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An efficient robust cost optimization procedure for rice husk ash concrete mix

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Abstract. As rice husk ash (RHA) is not produced in controlled manufacturing process like cement, its properties vary significantly even within the same lot. In fact, properties of Rice Husk Ash Based Concrete (RHABC) are largely dictated by uncertainty leading to huge deviations from their expected values. This paper proposes a Robust Cost Optimization (RCO) procedure for RHABC, which minimizes such unwanted deviation due to uncertainty and provides guarantee of achieving desired strength and workability with least possible cost. The RCO simultaneously minimizes cost of RHABC production and its deviation considering feasibility of attaining desired strength and workability in presence of uncertainty. RHA related properties have been modeled as uncertain-but-bounded type as associated probability density function is not available. Metamodeling technique is adopted in this work for generating explicit expressions of constraint functions required for formulation of RCO. In doing so, the Moving Least Squares Method is explored in place of conventional Least Square Method (LSM) to ensure accuracy of the RCO. The efficiency by the proposed MLSM based RCO is validated by experimental studies. The error by the LSM and accuracy by the MLSM predictions are clearly envisaged from the test results. The experimental results show good agreement with the proposed MLSM based RCO predicted mix properties. The present RCO procedure yields RHABC mixes which is almost insensitive to uncertainty (i.e., robust solution) with nominal deviation from experimental mean values. At the same time, desired reliability of satisfying the constraints is achieved with marginal increment in cost.

Keywords: robust cost optimization; rice husk ash based concrete; moving least squares method; uncertainty; metamodeling

1. Introduction

The priority agenda in concrete industry is to produce green concrete for the sake of environmental sustainability and economy (Suhendra 2014, Oti and Kinuthia 2015). In the country like India, where rice is considered to be main food crop with yearly rice production as high as 100 million MT, the RHABC provides a way of producing green concrete with solution to low cost housing. The main obstacles of wide use of RHABC are its severe batch to batch property variation produced even with same RHA source; sensitivity of RHABC to small variations in the constituent materials, mix proportions and other external factors; and also the absence of mix design guidelines in many countries including India.

The RHA is a material that is generally obtained as a waste product from rice mills. Property of RHA obtained from same rice mill varies significantly. Ewa *et al*. (2018) reported severe batch to batch variation of properties of RHA even after collecting RHA from same source. The similar variation is also observed from the research of de Sensale and Viacava (2018) on blended Portland cements containing residual RHA and limestone filler. This has been also verified in the present test program. 40 batches of RHABC cubes have been prepared of grade M25 from the same ingredients and mix constituents and tested for their compressive strength. The result is shown in Fig. 1. It can be clearly observed that there is significant batch to batch variation of RHABC even with same source of RHA. Moreover, distribution of the results are not normal, rather show non-uniform distribution. There remains 21% chance due to variability of RHA property that the compressive strength may fall below 20 MPa. Thus, application of some uncertainty measure in designing the mix that will make it robust to variation due to uncertainty is needed. Since, reducing uncertainty in this case is either impractical or add too much cost of production of RHABC, a mathematical optimization technique which considers uncertainty effect is supposed to be more viable.

The object of mix design is to attain desired strength and workability with minimum cost of production considering various uncertainty effects. The added problem of mix design of RHABC is its sensitivity to uncertainty related to property variation of ingredients, especially the RHA. This is not a pertinent problem for ordinary cement concrete, since properties of cement can be well tested and cement is produced in factories maintaining standard stipulations that render the uncertainty related to property of cement a

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minimal. Thus, in absence of well-defined data related to RHA and its influence on other ingredients, the problem of mix design of RHABC can be better investigated as an optimization problem under uncertainty.

Conventional Reliability Based Design Optimization (RBDO) approach for dealing optimization under uncertainty as expressed by Abbasnia *et al*. (2014) brings specified target reliability of desired performance; but the design may be still sensitive to input parameter variations. Moreover, when the probability density function is unavailable and stochastic parameters are modeled as uncertain but bounded type, Robust Optimization (RO) becomes an attractive alternate to RBDO. The RO is fundamentally concerned with minimizing the effect of uncertainty in the uncertain parameters to the variation of performance function and constraints. The RO has been successfully implemented in the recent past for stochastic mechanical systems (Beyer and Sendhoff 2007, Cheng *et al*. 2017). Thus, in the present paper, the RDO is applied for cost optimization of RHABC mix under uncertainty which is abbreviated here as Robust Cost Optimization (RCO) approach. The RCO simultaneously minimizes cost and standard deviation of cost in producing RHABC. At the same time, the guarantee of getting the required strength and workability under uncertainty is ensured by adding suitable penalty term(s) to the deterministic constraint(s) and then satisfying target reliability criteria. The RCO is supposed to yield a mix that will be least sensitive to the input parameter variations without reducing the sources of uncertainty. Nunes *et al*. (2013) presented robust mix design approach for self-compacting concrete. However, such study on RHABC has not yet been observed in the existing literature. But, severe batch-to-batch property variation due to material uncertainty is generally observed in case of RHABC, which implies the necessity of RCO study with RHABC and constitutes the objective of this paper.

It may be noted here that the RCO requires objective function and constraints in explicit functional form. Thus, a Response Surface Method (RSM) based metamodeling strategy is adopted in the present study. Nunes *et al*. (2013) applied polynomial RSM to approximate property for mix design of self-compacting concrete. Gazder *et al*. (2017) applied artificial Neural Network (ANN) to predict compressive strength of blended cement. Ozturk *et al*. (2018) applied ANN to approximate compressive strength of alkali-activated electric arc furnace slag. It is realized that a polynomial RSM will be particularly suitable for RCO, as this approach requires several repetitive evaluation of gradients of constraint function, which can be easily accomplished by the RSM. The RSM uncovers explicit functional relationship of compressive strength (or workability) as function of ingredient quantities. The conventional RSM is pivoted on the concept of Least Squares Method (LSM). In fact, Nunes *et al*. (2013) adopted the LSM based RSM when investigating robust mix design procedure. However, the LSM may be a major source of error in the RSM (Bhattacharjya and Chakraborty, 2011, Bhattacharjya *et al*. 2018). Hence, a comparatively newer Moving Least Squares Method (MLSM) based adaptive RSM is explored in the present paper. The application of the MLSM based RSM in RCO is not yet observed in the existing literature. Hence, this builds another uniqueness of the present study.

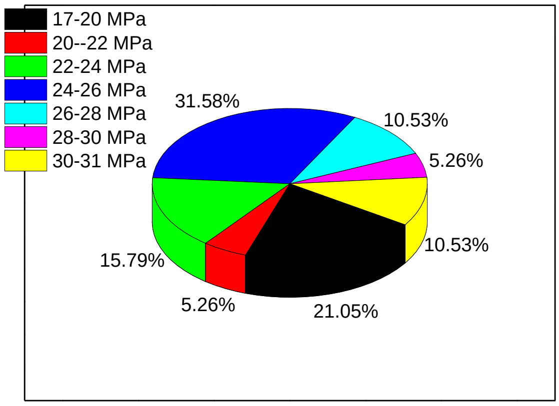

Fig. 1 Batch to batch variation of RHABC

The strength and workability of RHABC largely depends on the cement content (c), RHA content (r), super plasticizer content (s_p), water to binder ratio (w/b) and sand content (f_a) per m^3 of concrete. The binder refers to the total quantity of cement and RHA per m^3 of concrete. The RSM generates explicit equations for dependence of strength (and workability) on c, r, s_p and w/b. If r, c and w/b are known, quantity of water (w) can be evaluated as, $w=(w/b)\times(r+c)$, since $b=r+c$. Also, if specific gravity of coarse aggregate is γ_{ca}, the weight of coarse aggregate c_a is given by $(1.0\text{-}b\text{-}w\text{-}f_a\text{-}s_p)\times\gamma_{ca}$.

