0.08\sqrt{f_c} \\ < 0.20\sqrt{f_c} \end{matrix} \tag{11}$$

where $f_c \le 50$ MPa and for $A_{sw} \ge A_{sw,\min} = \frac{1}{16}\sqrt{f_c}\frac{b_w s}{f_{yw}}$ or $d < 400$ mm

$$k_d = 1.0 \tag{12}$$

If $A_v < A_{sw,\min} = \frac{1}{16}\sqrt{f_c}\frac{b_w s}{f_{yw}}$ and $d > 400$ mm

$$k_d = \left(\frac{400}{d}\right)^{0.25} \tag{13}$$

If $d < 200$ mm

$$V_c = \max\begin{cases} k_d(0.07 + 10\rho_l)\sqrt{f_c} b_w d \\ 0.17\sqrt{f_c} b_w d \end{cases} \tag{14}$$

For members with an effective depth between 200 mm and 400 mm, the value of V_c shall be found by linear interpolation.

3.3 Shear strength according to previous research studies

3.3.1 Gandomi et al. (2017)

Gandomi *et al*. (2017) gathered from the literature a large database of experimental data, containing 466 RC beams with shear reinforcement. Gene expression programming was developed to predict the shear strength of RC beams with stirrups and their model concluded to the following expression

$$V_{GEP}(kN) = \frac{\rho_l^2}{\rho_l - 6} - \left(\rho_w f_{yw} + \rho_l + \frac{a}{d} + 6\right)^2 + \sqrt[4]{\rho_l\left(f_c \rho_w f_{yw} \rho_l (d - b) - d\rho_l^2\right)^2} + 5\frac{a}{d} + b_w + 8 \tag{15}$$

where b_w (mm) is the smallest width of the cross-section in the tensile area, d (mm) is the effective depth of a cross-section, a/d is the shear span to depth ratio, f_c (MPa) is the concrete compressive strength, $\rho_l = A_{sl}/b_w d$ (%) is the longitudinal reinforcement ratio, $\rho_w f_{yw} = A_{sw} f_{yw}/b_w s$ (MPa) is the shear reinforcement contribution, A_{sl} and A_{sw} are the area of the longitudinal and shear reinforcement, respectively, s is the spacing of the stirrups and f_{yw} (MPa) is the yielding strength of the shear reinforcement.

3.3.2 Russo et al. (2013)

According to Russo *et al*. (2013), the shear capacity of a reinforced concrete beam is given by

$$V_u = 0.72\,\xi\left[\rho_l^{0.4} f_c^{0.39} + 0.5\rho_l^{0.83} f_y^{0.89}\left(\frac{a}{d}\right)^{-1.2-0.45\frac{a}{d}} + 0.075\, f_c^{0.5}\left(\rho_w f_{yw}\right)^{0.7}\right] \tag{16}$$

where ξ is a function that takes the size effect into consideration.

Fig. 3 presents a comparison between some of the aforementioned expressions for the evaluation of the

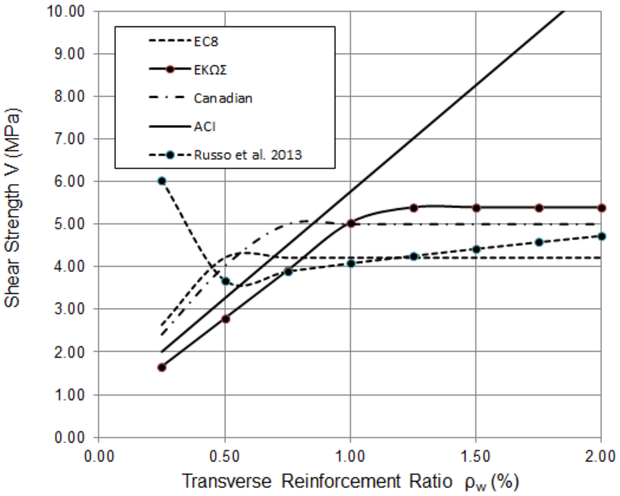

Fig. 3 Comparison of equations for the evaluation of shear strength of concrete beams

concrete beam shear strength in relation of transverse reinforcement ratio ρ_w. It is obvious that the concrete beam shear strength calculated based on these expressions shows considerable variation, revealing the need for further investigation and refinement of the proposals.

4. Artificial neural networks architectures

4.1 Back-propagation ANNs

The back-propagation neural network (BPNN) appears to be the simplest and most applicable network for the modeling of concrete structures. This has mainly to do with its ability to regulate the weights of all layers based on the inaccuracy present at the network results. A representative arrangement of BPNN model contains an input layer, one or more concealed layers and an output layer while every layer is made of several neurons (Armaghani *et al.* 2017).

Through a variety of procedures, the error is being fed through the network. On the basis of this information, the algorithm regulates the weights of each connection in order to reduce the error function value to a small percentage value. Having resumed this process for a sufficiently large number of training cycles, the network usually converges with a fairly low computational error. In order to adjust appropriately the weights, a generalized method for nonlinear optimization, called gradient descent, is applied. For the minimization of these errors, the derivative of the error function is calculated as a function of the network weights, while the weights are changed so as to reduce the error (downward path on the surface of the error function). For this reason, the backward algorithm is limited only to networks with productive functions. The back-spreading method of the error usually allows to achieve rapid convergence to local minimum errors for the networks.

BPNN is a multi-level and feed-forward network with specific structure where neurons are not connected within a plane, but participate in the neuron of that plane with all the neurons of the previous and subsequent levels. In this case,

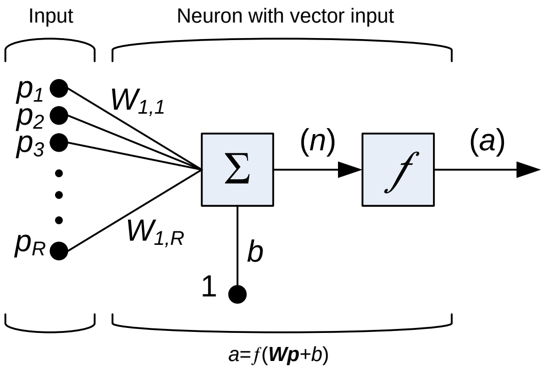

Fig. 4 Simple neuron with a simple input vector R

the neural network has the following structure

$$N - H_1 - H_2 - \cdots - H_{NHL} - M \quad (17)$$

where N the number of input parameters, H_v the number of neurons in N-hidden level for v=1, … , NHL where, NHL is the number of the hidden layers, and M the number of output parameters.

Based on the above a BPNN with a 5 entry neurons, two hidden levels of 4 and 3 neurons, respectively, and 2 output neurons is encoded as 5-4-3-2 BPNN.

Fig. 4 depicts the basic neural network architecture, which consists of a single neuron with multiple inputs (vector p). For each node, each value of the input vector p_1, p_2, …, p_R is multiplied by the corresponding weight $w_{1,1}$, $w_{1,2}$, …, $w_{1,R}$, and the weighted values feed the summation node. Then, the scalar product, $w{\cdot}p$, is computed by the vector-line $W = [w_{1,1}, w_{1,2}, \dots, w_{1,R}]$ and the vector-column $p = [p_1, p_2, \dots, p_R]^T$. This scalar product is added to another input product, which is always equal to the unit vector multiplied by its corresponding weight b. This last input into the adder is called bias, which has the ability of increasing or decreasing the input to the transfer function when it is positive or negative, respectively. This sum is described by the equation

$$n = w_{1,1}p_1 + w_{1,2}p_2 + \dots + w_{1,R}p_R + b = Wp + b \quad (18)$$

This sum, n, has to do with the input of activation function, which describes the output value of the network. The selection of activation function mainly affects the complexity and the effectiveness of neural networks. In this study, the Logistic Sigmoid as well as the Hyperbolic Tangent are adopted as transfer functions.

