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Static analysis of laminated reinforced composite plates using a simple first-order shear deformation theory

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Abstract. This paper aims to present an analytical model to predict the static analysis of laminated reinforced composite plates subjected to sinusoidal and uniform loads by using a simple first-order shear deformation theory (SFSDT). The most important aspect of the present theory is that unlike the conventional FSDT, the proposed model contains only four unknown variables. This is due to the fact that the inplane displacement field is selected according to an undetermined integral component in order to reduce the number of unknowns. The governing differential equations are derived by employing the static version of principle of virtual work and solved by applying Navier's solution procedure. The non-dimensional displacements and stresses of simply supported antisymmetric cross-ply and angle-ply laminated plates are presented and compared with the exact 3D solutions and those computed using other plate theories to demonstrate the accuracy and efficiency of the present theory. It is found from these comparisons that the numerical results provided by the present model are in close agreement with those obtained by using the conventional FSDT.

Keywords: static bending; simple FSDT; displacement field; cross-ply; angle-ply laminated plates

1. Introduction

Over the last several years, the composite materials represent a highly strategic research focus for many researchers that have become increasing important in different fields of engineering, including aeronautics, automotive components, submarine structures, civil and mechanical engineering structures, but also in the medical prosthetic devices and electronic circuit boards and other applications. This development resulted from many scientific programs with modern computational techniques, funded by the most important research laboratories in the world. These anisotropic materials are very much preferred, since they combine the best mechanical properties of various materials resulting from the most up-to-date technologies. It also offers a number of key advantages over conventional isotropic materials, such as *very high strength and stiffness* coupled with a *very low density*, resistance to chemicals, thermal and electrical insulation properties, making composites more attractive. Thus, the rapid growth in the use of composite materials in the designing of structures and industrial processes has required the development of structure mechanics to model and analyze more accurately the static and dynamic behaviours of structural components made from composite materials, such as laminates or sandwichs beams, plates and shells (Rezaiee-Pajand *et al.* 2012, Sayyad and Ghugal 2014, Behera and Kumari 2018, Narwariya *et al.* 2018).

In a general sense, a composite material at a macroscopic level is formed by the combination of two or more constituent materials with significantly different physical or chemical properties, without dissolving or blending them into each other "*but having a high adhesion capacity*", which blend to obtain a new material system with properties superior to those of the individual monolithic constituents, such as steel, aluminum and other types of metal. Due to its complexity" *i.e., their properties are not the same in all directions*"and the large-scale applications, several researchers of composite materials systems have been involved to the development of many plate theories and solution procedures based on considering the transverse shear deformation effect, and each of them, in their own way, contributes to predict correctly the bending, buckling and vibration behaviours of sandwichs and laminated composite plates.

The oldest and simplified theory, commonly called *classical plate theory* "CPT" is originally developed for homogeneous isotropic structures, based on the famous Kirchhoff hypotheses (Kirchhoff 1850), in which the

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normal to the mid-plane of the plate before deformation remains straight and normal to the deformed geometric mid-plane after loading. This means that transverse shear and transverse normal strains are ignored with respect to the other strains. Several studies have been carried out on anisotropic multilayered structures by extension of the CPT to the *classical laminated plate theory* "CLPT" (see, for example, Jones 1975, Christensen 1979, Whitney 1987, Noor and Burton 1989 and Reddy 2004). Therefore, it has been confirmed in the previous studies that this theory is not accurate for the mechanical response of moderately thick and thick plates, but gives acceptable results only for thin plates. Since the transverse shear deformation effects are more significant in thick plates, Reissner (1945) and Mindlin (1951) have been developed the conventional *first-order shear deformation theory* "FSDT" for homogeneous and isotropic plates to overcome the shortcoming regarding using CPT. According to this theory, the transverse shear strain is assumed to be constant in thickness coordinate and it needs a shear correction factor to correct for unrealistic variation of the shear stress across the thickness. The Reissner-Mindlin's theory has been extended to the case of laminated and functionally graded composite plates by several authors. Yang *et al*. (1966) investigated in detail the elastic wave propagation in heterogeneous laminates plates consisting of an arbitrary number of bonded anisotropic layers in a manner suggested by Mindlin (1951) for the homogeneous plates, wherein the effects of both transverse shear stresses and rotary inertia are taken into account. A similar method was used later by Whitney and Pagano (1970) for static bending and free vibration analysis of symmetric and antisymmetric laminated composite plates in which various values of shear correction factors have been used to compare the obtained results with the corresponding exact solutions. Reddy and Chao (1980) developed finite element model based on FSDT for the analysis of single-layer and two-layer cross-ply, simply supported rectangular compositeplates subjected to sinusoidal and uniform distributed normal pressure. Fares *et al*. (2000) have presented a refined nonlinear first-order thermal plate theory using a modified version of Reissner's mixed variational formula, including thermoelastic effects for the bending analysis of cross-ply laminated plates. A refined FSDT models, based on new mixed variational formulations are developed by Auricchio and Sacco (2003) for the bending analysis of composite laminates, introducing suitable representation forms of the shear stresses in the plate thickness. This approach does not require shear correction factors as well as the out-of-plane shear stresses can be derived without post-processing procedures. Akavci *et al*. (2007) presented the analytical solutions for bending analysis of symmetric cross-ply rectangular thick laminated plates resting on elastic foundation by using first-order shear deformation theory. To verify the computer program of this study, the obtained results have been compared with those obtained from the finite element method and a good agreement has been found. A generalized differential quadrature method has been proposed by Tornabene and Viola (2009) using the FSDT to study the free vibration analysis of functionally graded thick shells and panels of revolution. The numerical results for bending and free vibration analysis of functionally graded square plates were obtained by Thai and Choi (2013) using a simple FSDT and the Hamilton's principle. Sadoune *et al.* (2014) studied the bending and free vibration responses of simply supported laminated composite plates by using a new simple FSDT with only four unknown displacement functions and four governing differential equations. A simple and accurate FSDT which eliminates the use of a shear correction factor was proposed by Thai *et al*. (2014) for bending, buckling and free vibration analysis of simply supported functionally graded sandwich plates composed of FG face sheets and an isotropic homogeneous core "as ceramic". Heydari *et al.* (2014) presented an exact solution for transverse bending analysis of embedded laminated Mindlin plate. Avcar (2015) used FSDT to study effects of rotary inertia shear deformation and non-homogeneity on frequencies of beam. Recently, Mantari and Granados (2015) used an original FSDT with four unknowns for the free vibration analysis of functionally graded sandwich and single plates, in which the material properties of the plates are adopted to vary gradually in the thickness direction according to a power law distribution or Mori-Tanaka homogenization method in terms of the volume fractions of the components.

