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Development of an analytical method for optimum design of reinforced concrete beams considering both flexural and shear effects

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Abstract. Optimization is an important subject which is widely used in engineering problems. In this paper, an analytical method is developed for optimum design of reinforced concrete beams considering both flexural and shear effects. A closed-form formulation is derived for optimal height and rebar of beams. The total material cost of steel and concrete is considered as the objective function which is minimized during the optimization process. The ultimate flexural and shear capacities of the beam are considered as the main constraints. The ultimate limit state is considered for deriving the relations for flexural capacity of the beam. The design requirements are considered according to the item 9 of the Iranian National Building. Analytical formulas and some curves are proposed to be used for optimum design of RC beams. The proposed method can be used to perform the optimization of RC beams without the need of any prior knowledge in optimization. Also, the results of the studied numerical example show that the proposed method results in a better design comparing with the other methods.

Keywords: RC beam; optimization; lagrange multipliers method; LMM; INBR9

1. Introduction

Reinforced concrete (RC) is now widely used in a variety of structures owing to its versatility, high compressive strength, durability and resistance to fire and water damages. The wide usage of concrete structures increases the demand for economical and optimum design. In the structural optimization, the aim usually is to find the design parameters in such a way that the cost function be minimized and the design requirements get satisfied. Although structural optimization initially was developed during World War II for minimum weight design of the aircrafts (Cox and Smith 1943), it is still one of the most important research areas. Some of the recent important researches on the optimum design of concrete structures are as the following.

Mergos (2016) studied the optimum seismic design of reinforced concrete frames according to Eurocode 8 (EC 8) and *fib* Model Code 2010 (MC2010). Genetic algorithm (GA) was used to perform optimization and the optimum results obtained according to EC 8, and MC 2010 were compared. It was concluded that MC2010 provides enhanced structural damage control in RC frames.

Habibi *et al*. (2016) developed optimum design curves for RC beams according to INBR9 regulations. Lagrange Multipliers Method (LMM) was utilized to obtain closed-form solutions for design parameters. Minimization of concrete and steel material cost were considered as the objective of the optimization, while the ultimate flexural capacity of the beam was considered as the main constraint.

Gharebaghi *et al*. (2016) proposed a method for design optimization of RC frames subjected to earthquake loading. The method was an automated design procedure consisting of a Tree Classification Method (TCM) and a real-valued model of Particle Swarm Optimization (PSO).

Nigdeli and Bekdaş (2017), studied the Optimum design of RC continuous beams considering unfavorable live-load distributions. A Random Search Technique (RST) was utilized to find optimum cross-section dimensions and reinforcement of continuous RC beams. Design constraints were considered according to ACI 318.

Fedghouche (2017) studied the cost optimization of doubly reinforced high strength concrete (HSC) T-beams. The considered cost function consisted of HSC, rebar, and formwork costs. Also, the constraints were considered in accordance with Eurocode 2 (EC-2). The optimization problem solved using Generalized Reduced Gradient algorithm.

Chutani and Singh (2017) developed a method for design optimization of RC beams using Particle Swarm Optimization (PSO). The total material cost of concrete and rebars were considered as the objective function, while the main constraints were moment capacity, lateral stability, and deflection according to Indian Standard (IS).

Alghamdi and Ahmad (2018), developed an optimum design methodology for RC beams and columns exposed to chloride. An Excel program was developed to achieve structural durability of beams for desired service life and corrosive environment.

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Esfandiari *et al*. (2018), studied the Optimum design of 3D reinforced concrete frames using a hybrid algorithm called DMPSO. The hybrid algorithm was constructed by combining multi-criterion Decision Making (DM) and Particle Swarm Optimization (PSO) algorithms.

Uz *et al*. (2018) studied layout optimization of multi-span RC beams subjected to dynamic loading. Charged system search (CSS) algorithm was used to find optimum span ratios in continuous beams with several spans.

Gharebaghi *et al*. (2018) studied the optimum seismic design of reinforced concrete frame structures. The proposed procedure aimed to minimize construction cost and uniform damage distribution over the height of structures. The results showed that the proposed method is capable of resulting in designs with less damage under severe earthquakes.

Silva and Amilton (2018) studied the optimization of reinforced concrete polygonal sections under biaxial bending with axial force. The considered variables were location, diameter and number of steel rebars in the section. Sequential Linear Programming (SLP) algorithm was used as the optimization technique to minimize the rebar area of the section.