4.2 Optimal artificial neural network architecture

In order to find the optimal architecture of an ANN, it is enough to calculate the number of hidden neurons, since the number of input and output parameters is already known. At this point, it is worth noting that the best solution should avoid the problem that the optimal solution leads to over-fitting. The phenomenon of over-fitting arises when a model is too complex, e.g., when it has too many parameters in relation to the number of observations. Similarly, in the case where the training data do not cover the whole range of the input parameters of the problem and

Table 1 Database statistics

No	Source		Number of Beams			Shear Strength	Main Parameters	
	Authors	Year	Total	With Stirrups	Without Stirrups	MPa	f_c MPa	a/d
1	Ismail	2009	22	13	9	2.78-11.33	30.50-88.10	0.91-1.67
2	Zararis *et al.*	2009	11	4	7	2.55-6.20	16.80-24.00	0.61-0.83
3	Yoon *et al.*	1996	12	9	3	1.01-2.94	36.00-87.00	3.28
4	Angelakos *et al.*	2001	21	6	15	0.59-1.63	21.00-99.00	2.92
5	Londhe	2011	21	8	13	1.60-8.80	24.44-36.67	1.07-1.78
6	Clark	1951	58	48	10	2.26-6.59	13.79-47.58	1.16-2.43
7	Placas and Regan	1971	22	17	5	1.08-4.63	12.76-48.13	3.36-5.05
8	Fukuhara and Kokusho	1982	43	41	2	1.56-8.35	18.54-30.19	1.18-2.35
9	Xie *et al.*	1994	15	9	6	1.34-12.85	37.73-103.23	1.00-4.00
10	Kani	1967	42	-	42	0.87-7.78	24.75-30.75	1.00-9.05
11	Ghannoum	1998	16	-	16	0.89-1.78	34.20-58.60	2.50
12	Feldman & Siess	1955	11	-	11	1.21-4.28	21.03-36.68	2.01-7.04
13	Tompos & Frosh	2002	6	5	1	0.98-2.84	35.85-42.75	3.00
		Total	**300**	**160**	**140**	**0.59 - 12.85**	**12.75-103.23**	**0.61-9.05**

especially when the number of parameters is equal to or exceeds the number of observations. On the other hand, a simple model can predict training data by memorizing them, but fails to predict new ones because it does not learn to be generalized. In order to avoid the problem of over-fitting, various techniques, algorithms and criteria have been proposed (e.g., see Blum 1992, Boger and Guterman 1997, Berry and Linoff 1997, Papadopoulos *et al*. 2012, Lamanna *et al*. 2012, Chen 2013, Giovanis and Papadopoulos 2015, Asteris *et al*. 2016a, b, Cavaleri *et al*. 2016, 2017, 2019, amongst others) .

In the present paper, a simple heuristic algorithm is proposed which reliably leads to an optimal ANN architecture. The steps of this algorithm are as follows:

Step 1. Development and training of many neural networks: The development and training of NN takes place for a series of hidden levels ranging from 1 to 2 and with a number of neurons ranging from 1 to 30. Also, each neural network is trained for various activation functions.

Step 2. Determining the mean square error (MSE): For each of the above NNs, the average square error for the validation data that is not used during the training data process of the ANNs.

Step 3. Determine upper and lower limits: Enter upper and lower bounds for output parameter (shear capacity) based on experimental or numerical data, as well as reasonable estimates by users. For the work presented herein a lower limit for all the values of shear strength for the case of concrete beams with Shear span to Effective depth of beam ratio greater or equal to 5.00 $\left({}^{a}/_{d} \geq 5.00\right)$ is proposed in a next section.

Step 4. Choose optimal architecture: The optimal architecture is the one that gives the minimum average square error.

The main advantage of ANN, in comparison with other evaluation strategies based on computational methods, is reduced computation effort (Plevris and Asteris 2014a, b, 2015, Giovanis and Papadopoulos 2015, Asteris and Plevris 2013, 2017, Asteris *et al*. 2016a, b, 2017, Cavaleri *et al*. 2016, 2017).

5. Materials and method

5.1 Database

The database examined here consists of the data and results of 300 experiments on reinforced concrete beams with stirrups (synthesized from data published in thirteen (13) pertinent research studies as shown in Table 1. Despite the fact that a large number of databases are available in relevant literature, it was decided to compile a new database, in contrast to complementing an existing database. This decision was made for the following reasons: (i) the existing databases took into account different input parameters than the current research, (ii) quite frequently different values are given for the same experiments (incorrect transliteration of the original experimental data), but above all, because (iii) the datasets are not distributed as to cover the full range of input parameter values, but only a certain area of the whole range. It is worth noting that the number of a database's datasets is not enough to ensure its reliability; the distribution of input parameter values taken into account, however, plays a crucial role in this procedure. That is, if the model can represent and manage the knowledge available through experiments for the totality of parameters and, if possible, for the whole range of their values.

In light of the above, the database used herein, was compiled, abiding by the following principles:

- Datasets include experimental data from reinforced concrete beams both with and without stirrups (160 and 140 datasets respectively). This selection was intentional, despite the fact that the majority of researchers, in the field of soft computing technique, investigate concrete beams only with stirrups (Sanad and Saka 2001, Cladera and Marí 2004b) or without stirrups (Oreta 2004, Cladera and Marí 2004a, Seleemah 2005, 2012, Iruansi *et al*. 2010). By studying both cases the database is more complete and, furthermore, the values of shear strength of the beams with stirrups serve as the upper limit of the shear strength of the beams without stirrups, while the opposite is also true; the shear stress of the beams without stirrups serves as the lower limit of beams with stirrups. Thus, the aforementioned selection assists the neural network to reveal the natural laws by which the phenomenon under examination abides by, as will be illustrated in the following sections.

• It has been decided not to include beams with inclined stirrups in the compiled database, but only beams with vertical stirrups. Despite the fact that shear strength is largely affected by the angle of inclination of web reinforcement (Robinson 1968, Placas and Regan 1971), the number of available experimental data in relevant research dealing with inclined stirrups is too scarce to allow them to be included.

• All the datasets included in the database are referred to reinforced concrete beams with rectangular cross section.

• Only experimental data based on beams under concentrated loads have been used.

• For data which corresponded to the same experiment, with the same mechanical and geometric characteristics, however with different strength values, the average of the measured mechanical strength of the different experiments was used as output value for the respective dataset.

Suitable collection of participation features is vital for precise evaluation of shear capacity of reinforced concrete beams using ANN models. Nine parameters (Table 2) are selected for the training of the neural network based on experience from past experimental studies and which are also generally acknowledged as crucial variables in determining shear capacity of reinforced concrete beams. These relate to the mechanical and geometric characteristics of the beams, while the output parameter is the value of their experimental shear strength. Table 2 shows these parameters and their distributions, i.e., the minimum, maximum, average and standard deviation.

Table 2 Parameters of database

Code	Variable	Unit	Data used in NN models			
			Min	Average	Max	STD
01	Width of beam (b)	mm	80.00	195.12	612.14	97.49
02	Effective depth of beam (d)	mm	132.08	414.64	1097.28	221.57
03	Cylinder compressive strength of concrete (fc)	MPa	12.76	36.29	103.23	19.99
04	Yield stress of longitudinal reinforcement (fy)	MPa	0.00	418.18	910.11	135.54
05	Yield stress of transverse reinforcement (fyw)	MPa	0.00	284.72	1414.17	365.18
06	Shear span /Effective depth of beam (a/d)	-	0.61	2.39	9.05	1.33
07	Longitudinal reinforcement ratio (ρl)	(%)	0.00	2.41	4.54	1.10
08	Transverse reinforcement ratio (ρw)	(%)	0.00	0.28	2.25	0.38
09	Effective span / depth (L/d) of beam	-	1.22	5.86	20.01	3.19
10	Shear strength (V)	MPa	0.59	3.46	12.85	2.27

5.2 Training algorithms

Numerous training back-propagation neural network algorithms have been proposed in the past, such as: quasi-Newton, Resilient, One-step secant, Gradient descent with momentum and adaptive learning rate, and Levenberg-Marquardt. Perhaps, the best prediction algorithm for non-linear behavior of shear capacity of reinforced concrete beams with stirrups appears to be the Levenberg-Marquardt (Lourakis 2005), which seems to be the fastest one for medium-sized neural network training with feed forward (up to several hundreds of weights) as well as for non-linear problems.

5.3 Normalization of data

The normalization of data is a pre-processing phase which has been proved to be a crucial step in the field of soft computing techniques such as the artificial neural networks techniques. In the present study, during the pre-processing stage, the Min-Max (Delen *et al*. 2006) and the ZScore normalization methods are used. In particular, the nine input parameters (Table 2) as well as the single output parameter have been normalized using the Min-Max normalization method. As stated in Iruansi *et al*. 2010, in order to avoid problems associated with low learning rates of the ANN, the normalization of the data should be made within a range defined by appropriate upper and lower limit values of the corresponding parameter. In this work, the input and output parameters have been normalized in the range [0.10, 0.90]. Detailed and in-depth reports on normalization techniques can be found in works by Asteris *et al*. (2019b), Asteris and Nikoo (2019) and Cavaleri *et al*. (2017).