It may be noted that all the aforementioned quantities are uncertain. Thus, quantification of uncertainty is an important issue in RCO. It is most likely that during preparation of RHABC one will attempt to mix the design yielded quantities. However, there may be variations due to uncertainty arising out from manual error, loss during mixing, transportation, measuring error, etc. The quantities (except r and w/b) may be assumed as normally distributed (IS:10262 2009), with design yielded value as the mean and maximum 20% variation (i.e., Coefficient of Variation (COV)=0.2) with respect to the mean. However, uncertainty with RHA is not that straightforward. Due to several impurities and variation in fineness modulus, there may be substantial property variation of RHA even with same quantity of RHA (Fig. 1). The presence of impurity affects the property of RHABC significantly which has been also reported in de Sensale and Viacava (2018), Ewa *et al*. (2018). The type, chemical composition, grain size of impurity vary substantially among the same batch of RHA leading to considerable property variations of resulting RHABC. Since RHA is not produced in a controlled manufacturing process like cement, the fineness and grain size of RHA vary significantly (Ewa *et al*. 2018). Thus, it is more logical to assign RHA quantity a worst uncertainty case, which is assumed as Uncertain but Bounded (UBB) type in the present paper. With the UBB uncertainty, the RHA quantity will assume any possible values within the range of variation which is equally likely. It has been observed during the detailed experimentation that due to uncertainty in impurities and fineness (which induces heterogeneity in the mix), property of RHABC can vary

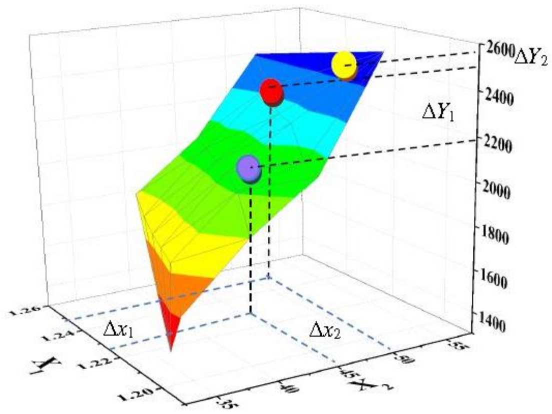

Fig. 2 Difference between RCO and conventional mix design

maximum 20%, if RHA of same rice mill and brand is used. Thus, the dispersion of RHA content is assumed to be 20% in either side of the nominal value. Since, RHA content (r) is modeled as UBB type, the *w/b* is also UBB type. Maximum 20% dispersion is assumed for *w/b*. This UBB assumption is likely to incorporate the effect of uncertainty related to RHA heterogeneity directly in the RCO process. However, except r and (w/b), other constituents quantities (c,w,s_p,f_a,c_a) are assumed to be normally distributed (IS:10262, 2009) with design yielded value as the mean and COV as 20% with respect to mean. The COV of cost related parameters is also taken as 20%.

There may be significant property variation of RHABC if two or more of these uncertain material quantities vary simultaneously. The situation can be demonstrated with the help of a response surface in Fig. 2. Say, the compressive strength of RHABC is considered as (Y) and input material constituents are represented by X_1 and X_2. Let, a typical (conventional) mix design is denoted by a red circle in Fig.2. Now, if $X_1$and X_2 both vary by the amounts of ΔX_1 and ΔX_2, the property of the RHABC will shift to the violet circle which is largely deviated from the original red circle. This shift in compressive strength (ΔY_1) causes significant reduction in compressive strength of RHABC, yielding an undesired (or unsafe) mix. Thus, there should be a prudent mix design approach which will yield robust mix that will perform well even in presence of considerable input uncertainty. This task can be accomplished by the RCO. Let, an RCO yields a mix denoted by yellow circle in Fig. 2. An RCO basically track the optimal solution in a comparatively flatter portion of response surface, which makes the design least sensitive to variation due to uncertainty. It may be clearly observed that even if the ΔX_1 and ΔX_2 are dispersed by the same amount in opposite side of red circle, the shift in compressive strength (ΔY_2) is substantially lesser than ΔY_1. This indicates the RCO yields mix design which is less dispersed or in other words less sensitive to input parameter variation due to uncertainty (note that the gradients at this yellow circle are substantially less). In fact, in design of products in mechanical engineering and design of safe structures such notion is already used (Bhattacharjya *et al*. 2015, Abhiram *et al*. 2018). However, this concept is not yet applied on RHABC and builds the scope of this study.

Thus, the primary contribution of this study is to propose a new RCO approach of RHABC in the framework of the MLSM based RSM. The RHA related properties are considered as UBB type. These considerations have not been yet observed in the existing literature and constitutes the uniqueness of the present study. The MLSM based adaptive RSM is explored in RCO in place of conventional LSM based RSM to ensure accuracy. A detailed experimental program has been also carried out to validate the proposed RCO approach.

2. Development of the RCO scheme

2.1 The Deterministic Design Optimization (DDO)

The performance of an optimal design depends on Design Variables (DVs) and Design Parameter (DPs). The DVs are the specific parameters need to optimize to achieve the desired performance(s). The DPs are those, which cannot be controlled or are difficult and expensive to control. In the present problem DVs (**x**) are c, r, s_p, (w/b), f_a per m^3 of concrete. The DPs (**z**) are cost of cement, cost of RHA, cost of super plasticizer, cost of water, cost of aggregate per m^3 of concrete, and specific gravities of the ingredients. The cost of mixing and placing per m^3 of concrete are also considered as DP. The nominal values of DPs are considered as \$114.3, \$ 14.3, \$ 22.5 and \$ 12 per MT of cement, RHA, coarse aggregates and fine aggregates, respectively; \$1.07 per kg of super plasticizer, \$ 0.71 per 100 kg of water, and \$ 7.14 as mixing and placing cost per m^3 of concrete. Here, manual mixing and placing is considered using conventional mixer machine. The electricity charge is included. It may be noted here that the cost reported in the paper is based on the Public Works Department rate schedule (PWD WB, 2017) prevailing in Bardhaman District of India. For foreign countries, these rates may substantially vary. However, the implementation procedure of RCO of RHABC will remain same for other countries and other currencies, as well. The DDO problem is formulated to find the optimal DVs, which will minimize the cost satisfying compressive strength and slump requirement (workability) criteria as

$$\begin{aligned} &\text{minimize} && f(\mathbf{x},\mathbf{z}):\text{cost} \\ &\text{subjected to} && g_1(\mathbf{x},\mathbf{z}):\ \sigma_c^t\text{-}\sigma(\mathbf{x},\mathbf{z})\le 0 \\ &&& g_2(\mathbf{x},\mathbf{z}):\ s_c^t\text{-}s(\mathbf{x},\mathbf{z})\le 0 \\ &&& x_i^L \le x_i \le x_i^U,\quad i=1,2,\ldots\ldots,K. \end{aligned} \tag{1}$$

In the above, x_i^L and x_i^U are the lower and the upper bounds of the i^{th} DV, respectively. σ_c^t, $\sigma(\mathbf{x},\mathbf{z})$, s_c^t and $s(\mathbf{x},\mathbf{z})$ are the target compressive strength, obtained compressive strength, the target slump and obtained slump, respectively. In the present study, s_c^t is taken as 25 MPa and 35 MPa in two separate cases. It may be noted that $\sigma(\mathbf{x},\mathbf{z})$ and $s(\mathbf{x},\mathbf{z})$ are implicit function of [**x z**], which will be explicitly approximated by the MLSM based RSM in the present paper.