In order to avoid the use of shear correction factors and satisfying the zero shear strain boundary conditions on the top and bottom surfaces of the plate, many studies have been carried out using *higher-order shear deformation theories* "HSDTs" for static, buckling and free vibration analysis of structures (e.g., Pandit *et al.* 2010, Xiang *et al.* 2011, Mantari *et al.* 2012, Grover *et al.* 2013, Swaminathan and Fernandes 2013, Zenkour2014, Sayyad and Ghugal 2014, Sayyad *et al.* 2016, Draiche *et al.* 2016, Sarangan and Singh 2016, Sahoo *et al.* 2016, Chikh *et al.* 2017, Swain *et al.* 2017, Zenkour and Radwan 2018, Bourada *et al.* 2019, Chaabane *et al.* 2019).

In the present paper, a novel FSDT recently developed by Mantari and Granados (2015) for dynamic analysis of functionally graded plates is used to predict the bending response of simply supported laminated composite plates subjected to sinusoidal and uniform loads. The displacement field of the proposed analytical model involves an undetermined integral component in order to reduce the number of unknowns and governing equations, which is greater in the conventional FSDT. This approach has some benefits due to its simplicity and low computational cost. The governing equations associated with the present theory are obtained using the principle of virtual work. Navier-type analytical procedure is obtained for antisymmetric cross-ply and angle-ply laminated composite plates. Numerical results are presented and compared with the analytical solution of conventional FSDT and those computed using other plate theories. Therefore, the capacity of the current first-order shear deformation theory is validated.

2. Theoretical formulation of the present theory

2.1 Plate under consideration

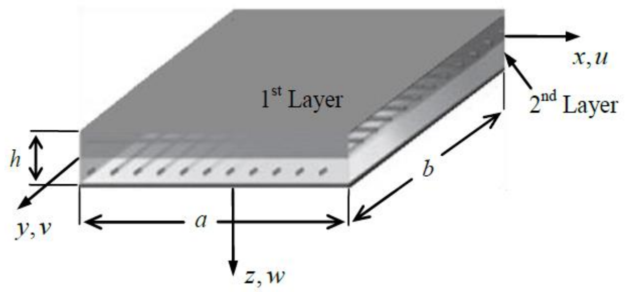

Fig. 1 Coordinate system and geometry of laminated composite plate

In this investigation, a simply supported laminated composite plate made of orthotropic fibrous composite material is considered as shown in Fig. 1. The constant thickness of the plate is denoted by h while its lateral dimensions, along the x and y directions, are denoted by a and$b$, respectively. The plate occupies the region $0 \le x \le a$, $0 \le y \le b$, $-h/2 \le z \le h/2$ in Cartesian coordinate system. The downward z-direction is assumed as positive. Let the plate be subjected to a mechanical load $q(x, y)$ acting normally at the upper surface ($z=-h/2$).

2.2 Displacement field and constitutive relations

It is evident from the literature that the conventional *first-order shear deformation theory* (FSDT) was developed by Whitney and Pagano (1970) for the bending and free vibration analysis of anisotropic laminated plates, in which the effect of the shear deformation and rotary inertia in the same manner as Mindlin's theory for isotropic homogeneous plates. The displacement field of the conventional FSDT is defined by

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\varphi_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\varphi_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

Where $u_0(x,y)$, $v_0(x,y)$ and $zv_0(x,y)$ denote the unknown displacement functions of the middle surface of the anisotropic plate, $\varphi_x(x,y)$ and $\varphi_y(x,y)$ represents the rotations about the y and x axes, respectively. In this study, in order to reduce the number of unknown variables, the previous displacement field is modified by introducing some simplifying suppositions and can be rewritten only with four unknowns in a simpler form as follows

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - zk_1\int \theta(x,y)dx \\ v(x,y,z) &= v_0(x,y) - zk_2\int \theta(x,y)dy \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

where $\theta(x,y)$ is the last unknowns displacement function whereas the constants k_1 and k_2 depends on the geometry. The normal and shear strains associated with the displacement field are obtained using the relationships of the linear theory of elasticity

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial v}{\partial z} + \dfrac{\partial w}{\partial y} \\ \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x} \end{Bmatrix} = \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{3}$$

so that virtual strains are known in terms of the virtual displacements

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_x^1 \\ \varepsilon_y^1 \\ \gamma_{xy}^1 \end{Bmatrix} = \begin{Bmatrix} -k_1\theta \\ -k_2\theta \\ -k_1\dfrac{\partial}{\partial y}\int \theta dx - k_2\dfrac{\partial}{\partial x}\int \theta dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_0}{\partial y} - k_2\int \theta dy \\ \dfrac{\partial w_0}{\partial x} - k_1\int \theta dx \end{Bmatrix} \tag{4}$$