5.4 Lower limit of shear strength

As already mentioned in Step 3 of the proposed heuristic algorithm, which was presented in the previous section for the development and selection of the optimum ANN, it is recommended, when possible, to implement upper and lower limits in order to control the value of the parameter which the ANN model is requested to predict. Specifically, it is proposed to select the optimum neural network, not only through the use and evaluation of statistical indexes, but also based on whether the predicted values fulfill additional criteria, based on conditions, benchmarks and principles relating to the natural phenomenon under investigation.

To this end, an effort is made to formulate such criteria in the current section, relating to the lowest value of shear stress of reinforced concrete beam for specific cases of different beam geometries and location of concentrated load (Fig. 5). In particular, based on the fact that flexural failure occurs for a/d values equal or higher than 5.00, it can be stated that a beam's resistance capability is based solely on the flexural capacity provided by its longitudinal bars.

Based on equilibrium of internal forces (Fig. 5) the following equation applies:

$$M - zF_y = 0 \tag{19}$$

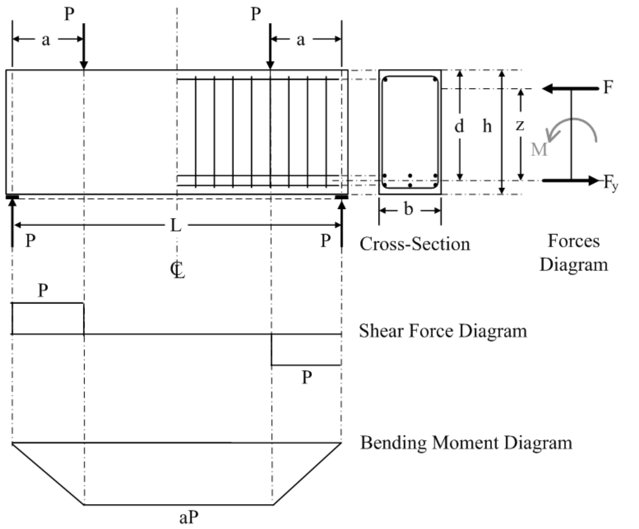

Fig. 5 Simply supported reinforced concrete beam under concentrated loads

where

M is the maximum applied bending moment due to external forces defined as

$$M=aP \tag{20}$$

P is the applied concentrated load

a is the is the distance between the concentrated load and the support (shear span)

z is the lever arm of internal forces, approximately equal to $0.9d$

d is the effective depth of the beam

F_y is the yield force of the reinforcement defined by the equation

$$F_y = f_y A_{sl} \tag{21}$$

f_y is the yield strength of reinforcement

A_{sl} is the tensile reinforcement area defined by

$$A_{sl} = \rho_l bd \tag{22}$$

ρ_l is the ratio of the longitudinal reinforcement

b is the width of the cross section

Based on the above equations the shear strength at failure can be calculated with the following equation in stress terms

$$v = \frac{P}{bd} = 0.90\frac{\rho_l}{a/d} f_y \tag{23}$$

The proposed equation above, consists a criterion (limit) regarding the value of shear stress. This limit will be used during the evaluation and selection of the optimum NN among the trained and developed NN models for the prediction of the value of shear strength of reinforced concrete beams.

5.5 Performance evaluation

For the assessment of the developed NN models the following four performance statistical indices have been used (Johnsson 2018)

- The Pearson's correlation coefficient R

$$R = \sqrt{1 - \frac{\sum_{i=1}^{N}\left(X_{imes} - X_{ipred}\right)^2}{\sum_{i=1}^{N}\left(X_{imes} - \overline{x}\right)^2}} \tag{24}$$

- Root Mean Square Error (*RMSE*)

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(X_{ipred} - X_{imes}\right)^2} \tag{25}$$

- Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{100}{N}\sum_{i=1}^{N}\frac{\left|X_{ipred} - X_{imes}\right|}{X_{imes}} \tag{26}$$

- Variance Account For (*VAF*)

$$VAF = \left[1 - \frac{var\left(X_{imes} - X_{ipred}\right)}{var\left(X_{imes}\right)}\right] \times 100 \tag{27}$$

where the parameters X_{imes}, X_{ipred} and $\overline{x}$ represent measured, predicted and mean values, respectively. N denotes the total number of data. In theory, the lower the *RMSE*, the more accurate is the evaluation. The Pearson's correlation coefficient R measures the variance that is interpreted by the model, which is the reduction of variance when using the model. R values ranges from 0 to 1 while the model has healthy predictive ability when it is near to 1 and is not analyzing whatever when it is near to 0. These performance metrics are a good measure of the overall predictive accuracy.

Furthermore, the following new engineering index, the a20-inex, has been recently proposed for the reliability assessment of the developed ANN models (Apostolopoulou *et al*. 2019, Armaghani *et al*. 2019, Chen *et al*. 2019, Asteris *et al*. 2019c, Asteris *et al*. 2019d, Xu *et al*. 2019)

$$a20 - index = \frac{m20}{M} \tag{28}$$

where M is the number of dataset sample and $m20$ is the number of samples with value of rate Experimental value/Predicted value between 0.80 and 1.20. Note that for a perfect predictive model, the values of a20-index values are expected to be unity. The proposed a20-index has the advantage that their value has a physical engineering meaning. It declares the amount of the samples that satisfies predicted values with a deviation ±20% compared to experimental values.

6. Results and discussion

6.1 Development of artificial neural networks

Based on the above mentioned algorithm as well as on the parameters presented in Table 3, 982800 models of back-propagation neural networks were designed and trained. Namely, NNs architectures for the following four cases have been trained and developed

Case I: 5400 architectures of NN models with one hidden layer and without any preprocess,

Case II: 5400 architectures of NN models with one

Table 3 Training parameters of ANN models

Parameter	Value
Training Algorithm	Levenberg-Marquardt Algorithm
Normalization	Minmax in the range 0.10 - 0.90
Number of Hidden Layers	1; 2
Number of Neurons per Hidden Layer	1 to 30 by step 1
Control random number generation	rand (seed, generator) where generator range from 1 to 10 by step 1
Training Goal	0
Epochs	250
Cost Function	MSE; SSE
Transfer Functions	Tansig (T); Logsig (L); Purelin (P)

Note:
MSE: Mean Square Error; SSE: Sum Square Error
Tansig (T): Hyperbolic Tangent Sigmoid transfer function
Logsig (L): Log-sigmoid transfer function
Purelin (P): Linear transfer function

hidden layer and with preprocess based on minmax normalization technique (the input and output parameters have been normalized in the range [0.10, 0.90]),
Case III: 486000 architectures of NN models with two hidden layers and without any preprocess,
Case IV: 486000 architectures of NN models with two hidden layer and with preprocess based on minmax normalization technique (also the input and output parameters have been normalized in the range [0.10, 0.90]).

Each model was trained through 180 data (out of a total of 300, i.e., 60%) and the reliability of the results was validated via 60 data values (20%) and tested against the remaining 60 data values (20 % of the total).

All the 982800 developed ANN models have been classified based on the *RMSE* index and the optimum