It can be noted here that the DDO problem as described by Eq. (1) does not consider the effect of uncertainty in [**x z**]. But, the performance function and the constraints are the function of [**x z**]. Thus, the uncertainty in [**x z**] is expected to propagate at the system level, influencing the performance function and the constraints of the related optimization problem. The RCO approach under uncertainty is discussed in the next section.

2.2 The Robust Cost Optimization (RCO)

2.2.1 Robustness of the objective function

The robustness of the objective function is generally expressed in terms of the dispersion of the performance function from its mean value. The objective of an ideal design is to achieve the optimal performance as well as less sensitivity of the performance function with respect to the variation in the DVs and DPs due to uncertainty. Thus, one needs to optimize the objective function as well as its dispersion (standard deviation for normal random parameters). Hence, the RDO problem is posed as the minimization problem of the mean and standard deviation of the objective function, leading to a two criteria RDO problem which can be expressed as

$$\text{Find x, to minimize } [\mu_f, \sigma_f]. \tag{2}$$

In the above, μ_f and σ_f are the mean and the standard deviation of the performance function respectively. Normally, minimization of the mean and variance of the performance are sought leading to a set of Pareto-optimal solution as shown by Deb *et al*. (2002). The Weighted Sum Method (WSM) is an easy, computationally efficient and popular way to deal with the trade-offs between conflicting objectives (Doltsinis *et al*. 2005) and is adopted in the present study. Applying the WSM the multi objective function is converted to an equivalent single objective function as

$$\phi(\mathbf{u}) = (1-\alpha)\,\mu_f \big/ \mu_f^* + \alpha\,\sigma_f \big/ \sigma_f^* ; \qquad 0 \le \alpha \le 1 \tag{3}$$

where, $\phi(\mathbf{u})$ is a new objective function, called desirability function and the parameter α serves as a weighting factor; μ^*_f and σ^*_f are the optimal values of the mean and the standard deviation obtained for α equals to 0.0 and 1.0, respectively. The maximum robustness will be achieved when α becomes 1.0. In the present case, two types of uncertain variables are involved in the RCO, i.e., normal random and UBB. Let us denote **u**=[**x z**]. By using first order perturbation approach, the mean and standard deviation of objective function can be obtained for normal random parameters as (Doltsinis *et al*. 2005)

$$\mu_{f_1}(\mathbf{u}) \approx f_1(\bar{\mathbf{u}}), \quad \sigma^2_{f_1} \approx \sum_{i=1}^{N} \left(\left.\frac{\partial f_1}{\partial u_i}\right|_{\bar{u}_i} \right)^2 \sigma^2_{u_i} \tag{4}$$

Similarly, for UBB uncertainty, using worst case propagation concept, the nominal value $\bar{f}$ (i.e., mean for normal random case) and dispersion Δf (i.e., standard deviation for normal random case) can be obtained as (Lee and Park 2001)

$$\bar{f}_2 = f(\bar{\mathbf{u}}), \qquad \Delta f_2 = \sum_{i=1}^{N} \left|\partial f / \partial u_i\right| \Delta u_i \tag{5}$$

In the above, $\bar{\mathbf{u}}$ denotes nominal value of **u,** i.e., $\bar{\mathbf{u}} = (\mathbf{u}^L+\mathbf{u}^U)/2$. Finally, for a mixed system of UBB and random parameters, the resulting nominal value and dispersion of objective function can be obtained as

$$\mu_f = \mu_{f_1} + \bar{f}_2, \qquad \sigma_f = \sigma_{f_1} + \Delta f_2 \tag{6}$$

The formulation presented above is valid for comparatively smaller levels of uncertainty (up to 25% level) in the **u**. However to deal with non-normal variables, the Monte Carlo Simulation (MCS) approach may be used in estimating mean and standard deviation values.

2.2.2 Robustness of the constraints

Due to uncertainty in [**u**], the optimal solution obtained by using the deterministic constraint functions, is expected to vary. Even, the final desired performance obtained by such deterministic constraints may become infeasible in the presence of uncertainty in **u** as shown by Cheng *et al*. (2017). Addressing the feasibility of constraints under uncertainty, Venanzi *et al*. (2015) developed a general probabilistic feasibility formulation for the j^{th} constraint g_j as

$$P\,[g_j(\mathbf{u}) \le 0] \ge P_{0j}, \qquad j = 1, \ldots\ldots\ldots, J \tag{7}$$

where, P_{oj} is the desired probability to satisfy the j^{th} constraint. To reduce the computational involvement of probabilistic feasibility evaluation, and assuming $g_j(\mathbf{u})$ as normally distributed, the probabilistic feasibility of the constraint can be approximated as (Lee and Park 2001)

$$\mu_{g_j} + k_j \sigma_{g_j} \le 0. \tag{8}$$

In the above, μ_{g_j} and σ_{g_j} are the mean and the standard deviation of g_j, respectively and are evaluated by the first order perturbation approach (see Eq. (4)). The designer specified penalty factor, k_j is used to enhance the feasibility of the j^{th} constraint and can be obtained from, $k_j=\Phi^{-1}(P_{Oj})$, where $\Phi^{-1}(.)$ is the inverse of the cumulative density function of standard normal distribution. Thus, in other words, k_j denotes the target reliability Index. However, for UBB parameters the dispersion of constraint should be obtained by worst case uncertainty propagation approach similar to Eq. (5). Thus, to consider the mixed system of random and UBB parameters equivalent mean μ_{g_j} and equivalent standard deviation σ_{g_j} is obtained as

$$\mu_g = g_j\left(\mu_{\mathbf{x}_R}, \mu_{\mathbf{z}_B}, \bar{\mathbf{x}}_R, \bar{\mathbf{z}}_B\right); \tag{9a}$$

$$\sigma^2_{g_j} = \left\{ \sum_{i=1}^{N} \left(\left.\frac{\partial g_j}{\partial \mathbf{x}_R}\right|_{\bar{u}_i} \right)^2 \sigma^2_{\mathbf{x}_{Ri}} + \sum_{i=1}^{N} \left(\left.\frac{\partial g_j}{\partial \mathbf{z}_R}\right|_{\bar{u}_i} \right)^2 \sigma^2_{\mathbf{z}_{Ri}} \right\}$$

$$+\left\{\sum_{i=1}^{N}\left|\frac{\partial g_j}{\partial \mathbf{x}_{Bi}}\right|_{\bar{u}_i}\Delta\mathbf{x}_{Bi}+\sum_{i=1}^{N}\left|\frac{\partial g_j}{\partial \mathbf{z}_{Bi}}\right|_{\bar{u}_i}\Delta\mathbf{z}_{Bi}\right\} \tag{9b}$$

In Eq. (9), $\mathbf{x}_R$, $\mathbf{x}_B$, $\mathbf{z}_R$ and $\mathbf{z}_B$ represent the random DVs, UBB DVs, random DPs and UBB DPs, respectively. A close examination of Eq. (9a) will reveal that the equivalent mean of constraint function has been evaluated at the mean values of probabilistic parameters and nominal values of UBB parameters. Similarly, the first part of Eq. (9b) refers to treatment to probabilistic parameters and the second part of Eq. (9b) depicts the consideration to UBB parameters for both the DVs and DPs. In the first part, first order perturbation approach for probabilistic parameters (see Eq. (4)) has been used and the second part uses worst case propagation principle for UBB parameters (see Eq. (5)).