The integrals adopted in the previous relations shall be resolved by a Navier solution and can be determined by

$$\frac{\partial}{\partial y}\int \theta dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \quad \frac{\partial}{\partial x}\int \theta dy = B'\frac{\partial^2\theta}{\partial x\partial y}, \quad \int \theta dx = A'\frac{\partial\theta}{\partial x}, \quad \int \theta dy = B'\frac{\partial\theta}{\partial y} \tag{5}$$

where A' and B' are defined according to the type of solution employed, in this case via Navier. Thus, the parameters A' and B' are expressed by

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{6}$$

where the parameters α and β are defined as

$$\alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b} \tag{7}$$

Each layer in the laminate is assumed to be in a two-dimensional stress state so that thelinear constitutive relations for the k^{th} orthotropic layer are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^k, \quad \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^k = \begin{bmatrix} \overline{Q}_{55} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{44} \end{bmatrix}^k \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^k \tag{8}$$

Where $\{\sigma\}^{\boldsymbol{k}}$ and $\{\varepsilon\}^{\boldsymbol{k}}$ are the stress and strain vectors, respectively. Whereas $\overline{Q}_{ij}$ are called the transformed reduced stiffnesses and they are defined by

$$\begin{aligned}
\overline{Q}_{11}^k &= Q_{11}\cos^4\theta_k + 2(Q_{12} + 2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\sin^4\theta_k \\
\overline{Q}_{12}^k &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{12}(\sin^4\theta_k + \cos^4\theta_k) \\
\overline{Q}_{16}^k &= (Q_{11} - Q_{12} - 2Q_{66})\sin\theta_k\cos^3\theta_k + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta_k\cos\theta_k \\
\overline{Q}_{22}^k &= Q_{11}\sin^4\theta_k + 2(Q_{12} + 2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\cos^4\theta_k \\
\overline{Q}_{26}^k &= (Q_{11} - Q_{12} - 2Q_{66})\sin\theta_k\cos^3\theta_k + (Q_{12} - Q_{22} + 2Q_{66})\cos\theta_k + \sin^3\theta_k \\
\overline{Q}_{66}^k &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{66}(\sin^4\theta_k + \cos^4\theta_k) \\
\overline{Q}_{44}^k &= Q_{44}\cos^2\theta_k + Q_{55}\sin^2\theta_k \\
\overline{Q}_{45}^k &= (Q_{55} - Q_{44})\cos\theta_k\sin\theta_k \\
\overline{Q}_{55}^k &= Q_{55}\cos^2\theta_k + Q_{44}\sin^2\theta_k
\end{aligned} \tag{9}$$

where θ_k denotes the orientation of the k^{th} layer with respect to the global coordinate system and Q_{ij} are the reduced stiffness coefficients, which are related to the engineering constants as follows

$$Q_{11}=\frac{E_1}{1-\nu_{12}\nu_{21}},\ Q_{22}=\frac{E_2}{1-\nu_{12}\nu_{21}},\ Q_{12}=\frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},\ Q_{66}=G_{12},\ Q_{44}=G_{23},\ Q_{55}=G_{13} \quad (10)$$

where E_i, G_{ij} and ν_{ij} are the Young's modulus, shear modulus and Poisson's ratio, respectively.

2.3 Governing equations

In the proposed model of this theory, the governing equations and associated boundary conditions are derived using static version of principle of virtual work. This principle is applied in the following analytical form

$$\int_{-h/2}^{h/2}\int_A \left(\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\right)dAdz-\int_A q(x,y)\delta w dA=0 \quad (11)$$

where δ denotes the variational operator, A is the top surface of the plate and $q(x,y)$ is the transverse load. By substituting the expressions for virtual strains given in Eq. (3) into Eq. (11), the principle of virtual work can be rewritten as (Zarga *et al.* 2019)

$$\begin{aligned}\int_A\Big[&N_x\delta\varepsilon_x^0+N_y\delta\varepsilon_y^0+N_{xy}\delta\gamma_{xy}^0+M_x\delta\varepsilon_x^1+M_y\delta\varepsilon_y^1+M_{xy}\delta\gamma_{xy}^1\\&+S_{yz}\delta\gamma_{yz}^0+S_{xz}\delta\gamma_{xz}^0-q\delta w_0\Big]dA=0\end{aligned} \quad (12)$$

where N, M and S are the stress resultants acting on the cross section of the laminate, defined as follows

$$\begin{Bmatrix}N_x, & N_y, & N_{xy}\\ M_x, & M_y, & M_{xy}\end{Bmatrix}=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}(\sigma_x,\sigma_y,\tau_{xy})\begin{Bmatrix}1\\z\end{Bmatrix}dz,\ \ (S_{xz},S_{yz})=\sum_{k=1}^{N}K\int_{h_k}^{h_{k+1}}(\tau_{xz},\tau_{yz})dz \quad (13)$$

where K denotes the shear correction factor. By substituting the constitutive relations of Eq. (8) into the Eq. (13), the stress resultants are obtained in terms of strains as following form

$$\begin{aligned}
N_x&=A_{11}\varepsilon_x^0+A_{12}\varepsilon_y^0+A_{16}\gamma_{xy}^0+B_{11}\varepsilon_x^1+B_{12}\varepsilon_y^1+B_{16}\gamma_{xy}^1,\\
N_y&=A_{12}\varepsilon_x^0+A_{22}\varepsilon_y^0+A_{26}\gamma_{xy}^0+B_{12}\varepsilon_x^1+B_{22}\varepsilon_y^1+B_{26}\gamma_{xy}^1,\\
N_{xy}&=A_{16}\varepsilon_x^0+A_{26}\varepsilon_y^0+A_{66}\gamma_{xy}^0++B_{16}\varepsilon_x^1+B_{26}\varepsilon_y^1+B_{66}\gamma_{xy}^1,\\
M_x&=B_{11}\varepsilon_x^0+B_{12}\varepsilon_y^0+B_{16}\gamma_{xy}^0+D_{11}\varepsilon_x^1+D_{12}\varepsilon_y^1+D_{16}\gamma_{xy}^1,\\
M_y&=B_{12}\varepsilon_x^0+B_{22}\varepsilon_y^0+B_{26}\gamma_{xy}^0+D_{12}\varepsilon_x^1+D_{22}\varepsilon_y^1+D_{26}\gamma_{xy}^1,\\
M_{xy}&=B_{16}\varepsilon_x^0+B_{26}\varepsilon_y^0+B_{66}\gamma_{xy}^0+D_{16}\varepsilon_x^1+D_{26}\varepsilon_y^1+D_{66}\gamma_{xy}^1,\\
S_{yz}^s&=KA_{44}^s\gamma_{yz}^0+KA_{45}^s\gamma_{xz}^0,\\
S_{xz}^s&=KA_{45}^s\gamma_{yz}^0+KA_{55}^s\gamma_{xz}^0
\end{aligned} \quad (14)$$