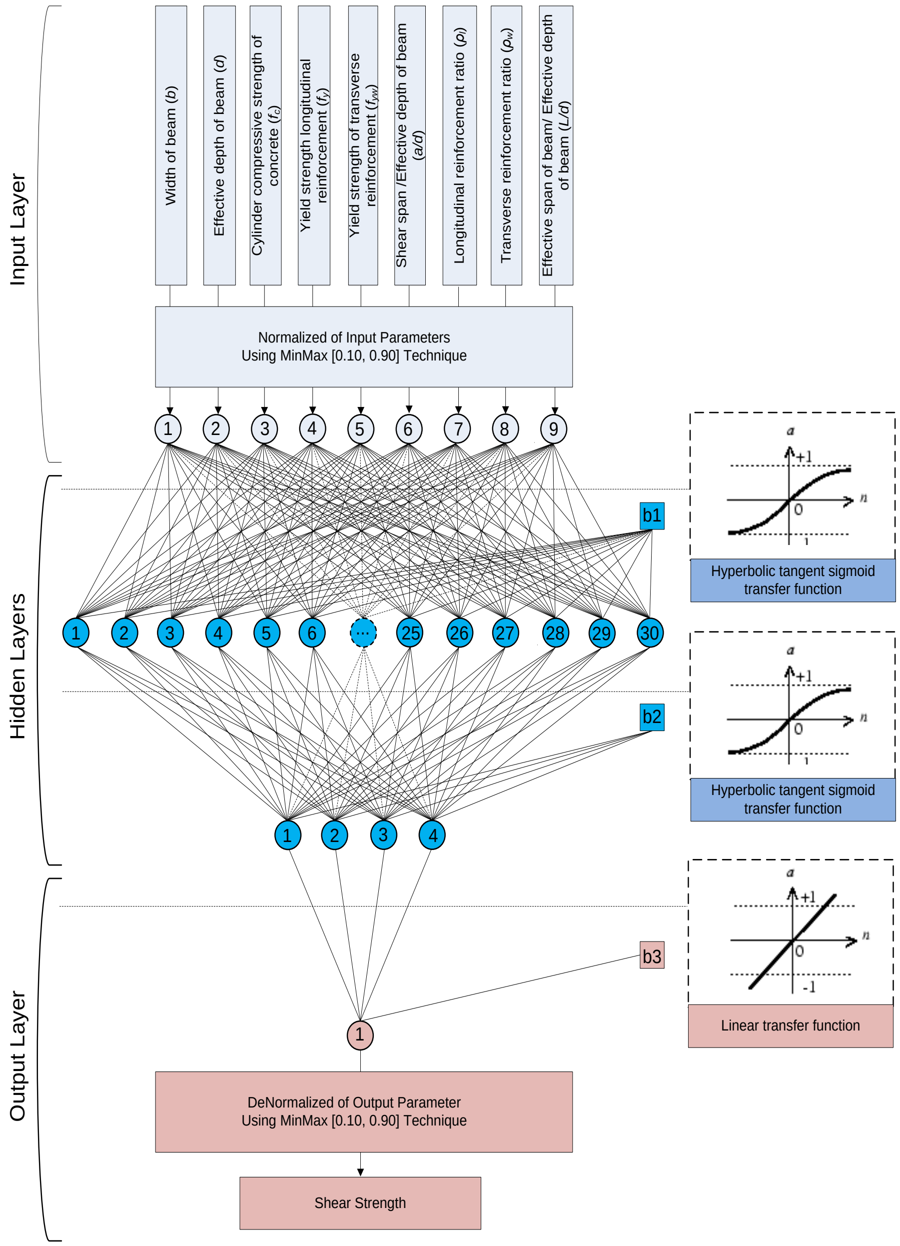

Fig. 6 Architecture of the optimum BPNN 9-30-4-1

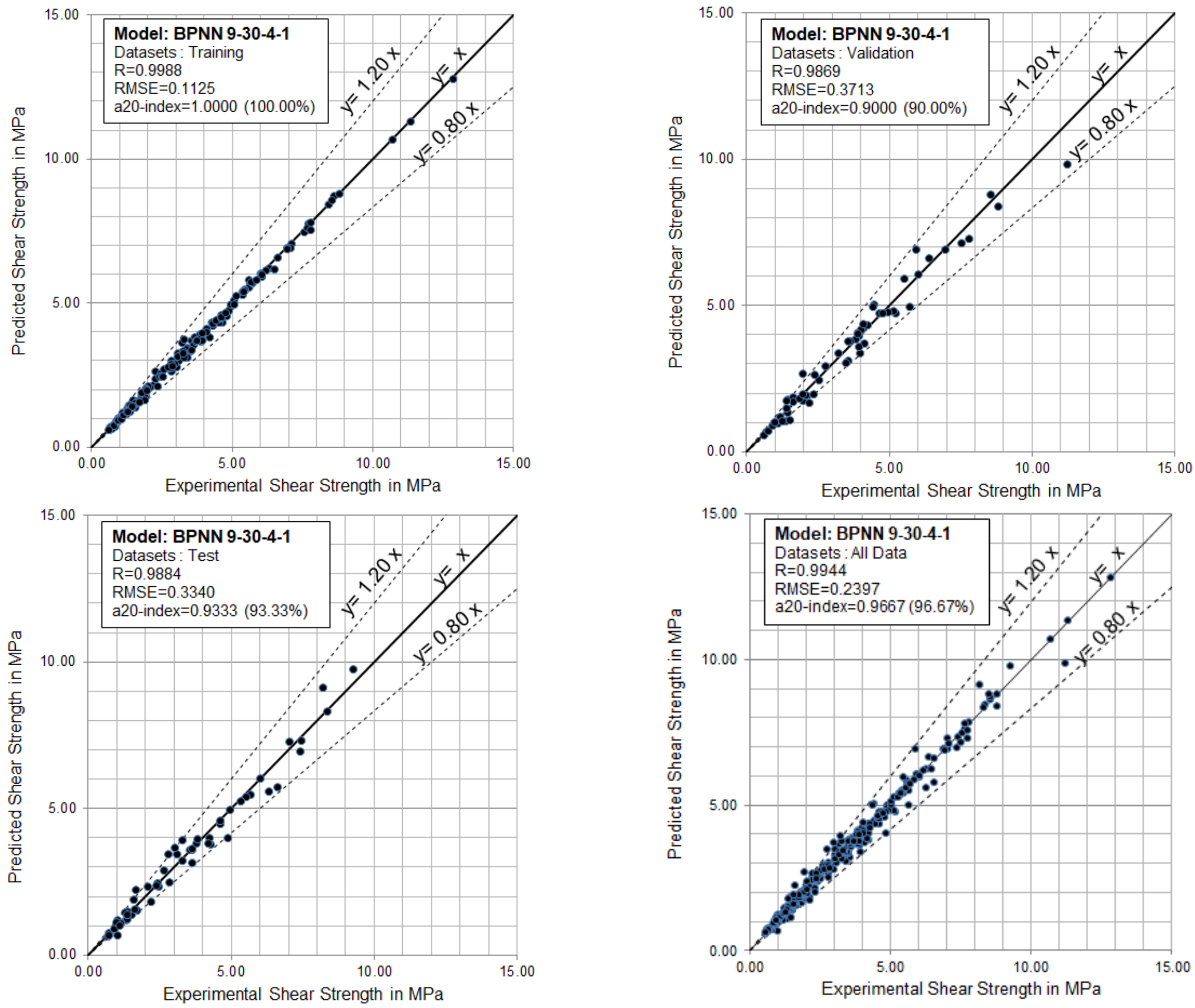

Fig. 7 Comparison of experimental values of shear strength with those predicted by the optimum BPNN 9-30-4-1

Table 4 Statistical Indexes of the optimum BPNN models

Case	Model	Normalization	DataSet	Statistical Indices				
				a20-index	R	RMSE	MAPE	VAF
I	9-23-1	NoPreprocess	Training	1.0000	0.9984	0.1304	0.0375	99.6726
II	9-21-1	MinMax [0.10, 0.90]		0.9722	0.9964	0.1930	0.0589	99.2839
III	9-27-7-1	NoPreprocess		0.9944	0.9985	0.1265	0.0411	99.6974
IV	9-30-4-1	MinMax [0.10, 0.90]		1.0000	0.9988	0.1125	0.0307	99.7566
I	9-23-1	NoPreprocess	Test	0.8500	0.9788	0.4534	0.1109	95.7530
II	9-21-1	MinMax [0.10, 0.90]		0.8167	0.9838	0.3968	0.1106	96.7576
III	9-27-7-1	NoPreprocess		0.9167	0.9849	0.3907	0.0953	96.8833
IV	9-30-4-1	MinMax [0.10, 0.90]		0.9333	0.9884	0.3340	0.0792	97.6924

architecture for each one from the four cases are shown in Table 4. Based on these results, the optimal model of neural networks appears to be the 9-30-4-1 (see also Fig. 6) with a RMSE value equal to 0.3340 for the case of test datasets. This network represents the case of architectural neural networks with two hidden levels and without the use of any normalization technique. Furthermore, as it is presented in Fig. 6 the transfer functions are the Hyperbolic Tangent Sigmoid transfer function (tansig) for the first and second hidden layer and the Linear transfer function (purelin) for the output layer.

Fig. 7 depicts the experimental values of shear strength relative to those predicted by the optimum neural network 9-30-4-1. It is observed that the neural network evaluates adequately the shear capacity of the reinforced concrete beams.