2.2.3 The RCO formulation

Combining Eqs. (3) and (8) to meet the requirements of the performance and the constraints feasibility under uncertainty in **u**, the RCO problem is formulated as

$$\begin{aligned}&\text{minimize:} && \phi(\mathbf{u})=(1-\alpha)\frac{\mu_f}{\mu_f^*}+\alpha\frac{\sigma_f}{\sigma_f^*}, && 0\le\alpha\le 1\\ &\text{subjected to:} && \mu_{g_j}+k_j\sigma_{g_j}\le 0 && j=1,2,......,J\\ & && x_i^L\le x_i\le x_i^U, && i=1,2......,K.\end{aligned} \tag{10}$$

It may be noted here that the individual gradient of the performance function and the constraints are required to be evaluated at each updated design point during the optimization process. For simple explicit performance function and constraints, one can directly evaluate the gradients. In cases of implicit functions involving complex response evaluations, the sensitivity gradients can be obtained by metamodeling (Bhattacharjya and Chakraborty 2011). In the present study, an efficient MLSM based adaptive RSM of metamodeling approach has been adopted which is detailed in the next section.

2.3 Efficient RSM in RCO

Conventional RSM is hinged on the concept of the LSM. The fundamentals of the LSM based RSM is detailed in the subsection 2.3.1 followed by its fundamental difference with the MLSM in subsection 2.3.2.

2.3.1 The LSM based RSM

The object of the RSM is to obtain an explicit mathematical function $\hat{y}=\Phi(\mathbf{u})$ which best fits to a given set of experimental data (**u**) and associated output (**Y**), i.e.

$$\hat{y}=\Phi(\mathbf{u})=\beta_0+\sum_{i=1}^{K}\beta_i u_i+\sum_{i=1}^{K}\sum_{j=1}^{K}\beta_{ij}u_i u_j=\mathbf{Q\beta} \tag{11}$$

where, $\mathbf{Q}$ is known as the design matrix and $\boldsymbol{\beta}$ is the unknown coefficient vector to be evaluated by the LSM. It may be noted that actual response vector is **Y** and predicted response is $\hat{\mathbf{y}}$. The input experimental data are chosen following well established judicious methods, referred as Design of Experiment (DOE). Thus, at first, based on the selected DOE scheme, the input constituents for experiment are decided. Then, the experiments are physically conducted to obtain actual response vector, **Y** required for the RSM. Let us denote an error residual $\boldsymbol{\varepsilon}=\mathbf{Y}-\hat{\mathbf{y}}=\mathbf{Y}-\mathbf{Q\beta}$. The sum of the squares of error residual ($\boldsymbol{\varepsilon}^T\boldsymbol{\varepsilon}$) is minimized in the conventional LSM as below

$$\text{Minimize}\ \ \mathbf{L_y}(\mathbf{x})=\boldsymbol{\varepsilon}^{\mathbf{T}}\boldsymbol{\varepsilon}=(\mathbf{Y}-\mathbf{Q\beta})^{\mathbf{T}}(\mathbf{Y}-\mathbf{Q\beta}) \tag{12}$$

Then, the following matrix operations yields (Myers and Montgomery 1995)

$$\boldsymbol{\beta}=\left[\mathbf{Q}^{*\mathbf{T}}\mathbf{Q}^{*}\right]^{-1}\mathbf{Q}^{*T}\mathbf{Y} \tag{13}$$

In the above, $\mathbf{Q}^*$ is the design matrix evaluated at the DOE points. Once, the unknown coefficients are evaluated, the response surface is explicitly determined in polynomial functional form as $\hat{y}=\Phi(\mathbf{u})$ (Eq. (11)).

Though, the LSM based RSM is a widely used conventional method, possibility of inclusion of error by the LSM has been reported by various researchers (Datta *et al*. 2017, Li *et al*. 2018, Bhattacharjya *et al*. 2018). This may be due to characteristics of global approximation by the LSM. In this regard, a comparatively new MLSM, which is based on moving and local approximations, seems to be more elegant and explored in the present study. The concept of the MLSM based RSM is presented in the next section.

2.3.2 The MLSM based RSM

The MLSM based RSM is a weighted LSM which has varying weight functions based on the position of approximation (Taflanidis 2012). The weight corresponding to a particular sampling point $\mathbf{u_i}$ decays as the prediction point **u** moves away from $\mathbf{u_i}$. The weight function is defined around the prediction point **u** and its magnitude changes with **u**. The modified error norm $\mathbf{L'_y}(\mathbf{u})$ is defined as the sum of the weighted errors (Taflanidis 2012)

$$\mathbf{L'_y}(\mathbf{u})=\boldsymbol{\varepsilon}^{\mathbf{T}}\mathbf{W}(\mathbf{u})\boldsymbol{\varepsilon}=(\mathbf{Y}-\mathbf{Q\beta})^{\mathbf{T}}\ \mathbf{W}(\mathbf{u})(\mathbf{Y}-\mathbf{Q\beta}) \tag{14}$$

In the above equation, **W(u)** is a diagonal matrix of the weight function and it depends on the location of the associated approximation point of interest (**u**). **W**(**u**) is obtained as,

$$\mathbf{W(u)}=\begin{bmatrix}\mathbf{w(u-u_1)} & 0 & \cdots & 0\\ 0 & \mathbf{w(u-u_2)} & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & \cdots & \mathbf{w(u-u_n)}\end{bmatrix} \tag{15}$$

$$\text{where,}\quad \mathbf{w(u-u}_i\mathbf{)}=\mathbf{w(d)}=\exp(\mathbf{-d}/R_I)$$

In the above, R_I denotes a hyper ellipsoidal space with prediction point (**u**) as centre. If a DOE point is located inside the space, the weight defined in Eq. (15) is assigned for the DOE point. Otherwise, for the DOE points located beyond this hyper ellipsoidal space, the weight is assigned as zero. Also, more weight is assigned to those DOE points which are more close to the prediction point. **d** is the Euclid distance between sampling point and the prediction point. R_I is calculated as the Euclid distance between the

