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Analytical solution for buckling analysis of micro sandwich hollow circular plate

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Abstract. In this paper, the buckling of micro sandwich hollow circular plate is investigated with the consideration of the porous core and piezoelectric layer reinforced by functionally graded (FG)carbon nano-tube. For modeling the displacement field of sandwich hollow circular plate, the high-order shear deformation theory (HSDT) of plate and modified couple stress theory (MCST) are used. The governing differential equations of the system can be derived using the principle of minimum potential energy and Maxwell's equation that for solving these equations, the Ritz method is employed. The results of this research indicate the influence of various parameters such as porous coefficients, small length scale parameter, distribution of carbon nano-tube in piezoelectric layers and temperature on critical buckling load. The purpose of this research is to show the effect of physical parameters on the critical buckling load of micro sandwich plate and then optimize these parameters to design structures with the best efficiency. The results of this research can be used for optimization of micro-structures and manufacturing different structure in aircraft and aerospace.

Keywords: buckling analysis; micro sandwich hollow circular plate; porous core; piezoelectric layer; HSDT

1. Introduction

Sandwich structures according to high weight strength are used in various industries such as aerospace, shipbuilding and automobile. The purpose of this study becomes to analyze the critical buckling load of micro sandwich hollow circular plate, among the studies that have been done in this field could be mentioned the following cases:

Feldman and Aboudi (1997) analyzed the buckling of functionally graded (FG) plates under in-plane compressive loading. Javaheri and Eslami (2002) used the classical plate theory (CPT) to analyze the thermal buckling of functionally graded plates. Tornabene (2009) studied the dynamic behavior of moderately thick functionally graded conical, cylindrical shells and annular plates. Using finite element method, Koukouselis and Mistakidis (2015) presented the buckling behavior of thin ferrocement stiffened plates. Tornabene and Reddy (2013) analyzed the mechanical behavior of functionally graded material (FGM) and laminated doubly-curved shells and panel resting on nonlinear and linear elastic foundation. Ma and Wang (2004) studied the axisymmetric bending and buckling of FGM circular plates based on third-order shear deformation plate theory and classical plate theory. Chan *et al*. (2019) investigated the nonlinear buckling of stiffened FGM truncated conical shells resting on an elastic foundation and subjected to a uniform axial compressive load. Based on the classical nonlinear von Karman plate theory. Ma and Wang (2003) investigated nonlinear bending and post-buckling of a functionally graded circular plate under mechanical and thermal loadings. The effect of small scale on the buckling/thermal buckling of carbon nanotubes is investigated by Arani *et al*. (2011, 2012a), and Mohammadimehr *et al*. (2011). Tornabene *et al*. (2011) used the generalized differential quadrature (GDQ) method and the first-order shear deformation theory (FSDT) to studied the dynamic behavior of functionally graded materials (FGMs) and laminated doubly curved shells and panels of revolution with a free-form meridian. Sobhy and Zenkour (2019) studied the porosity and inhomogeneity effects on the buckling and vibration of double-FGM nanoplates. Rajabi and Mohammadimehr (2019) presented the bending analysis of a micro sandwich skew plate using extended Kantorovich method based on Eshelby-Mori-Tanaka approach. Tornabene *et al*. (2015) analyzed recovery of through-the-thickness transverse normal and shear strains and stresses in statically deformed functionally graded (FG) doubly-curved sandwich shell structures and shells of revolution. By using differential quadrature method (DQM), Jafarian Arani and Kolahchi (2016) analyzed the buckling of embedded concrete columns armed with carbon nanotubes. Their results indicated the influences of volume percent of SWCNTs, geometrical parameters, elastic foundation and boundary conditions on the buckling of column. Liu *et al*. (2017) investigated the static bending, free vibration, and buckling of functionally graded (FG) moderately thick microplates. Their results showed the deflection decreases while the frequency and buckling load increase with decreasing the plate thickness. Frikha *et al*. (2018) presented the dynamic behavior of functionally graded carbon nanotubes reinforced composite shell structures (FG-CNTRC) via forced vibration. Liu *et al*. (2018) studied the mechanical behavior of functionally