Shariat *et al*. (2018) studied design optimization and sensitivity analysis of RC beams using Lagrangian Multipliers Method (LMM). Total material cost of steel and concrete was considered as the objective function and flexural capacity of beams as the main constraint. By performing sensitivity analysis, the optimum results obtained according to three design codes of ACI, BS, and Iranian concrete standard (ICS) were compared.

The Lagrange Multipliers Methods (LMMs) have been successfully used in constrained engineering problems (Arora *et al*. 1994). In the LMMs the constrained problem is transformed to an unconstrained one and the final solution is obtained through a series of unconstrained optimization sub-problems. This method has been successfully applied in the optimization of singly and doubly RC beams (Habibi *et al*. 2016, Shariat *et al*. 2018). Ceranic and Fryer (2000) applied LMMs in the optimization of RC beams in accordance with British Standard requirements. Barros *et al*. (2005) utilized LMMs for design optimization of singly and doubly reinforced concrete beams based on the EC2-2001 design criteria. Also, in some researches the LMMs are combined with the other optimization techniques. For instance, Adamu *et al*. (1994) proposed an application of Continuum-type Optimality Criteria (COC) method for optimum design of RC beams, where the minimality conditions were derived using the Augmented Lagrangian method. The main advantage of using LMM in the design optimization of RC beams is that analytical equations can be developed to obtain optimum values of the design parameters. These analytical equations can then be used by practicing engineers easily.

The main objective of the present study is to develop an analytical approach based on LMM for optimum design of RC beams, in accordance with the INBR9 criteria. In the present study, both shear and flexural capacities are considered as the main constraints. This is the main difference in the optimization model of the present study with the previous works done by Habibi *et al*. (2016) and Shariat *et al*. (2018), where only the flexural capacity had been considered. Considering shear capacity as a constraint leads to have a better optimum design formulation because, in practical designs, the tensile reinforcements are used together with the shear reinforcements. As it is shown through the paper, this assumption will reduce the ratio of reinforcements (ρ), and this will result in lower costs. The optimum design equations and curves achieved in this study can be used for the minimum cost design of RC beams without the need of any prior knowledge of optimization.

2. Proposed method

In the present work, the LMM method is employed to obtain the analytical formulation for optimum design of RC beams. The objective is to minimize the total material cost of the RC beams. In general, the objective function can be written as below

$$Z = f(x_1, x_2, x_3, \dots, x_n) \tag{1}$$

Subject to constraints

$$g_i(x_1, x_2, x_3, \dots, x_n) \leq 0 \qquad i = 1, 2, \dots, P \tag{2}$$

Where $\boldsymbol{x_1}$ to $\boldsymbol{x_n}$ are the design variables, and $\boldsymbol{g_1}$ to $\boldsymbol{g_p}$ are the constraints.

In this study, the main design constraints of the problem are assumed to be active. Accordingly, the unconstrained Lagrangian function can be written as follows

$$\begin{aligned} & L\,(x_1, x_2, \dots, x_n, \lambda_1, \lambda_2, \dots, \lambda_P) \\ & = f(x_1, x_2, \dots, x_n) + \sum_{i=1}^{P} \lambda_i g_i(x_1, x_2, \dots, x_n) \end{aligned} \tag{3}$$

Where $\boldsymbol{\lambda_i}$ are the Lagrange multipliers. The necessary conditions can be written as follows

$$\frac{\partial L}{\partial x_k} = \frac{\partial f}{\partial x_k} + \sum_{i=1}^{P} \lambda_i \frac{\partial g_i}{\partial x_k} = 0 \quad k = 1, 2, \dots, n \tag{4}$$

$$\frac{\partial L}{\partial \lambda_i} = g_i = 0 \qquad i = 1,2, \dots, P \tag{5}$$

Eqs. (4) and (5) taken together form a system of $\boldsymbol{n+p}$ equations in $\boldsymbol{n+p}$ unknowns and its solution will yield stationary values for $\boldsymbol{x_1, x_2}, \dots, \boldsymbol{x_n}$ and $\boldsymbol{\lambda_1, \lambda_2}, \dots, \boldsymbol{\lambda_p}$ from which an optimum design can be achieved.