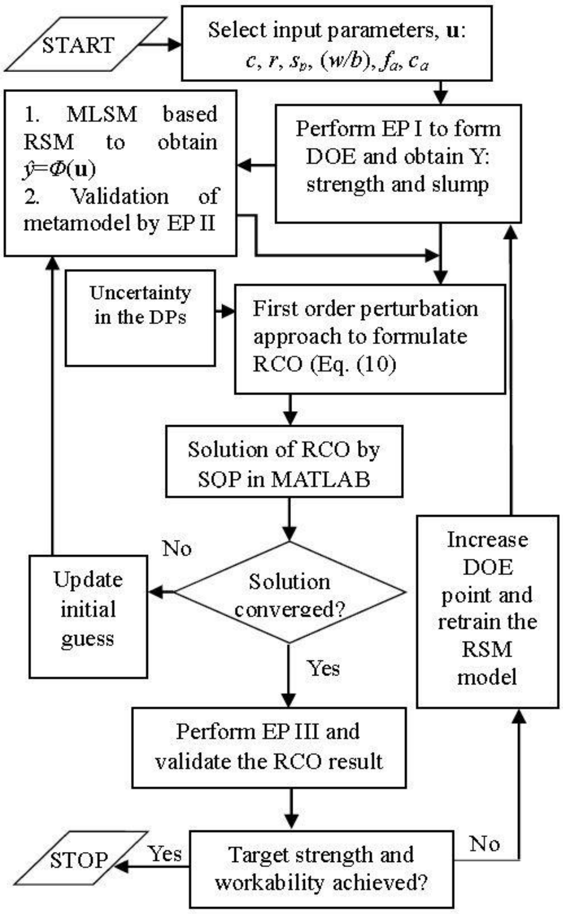

Fig. 3 Implementation of the proposed RCO in the MLSM based RSM framework

prediction point and a point located at twice the standard deviation distance of individual variable (Bhattacharjya and Chakraborty 2011). Now, the coefficient vector, $\boldsymbol{\beta}(\mathbf{u})$ which is also a function of $\mathbf{u}$, can be obtained by the matrix operation analogous to Eq. (13) as (Kim *et al*. 2005)

$$\boldsymbol{\beta}(\mathbf{u}) = \left[\mathbf{Q}^{*\mathrm{T}}\mathbf{W}(\mathbf{u})\mathbf{Q}^{*}\right]^{-1}\mathbf{Q}^{*\mathrm{T}}\mathbf{W}(\mathbf{u})\mathbf{Y} \tag{16}$$

Thus, unlike the LSM, the response surface approximation by the MLSM changes for every realization of $\mathbf{u}$ to capture minute localized variations of constraint function. The implementation procedure of the RCO in the MLSM based RSM framework is presented by a flowchart in Fig. 3.

Three experimental programs (EPs) are required to complete the RCO procedure. EP I is executed to construct the DOE for the RSM (i.e., for training of metamodel). Once, the metamodel is developed, it is validated with a second round of experiment (EP II). This validation is done by comparing experimental results with the RSM predicted values. If the accuracy by the RSM is not up to the mark, a second training is performed by repeating EP I in order to gather some more data for increasing DOE sample size. In this way, these steps are repeated till the validation of the metamodel is satisfactory. Thereafter, the RCO is executed in the MLSM based RSM framework and a RHABC mix is recommended. The optimization is accomplished in MATLAB using available subroutine *fmincon* for the Sequential Quadratic Programming (SQP). Then, the third round of experiment program (i.e., EP III) is conducted to ascertain whether the proposed RCO yields the target performance (slump and strength) of RHABC. If not, training of metamodel is to be processed with those failed experimental points again. In the subsequent sections, details of EP I, validation of metamodel by EP II, The RCO results and validation of the proposed RCO by EP III are explained.

Table 1 Details of the materials used in the experimental investigation

Cement	Ordinary Portland Cement (OPC) 43 grade conforming to IS: 8112 (2013), Specific gravity 3.15.
RHA	Procured from Bardhaman District of India, Silica (SiO_2) content is around 90% by weight. Specific gravity 2.1.
Coarse Aggregate	Pakur variety of 20 mm graded down as per (IS) 383 (1970). Bulk density: 1454 kg/m^3.
Fine Aggregate	Air-dried River sand obtained from Bardhaman District of India conforming to grading zone II as per (IS) 383 (1970); Bulk density: 1466 kg/m^3; Fineness modulus: 3.02.
Superplasticizer	High Range Water Reducer BASF MasterPel 777, having polymer base complying with (IS) 9103-99 (2007), ASTM C 494 Types A and D. Specific gravity:1.05; Color : dark brown.
Water	Normal tap water available from the Institute water supply system

3. Experimental programme I

Before starting EP I, 300 random mix compositions are theoretically generated using Latin Hypercube Sampling assuming *r* and (*w/b*) as UBB type and other parameters as normal random. Then, 24 best candidate mixes out of these are selected which are promising. Here ‘promising mix’ indicates the mixes which are practicable and supposed to (based on user’s experience) yield target slump and strength. These 24 points constitute the DOE. Six cubes are casted for each of these 24 mixes. Cubes were of 150 mm size. The details of various materials used in this experimental investigation are mentioned in Table 1. Necessary physical tests for cement, RHA, superplasticizer and aggregates were conducted in the Institute laboratory to obtain free moisture, water absorption, specific gravity and grading of the ingredients. To determine the workability of concrete, slump test was conducted using standard slump cone following the procedure as per (IS) 1199 (1959). The 24 DOE points considered during EP I are shown in Table 2 along with the obtained average slump values. In this table ‘C80%, R20%’ implies *c*:*r*=0.8:0.2 by weight. It can be observed from Table 2 that the weight of concrete cubes decreases as the RHA content increases. The decrease of weight and workability of concrete with increase in RHA content is due to the low specific gravity of RHA compared to cement.

All specimens were water cured until the age of testing

(a) Mixing of ingredients

(b) Compressive strength test set up

(c) Failure of concrete cube

Fig. 4 Experimental works at Civil Engineering Laboratory

at 28 days. The cubes were tested for compressive strength using 200 t capacity electrical compression testing machine following the procedure of IS: 516 (1959). The load is applied without shock and increased continuously at a rate of approximately 140 kg/cm^2/min. Mixing of ingredients, compression test set up and a sample failure pattern of cube are shown in Fig. 4(a), Fig. 4(b) and Fig. 4(c), respectively.

The variation of compressive strength as obtained during EP I for different RHA replacement levels and total binder contents (c+r) are presented in Fig. 5. It can be observed from this figure that in all the considered cases compressive strength increases with increasing RHA replacement levels up to an optimum dose. Thereafter, there is a drop in compressive strength. The maximum compressive strength is attained at an optimum RHA replacement levels of 20%, 20% and 10% for total binder content of 350 kg/m^3 and 380 kg/m^3 and 450 kg/m^3, respectively. For total binder content of 410 kg/m^3 the maximum compressive strength is obtained in three optimum RHA replacement levels (i.e., 10%, 15% and 20%). However, beyond 20% RHA replacement level, a drop in the compressive strength is noted for all the considered cases and the reduced compressive strength becomes even less than that of 0% RHA replacement case.

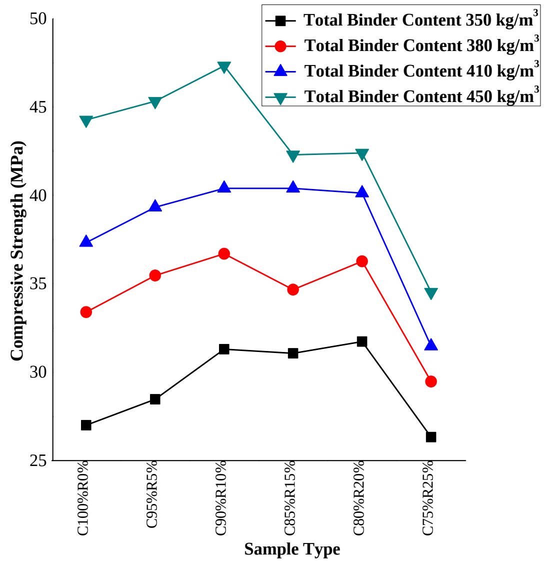

Fig. 5 Compressive strength of RHABC at different RHA replacement levels

It can be further noted from Fig. 5 that the variation of compressive strength is not a smooth and monotonous function in terms of variables of optimization problem.