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graded (FG) microplates with defects or cracks by effective numerical methods. Eslami *et al*. (2016) studied the effects of different porosity types, including symmetric and asymmetric distributions on the critical buckling load of the circular plate, by using the higher order shear deformation theory (HSDT). Their results indicated that the asymmetric distribution of porosity relative to the symmetric distribution reduces critical buckling load and also decreases buckling load with increasing porosity coefficient. Also in recent years, the use of high-order plate theories to analyze the mechanical behavior of various structures has been more prosperous, Vu *et al*. (2018), Bui *et al*. (2016), Yin (2016), Do *et al*. (2017a). They used new third order shear deformation theory (TSDT) without any requirement of shear correction factors. Their results showed the influence of using new plate theory on mechanical behavior of different structure. Mohammadimehr *et al*. (2018b) investigated bending, buckling, and free vibration analyses of carbon nanotube reinforced composite beams and experimental tensile test to obtain the mechanical properties of nanocomposite. Zghal *et al*. (2018) investigated the mechanical buckling behavior of functionally graded materials and carbon nanotubes-reinforced composite plates and curved panels. Sobhy and Zenkour (2018) investigated the influence of magnetic field on buckling and vibration of sandwich nanobeams with CNT reinforced face sheets. Their results showed the effect of different parameter such as viscoelastic damping for the structure and the foundation, the magnetic field parameter, humidity concentration on critical buckling load. Tornabene *et al*. (2016) presented the free vibrations of laminated composite doubly-curved shells and panels reinforced by CNTs. Hajmohammad *et al*. (2018) investigated the effects of radial compressive load and uniformly distributed load on the critical buckling load of micro circular plate based on the nonlinear boundary layer theory. The governing equations of sandwich rectangular plate by using first order shear deformation theory for different boundary conditions is obtained by Kiani (2016). His results showed by increasing the temperature, the critical buckling load of rectangular sandwich plate increases. By using doublet mechanics theory, Aydogdu and Gul (2018) analyzed the buckling of double nanofibers embedded in an elastic matrix based on an Euler-Bernoulli beam model. Mohammadimehr *et al*. (2018a) investigated the free vibration and buckling of micro sandwich plate by using strain gradient theory subject to different loads such as electric, magnetic and mechanic. Bending and buckling of perfect functionally graded solid circular plates is studied by Saidi *et al*. (2009). Liu *et al*. (2019) employed the non-classical Kirchhoff plate theory and modified couple stress theory for studying the static bending and buckling behaviors of nanoplates. Karamanli and Aydogdu (2019) presented the elastic buckling behavior of isotropic, laminated composite and sandwich beams subjected to various axially varying in-plane loads and boundary conditions (BCs). Yu *et al*. (2019a) used the modified couple stress theory (MCST) to analysis the mechanical behavior of functionally graded (FG) microbeams. Safari Bilouei *et al*. (2016) studied the buckling of concrete columns retrofitted with nano-fiber reinforced polymer (NFRP) by using Euler-Bernoulli beam (EBB) theory. Some researchers worked about vibration analysis of micro sandwich structures including cylindrical shells (Yazdani *et al*. 2019), annular circular plate (Emdadi *et al*. 2019), vibration analysis of viscoelastic tapered micro-rod (Mohammadimehr *et al*. 2015), electro-thermal non-local vibration analysis of embedded double-walled boron nitride nanotubes (DWBNNTs) (Arani *et al*. 2012b). Trabelsi *et al*. (2019) investigated the thermal buckling behavior of functionally graded plates and cylindrical shells. Their results indicated the effect of material compositions, power law index, thermal loading, boundary conditions and geometrical parameters of shells on the thermal buckling behavior of FGM structures. Akhavan *et al*. (2019) considered active control of micro Reddy beam integrated with functionally graded nanocomposite sensor and actuator based on linear quadratic regulator method. Yu *et al*. (2019b) investigated the effect of material gradient factors along the axial and thickness directions, material length scale factor, boundary condition, and other aspect ratios of two-directional FG on mechanical behavior. Recent studies have been done on the mechanical behavior of sandwich structures, Bui *et al*. (2013), Do *et al*. (2017b). Their results carried out to demonstrate the influence of temperature, power law index of FG core and geometric parameter on mechanical behavior of sandwich plate.