In this study, the effects of shear force ($\boldsymbol{Vu}$) and bending moment ($\boldsymbol{Mu}$) are simultaneously considered. As mentioned in the previous section, first, a cost function is defined as the objective function, which will be minimized during the optimization process. The shear and the bending moment capacity are considered as the main constraints. Finally, according to Eq. (3) the Lagrangian function can be developed to obtain the minimum cost solution through Eqs. (4) and (5).

All of these steps are performed as explained in the following:

The total cost per unit length of a beam depends on its material cost, geometry and reinforcement area. In the present study the cost function is calculated as follows

$$c = (c_s + c_{sv} + c_c)L \tag{6a}$$

$$c = \left[c_s\left(\rho bd + \frac{A_{sv}(2b + 2d + 4d')L}{s}\right) + c_c(bh)\right]L \tag{6b}$$

where $\boldsymbol{C}$ is the total material cost of the beam, $\boldsymbol{C_s}$ is the cost of the longitudinal reinforcements, $\boldsymbol{C_{sv}}$ is the cost of the transverse reinforcements (stirrups), $\boldsymbol{C_c}$ is the cost of concrete, and $\boldsymbol{L}$ is the length of the beam. Also, $\boldsymbol{b}$ and $\boldsymbol{d}$ are the width and the effective depth of the given beam. $\boldsymbol{\rho}$ is the reinforcement ratio which equals to $\boldsymbol{As/bd}$, and $\boldsymbol{As}$ is the tensile rebar area. $\boldsymbol{A_{sv}}$ is the transverse reinforcement area and $\boldsymbol{s}$ is the distance between the shear reinforcements which is obtained from Eq. (7b). In the present study, it is assumed that $\boldsymbol{b}$ and concrete cover ratio $\boldsymbol{r}$ are constant and do not change during the design process. It is obvious that the length of each shear reinforcement (stirrup) equals to $(\boldsymbol{2b + 2d + 4d'})$.

The material cost ratio can be introduced as $\boldsymbol{q} = \boldsymbol{C_c} / \boldsymbol{C_s}$, where $\boldsymbol{C_c}$ and $\boldsymbol{C_s}$ are concrete and steel cost per unit volume, respectively. The cost objective function of the beam can be defined as Eq. (6c), where $\boldsymbol{r}$ is concrete cover ratio with respect to the effective depth of the beam $\boldsymbol{d}$.

$$c = c_s\left(\rho bd + \frac{A_{sv}(2b + 2d + 4d')}{s} + (1 + r)bdq\right)L \tag{6c}$$

The ultimate shear force is considered equal to the shear resistance of the beam as follows

$$V_u = V_r = v_c + v_s \tag{7a}$$

Where $\boldsymbol{v_s}$ and $\boldsymbol{v_c}$ are shear strengths that $\boldsymbol{v_s}$ is supplied by stirrups and $\boldsymbol{v_c}$ is supplied by concrete. These strengths can be calculated using the INBR9 equations as follows

$$v_s = dA_{sv}f_{yd}/s,\ \ v_c = 0.2bd\phi_c\sqrt{f_c} \tag{7b}$$

Also the distance between the stirrups can be calculated using the Eq. (7c)

$$V_u = \left(dA_{sv}f_{yd}/s\right) + 0.2bd\phi_c\sqrt{f_c} \ \rightarrow\ s = \frac{dA_{sv}f_{yd}}{V_u - 0.2bd\phi_c\sqrt{f_c}} \tag{7c}$$

By substituting Eq. (7c) in Eq. (6c), the following equation can be obtained.

$$c = \frac{1}{f_{yd}}(c_s b)\Big[\rho df_{yd} + \frac{2V_u}{b} + \frac{2V_u}{d} - \frac{4V_u r}{b} + 0.4\phi_c\sqrt{f_c}d(2r - 1) - 0.4\phi_c\sqrt{f_c}b + (1 + r)dqf_{yd}\Big]L \tag{8}$$

Where $\frac{1}{f_{yd}}(\boldsymbol{c_s b})$ has been considered to be a constant (one). Similar equation can be defined for total cost of a Doubly Reinforced Beam (DRB) per unit length using Eq. (9), where length of the beam is assumed to be one (*L*=1).

$$c' = \Big[\rho df_{yd} + \frac{2V_u}{b} + \frac{2V_u}{d} - \frac{4V_u r}{b} + 0.4\phi_c \sqrt{f_c}d(2r - 1) - 0.4\phi_c\sqrt{f_c}b + (1 + r)dqf_{yd}\Big] \tag{9}$$