6.2 Neural network results assessment

In order to assess the results of the neural network, a detailed comparison between the shear capacity values predicted by the neural network with those provided by

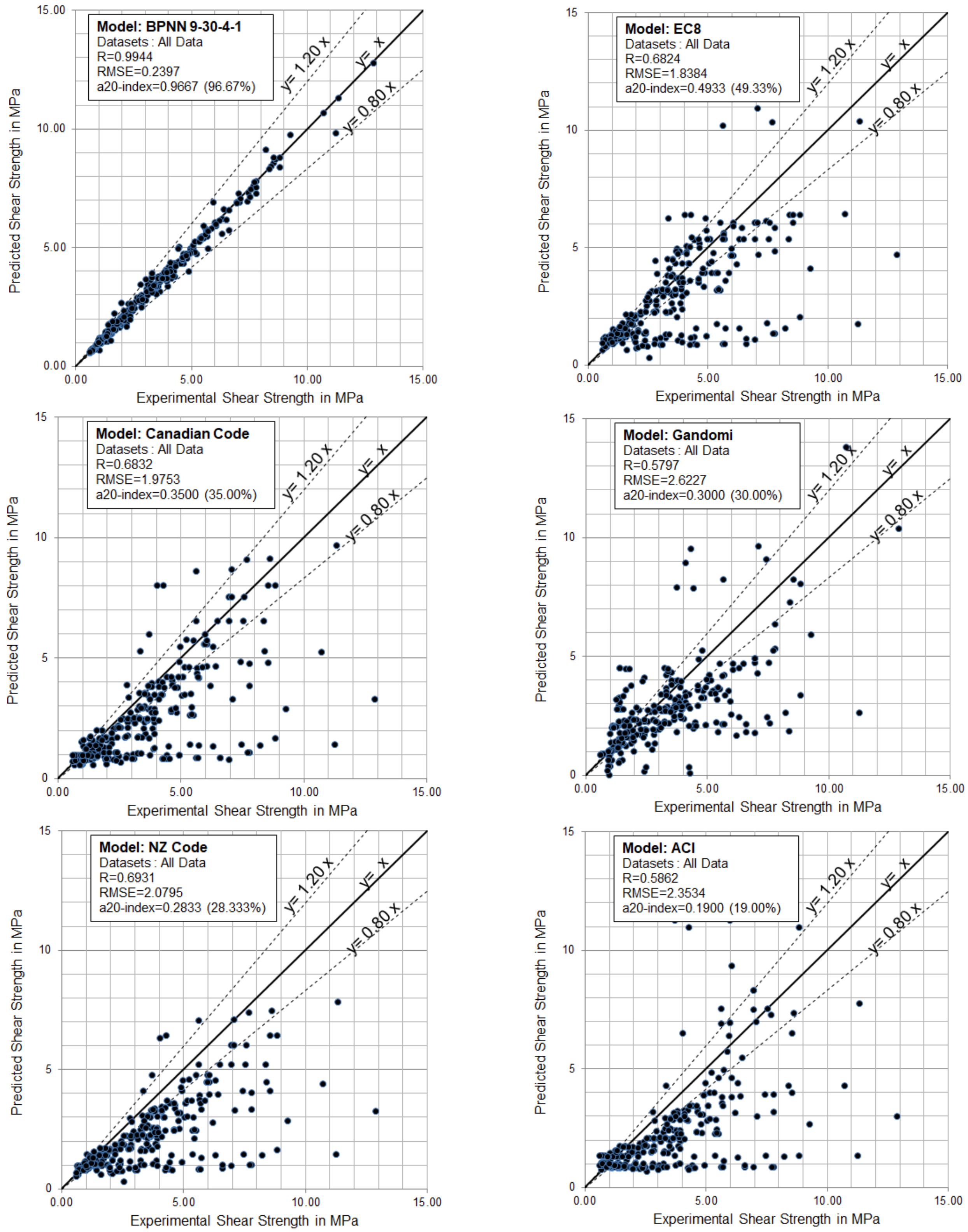

Fig. 8 Comparison of experimental test results of shear capacity with those provided by the Artificial Neural Network and previous studies from the pertinent literature

code regulations and other researchers is shown in Table 5 as well in the following figures, examining the total gamut of 300 experimental data.

Fig. 8 depicts the experimental values of shear capacity predicted by both the ANN method and the pertinent regulations of codes. This figure demonstrates the wide dispersion of the values provided by the regulations as well as the much better evaluation by the proposed neural network approach. Furthermore, the Pearson correction factors - alpha factors - are provided.

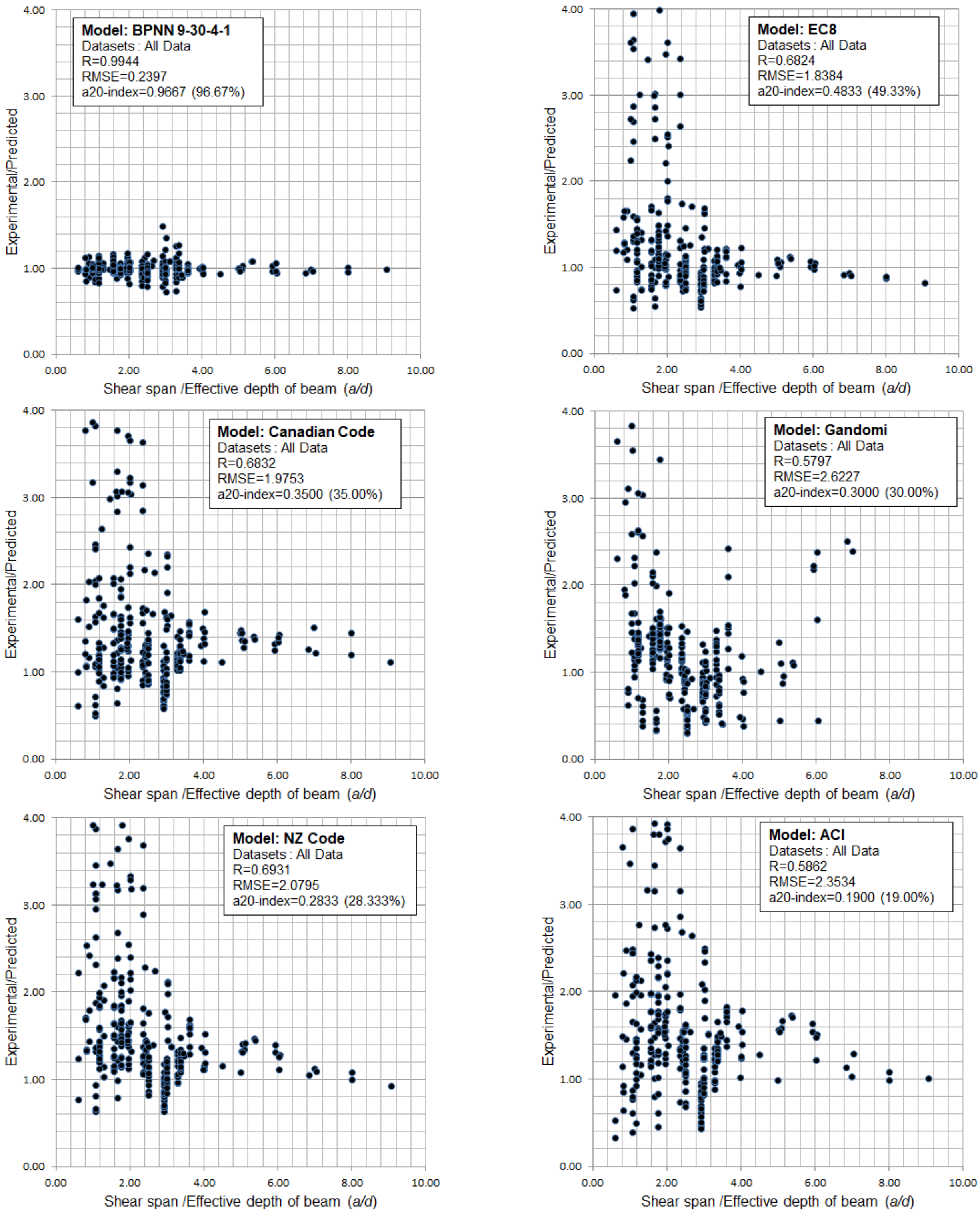

Fig. 9 Variation in the experimental tests values of shear capacity to those predicted by the Artificial Neural Network and by previous research studies, based on the ratio *a/d*

Figs. 9 and 10 show the ratio of the predicted shear capacity versus the shear span/effective depth of beam (*a/d*) and shear reinforcement ratio (*ρw*), respectively. It is obvious that the neural network approach provides much better predictions than the other methods. Their accuracy in descending order is: Proposed Neural Network method, EC2, Canadian Code, Gandomi *et al*. (2017), New Zealand Code and ACI. It is worth noting that EC2 for *a/d* ratios greater than 2.0 provides very good estimated values for the shear capacity.