where A_{ij}, B_{ij}, D_{ij} and A_{ij}^s are the plate stiffness coefficients given by

$$\left(A_{ij},B_{ij},D_{ij}\right)=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}\left(1,z,z^2\right)dz,\ \ i,j=1,2,6 \quad (15a)$$

$$A_{ij}^s=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}dz,\ \ i,j=4,5 \quad (15b)$$

Substituting Eqs. (4) and (8) into Eq. (12) and integrating the resulting expressions by parts and collecting the coefficients of δu_0, δv_0, δw_0 and $\delta\theta$, the governing differential equations in terms of stress resultants are obtained as follows

$$\begin{aligned}
\delta u_0:&\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0\\
\delta v_0:&\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0\\
\delta w_0:&\ \frac{\partial S_{xz}}{\partial x}+\frac{\partial S_{yz}}{\partial y}+q=0\\
\delta\theta:&\ k_1M_x+k_2M_y+\left(k_1A'+k_2B'\right)\frac{\partial^2M_{xy}^s}{\partial x\partial y}-k_1A'\frac{\partial S_{xz}}{\partial x}-k_2B'\frac{\partial S_{yz}}{\partial y}=0
\end{aligned} \quad (16)$$

3. Analytical solutions for laminated composite plates using Navier solution

Analytical solutions of the governing differential equations in Eq. (16) for simply supported laminated composite platesare obtained using Navier solution procedure. The plate is subjected to transverse mechanical loadings $q(x,y)$ acting in the downward z-direction. In this study two different types are considered, cross-ply and angle-ply laminated plates. For the first type, the following stiffness components are identically zero

$$A_{16}=A_{26}=B_{12}=B_{16}=B_{26}=B_{66}=D_{16}=D_{26}=A_{45}^s=0 \quad (17)$$

Based on the Navier procedure, in which the displacement components are expanded in a double trigonometric series of unknown variables, the boundary conditions of simply supported cross-ply laminated plates are satisfied by the following forms of the variables u_0, v_0, w_0 and θ (Meksi *et al.* 2019, Boussoula *et al.* 2019).

$$\begin{Bmatrix}u_0\\v_0\\w_0\\\theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}\cos(\alpha\,x)\sin(\beta\,y)\\V_{mn}\sin(\alpha\,x)\cos(\beta\,y)\\W_{mn}\sin(\alpha\,x)\sin(\beta\,y)\\\Phi_{mn}\sin(\alpha\,x)\sin(\beta\,y)\end{Bmatrix} \quad (18)$$

For angle-ply laminated plates, the following stiffness components are identically zero

$$A_{16}=A_{26}=B_{11}=B_{12}=B_{22}=B_{66}=D_{16}=D_{26}=A_{45}^s=0 \quad (19)$$

And the displacement variables which automatically satisfy the boundary conditions can be expressed in the following forms

$$\begin{Bmatrix}u_0\\v_0\\w_0\\\theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}\sin(\alpha\,x)\cos(\beta\,y)\\V_{mn}\cos(\alpha\,x)\sin(\beta\,y)\\W_{mn}\sin(\alpha\,x)\sin(\beta\,y)\\\Phi_{mn}\sin(\alpha\,x)\sin(\beta\,y)\end{Bmatrix} \quad (20)$$

where U_{mn}, V_{mn}, W_{mn} and $\Phi_{mn}$are unknown coefficients, whereas the parameters α and β are already defined in Eq. (7). We assume that the transverse load $q(x,y)$ can also be expanded in the double-Fourier sine series as

$$q(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} q_{mn}\sin(\alpha x)\sin(\beta y) \tag{21}$$

where m and n are positive integers and the coefficients q_{mn} are given for sinusoidal and uniform loads as follows

$$q_{mn}=\begin{cases} q_0 & \text{For a sinusoidal load } (m=n=1) \quad \text{(22a)} \\ \dfrac{16q_0}{mn\pi^2} & \text{For a uniform load } (m=n=1,3,5,\ldots) \quad \text{(22b)} \end{cases}$$

where q_0 is the maximum intensity of the load at the centre of a plate. Substitution the solution of Eqs. (18), (20) and (21) into the governing equations Eq. (16), the bending analysis of simply supported laminated composite plates can be obtained from the following matrix form