Then, the ultimate flexural capacity equation is derived which is considered as a constraint. The ultimate limit state

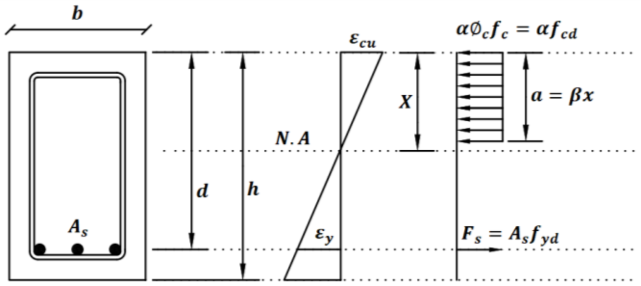

Fig. 1 Reinforced section with rectangular stress block based on the INBR

methodology with the equivalent concrete stress block is used (Fig. 1). The partial strength reduction factors for steel and concrete are considered based on the INBR9. By taking moment about the rebar's axis the flexural capacity of the beam can be obtained as Eq. (10).

$$\frac{M_u}{bd^2} = \rho f_{yd}\left(1 - \frac{\rho f_{yd}}{2\alpha f_{cd}}\right) \tag{10}$$

Where $\boldsymbol{M_u}$ is the ultimate applied moment, $\boldsymbol{f_{yd}}$ is reduced yield strength of steel which equals to $\boldsymbol{\emptyset_y f_y}$, and, $\boldsymbol{f_{cd}}$ is the reduced compressive strength of concrete which equals to $\boldsymbol{\emptyset_c f_c}$. According to INBR9 $\boldsymbol{\emptyset_y}$ and $\boldsymbol{\emptyset_c}$ are taken as 0.85 and 0.65, respectively. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the coefficients which are used to define the equivalent concrete stress block. According to INBR 9 these coefficients are calculated as $\alpha = 0.85 - 0.0015fc$, $\beta = 0.97 - 0.0025fc$ where $\boldsymbol{fc}$ is the characteristic strength of concrete.

The final step includes formulating the Lagrangian function of the problem and solving it to obtain the optimum solution. Using Eq. (3) and based on the proposed method the unconstrained problem can be defined as follows

$$L = \rho df_{yd} + \frac{2V_u}{b} + \frac{2V_u}{d} - \frac{4V_u r}{b} + 0.4\phi_c\sqrt{f_c}d(2r - 1) - 0.4\phi_c\sqrt{f_c}b + (1 + r)dqf_{yd} + \lambda\left[M_u - \rho bd^2 f_{yd}\left(1 - \frac{\rho f_{yd}}{2\alpha f_{cd}}\right)\right] \tag{11}$$

Eq. (11) is the Lagrangian function for as Singly Reinforced Beam (SRB). By taking partial derivatives of the Lagrangian function and equating to zero, the optimum reinforcement ratio will be obtained

$$\frac{\partial L}{\partial d} = 0 \rightarrow \rho f_{yd} - \frac{2V_u}{d^2} + 0.4\phi_c\sqrt{f_c}(2r - 1) + (1 + r)qf_{yd} - \lambda\left(2\rho bdf_{yd}\left(1 - \frac{\rho f_{yd}}{2\alpha f_{cd}}\right)\right) = 0 \tag{11a}$$

$$\frac{\partial L}{\partial \rho} = 0 \longrightarrow df_{yd} - \lambda\left(bd^2 f_{yd} + \frac{\rho bd^2 {f_{yd}}^2}{\alpha f_{cd}}\right) = 0 \tag{11b}$$

$$\frac{\partial L}{\partial \lambda} = 0 \longrightarrow$$
$$M_u - \rho b d^2 f_{yd}\left(1 - \frac{\rho f_{yd}}{2\alpha f_{cd}}\right) = 0 \tag{11c}$$