6.3 Weights for FF-NN model

In most of previous research studies that examined

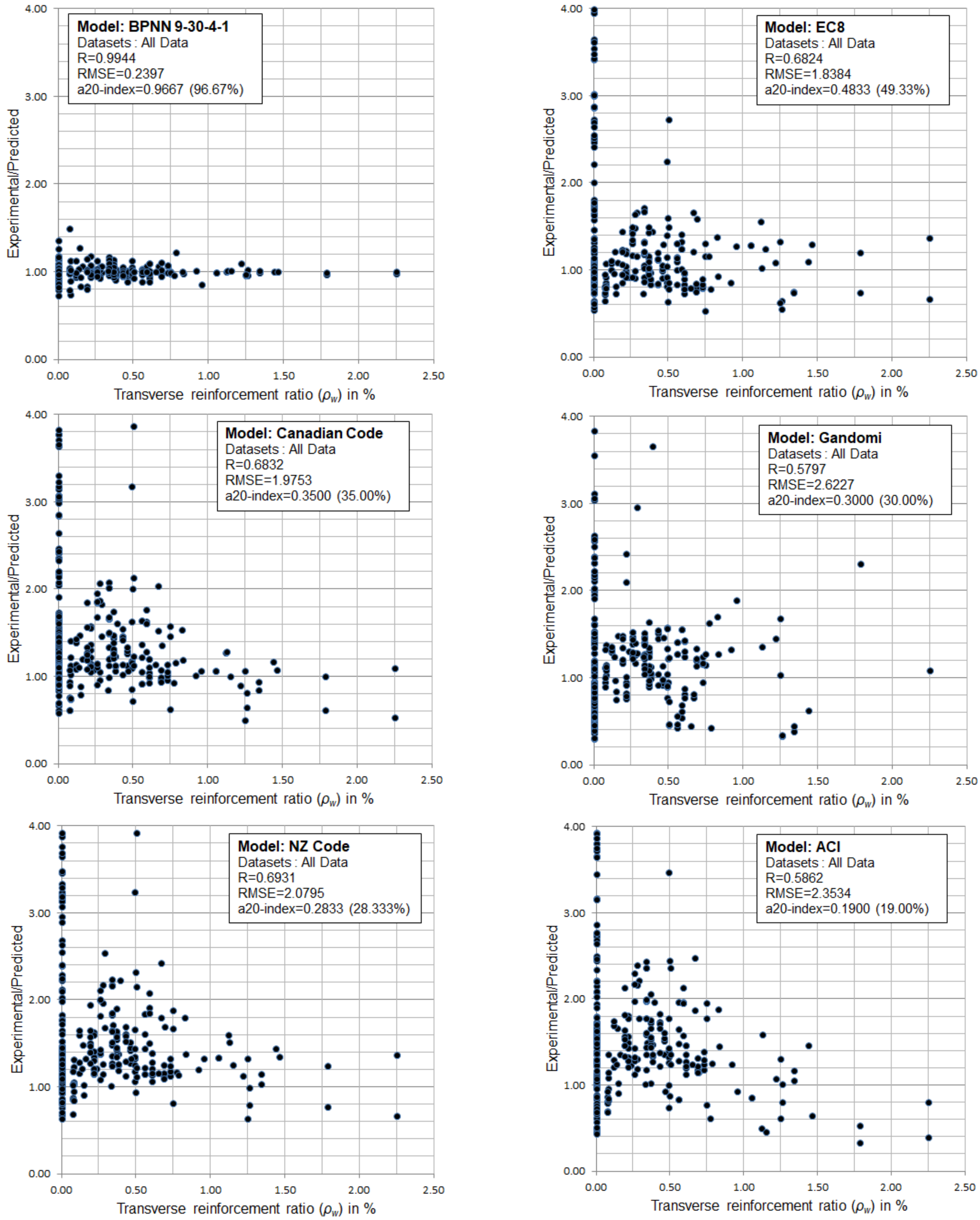

Fig. 10 Variation in the experimental tests values of shear capacity to those predicted by the Artificial Neural Network and by previous research studies, based on the ratio *ρw*

artificial neural networks, information concerning the values of ANN weights is missing while the main topic of the studies has to do with the proposed architecture of the optimum NN model. In such cases, the proposed methods have limited applicability as it is difficult for other researchers and practicing engineers to reproduce the results. To address this issue, the (quantitative) values of weights are given so that the proposed ANN scheme can be readily implemented in a spreadsheet and be accessible to everyone interested in the procedure of simulation. More specifically, in this paper, the final weights' values and biases for the optimal with two hidden layers BPNN 9-30-

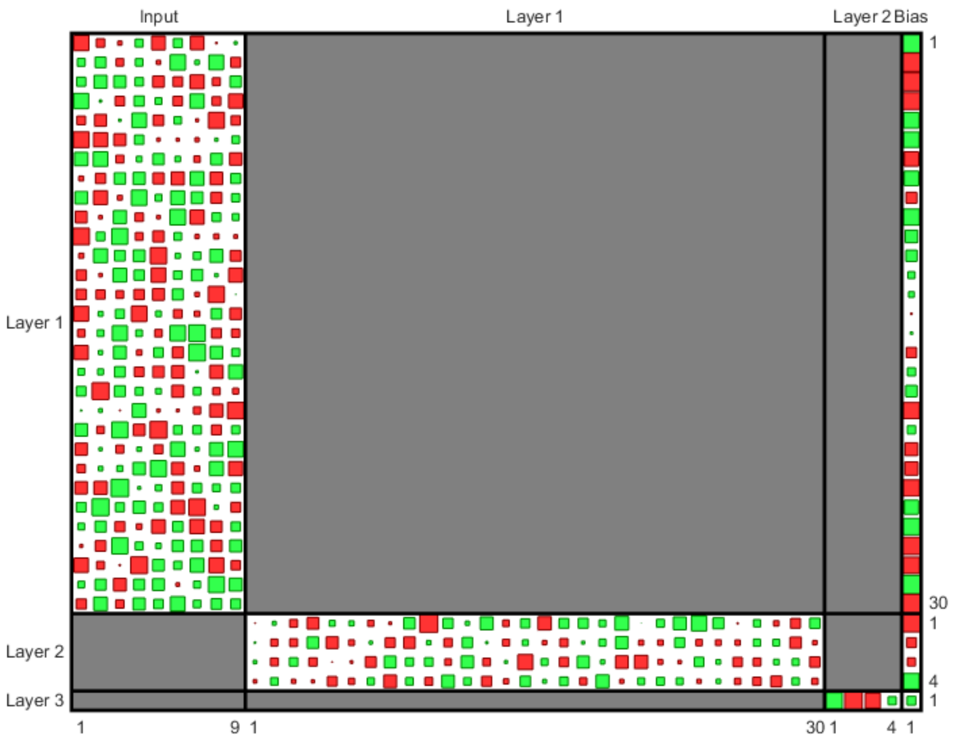

Fig. 11 Weights and Bias values of the optimum with two hidden layers BPNN 9-30-4-1 for the evaluation of shear capacity of concrete beams (The values are presented in Table 6)

Table 5 Statistical Indexes of the optimum BPNN models

Ranking Based on		Model	Statistical Indices				
a20-index	*R*		a20-index	*R*	*RMSE*	*MAPE*	*VAF*
1	1	9-30-4-1	0.9667	0.9944	0.2397	0.0519	98.8798
2	4	EC8	0.4933	0.6824	1.8384	0.6259	42.9313
3	3	Canadian Code	0.3500	0.6832	1.9753	0.8064	43.8300
4	6	Gandomi *et al.* 2017	0.3000	0.5799	2.6227	0.9071	-31.9055
5	2	NZS	0.2833	0.6931	2.0795	0.8612	47.9592
6	5	ACI	0.1900	0.5862	2.3534	0.9600	15.1939

4-1 for the prediction of shear capacity of concrete beams are explicitly reported in Fig. 11 and in the Table 6. Adopting the provided values of weights and biases an ANN-based estimator can be straightforwardly constructed for the shear capacity of concrete beams.

6.4 Sensitivity analysis

Based on the proposed optimum neural network, a sensitivity analysis has been performed in order to further assess its reliability as well as to reveal the dependence of shear strength on the mechanical and geometrical parameters of concrete beams. More specifically, in addition to the aforementioned standard neural network reliability test performed above, and based on the capability for the neural network to predict reliably the shear strength for 60 experimental tests (out of a total of 300 databases), three additional investigations have been conducted.