Piezoelectric materials applications, nanotechnology and micro scale analysis for smart structures were reviewed in the introduction. According to reviewed literatures, a combination of these topics for smart structures is a lack of the previous study. By reviewing the literature, it can be seen that up to date there are no researches about the influence of various parameters on critical buckling load of micro sandwich hollow circular plate with considering porous core and piezoelectric layer reinforced by FG carbon nano-tube using Ritz method. For obtaining the governing equation of equilibrium, high-order shear deformation theory and modified couple stress theory are employed. The Ritz method is applied to solve the differential equations. The results of this research can be used to optimize the micro sandwich plate for various applications.

2. Governing equations of equilibrium

Consider a micro sandwich hollow circular plate with

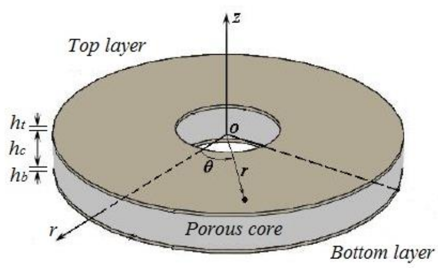


Fig. 1 A schematic view of a micro sandwich hollow circular plate with considering porous core integrated with piezoelectric layers

the inner radius a, outer radius b, the thickness of porous core h_c, the thickness of bottom piezoelectric layer h_b and the thickness of top piezoelectric layer h_t (see Fig. 1).

2.1 Displacement field of sandwich circular micro sandwich plate

High-order shear deformation theory (HSDT) of plate is employed to model the micro sandwich hollow circular plate, displacement fields are given as Nguyen-Xuan *et al*. (2013)

$$u(r,\theta,z)=u_0(r,\theta)-z\frac{\partial w_0(r,\theta)}{\partial r}+f(z)\varphi_r(r,\theta) \quad (1)$$

$$v(r,\theta,z)=v_0(r,\theta)-\frac{1}{r}\frac{\partial w_0(r,\theta)}{\partial \theta}+f(z)\varphi_\theta(r,\theta) \quad (2)$$

$$w(r,\theta,z)=w_0(r,\theta) \quad (3)$$

where $f(z)=(z-\frac{4z^3}{3H^2})$, $H=h_t+h_c+h_b$, $u_0(r,\theta)$, $v_0(r,\theta)$ and $w_0(r,\theta)$ are the displacements at the mid-surface, $\varphi_r(r,\theta)$ and $\varphi_\theta(r,\theta)$ are the rotation of a transverse normal about the axial and circumferential directions, respectively.

To increase the accuracy of problem solutions and to improve the proposed model, high-order shear deformation theory (HSDT) with the five degrees of freedom is used. Also, by considering that the proposed model is related to a thick plate, the use of HSDT is necessary.

2.2 The modified couple stress theory for micro sandwich hollow circular plate

Based on the modified couple stress theory (MCST), the components of the symmetric curvature tensor (χ_{ij}) are defined as Rostami *et al*. (2018)

$$\chi_{ij}^{(s)}=\frac{1}{2}\left(e_{ipq}\frac{\partial \varepsilon_{qj}}{\partial x_p}+e_{jpq}\frac{\partial \varepsilon_{qi}}{\partial x_p}\right) \quad (4)$$

$$m_{ij}^{(s)}=2GL_2^2\chi_{ij}^k \quad (5)$$

where L_2 and G are the material length scale parameter and the shear modulus, respectively.

2.3 The constitutive equations for porous core and piezoelectric layers

The linear constitutive equations for porous core are defined as Mohammaimehr *et al*. (2016)

$$\begin{Bmatrix}\sigma_{rr}\\\sigma_{\theta\theta}\\\sigma_{r\theta}\\\sigma_{rz}\\\sigma_{z\theta}\end{Bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&0&0&0\\Q_{12}&Q_{11}&0&0&0\\0&0&Q_{44}&0&0\\0&0&0&Q_{44}&0\\0&0&0&0&Q_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_{rr}\\\varepsilon_{\theta\theta}\\\gamma_{r\theta}\\\gamma_{rz}\\\gamma_{z\theta}\end{Bmatrix} \quad (6)$$