By solving these three equations as a simultaneous system of equations, the optimum reinforcement ratio is obtained as follows

$$\rho_{\,opt} = \rho_1 - \rho_2 + \rho_3 \tag{12}$$

where

$$\rho_1 = \frac{\alpha f_{cd}}{f_{yd}} \tag{12a}$$

$$\begin{gathered}\rho_2 = (0.4199(-9b^2\alpha^2 f_{cd}^2 f_{yd}^4 v_u^2 \\ +3b\alpha f_{cd} f_{yd}^4 v_u(\alpha f_{cd} m_u + q f_{yd} m_u + q r f_{yd} m_u \\ +2b\alpha f_{cd} v_u - 0.4\sqrt{f_c} m_u \phi_c + 0.8r\sqrt{f_c} m_u \phi_c)))/ \\ (b f_{yd}^3 v_u(-27b^2\alpha^3 f_{cd}^3 f_{yd}^6 m_u v_u^2 + A^*)^{1/3})\end{gathered} \tag{12b}$$

$$\rho_3 = \frac{0.2645(-27b^2\alpha^3 f_{cd}^3 f_{yd}^6 m_u v_u^2 + A^*)^{1/3}}{b f_{yd}^3 v_u} \tag{12c}$$

In the above equations, A^* equals to

$$\begin{gathered}A^* = \sqrt{(}729b^4\alpha^6 f_{cd}^6 f_{dy}^{12} m_u^2 v_u^4 \\ +4(-9b^2\alpha^2 f_{cd}^2 f_{dy}^4 v_u^2 + 3b\alpha f_{cd} f_{dy}^4 v_u[\alpha f_{cd} m_u \\ +q f_{dy} m_u + q r f_{dy} m_u + 2b\alpha f_{cd} v_u \\ -0.4\sqrt{f_c} m_u \phi_c + 0.8r\sqrt{f_c} m_u \phi_c])^3)\end{gathered} \tag{12d}$$

Also, the optimal effective depth can be obtained using Eq. (13)

$$d_{\,opt} = \sqrt{\frac{M_u}{\rho_{\,opt}\;\; f_{yd} b\left(1 - \frac{\rho_{\,opt}\;\, f_{yd}}{2\alpha f_{cd}}\right)}} \tag{13}$$

It is necessary to define the maximum tensile reinforcement $\boldsymbol{\rho_{max}}$, i.e., when the reinforcement ratio is greater than $\boldsymbol{\rho_{max}}$, the double reinforced design is required. In accordance with the INBR9 the maximum tensile reinforcement ratio is limited to lowest value of $\boldsymbol{\rho_b}$ and 0.025, where $\boldsymbol{\rho_b}$ is the balanced reinforcement ratio

The balanced reinforcement ratio is obtained by Eq. (14a). The compressive reinforcement ratio must be bounded by the allowable minimum reinforcement ratio as per INBR9 regulations (Eq. (14b)).

$$\rho_b = \frac{\alpha f_{cd}}{f_{yd}}\left(\frac{700\beta}{700 + f_y}\right) \tag{14a}$$

$$\rho_{min} = \max\left\{\frac{1.4}{f_y}, \frac{0.25\sqrt{f_c}}{f_y}\right\} \tag{14b}$$

In a continuous beam with distributed loading, by increasing the length of the beam, the ratio of $\boldsymbol{M_u/V_u}$ will also increase. Consequently, the ratio of $\boldsymbol{\rho_{\,opt}}$ will be increased to compensate the bending capacity of the beam. This is shown in Figs. 2 to 7. For deriving these curves, the width of the beam is considered as 0.3 meters. In these figures, $\boldsymbol{\rho_{min}}$ and $\boldsymbol{\rho_{max}}$ are the lower and upper bound limits for $\boldsymbol{\rho}$ where have been calculated according to