Thus, in place of conventional LSM, the MLSM based RSM is adopted in the present paper to accurately capture the trends of compressive strength and slump as functions of the DVs. Based on the test results of EP I, response surfaces for compressive strength and slump have been developed by both the LSM based RSM and the MLSM based RSM in separate modules. Type III RSM polynomial is adopted in the present study for developing the metamodels. These metamodels are now validated with actual test results before using these in RCO, which is described in the next section.

4. Validation of the metamodel

Once the metamodels for slump and compressive strength are developed, the accuracy of these metamodels are checked by another round of experiments (EP II). These checking experimental points are distinctly different than the test points of EP I used for training the metamodels. The new 24 checking points are shown in Table 3.

The predictions by the conventional LSM based RSM and the present MLSM based RSM are compared with the experimental results of round EP II in Table 4. It may be clearly observed that the conventional LSM based RSM fails to predict the experimental trends in most of the cases; whereas, the proposed MLSM based RSM consistently captures the experimental results of EP II. The coefficient of determination (R^2) values for the metamodel of slump are 0.79 and 0.967 by the LSM and the MLSM, respectively. The R^2 values for compressive strength are 0.83 and 0.971 by the LSM and the MLSM, respectively. Thus, the R^2 values by the MLSM are more than 0.95 in both the cases, which further attests the accuracy and acceptability of the MLSM based RSM predictions. On the other hand, the

Table 2 Composition of concrete mixes for the 24 DOE points in the EP I round and Test Results

Sample Number	Sample Type	Ingredients (kg/m^3)						w/b	Slump (mm)	28 Days Weight (kg/m^3)
		c	r	w	c_a	f_a	s_p			
1	C100%, R0%	350	0	122.5	1280	730	2.2	0.35	63	2506
2	C95%, R5%	332.5	17.5	133	1280	730	2.4	0.38	67	2506
3	C90%, R10%	315	35	140	1280	730	2.75	0.40	66	2506
4	C85%, R15%	297.5	52.5	150.5	1280	730	2.75	0.43	60	2480
5	C80%, R20%	280	70	157.5	1280	730	2.75	0.45	75	2472
6	C75%, R25%	262.5	87.5	168	1280	730	3.1	0.48	68	2470
7	C100%, R0%	380	0	133	1270	720	2.4	0.35	65	2518
8	C95%, R5%	361	19	144.4	1270	720	2.75	0.38	70	2518
9	C90%, R10%	342	38	152	1270	720	3.1	0.40	62	2510
10	C85%, R15%	323	57	163.4	1270	720	3.1	0.43	60	2494
11	C80%, R20%	304	76	171	1270	720	3.1	0.45	64	2482
12	C75%, R25%	285	95	182.4	1270	720	3.3	0.48	66	2472
13	C100%, R0%	410	0	143.5	1260	710	2.75	0.35	60	2548
14	C95%, R5%	389.5	20.5	155.8	1260	710	3.1	0.38	64	2548
15	C90%, R10%	369	41	164	1260	710	3.3	0.40	60	2520
16	C85%, R15%	348.5	61.5	176.3	1260	710	3.3	0.43	62	2500
17	C80%, R20%	328	82	184.5	1260	710	3.3	0.45	62	2504
18	C75%, R25%	307.5	102.5	196.8	1260	710	3.7	0.48	68	2490
19	C100%, R0%	450	0	157.5	1250	700	3.1	0.35	62	2556
20	C95%, R5%	427.5	22.5	171	1250	700	3.3	0.38	60	2548
21	C90%, R10%	405	45	180	1250	700	3.5	0.40	65	2534
22	C85%, R15%	382.5	67.5	193.5	1250	700	3.5	0.43	65	2522
23	C80%, R20%	360	90	202.5	1250	700	3.7	0.45	64	2530
24	C75%, R25%	337.5	112.5	216	1250	700	3.8	0.48	60	2522

Table 3 Composition of RHABC mixes used for validation of the metamodels

Sample Number	Sample Type	Ingredients (kg/m^3)						w/b
		c	r	w	c_a	f_a	s_p	
25	C97.5%, R2.5%	341.25	8.75	122.5	1280	730	2.2	0.35
26	C92.5%, R7.5%	323.75	26.25	133	1280	730	2.4	0.38
27	C87.5%, R12.5%	306.25	43.75	140	1280	730	2.75	0.40
28	C82.5%, R17.5%	288.75	61.25	150.5	1280	730	2.75	0.43
29	C78.5%, R22.5%	271.25	78.75	157.5	1280	730	2.75	0.45
30	C72.5%, R27.5%	253.75	96.25	168	1280	730	3.1	0.48
31	C97.5%, R2.5%	370.5	9.5	133	1270	720	2.4	0.35
32	C92.5%, R7.5%	351.5	28.5	144.4	1270	720	2.75	0.38
33	C87.5%, R12.5%	332.5	47.5	152	1270	720	3.1	0.40
34	C82.5%, R17.5%	313.5	66.5	163.4	1270	720	3.1	0.43
35	C78.5%, R22.5%	294.5	85.5	171	1270	720	3.1	0.45
36	C72.5%, R27.5%	275.5	104.5	182.4	1270	720	3.3	0.48
37	C97.5%, R2.5%	399.75	10.25	143.5	1260	710	2.75	0.35
38	C92.5%, R7.5%	379.25	30.75	155.8	1260	710	3.1	0.38
39	C87.5%, R12.5%	358.75	51.25	164	1260	710	3.3	0.40
40	C82.5%, R17.5%	338.25	71.75	176.3	1260	710	3.3	0.43
41	C78.5%, R22.5%	317.75	92.25	184.5	1260	710	3.3	0.45
42	C72.5%, R27.5%	297.25	112.75	196.8	1260	710	3.7	0.48
43	C97.5%, R2.5%	438.75	11.25	157.5	1250	700	3.1	0.35
44	C92.5%, R7.5%	416.25	33.75	171	1250	700	3.3	0.38
45	C87.5%, R12.5%	393.75	56.25	180	1250	700	3.5	0.40
46	C82.5%, R17.5%	371.25	78.75	193.5	1250	700	3.5	0.43
47	C78.5%, R22.5%	348.75	101.25	202.5	1250	700	3.7	0.45
48	C72.5%, R27.5%	326.25	123.75	216	1250	700	3.8	0.48

LSM predictions have significantly lesser R^2 values which denotes application of LSM in RCO may result in erroneous and unsafe results.

5. RCO results and discussion

After validation of metamodels, the response surface

Table 4 Validation of metamodels for Slump and 28 days Compressive strength

Sample Number	Slump (mm) MLSM Based RSM	Slump (mm) Experimental	Slump (mm) LSM Based RSM	Compressive Strength (MPa) MLSM Based RSM	Compressive Strength (MPa) Experimental	Compressive Strength (MPa) LSM Based RSM
25	65	63	69	26.73	27.0	30.47
26	63	65	70	31.87	33.4	30.69
27	62	60	65	38.22	37.33	35.17
28	62	62	66	46.61	44.3	48.59
29	66	67	63	29.27	28.5	26.60
30	67	70	62	34.73	35.5	33.59
31	65	64	61	40.32	39.33	42.85
32	62	60	66	47.31	45.33	48.64
33	64	66	62	31.85	31.31	28.49
34	61	62	58	36.51	36.71	34.05
35	61	60	56	40.83	40.49	43.32
36	67	65	60	46.96	47.33	44.77
37	60	60	65	32.05	31.07	29.38
38	61	60	64	35.22	34.77	37.90
39	61	62	65	39.96	40.41	43.09
40	63	65	60	45.19	42.32	46.39
41	73	75	68	31.68	31.73	29.65
42	63	64	60	35.25	36.27	34.91
43	63	62	63	38.31	40.13	36.80
44	65	64	60	42.89	42.4	40.13
45	70	68	62	28.24	26.33	30.84
46	65	66	63	32.32	29.47	26.49
47	66	68	59	33.86	31.47	26.85
48	63	60	55	37.47	34.53	29.41