Q_{ij} is the stiffness components of the porous core that is expressed as follows

$$Q_{11}=\frac{E^{Core}(z)}{1-\upsilon_c^2} \quad (7)$$

$$Q_{12}=\frac{\nu_c E^{Core}(z)}{1-\upsilon_c^2} \quad (8)$$

$$Q_{44}=\frac{E^{Core}(z)}{2(1+\upsilon_c)} \quad (9)$$

where

$$E^{core}(z)=E_c(1-e_0\psi(z)),\qquad \psi(z)=\cos(\frac{\pi z}{h_c}) \quad (10)$$

The linear constitutive equations for piezoelectric layer reinforced by FG carbon nano-tube are defined as Rostami *et al*. (2019)

$$\begin{Bmatrix}\sigma_{rr}\\\sigma_{\theta\theta}\\\sigma_{r\theta}\\\sigma_{rz}\\\sigma_{z\theta}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&0&0&0\\C_{12}&C_{22}&0&0&0\\0&0&C_{66}&0&0\\0&0&0&C_{44}&0\\0&0&0&0&C_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{rr}\\\varepsilon_{\theta\theta}\\\gamma_{r\theta}\\\gamma_{rz}\\\gamma_{z\theta}\end{Bmatrix}-\begin{bmatrix}0&0&e_{31}\\0&0&e_{31}\\0&0&0\\e_{15}&0&0\\0&e_{15}&0\end{bmatrix}\begin{Bmatrix}E_r\\E_\theta\\E_z\end{Bmatrix} \quad (11)$$

$$\begin{Bmatrix}D_r\\D_\theta\\D_z\end{Bmatrix}=\begin{bmatrix}0&0&0&e_{15}&0\\0&0&e_{15}&0&0\\e_{31}&e_{31}&0&0&0\end{bmatrix}\begin{Bmatrix}\varepsilon_{rr}\\\varepsilon_{\theta\theta}\\\gamma_{\theta z}\\\gamma_{rz}\\\gamma_{r\theta}\end{Bmatrix}+\begin{bmatrix}k_{11}&0&0\\0&k_{11}&0\\0&0&k_{33}\end{bmatrix}\begin{Bmatrix}E_x\\E_\theta\\E_z\end{Bmatrix} \quad (12)$$

where $\{D_r,D_\theta,D_z\}^T$, $\{E_r,E_\theta,E_z\}^T$, e_{ij} and k_{ij} are electric field intensity, electric displacement vectors, piezoelectric coefficients and dielectric coefficients, respectively.

$$C_{11}=\frac{E_{11}}{1-\nu^2} \quad (13)$$

$$C_{22}=\frac{E_{22}}{1-\nu^2} \quad (14)$$

$$C_{12}=\frac{E_{11}\nu}{1-\nu^2} \quad (15)$$

$$C_{44}=C_{55}=\frac{E_{22}}{2(1+\nu)} \quad (16)$$

$$C_{66}=\frac{E_{11}}{2(1+\nu)} \quad (17)$$

E_{11}, E_{22} and υ_{12} are Young's modulus and Poisson's ratio as Shen and Xiang (2012), respectively.

$$E_{11} = \eta_1 V_{CN} E_{11}^{CN} + V_m E^m \tag{18}$$

$$\frac{\eta_2}{E_{22}} = \frac{V_{CN}}{E_{22}^{CN}} + \frac{V_m}{E^m} \tag{19}$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{CN}}{G_{12}^{CN}} + \frac{V_m}{G^m} \tag{20}$$

$$\nu_{12} = V_{CN} \nu_{12}^{CN} + V_m \nu^m \tag{21}$$

where E_{11}^{CNT}, E_{22}^{CNT}, G_{12}^{CNT}, E^m and G^m are Young's moduli, shear moduli of carbon nanotube and the matrix, respectively. V_{CNT} and V_m are volume fraction of carbon nano-tube and the matrix, respectively.