Investigation 1:

Using the proposed neural network, the behavior of eight beams, which were experimentally tested by Lodhe (2011) are investigated. These experimental results have been used toward the neural network training. The data for these beams, the corresponding experimental values, and the values predicted by the neural network are shown in Table 7 and in Fig. 12. It is evident that the proposed neural network predicts accurately the experimental values (R=0.9993). Moreover, the proposed neural network identifies the engineering laws that apply to intermediate values of the percentage of longitudinal reinforcement, especially for values between ρ1=1.20 and ρ1=1.80%. In addition, the smoothness of the curvatures of the respective curves indicate that the proposed heuristic algorithm successfully addresses the frequently encountered over-fitting problem during the training and development of ANN models.

Investigation 2:

In the second investigation, the behavior of 6 beams that were experimentally tested by Wafa *et al.* (1994) are examined. These experimental results, unlike the first investigation, have not been used in neural network training (i.e., they were not included in the database). The data for these beams, the corresponding experimental results, and the values predicted by the neural network are shown in Table 8 and Fig. 13. The neural network predicts well the experimental values (R=9954). As in the case of the first investigation, the neural network reveals the engineering laws that apply to shear strength. The smoothness of the curvatures is an indication that the proposed heuristic

Table 6 Final values of weights and bias of the optimum NN model 9-30-4-1

IW {1,1} (30×9)									$LW^T\{2,1\}$ (30×4)				B{1,1} (30×1)
-1.27	-0.39	-0.10	0.35	-1.03	0.44	-1.01	0.00	0.05	0.11	0.10	-0.05	0.09	2.04
0.34	0.63	-0.31	0.31	-0.09	1.31	0.12	1.20	-0.52	-0.07	-0.29	-0.20	0.19	-1.90
0.41	0.62	0.77	0.22	-0.77	-0.69	-1.19	-0.39	0.57	-0.28	-0.38	0.27	0.42	-1.76
0.80	0.24	-0.88	0.58	0.87	-0.46	0.80	-0.69	-0.51	0.19	0.13	0.23	0.18	-1.62
-0.49	-0.83	-0.04	1.02	-0.61	0.12	-0.21	-1.16	-0.65	0.35	-0.43	-0.11	-0.43	1.48
-1.36	-1.11	-0.86	0.49	0.03	-0.06	-0.11	0.12	0.29	0.20	0.03	-0.18	-0.34	1.34
0.96	1.10	-0.34	0.21	0.74	0.22	-0.19	0.79	-0.78	-0.25	0.15	-0.44	-0.10	-1.20
-0.06	-0.46	0.72	0.65	-0.70	-0.98	0.93	-0.70	0.46	-0.02	-0.43	0.07	-0.12	1.06
0.90	-0.56	-0.40	0.82	0.48	1.08	0.53	-0.60	0.42	0.16	-0.44	0.32	0.21	-0.92
-0.31	-0.01	1.05	-0.61	-0.28	0.89	-1.02	0.67	-0.49	-0.39	0.30	-0.35	-0.18	0.77
-1.59	0.36	0.87	-0.02	-0.70	0.49	0.01	-0.16	-0.08	-0.32	-0.15	0.19	0.39	0.63
-0.26	0.92	0.10	0.67	-0.85	0.62	0.54	0.93	-0.74	0.25	0.27	0.06	0.02	0.49
-0.59	0.16	1.01	0.50	-0.77	0.29	0.72	0.12	-1.15	0.38	0.33	0.14	-0.43	0.35
-0.75	-0.54	-0.26	-0.80	-0.94	0.48	-0.09	-1.22	-0.04	-0.23	-0.18	0.17	0.25	0.21
-0.93	-0.39	0.85	-0.96	0.71	0.14	-0.88	-0.05	-0.46	0.43	-0.20	-0.27	0.40	0.07
-0.25	0.33	0.32	0.30	-0.62	1.04	0.95	-1.12	-0.41	0.28	0.04	0.03	-0.05	-0.07
-0.89	0.22	0.93	-0.21	0.35	-0.71	1.03	0.28	0.81	0.00	-0.31	-0.47	0.40	-0.21
0.52	-0.13	0.53	-0.74	-0.83	-0.97	0.30	-0.84	0.79	0.35	-0.31	-0.02	0.22	0.35
0.81	-1.17	0.82	0.85	0.23	-0.68	0.37	-0.24	-0.20	-0.28	0.31	0.31	0.06	0.49
-0.27	-0.05	0.01	1.09	-0.20	-0.48	-0.80	-0.94	-1.06	0.37	0.31	-0.39	0.31	-0.63
0.57	0.01	0.82	-1.02	-1.02	0.54	0.77	-0.40	0.20	0.11	-0.14	-0.35	0.27	0.77
-0.88	0.04	-0.40	0.08	-0.26	0.85	0.21	1.14	1.05	0.35	-0.14	-0.02	0.30	-0.92
-0.20	0.17	0.13	0.89	1.00	-0.55	-0.52	0.87	-0.98	0.22	0.26	0.12	0.06	-1.06
-0.77	-0.75	1.05	-0.30	0.52	-0.57	0.28	1.05	0.20	0.19	-0.29	-0.02	0.44	-1.20
0.59	1.03	0.43	0.35	0.85	-0.60	-0.78	0.06	-0.87	0.38	-0.21	-0.36	0.28	1.34
0.29	0.77	-0.49	0.02	-0.84	0.67	-1.08	-0.64	0.72	-0.01	0.08	-0.36	-0.37	1.48
-0.11	-0.75	1.11	0.41	0.10	0.71	0.66	-0.62	0.93	-0.25	0.36	-0.45	-0.13	-1.62
-1.03	-0.33	0.25	-0.56	0.58	0.52	1.01	-1.00	-0.13	0.20	0.35	0.25	-0.06	-1.76
0.50	1.08	-0.86	0.33	0.79	-0.51	-0.38	0.83	0.44	-0.02	-0.21	0.13	0.18	1.90
-0.50	0.90	-0.34	0.81	0.60	1.08	0.32	0.71	0.47	0.43	-0.13	-0.44	-0.04	-2.04
LW{3,2} (1×4)									$B^T\{2,1\}$ (1×4)				B{3,1} (1×1)
0.82	-0.57	-0.73	0.47						-1.47	-0.49	-0.49	1.47	-0.63

Note
IW{1,1}=Weights values for Input Layer
LW{2,1}=Weights values for 1st hidden Layer , LW{3,2}=Weights values for 2nd hidden Layer
B{1,1}= Bias values for 1st hidden layer, B{2,1}= Bias values for 2nd hidden layer, B{3,1}= Bias values for output layer
T denotes the transpose of a matrix.

algorithm successfully addresses the frequently encountered over-fitting problem during the training and development of ANN models.

Investigation 3:

In this investigation, unlike the two previous investigations, the behavior of a number of beams has been studied; for these beams, the corresponding experimental results are not available, but they are estimated by the proposed neural network to investigate the *a/d* ratio for values from 1 to 7 and for 5 different concrete compressive strengths (20, 30, 40 50 and 60 MPa).

The geometrical and mechanical characteristics of the beams are shown in Table 9. These values correspond to 330 different cases of beams for which the proposed neural network 9-30-4-1 predicts the corresponding values of shear strength as shown in Fig. 14. The results confirm the fact that the shear strength changes with respect to both the *a/d* parameter and the compressive strength of the concrete. It should be mentioned that this diagram is a first indication of the fact that the shear strength varies with respect to two of the most important parameters involved in the problem. It is worth noticing that the number of experimental data with *a/d* ratio values greater than 4.00 is relatively small and a larger number of experimental data is required. Toward this direction, an extensive database of experimental data has already begun to be developed for the training and development of a new neural network that will cover a larger range of data.