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_{mn} \\ 0 \end{Bmatrix} \tag{23}$$

$$\begin{aligned}
&S_{11}=\alpha^2A_{11}+\beta^2A_{66},\quad S_{12}=\alpha\beta\left(A_{12}+A_{66}\right),\; S_{13}=0,\\
&S_{22}=\beta^2A_{22}+\alpha^2A_{66},\quad S_{23}=0,\quad S_{33}=K\left(\beta^2A_{44}^s+\alpha^2A_{55}^s\right),\\
&S_{34}=-K\left(k_2B'\beta^2A_{44}^s+k_1A'\alpha^2A_{55}^s\right),\\
&S_{44}=k_1^2D_{11}+k_2^2D_{22}+\left(k_1^2A'^2+k_2^2B'^2+2k_1k_2A'B'\right)\alpha^2\beta^2D_{66}\\
&\qquad +2k_1k_2D_{12}+K\left(k_2^2B'^2\,\beta^2A_{44}^s+k_1^2A'^2\,\alpha^2A_{55}^s\right),
\end{aligned} \tag{24}$$

$$\left.\begin{aligned} S_{14}&=k_1\alpha B_{11}-\left(k_1A'+k_2B'\right)\alpha\beta^2B_{66},\\ S_{24}&=k_2\beta B_{22}-\left(k_1A'+k_2B'\right)\alpha^2\beta B_{66} \end{aligned}\right\}\ \text{For antisymmetric cross-ply}$$

$$\left.\begin{aligned} S_{14}&=k_1\beta B_{16}+k_2\beta B_{26}-\left(k_1A'+k_2B'\right)\alpha^2\beta B_{16},\\ S_{24}&=k_1\alpha B_{16}+k_2\alpha B_{26}-\left(k_1A'+k_2B'\right)\alpha\beta^2B_{26} \end{aligned}\right\}\ \text{For antisymmetric angle-ply}$$

4. Numerical results and discussions

In this study, the validity and effectiveness of the proposed theory is proved in predicting the displacements and stresses of simply supported antisymmetric cross-ply and angle-ply laminated composite plates. Various numerical examples are presented and compared with the results of exact 3D solutions and those calculated using different plate theories. Through all the examples, the value of shear correction factor is taken as 5/6 for the proposed model and the conventional FSDT.

The following material properties are used to obtain the numerical results (Reddy2004)

$$E_1=25E_2,\; G_{12}=G_{13}=0.5E_2,\; G_{23}=0.2E_2,\; \nu_{12}=0.25 \tag{25}$$

The following non-dimensional parameters are used for the purpose of presenting the numerical results of the displacements and stresses

$$\begin{gathered}
\bar{u}\left(0,\frac{b}{2},z\right)=\frac{100h^3E_2}{q_0a^4}u,\quad \bar{w}\left(\frac{a}{2},\frac{b}{2},z\right)=\frac{100h^3E_2}{q_0a^4}w,\quad \bar{\sigma}_{x,y}\left(\frac{a}{2},\frac{b}{2},z\right)=\frac{h^2}{q_0a^2}\sigma_{x,y},\\
\bar{\tau}_{xy}(0,0,z)=\frac{h^2}{q_0a^2}\tau_{xy},\quad \bar{\tau}_{xz}\left(0,\frac{b}{2},z\right)=\frac{h}{q_0a}\tau_{xz},\quad \bar{\tau}_{yz}\left(\frac{a}{2},0,z\right)=\frac{h}{q_0a}\tau_{yz}
\end{gathered} \tag{26}$$

Table 1 Comparison of non-dimensional transverse displacement $\bar{w}$ for antisymmetric cross-ply (0°/90°) laminated square plate subjected to sinusoidal loads

Theory	*a/h*				
	2	5	10	20	100
Exact 3D [(a)]	4.9362	1.7287	1.2318	1.1060	1.0742
TSDT	4.5619	1.6670	1.2161	1.1018	1.0651
FSDT	5.4059	1.7584	1.2373	1.1070	1.0653
CLPT	1.0636	1.0636	1.0636	1.0636	1.0636
Present	5.4059	1.7584	1.2373	1.1070	1.0653

Example1: Bending analysis of cross-ply (0°/90°) laminated composite plates

The first example is carried out for the non-dimensional transverse displacement of simply supported antisymmetric cross-ply (0°/90°) laminated composite square plates subjected to sinusoidal loads for different values of side-to-thickness ratio *a/h*, ranging from 2 to 100 "*corresponding to from very thick to thin plates*", as presented in Table 1.

(a) Results taken from reference of Pagano(1970)

The numerical results are compared with those obtained by the exact 3D solutions given by Pagano (1970) and those computed using *classical laminated plate theory* "CLPT", conventional *first-order shear deformation theory* "FSDT" of Mindlin and *third-order shear deformation theory* "TSDT" of Reddy. From the examination of Table 1, it can be observed that the numerical results of transverse displacements obtained by using the present model are exactly matching with the results of the conventional FSDT for all side-to-thickness ratios. We also noted that the increment in the side-to-thickness ratio leads to the decrease in the transverse displacements of antisymmetric cross-ply laminated plates. Moreover, the CLPT gives acceptable results only for thin square laminated plate ($a/h\geq 20$) and underestimates the results of transverse displacement as compared to other theories due to neglect of transverse shear strains.

Example 2: Bending analysis of cross-ply $(0°/90°)_n$ laminated composite plates

To further assure the accuracy of the present novel theory, the next example is checked for bending analysis of simply supported multilayered antisymmetric $(0°/90°)_n$ laminated composite square plates subjected to two different loading conditions, i.e., sinusoidal and uniform loads. The comparison of non-dimensional transverse displacement for different values of side-to-thickness ratio (*a/h*) and numbers of layers is listed in Tables 2 and 3.

The numerical results generated for this example are compared with those computed using CLPT, FSDT and TSDT. It can be seen again that the obtained results are in excellent agreement with those presented by the conventional FSDT with each other for different values of thickness ratio and numbers of layers.