$$V_m + V_{CNT} = 1 \tag{22}$$

The relations of volume fraction by considering the distribution of carbon nano-tube along the thickness direction of piezoelectric layers expressed as Dinh Duc *et al*. (2017)

$$V_{CNT} = V_{CNT}^* \tag{23}$$

$$(1 + \frac{2z}{h})V_{CNT}^* \tag{24}$$

$$2(1 - \frac{2|z|}{h})V_{CNT}^* \tag{25}$$

$$\frac{4|z|}{h} V_{CNT}^* \tag{26}$$

2.4 The electric field in piezoelectric layers

The electric field of piezoelectric layers defined as Rouzegar and Abad (2015)

$$E_r = -\frac{\partial \phi}{\partial r},\ E_z = -\frac{\partial \phi}{\partial z},\ E_\theta = -\frac{\partial \phi}{r \partial \theta} \tag{27}$$

$$\phi(r,\theta,z) = \begin{cases} \phi(r,\theta)\sin(\dfrac{\pi(z-(h+h_i))}{h_i}) \\ +(\dfrac{2z-(h_i+2h)}{2h_i})v_0 \to top \\ \phi(r,\theta)\sin(\dfrac{-\pi(z+(h+h_b))}{h_b}) \\ +(\dfrac{2z+(h_b+2h)}{2h_b})v_0 \to bottom \end{cases} \tag{28}$$

2.4 Hamilton's principle

The governing equations of motion for micro sandwich hollow circular plate are derived by using Hamilton's principle as Sahmani *et al*. (2013)

$$\int_0^t (\delta U + \delta W)dt = 0 \tag{29}$$

The variation form of strain energy and external work are defined as Mohammadimehr and Rostami (2018)

$$\delta U = \iiint (\sigma_{ij}\delta\varepsilon_{ij} + m_{ij}^{(s)}\delta\chi^{(s)}_{ij} - D_i \delta E_i) dz d\theta r dr \tag{30}$$

$$\delta W = -\iint N_{rr0} w_{0,r} \delta w_{0,r} r dr d\theta \tag{31}$$

The equilibrium equations of micro sandwich hollow circular plate considering porous core and piezoelectric layers can be derived by substituting the Eqs. (30) and (31) into Eq. (29) as follows

$$\delta u_0 : -\frac{\partial}{\partial r}(rN_{rr}) = 0 \tag{31}$$

$$\begin{aligned} \delta v_0 : & -\frac{1}{2}\frac{\partial}{\partial r}(A_{\theta\theta}) - \frac{1}{2r}(A_{\theta\theta}) + \frac{1}{2}\frac{\partial}{\partial r}(A_{zz}) + \frac{1}{4r}(A_{zz}) \\ & -\frac{1}{2}(N_{\theta z}) - \frac{1}{2}\frac{\partial}{\partial r}(rN_{r\theta}) + \frac{1}{4}\frac{\partial^2}{\partial r^2}(A_{rz}) \end{aligned} \tag{32}$$

$$\begin{aligned} \delta w_0 : & -\frac{\partial^2}{\partial r^2}(rM_{rr}) + N_{\theta\theta} - \frac{1}{2}\frac{\partial}{\partial r}(A_{\theta Z}) - \frac{1}{2r}(A_{\theta Z}) \\ & -\frac{1}{2}\frac{\partial^2}{\partial r^2}(rA_{r\theta}) + \frac{\partial}{\partial r}(rN_{rr0}w_{0,r}) = 0 \end{aligned} \tag{33}$$

$$\delta\varphi_r : -\frac{\partial}{\partial r}(rN_{rr}^f) + \frac{r}{4}(A_{\theta z}^{(2)}) - \frac{1}{4}\frac{\partial}{\partial r}(rA_{r\theta}^{(1)}) + \frac{1}{2}r(N_{rz}^{(1)}) = 0 \tag{34}$$

$$\begin{aligned} \delta\varphi_\theta : & \frac{1}{2}\frac{\partial}{\partial r}(rA_{rr}^{(1)}) - \frac{1}{2}\frac{\partial}{\partial r}(A_{\theta\theta}^f) - \frac{1}{2}\frac{\partial}{\partial r}(rA_{zz}^{(1)}) + \frac{1}{2}\frac{\partial}{\partial r}(A_{zz}^f) \\ & + \frac{1}{2}r(N_{\theta z}^{(1)}) - \frac{1}{2}(N_{\theta z}^f) - \frac{1}{2}\frac{\partial}{\partial r}(rM_{r\theta}) \\ & + \frac{1}{4}\frac{\partial^2}{\partial r^2}(rA_{rz}^f) - \frac{r}{4}(A_{rz}^{(2)}) = 0 \end{aligned} \tag{35}$$