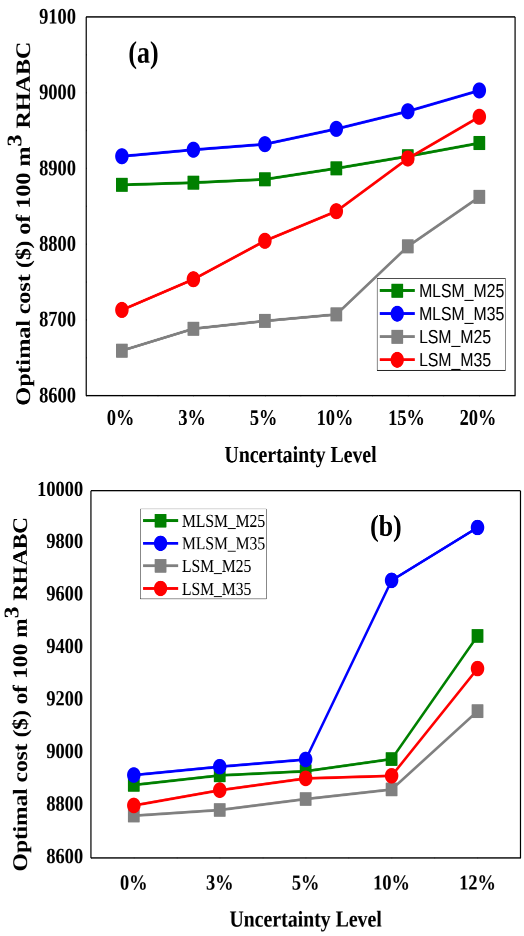

Fig. 6 The optimal cost by the RCO for varying uncertainty levels and concrete grade for (a) k_j=2, (b) k_j=3

expressions of strength and slump are used to formulate the constraint functions of Eq. (1). Subsequently, the gradients of the constraint functions are evaluated and the RCO problem of Eq. (10) is mathematically casted.

The RCO is solved by the SQP. The RCO results are presented in this section. At first a parametric study has been made by varying uncertainty levels in the DVs and DPs, k_j and α. Then, the efficiency of the proposed RCO procedure is validated by a third round of experimental programme (EP III).

The optimal cost (in USD) obtained by the RCO for producing 100 m^3 of RHABC is presented in Figs.6 for varying uncertainty levels in the DVs and DPs. Results are presented for M25 and M35 grade of concrete with 50 mm target slump. α is considered as 0.5 to develop these figures. Fig. 6(a) depicts the RCO results for k_j=2 and Fig. 6(b) presents the robust optimal cost for k_j= 3. It may be noted here that the assumption of k_j=2 is associated with 2.275% probability of failure. k_j=3 implies 0.135% probability of failure. The prevailing code of practice in India (IS: 10262 2009) considers 5% probability of failure, which corresponds to k_j=1.65. Thus, the present RCO solutions provide sufficient reliability of attaining desired strength and slump in presence of uncertainty especially when k_j is taken as 3.0. The results by the MLSM based RCO and the LSM based RCO are shown in the same figures for comparison. It may be clearly observed that the MLSM based RCO results are distinctly different than the LSM based RCO results. The LSM predicts comparatively lesser cost than the MLSM case. However, the possibility of error inclusion by the LSM has been already pointed out in the previous section. Hence, it may be inferred that the optimal cost predicted by the LSM may not be practically possible to attain. It may be further observed from Fig. 6(b) that there is a jump in optimal cost over 5% uncertainty level in case of M35 grade concrete when k_j= 3. This is due to the fact that robust constraint conditions become very stringent in such cases requiring substantially higher optimal cost.

The COV of the optimal cost of M25 and M35 grade RHABC yielded by both the LSM based RCO and the MLSM based RCO for varying uncertainty level of the DVs and DPs are presented in Figs. 7(a) and 7(b) for k_j=2 and 3, respectively. Interestingly, the COV remains almost constant at maximum 5.6% level (for the MLSM case), even when the uncertainty is varied to higher extent. Thus, the RCO yields optimum mixes which are insensitive to the variation of input uncertainty (i.e., robust solutions). The deviation of conventional LSM based RCO results in comparison to that by the proposed MLSM based RCO approach can be clearly envisaged from these figures.

The LSM based RCO yields more COV of optimal cost (i.e., this solutions have lesser robustness compare to the proposed MLSM based RCO approach), which clearly establishes the efficiency of the proposed MLSM based RCO approach. It may be also noted that the LSM based

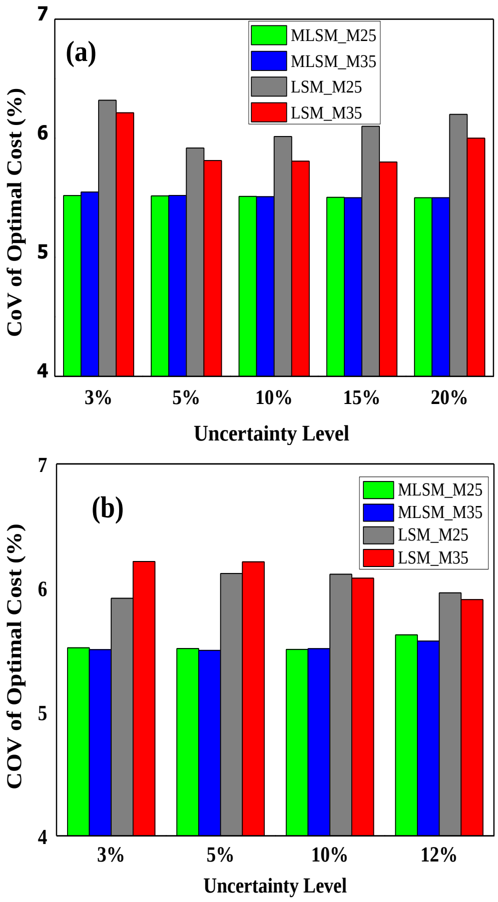

Fig. 7 COV of the robust optimal cost by the RCO for (a) k_j=2 and (b) k_j=3

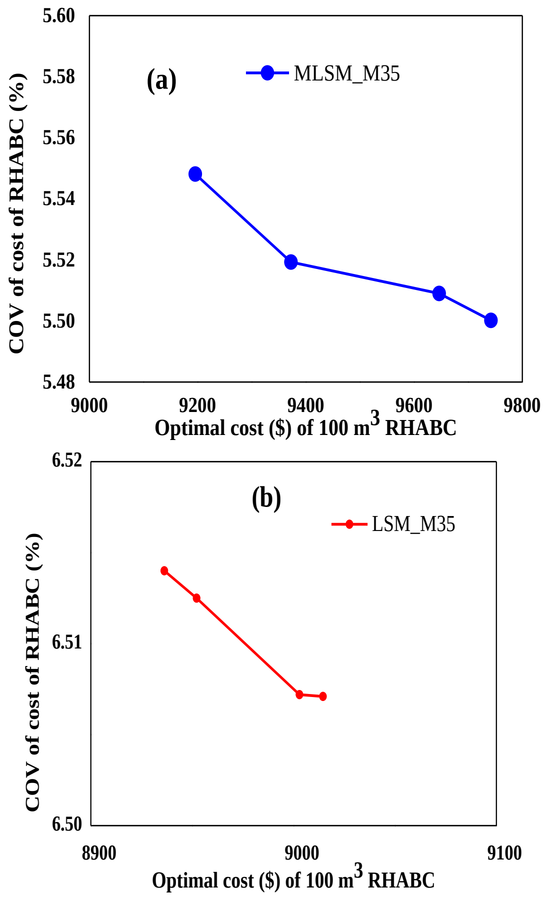

Fig. 8 The Pareto-fronts by (a) the MLSM based RCO and (b) the LSM based RCO

predictions are erroneous (Section 4) and hence one should not resort on the LSM based RCO results.