The variations of non-dimensional transverse displacement $\bar{w}$ with respect to side-to-thickness ratio *a/h* and modulus ratio E_1/E_2 of two-layer and six-layer

Table 2 Comparison of non-dimensional transverse displacement $\bar{w}$ for antisymmetric cross-ply $(0°/90°)_n$ laminated square plate subjected to sinusoidal loads

Lay-ups	Theory	*a/h*			
		4	10	20	100
$(0°/90°)_1$	TSDT	1.9985	1.2161	1.1018	1.0651
	FSDT	2.1492	1.2373	1.1070	1.0653
	CLPT	1.0636	1.0636	1.0636	1.0636
	Present	2.1492	1.2373	1.1070	1.0653
$(0°/90°)_2$	TSDT	1.6093	0.6865	0.5517	0.5083
	FSDT	1.5921	0.6802	0.5500	0.5083
	CLPT	0.5065	0.5065	0.5065	0.5065
	Present	1.5921	0.6802	0.5500	0.5083
$(0°/90°)_3$	TSDT	1.5411	0.6382	0.5060	0.4635
	FSDT	1.5473	0.6354	0.5053	0.4635
	CLPT	0.4617	0.4617	0.4617	0.4617
	Present	1.5473	0.6354	0.5052	0.4635
$(0°/90°)_4$	TSDT	1.5168	0.6229	0.4918	0.4496
	FSDT	1.5335	0.6216	0.4913	0.4496
	CLPT	0.4479	0.4479	0.4479	0.4479
	Present	1.5335	0.6216	0.4913	0.4496

Table 3 Comparison of non-dimensional transverse displacement $\bar{w}$ for antisymmetric cross-ply $(0°/90°)_n$ laminated square plate subjected to uniform loads

Lay-ups	Theory	*a/h*			
		4	10	20	100
$(0°/90°)_1$	TSDT	3.0706	1.9173	1.7509	1.6977
	FSDT	3.2709	1.9468	1.7582	1.6980
	CLPT	1.6955	1.6955	1.6955	1.6955
	Present	3.2741	1.9480	1.7586	1.6980
$(0°/90°)_2$	TSDT	2.4282	1.0693	0.8737	0.8111
	FSDT	2.3837	1.0596	0.8712	0.8111
	CLPT	0.8085	0.8085	0.8085	0.8085
	Present	2.3872	1.0611	0.8717	0.8111
$(0°/90°)_3$	TSDT	2.3243	0.9929	0.8010	0.7397
	FSDT	2.3123	0.9882	0.7998	0.7396
	CLPT	0.7371	0.7371	0.7371	0.7371
	Present	2.3158	0.9897	0.8003	0.7396
$(0°/90°)_4$	TSDT	2.2879	0.9686	0.7784	0.7176
	FSDT	2.2902	0.9660	0.7776	0.7175
	CLPT	0.7150	0.7150	0.7150	0.7150
	Present	2.2936	0.9676	0.7782	0.7175

antisymmetric cross-ply laminated square plate subjected to sinusoidal loads are plotted in Figs. 2 and 3, respectively.

It can be seen that the variation of transverse displacement obtained by present theory are in close agreement with those calculated using the conventional FSDT. Whereas CLPT underestimates transverse displacement of thick laminated plate with *a/h*<20 due to neglecting shear deformation effects (see Fig. 2).On the other hand, the variations and the associated values of the in-plane displacement $\bar{u}$, in-plane normal stresses $\bar{\sigma}_x, \bar{\sigma}_y$ and transverse shear stresses $\bar{\tau}_{xy}, \bar{\tau}_{xz}, \bar{\tau}_{yz}$ through the

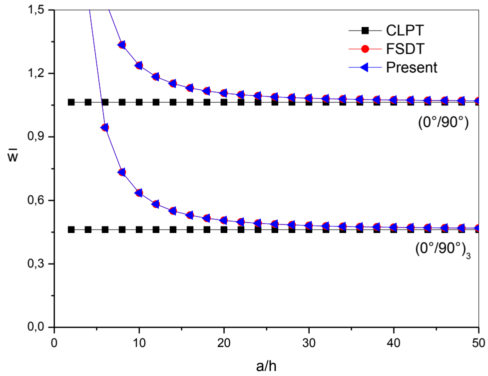

Fig. 2 Variation of non-dimensional transverse displacement $(\bar{w})$ of antisymmetric cross-ply $(0°/90°)_n$ laminated square plates under sinusoidal loads with respect to side-to-thickness ratio

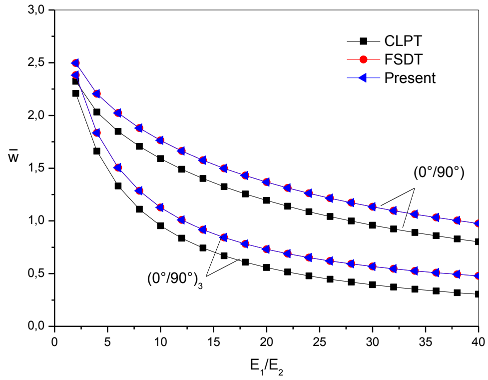

Fig. 3 Variation of non-dimensional transverse displacement $(\bar{w})$ of antisymmetric cross-ply $(0°/90°)_n$ laminated square plates under sinusoidal loads with respect tomodulus ratio, (*a/h*=10)

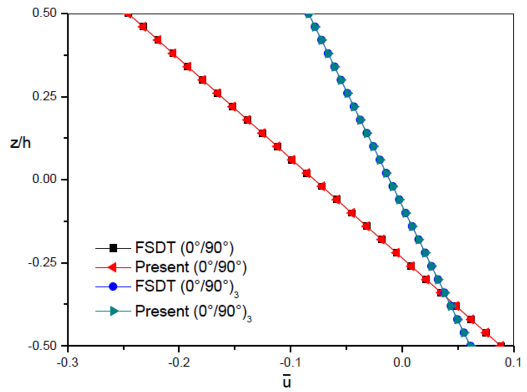

Fig. 4 Variation of non-dimensional axial displacement $(\bar{u})$ through the thickness of antisymmetric cross-ply $(0°/90°)_n$ laminated square plate under sinusoidal loads, (*a/h*=10)

thickness of a moderately thick multilayered antisymmetric $(0°/90°)_n$ laminated composite plates for the thickness ratio (*a/h*=10) are shown in Figs. 4-9 and presented in Tables 4 and 5 for sinusoidal and uniform loading conditions, respectively. It must be noted again that the present computations are in an excellent agreement with the conventional FSDT for allamination schemes.