$$\delta\phi : \frac{\partial}{\partial r}(rD_r^r) + \frac{\partial}{\partial z}(rD_z^z) = 0 \tag{36}$$

where the resultant forces and moments are defined as follows

$$\begin{aligned} N_{ij} &= \int \sigma_{ij} dz,\quad N_{ij}^{(k)} = \int \sigma_{ij} \frac{\partial^k}{\partial z^k} f(z) dz, \\ N_{ij}^f &= \int \sigma_{ij} f(z) dz \end{aligned} \tag{37}$$

$$\begin{aligned} A_{ij} &= \int m_{ij} dz,\quad A_{ij}^{(k)} = \int m_{ij} \frac{\partial^k}{\partial z^k} f(z) dz, \\ A_{ij}^f &= \int m_{ij} f(z) dz \end{aligned} \tag{38}$$

$$(M_{rr}, M_{r\theta}) = \int (\sigma_{rr}, \sigma_{r\theta}) z dz \tag{39}$$

$$D_r^r, D_z^z = \int (D_r, D_z) dz \tag{40}$$

3. Ritz method type solutions

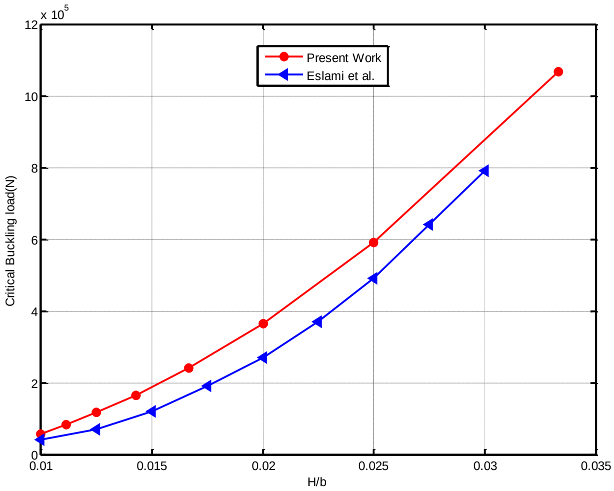


Fig. 2 Comparison of critical buckling load for a clamped supported isotropic homogeneous circular plate (p=0.001 N, H=0.5 μm, L=17.6 μm, e_0=0, T_1=300 K)

To examine the buckling of micro sandwich hollow circular plate under radial load the Ritz numerical method is used to solve the governing equations, the relationships used in this method are as Kiani (2016)

$$u_0(r) = R^{u_0}(r)\sum_{i=1}^{I} A_i^{u_0} r^i \tag{41}$$

$$v_0(r) = R^{v_0}(r)\sum_{i=1}^{I} F_i^{v_0} r^i \tag{42}$$

$$w_0(r) = R^{w_0}(r)\sum_{i=1}^{I} B_i^{w_0} r^i \tag{43}$$

$$\varphi_r(r) = R^{\varphi_r}(r)\sum_{i=1}^{I} H_i^{\varphi_r} r^i \tag{44}$$

$$\varphi_\theta(r) = R^{\varphi_\theta}(r)\sum_{i=1}^{I} G_i^{\varphi_\theta} r^i \tag{45}$$