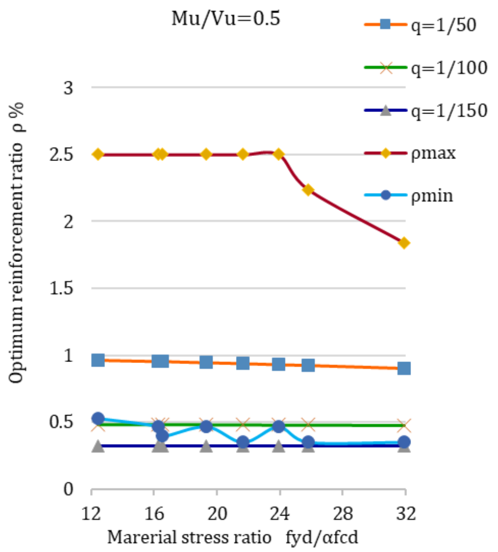

Fig. 2 Optimum tensile reinforcement ratio for *Mu/Vu*=0.5

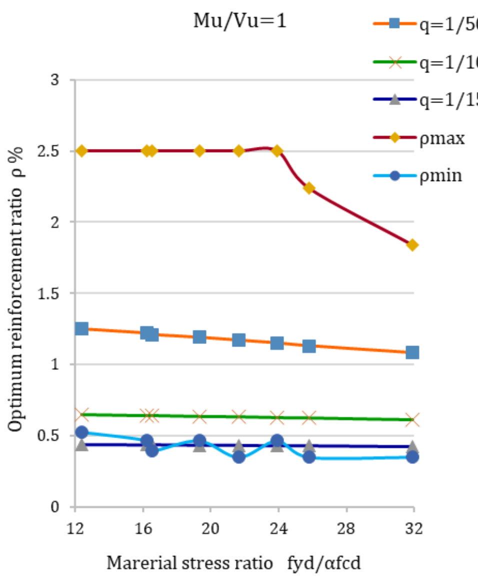

Fig. 3 Optimum tensile reinforcement ratio for *Mu/Vu*=1

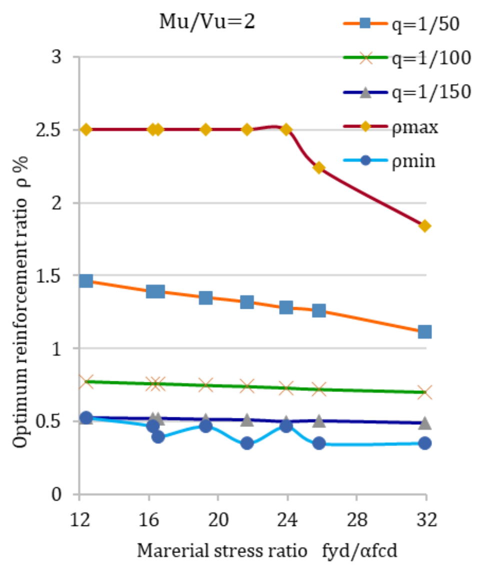

Fig. 4 Optimum tensile reinforcement ratio for *Mu/Vu*=2

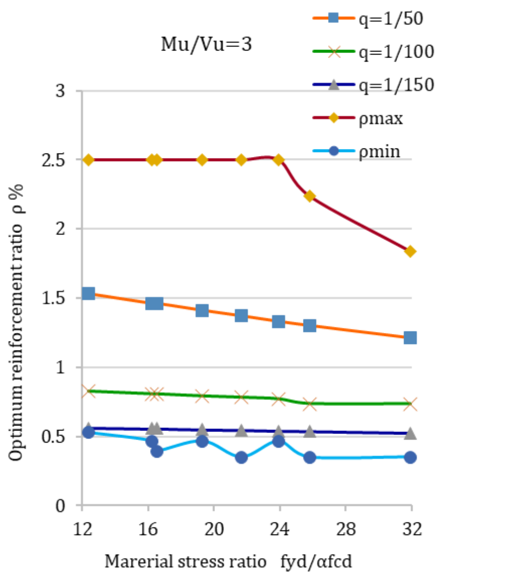

Fig. 5 Optimum tensile reinforcement ratio for *Mu/Vu*=3

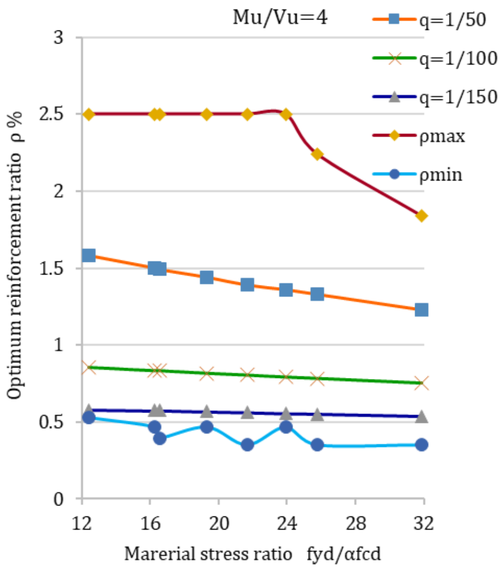

Fig. 6 Optimum tensile reinforcement ratio for *Mu/Vu*=4

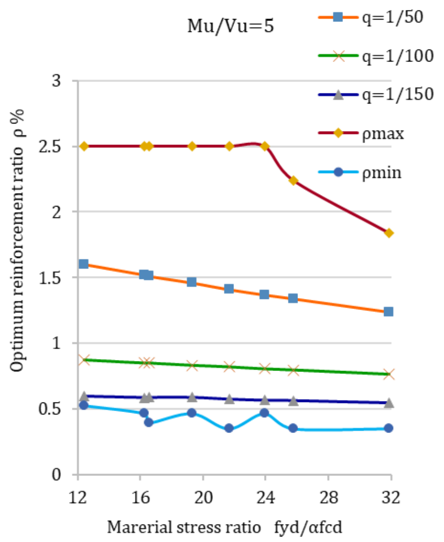

Fig. 7 Optimum tensile reinforcement ratio for *Mu/Vu*=5

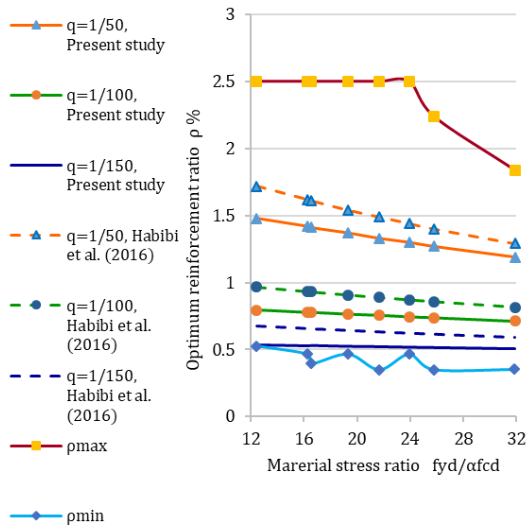

Fig. 8 Optimum tensile reinforcement ratio for reinforced concrete beams with and without stirrups

INBR9. It should be noted that these limitations are side constraints of the problem and the main constraints are the flexural and shear capacities of the beam. Similar graphs can be drawn for different values of $\boldsymbol{r}$, $\boldsymbol{b}$, $\boldsymbol{M_u}$ and $\boldsymbol{V_u}$.

For ratios larger than $\boldsymbol{M_u/V_u = 5}$, the diagram does not have tangible changes. For $\boldsymbol{\rho}$ smaller than $\boldsymbol{\rho_{min}}$, $\boldsymbol{\rho_{min}}$ is used to solve the problem.

It is observed that increasing the ratio of material cost $\boldsymbol{q = Cc\,/\,Cs}$ (i.e., increasing the cost of concrete or reducing the cost of steel) results in increasing the optimum reinforcement ratio. The reason for this, is that when the cost of concrete is increased, the cost function must remain minimum, thus, the effective depth will be reduced to decrease the concrete volume. This will lead to reducing of the bending strength of the beam. To compensate this decrease in the beam capacity, the amount of tensile rebar (reinforcement ratio) should be increased.