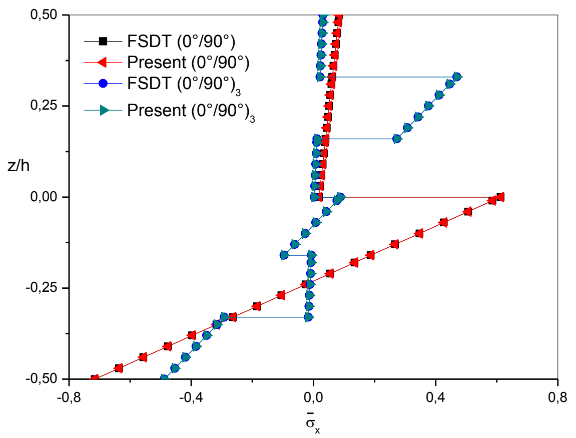

Fig. 5 Variation of non-dimensional in-plane normal stress $(\bar{\sigma}_x)$ through the thickness of antisymmetric cross-ply $(0°/90°)_n$ laminated square plates under sinusoidal loads, (a/h=10)

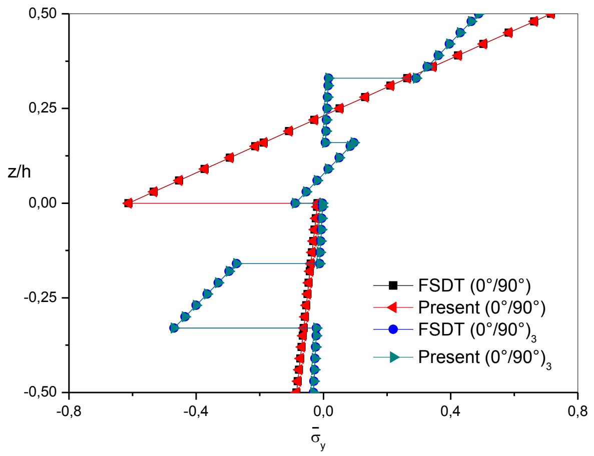

Fig. 6 Variation of non-dimensional in-plane normal stress $(\bar{\sigma}_y)$ through the thickness of antisymmetric cross-ply $(0°/90°)_n$ laminated square plates under sinusoidal loads, (a/h=10)

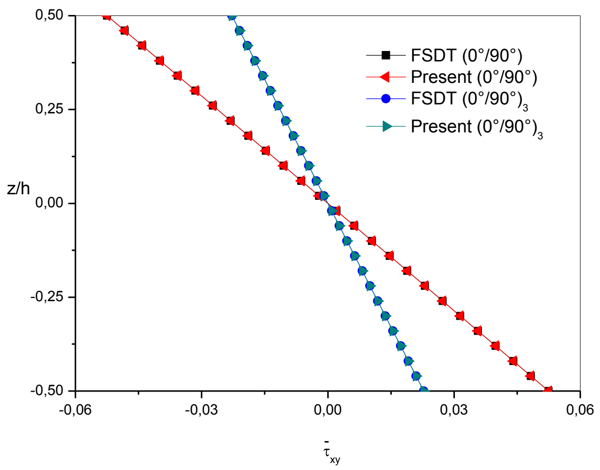

Fig. 7 Variation of non-dimensional in-plane shear stress $(\bar{\tau}_{xy})$ through the thickness of antisymmetric cross-ply $(0°/90°)_n$ laminated square plates under sinusoidal loads, (a/h=10)

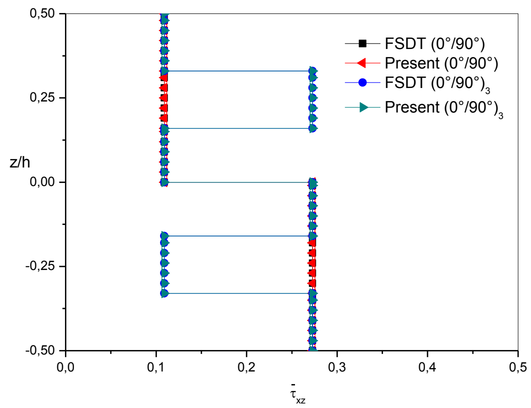

Fig. 8 Variation of non-dimensional transverse shear stress $(\bar{\tau}_{xz})$ through the thickness of antisymmetric cross-ply $(0°/90°)_n$ laminated square plates under sinusoidal loads, (a/h=10)

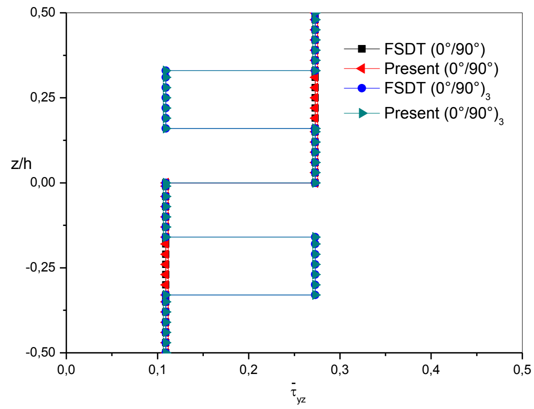

Fig. 9 Variation of non-dimensional transverse shear stress $(\bar{\tau}_{yz})$ through the thickness of antisymmetric cross-ply $(0°/90°)_n$ laminated square plates under sinusoidal loads, (a/h=10)

Table 4 Comparison of non-dimensional stresses for antisymmetric cross-ply $(0°/90°)_n$ laminated square platessubjected to sinusoidal loads, (a/h=10)