$$\phi(r) = R^{w_0}(r)\sum_{i=1}^{I} L_i^{\phi} r^i \tag{46}$$

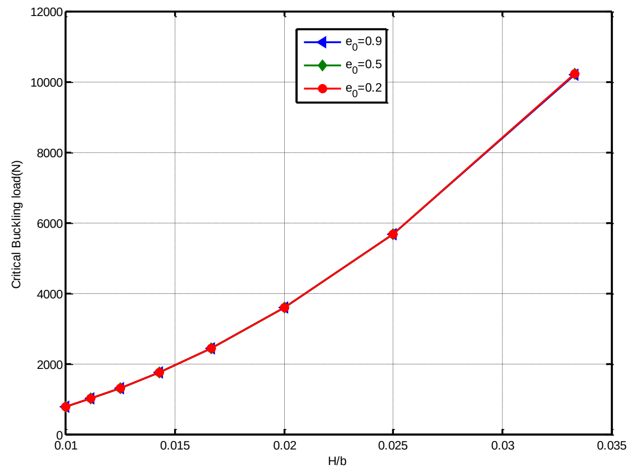
where

$$R^K(r) = (1-\frac{r}{a})^n (1-\frac{r}{b})^m \tag{46}$$

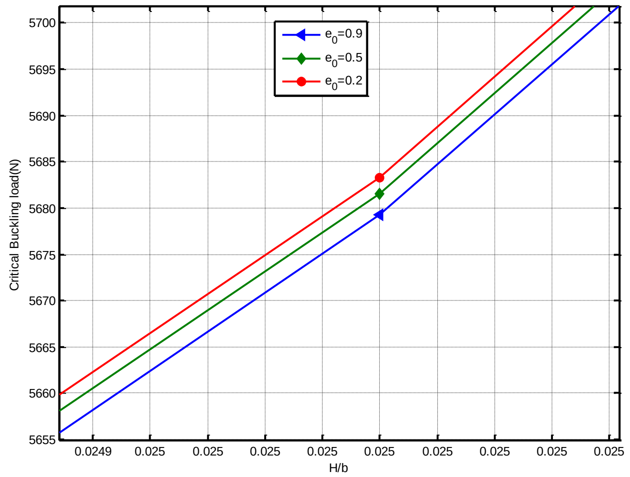
4. Numerical results and discussions

In this section, the buckling of micro sandwich hollow circular plate with the consideration of the porous core and piezoelectric layer reinforced by FG carbon nano-tube is studied. The effects of various parameters such porous coefficients, small length scale parameter, distribution of carbon nano-tube in piezoelectric layers and temperature on the critical buckling load are investigated.

4.1 Validations of results



(a)



(b)

Fig. 3 The effect of porous coefficients on critical buckling load of micro sandwich hollow circular plate H=0.3 mm, $V_{CNT}=V^*_{CNT}$, T_1=300 K, a=b/4, L=17.6 μm for (a) H/b=0.01-.035 (b) H/b=

To validate the analysis, the results for circular plate are compared with (Eslami *et al*. 2016) see Fig. 2. The comparison shows that the present results have a good agreement with those in the literature.

4.2 Buckling analysis of micro sandwich hollow circular plate

For analyzing the influence of different parameters on critical buckling load of micro sandwich hollow circular plate is plotted in Figs. 3-7 as follows. In Fig. 3 we can see the effects of porous coefficients on critical buckling load of micro sandwich hollow circular plate. With increasing porous coefficients, the stiffness of micro sandwich hollow circular plate decreases so that the critical buckling load of system decreases. Fig. 4 demonstrates the influence of small length scale parameter, from this figure could be found by increasing this parameter, the critical buckling load of micro sandwich hollow circular plate increases. Fig. 5 shows the effect of temperature on critical buckling load with respect to (H/b) of micro sandwich. With increasing

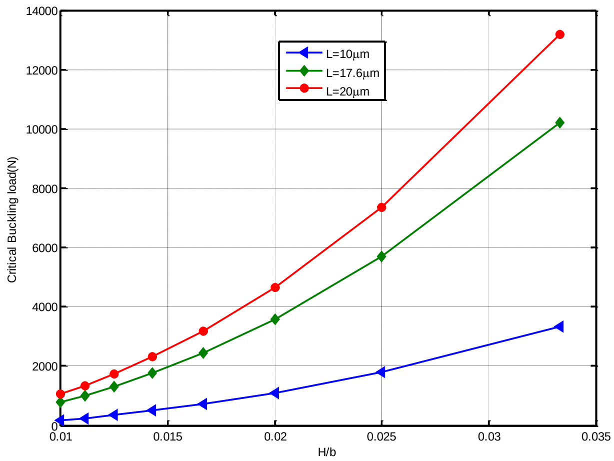


Fig. 4 The effect of small length scale parameter on critical buckling load of micro sandwich hollow circular plate (H=0.3 mm, V_{CNT}=V^*_{CNT}, T_1=300 K, a=b/4, L=17.6 μm)