7. Limitations

The proposed neural network is applied in the ten-dimensional space defined by the ten parameters which influence the development of shear strength of reinforced concrete beams with or without stirrups. In fact, the neural network is applicable for parameter values ranging between the lowest and highest values of each parameter, as presented in Table 2. Taking into account that included

Table 7 Experimental datasets by Lodhe (2011)

No	Input Parameters									Output Parameter		Vexp / Vpred	R
	b	d	f_c	f_y	f_{yw}	a/d	ρ_l	ρ_w	L/d	Vexp	Vpred		
	(mm)	(mm)	(MPa)	(MPa)	(MPa)	-	(%)	(%)	-	(MPa)	(MPa)	-	-
(1)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(15)	(16)	(17)	(18)
1	100	375.00	32.14	445.0	445.4	1.07	0.60	0.50	2.67	2.80	2.75	1.02	
2	100	375.00	32.19	445.0	445.4	1.07	0.60	0.75	2.67	3.33	3.40	0.98	
3	100	375.00	32.20	445.0	445.4	1.07	0.60	1.25	2.67	4.00	4.18	0.96	
4	100	375.00	32.16	445.0	445.4	1.07	0.60	2.25	2.67	4.27	4.34	0.98	0.9993
5	100	375.00	32.14	446.0	445.3	1.07	2.40	0.50	2.67	7.79	7.76	1.00	
6	100	375.00	32.19	446.0	445.3	1.07	2.40	0.75	2.67	8.40	8.37	1.00	
7	100	375.00	32.20	446.0	445.3	1.07	2.40	1.25	2.67	8.53	8.74	0.98	
8	100	375.00	32.16	446.0	445.3	1.07	2.40	2.25	2.67	8.80	8.79	1.00	

Table 8 Experimental datasets by Wafa *et al*. (1994)

No	Input Parameters									Output Parameter		Vexp / Vpred	R
	b	d	f_c	f_y	f_{yw}	a/d	ρ_l	ρ_w	L/d	Vexp	Vpred		
	(mm)	(mm)	(MPa)	(MPa)	(MPa)	-	(%)	(%)	-	(MPa)	(MPa)	-	-
(1)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(15)	(16)	(17)	(18)
1	125	215	94.8	460	0	1.00	2.84	0.00	4.33	7.10	9.50	0.75	
2	125	215	94.9	460	0	2.00	2.84	0.00	6.33	3.35	3.63	0.92	
3	125	215	93.7	460	0	2.50	2.84	0.00	7.33	2.07	2.47	0.84	0.9954
4	125	215	91.5	460	0	3.00	2.84	0.00	8.33	2.05	1.90	1.08	
5	125	215	92.3	460	0	4.00	2.84	0.00	10.33	1.90	1.48	1.28	
6	125	215	92.0	460	0	6.00	2.84	0.00	14.33	1.49	1.43	1.05	

Table 9 Parameters of beams

No.	Variable	Valuet
1	Width of beam (b)	200 mm
2	Effective depth of beam (d)	400 mm
3	Cylinder compressive strength of concrete (f_c)	20 to 60 MPa by step 10 MPa
4	Yield stress of longitudinal reinforcement (f_y)	500 MPa
5	Yield stress of transverse reinforcement (f_{yw})	500 MPa
6	Shear span /Effective depth of beam (a/d)	0.5 to 7.00 by step 0.10
7	Longitudinal reinforcement ratio (ρ_l)	3.00 (%)
8	Transverse reinforcement ratio (ρ_w)	0.50 (%)
9	Effective span / depth (L/d) of beam	8.00

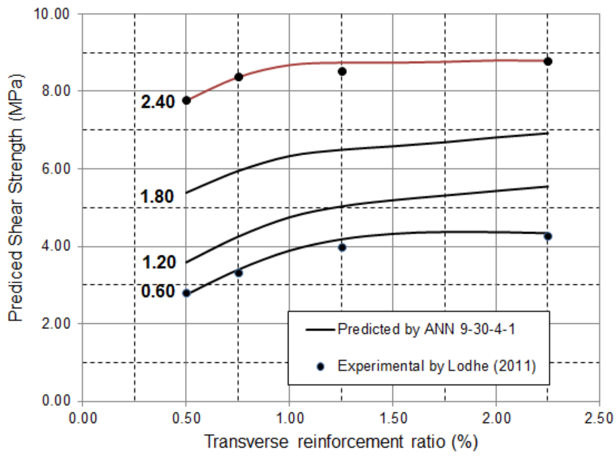

Fig. 12 Sensitivity of the shear strength to the transverse reinforcement ratio of beams based on the optimum BPNN 9-30-4-1 as well as comparisons with experimental data by Lodhe (2011)

experimental datasets for *a/d* ratios over 4.00 are sparse, it is proposed to limit the use of the NN model only for a/d ratios between 0.61 and 4.00.

8. Conclusions

In this paper, Artificial Neural Networks are developed to evaluate the shear capacity of reinforced concrete beams with shear reinforcement - stirrups. More specifically, a new algorithm is proposed for finding the optimal neural network architecture. The main conclusions of this study are the following:

- Artificial Neural Networks can be used to evaluate the shear capacity of reinforced concrete beams with the minimum computational effort, in comparison with other computational/numerical methods
- the proposed heuristic algorithm contributes to finding the optimal architecture of Artificial Neural Networks
- the proposed method gives the most reliable estimation for the shear capacity of reinforced concrete beams in the dataset, in comparison with available semi-empirical and/or analytical relations proposed others or used in international standards.
- Using the architecture of the proposed optimum neural network and the resulting values of final weights of the parameters (see supplementary materials) a useful tool is developed for researchers, engineers, and for supporting

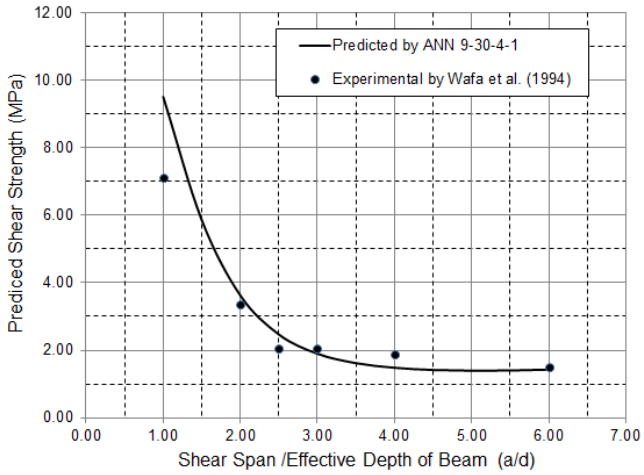

Fig. 13 Sensitivity of the shear strength to the effective depth of beam based on the optimum BPNN 9-30-4-1 as well as comparisons with experimental data by Wafa *et al.* (1994)

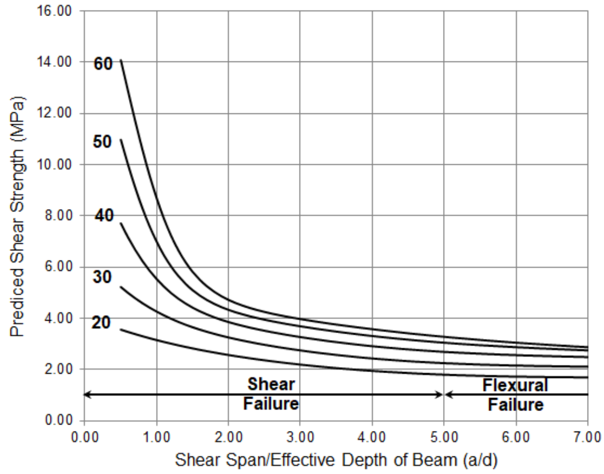

Fig. 14 Sensitivity of the shear strength to the shear span to effective depth of beam ratio as well to the compressive strength of concrete (20:60:10) based on the optimum BPNN 9-30-4-1

the teaching and interpretation of the behavior of reinforced concrete beams under shear loading.

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CC

Notations

The following symbols and abbreviations are used in this paper:

Latin letters

a	Shear span
ANNs	Artificial Neural Networks
A_c	area of concrete
A_{sl}	is the area of the longitudinal reinforcement
A_{sw}	is the area of the shear reinforcement
b	width of beam
b_w	the smallest width of the cross-section in the tensile area
B	vector of bias values
BP	Back Propagation
d	effective depth of beam
f_c	cylinder compressive strength of concrete
f_y	yield stress of longitudinal reinforcement
f_{yw}	yield stress of transverse reinforcement
F_c	concrete compressive force
F_y	longitudinal reinforcement tensile force
h	height of beam
IW	matrix of weights values for Input Layer
L	effective span of beam
LW	matrix of weights values for hidden Layer
M	bending moment
NZS	New Zealand Standards
s	is the spacing of the stirrups
V	shear force at section
$v = \frac{V}{bd}$	shear stress at section
V_c	the concrete shear contribution
V_s	the transverse reinforcement shear contribution
z	is the inner lever arm

Hellenic letters

ρ_l	longitudinal reinforcement ratio
ρ_w	transverse reinforcement ratio
$\tau = \frac{V}{bdf_c}$	dimensionless shear stress at section