For different ratios of material cost and materials stress, a comparison has been made between the results obtained using the developed method of the present study, and the method developed by Habibi *et al*. (2016). This comparison is shown in Fig. 8. It should be noted that in both methods, the ratio $\boldsymbol{r}$ is taken as 0.1 for drawing the curves shown in Fig. 8.

According to Fig 8, it is obvious that the reinforcement ratio obtained using the developed method of the present study is lower than the ratio obtained using the method developed by Habibi *et al*. (2016). The existence of the parameter $[-\frac{2V_u}{d^2}+0.4\phi_c\sqrt{f_c}(2r-1)]$ in Eq (11a) has led to an increase in effective depth and a decrease in reinforcement ratio. This parameter enters into calculations when the shear constraint is considered.

3. Numerical example

A numerical example is considered to evaluate the

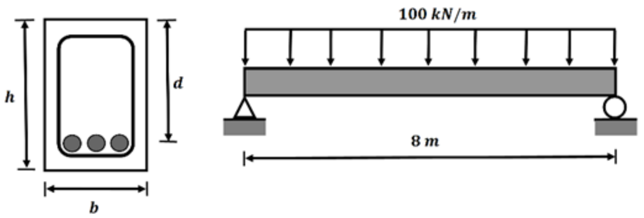

Fig. 9 The given RC beam

Table 1 Design results using optimal and conventional methods

Design number	Effective depth (mm)	Area of tensile reinforcement (mm^2)	Area of stirrups in a meter (mm^2)	Total material costs ($/$m^3$)
1 (Conventional)	1100	2422.6786	997.7107	0.7734×Cc
2 (Habibi *et al*. 2016)	1116.6	2375.9832	975.4588	0.7706×Cc
3 (Conventional)	1200	2169.3174	869.0540	0.7606×Cc
4 (Conventional)	1300	1968.3880	747.3843	0.7549×Cc
5 (Present Study)	**1363.2**	**1860.9806**	**673.4388**	**0.7540**×Cc
6 (Conventional)	1400	1804.2815	631.3689	0.7543×Cc
7 (Conventional)	1500	1667.1224	519.8822	0.7574×Cc
8 (Conventional)	1600	1550.4555	412.0704	0.7635×Cc