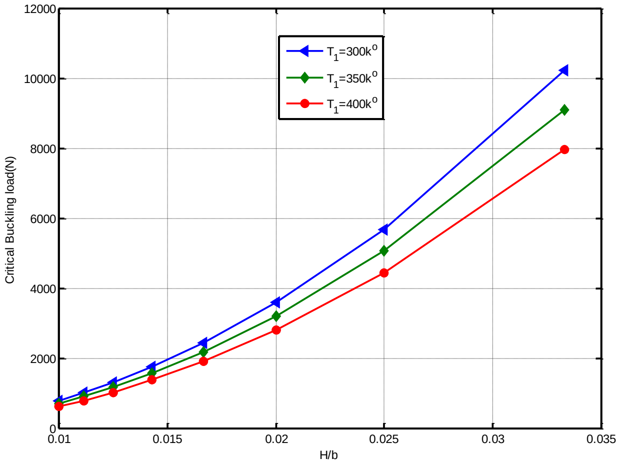


Fig. 5 The effect of temperature on critical buckling load of micro sandwich hollow circular plate (H=0.3 mm, V_{CNT}= V^*_{CNT}, T_1=300 K, a=b/4, L=17.6 μm)

temperature, the stiffness of micro structure decreases so that the critical buckling load of system decreases. The influence distribution of carbon nano-tube in piezoelectric layers is indicated in Fig. 6. By increasing (*H/b*) the critical buckling load of system decreases in FG-V case, vice versa in FG-O and UD distribution increases.

5. Conclusions

In this paper, the buckling analysis of micro sandwich hollow circular plate with the consideration of the porous core and piezoelectric layer reinforced by FG carbon nano-tube is studied. By applying the principle of minimum potential energy, high-order shears deformation theory and modified couple stress theory, the governing equations of equilibrium for micro sandwich hollow circular plate were derived. The Ritz method type solution is used to obtain the critical buckling load of micro sandwich hollow circular plate. The accuracy of the analytical method is more than the numerical method and also requires less time to solve,

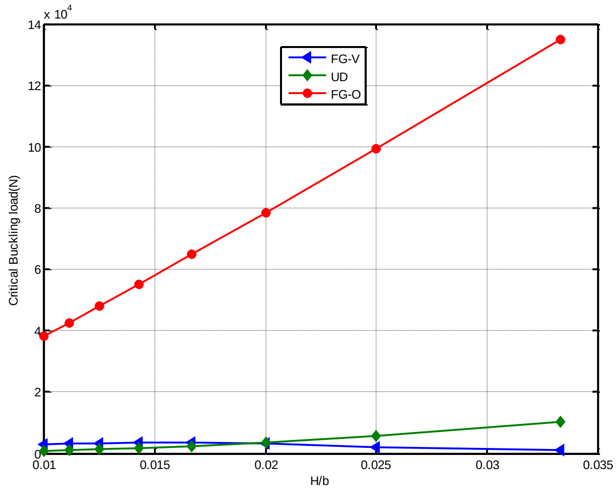


Fig. 6 The effect of distribution of carbon nano-tube in piezoelectric layers on critical buckling load of micro sandwich hollow circular plate (H=0.3 mm, V_{CNT}=V^*_{CNT}, T_1=300 K, a=b/4, L=17.6 μm)

thus the analytical method has been used, also, it is well known that analytical approaches are quite limited to practice. The results indicate that by increasing the porous coefficient of core the critical buckling of system decreases. Also, by increasing the temperature change, the stiffness of system and critical buckling decreases. Moreover, it is seen that the critical buckling load increases with an increase in the material length scale parameter. By increasing (H/b) the critical buckling load of system decreases in FG-V case, vice versa in FG-O and UD distribution increases. The results of this research can be used for optimization of micro-structures and manufacturing different structure in aircraft and aerospace.

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