Lay-ups	Theory	$\bar{\sigma}_x(h/2)$	$\bar{\sigma}_y(h/2)$	$\bar{\tau}_{xy}(-h/2)$	$\bar{\tau}_{xz}(-h/4)$
$(0°/90°)_1$	FSDT	0.0843	0.7157	0.0525	0.2728-
	Present	0.0843	0.7157	0.0525	0.2728
$(0°/90°)_2$	FSDT	0.0357	0.4868	0.0250	0.1091
	Present	0.0357	0.4868	0.0250	0.1091
$(0°/90°)_3$	FSDT	0.0312	0.4881	0.0228	0.1091
	Present	0.0312	0.4881	0.0228	0.1091
$(0°/90°)_4$	FSDT	0.0296	0.4950	0.0221	0.2728
	Present	0.0296	0.4950	0.0221	0.2728

Example 3: Bending analysis of angle-ply $(45°/\text{-}45°)_n$ laminated composite plates

The last example is devoted to the analysis of multilayered antisymmetric angle-ply $(45°/\text{-}45°)_n$ laminated composite square plates with simply supported boundary conditions subjected to sinusoidal and uniform loads. The comparison of non-dimensional transverse displacements

Table 5 Comparison of non-dimensional stresses for antisymmetric cross-ply $(0°/90°)_n$ laminated square plates subjected to uniform loads, (a/h=10)

Lay-ups	Theory	$\bar{\sigma}_x(h/2)$	$\bar{\sigma}_y(h/2)$	$\bar{\tau}_{xy}(-h/2)$	$\bar{\tau}_{xz}(-h/4)$
$(0°/90°)_1$	FSDT	0.1264	1.0715	0.0961	0.5772
	Present	0.1268	1.0762	0.0934	0.5770
$(0°/90°)_2$	FSDT	0.0536	0.7295	0.0472	0.2315
	Present	0.0541	0.7368	0.0442	0.2308
$(0°/90°)_3$	FSDT	0.0468	0.7313	0.0433	0.2315
	Present	0.0473	0.7392	0.0403	0.2308
$(0°/90°)_4$	FSDT	0.0444	1.7415	0.0420	0.5787
	Present	0.0449	0.7496	0.0391	0.5770

Table 6 Comparison of non-dimensional transverse displacement $\bar{w}$ for antisymmetric angle-ply $(45°/-45°)_n$ laminated square plate subjected to sinusoidal loads

Lay-ups	Theory	a/h			
		4	10	20	100
$(45°/-45°)_1$	TSDT	1.5497	0.8027	0.6919	0.6562
	FSDT	1.7403	0.8284	0.6981	0.6564
	CLPT	0.6547	0.6547	0.6547	0.6547
	Present	1.7403	0.8284	0.6981	0.6564
$(45°/-45°)_4$	TSDT	1.2931	0.4207	0.2900	0.2479
	FSDT	1.3317	0.4198	0.2896	0.2479
	CLPT	0.2462	0.2462	0.2462	0.2462
	Present	1.3317	0.4198	0.2896	0.2479

obtained by the present model and other plate theories of two-layer and eight-layer antisymmetric angle-ply laminate is reported in Tables 6 and 7 for different values of the thickness ratio (*a/h*). From the examination of Table 6, it is observed that the present theory is in excellent agreement while predicting the transverse displacements $\bar{w}$ as compared to those provided by using the conventional FSDT. Moreover, it can be noticed that the increase of the thickness ratio and numbers of layers have a significant effect on the decrease of the transverse displacement.

4. Conclusions

In this work, the static analysis of simply supported laminated composite plates subjected to sinusoidal and uniform loads is studied based on the novel *first-order shear deformation theory*, in which the displacement field contains a smaller number of unknowns with an undetermined integral component. The governing equations and its boundary conditions are derived by utilizing the principle of virtual work and solved using Navier's solution method. The numerical results of the transverse displacement and stresses for simply supported antisymmetric cross-ply and angle-ply laminated composite plates are presented and compared with the solutions calculated using different plate theories. Effects of the side-to-thickness ratio, numbers of layers and lamination scheme on the displacements and stresses as well as corresponding shapes of the loading conditions are studied. In conclusion,

Table 7 Comparison of non-dimensional transverse displacement $\bar{w}$ forantisymmetricangle-ply $(45°/-45°)_n$ laminated square plate subjected to uniform loads

Lay-ups	Theory	a/h			
		4	10	20	100
$(45°/-45°)_1$	TSDT	2.3346	1.2421	1.0817	1.0302
	FSDT	2.6034	1.2792	1.0907	1.0305
	CLPT	1.0280	1.0280	1.0280	1.0280
	Present	2.6067	1.2806	1.0912	1.0305
$(45°/-45°)_4$	TSDT	1.9197	0.6383	0.4490	0.3883
	FSDT	1.9613	0.6366	0.4483	0.3883
	CLPT	0.3858	0.3858	0.3858	0.3858
	Present	1.9644	0.6384	0.4489	0.3883

it can be said that the novel shear deformation theory with only four unknowns is not only more accurate but also simple than the conventional FSDT in predicting the bending response of thick laminated composite plates. An improvement of the present study will be considered in the future work to consider other type of materials (Benferhat *et al*. 2016, Kolahchi *et al*. 2016, Daouadji 2017, Eltaher *et al*. 2018, Ayat *et al*. 2018, Natanzi *et al*. 2018, Panjehpour *et al*. 2018, Bensaid *et al*. 2018, Selmi and Bisharat2018, Karami*et al*. 2018, Faleh*et al*. 2018, Bensattalah *et al*. 2019, Medani *et al*. 2019, Hussain and Naeem2019, Fadoun 2019, Hussain *et al*. 2019, Avcar 2019, Rajabi and Mohammadimehr 2019, Draoui *et al*. 2019, Boutaleb *et al*. 2019, Boukhlif *et al*. 2019, Mahmoudi *et al*. 2019 Boulefrakh *et al*. 2019).

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