proposed relations for optimum design of concrete beams accounting for flexural and shear effects. In this example, a rectangular RC beam with 300 mm width and 8 meters in length, as shown in Fig. 9, is subjected to an ultimate distributed load of 100 kN/m.

The ratio $\boldsymbol{r}$ is taken as 0.10, and the material cost ratio $\boldsymbol{q}$ is assumed as 1/150. Characteristic strength of concrete and steel yield stress are considered to be 20 and 400 MPa, respectively. According to the considered strengths for concrete and steel and using the INBR9 relations the values of $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ are calculated as 0.895 and 0.82, respectively. Also, $\boldsymbol{f_{cd}}$ and $\boldsymbol{f_{yd}}$ are obtained as 13 and 340 MPa, respectively. The maximum ultimate bending moment and the maximum effective ultimate shear force acting on the beam are assumed as 800 $\mathrm{kN.m}$ and 350 kN, respectively. By using Eq. (12), the optimal tensile reinforcement ratio is obtained as 0.0045. From Eq. (13a), the corresponding optimum effective depth is obtained as 1363.2 mm and the required rebar area is obtained as 1860.98 mm^2. The minimum material cost of the beam per unit length is also obtained from Eq. (8) as $0.754C_c$.

The design results obtained using the proposed method are compared with the results obtained using conventional design procedure, and the method proposed by Habibi *et al*. (2016). This comparison is shown in Table 1. In this table, design number 5 is obtained using the proposed method, design number 2 is obtained using the method proposed by Habibi *et al*. (2016), and the other designs in the table are obtained by conventional design procedure. The material cost results of Table 1 are also plotted in Fig. 10. According to Table 1 and Fig. 10, material costs of all designs including conventional method and optimum method developed by Habibi *et al*. (2016) are higher than those obtained using the proposed method of the present study.

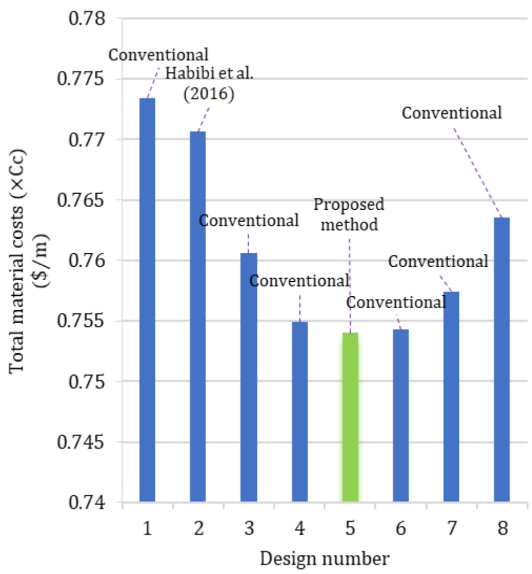

Fig. 10 Comparison between the total material costs of the RC beam

4. Conclusions

In this research, an analytical method was proposed and developed to obtain the minimum cost design of RC beams considering both flexural and shear effects. In the optimum design formulation, the total costs were considered as the sum of concrete, reinforcement and stirrups' costs. Cost of formwork and labor were not considered in the formulation. Moment and shear capacities of RC beam were considered as the main constraints. The minimum cost design was found using an LMM based method**.** The results were presented in closed form relations which can be used easily by practicing engineers. Also, graphical representations of the results were provided for better comprehension of the optimization process. Considering the shear capacity as a constraint made the $\boldsymbol{\rho_{opt}}$ equation more complicated and thus deriving the graphs from the analytical solution was not easy. To overcome this issue, by considering different ratios for $\boldsymbol{M_u/V_u}$, various graphs have been plotted. The results of this study show that by using the developed method and considering the shear capacity as a constraint, the optimum reinforcement ratio $(\boldsymbol{\rho})$ will be reduced. Consequently, this will result in lower material costs.

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