The RCO is a multi-objective optimization problem. The two objectives, i.e., the optimal cost and the associated dispersion (which is the measure of robustness) are conflicting in nature. In order to achieve more robustness in the mix one may have to incur more expenditure (i.e., optimal cost is more), or vice versa. This situation is generally studied in terms of Pareto-front (Deb *et al*. 2002), which is a plot between these two objective functions. The Pareto-front is obtained by varying α in Eq. (10). Two typical Pareto-fronts are presented in Fig. 8(a) and Fig. 8(b) by the MLSM based RCO and the LSM based RCO, respectively. These curves have been developed for k_j=2.0 and 20% uncertainty level in the DVs and DPs. The grade of concrete is M35. It can be clearly observed that the MLSM based RCO approach (Fig. 8(a)) requires more cost, but also provides more robustness (i.e., less COV of optimal cost) in comparison to the LSM based RCO approach (Fig. 8(b)). Thus, it can be inferred that the MLSM based RCO yields more robust Pareto-front than the LSM based RCO.

It has been observed that the cost of production of 100 m^3 of RHABC in India is approximately $ 8820 for M25 grade and $ 9475 for M35 grade of concrete. This is the average cost obtained based on the first round of experiment without using the RCO. After executing the proposed RCO, optimal cost of $ 9420 for M25 grade and $ 9855 for M35 grade RHABC have been yielded when k_j is 3.0. Thus, the RCO suggests 6.8% and 4% increment of cost for M25 and M35 grade of concrete, respectively. However, by sacrificing this marginal cost, the designer can have a guarantee of achieving desired slump and strength by ensuring low probability of failure (0.135%) and less deviation from expected performance criteria.

To validate the accuracy by the proposed RCO approach a third round of experimental programme (EP III) has been taken up. This experimentation will ascertain whether the desired properties of RHABC are really achieved or not. Twelve numbers of cubes were casted and tested for each MLSM based RCO predicted mix in this round to arrive at the mean, standard deviation and COV values of compressive strength and slump. The RHABC mix properties (slump and compressive strength) predicted by the RCO are compared with the experimental results obtained during EP III in Table 5 for different set up of k_j and uncertainty levels. It may be observed from this table that both the experimental compressive strength and slump values are in close conformity with the RCO predictions in all the cases. The errors by the RCO predictions with respect to the experimental results are also shown in Table 5. It can be observed that the error with the compressive strength is marginal. Though, error with slump values are little bit higher but acceptable since the target slump is achieved.

Table 5 Experimental validation of the RCO results

Set	Uncertainty in DVs and DPs	k_j	Theoretically obtained MLSM based RCO Result (c, r, s_p, w/b)	Compressive strength (MPa)				Slump (mm)			
				Proposed RCO	Experimental	Error (%)	COV (%) of Experimental Values	Proposed RCO	Experimental	Error (%)	COV (%) of Experimental Values
A	10%	3	370, 48.8, 2.6, 0.39	46.08	47.3	2.6	4.63	64	58	9.4	3.56
B	15%	3	420, 65.33, 2.6, 0.4	57.79	56.9	1.5	4.6	66	59	10.6	4.25
C	20%	1	350, 73.4, 2.2, 0.35	48.65	51.2	4.9	4.55	60	55	8.3	4.45
D	15%	2	401, 83, 2.2, 0.39	56.31	57.1	1.4	4.8	58	52	10.3	3.14

Note: 1.RCO yielded COV (%) of compressive strength=5%
2. RCO yielded COV (%) of slump=6.1%

To validate the actual robustness achieved by the RCO, the calculated COV values of slump and compressive strength associated with test results of EP III are compared with RCO predictions in Table 5. The RCO predicted COV values are mentioned in the footnote of Table 5. It can be observed that the experimental COV values are in well conformity with that predicted by the proposed RCO. The maximum COV of slump is only 4.25% based on experimental result, and is 6.1% by the proposed RCO. Similarly, the maximum COV of compressive strength is as small as 4.8%; whereas the proposed RCO predicts 5%. Lesser the COV of slump and compressive strength, more will be the robustness, because deviation from the expected value is lesser. Hence, it can be inferred that the proposed MLSM based RCO approach yields mix proportions which are optimum and robust as is clear from the experimental investigations.

6. Conclusions

An RCO of RHABC is presented considering uncertainty in mix constituents and cost related parameters. The RHA related uncertain parameters have been assumed as UBB type. To formulate the constraint function of RCO, metamodeling technique is adopted in the present study. In doing so, the MLSM based RSM is explored in RCO in place of conventional LSM approach to ensure accuracy by the metamodeling. The experimental results reveal that the MLSM has captured the trend of the experimental results even for the test points located outside the DOE, whereas, the error by the LSM is substantially high at those points. It has been observed that there is a nominal increment in the optimal cost by the RCO in comparison to the conventional deterministic optimization technique. However, by sacrificing this marginal cost increment, the present RCO ensures guarantee of achieving desired workability and strength of RHABC by limiting probability of failure to sufficiently low values. The COV of optimal cost remains almost constant over the varying uncertainty levels by the proposed RCO approach even for higher uncertainty levels. This indicates that the RCO yields solutions which are insensitive to the variation (or robust) in presence of uncertainty. The efficiency of the RCO approach is validated by an experimental study. The experimental results in terms of compressive strength, slump and the associated COV values are in good agreement with the proposed RCO prediction. The test results depict that the COV of both the slump and compressive strength are sufficiently low as has been predicted by the proposed RCO. This indicates lesser deviation of the mix properties from their desired values. Thus, it can be concluded that the proposed MLSM based RCO approach yields mix proportions which are optimum and robust for the present case.

It may be noted that the RCO presented in this study has been validated for M25 and M35 grades of concrete, with slump ranging between 50 to 75 mm. However, the proposed metamodeling based RCO procedure will be similar for other grades of concrete and other target slump of concrete, as well. Only, for other grades and slump requirement, a fresh metamodel has to be constructed by experimentation based on the influence of different ingredients.

It may be further noted that cost reported in the paper is based on the prevailing rates at Bardhaman District of India. For foreign countries, these rates may substantially vary. However, the implementation procedure of RCO of RHABC will remain same for other countries and other currencies, as well.

References

Abbasnia, R., Shayanfar, M. and Khodam, A. (2014), “Reliability based design optimization of structural systems using a hybrid genetic algorithm”, *Struct. Eng. Mech.*, **52**(6), 1099-1120. https://doi.org/10.12989/sem.2014.52.6.1099.

Abhiram, D.R., Ganguli, R., Harursampath, D. and Friedmann, P.P. (2018), “Robust design of small unmanned helicopter for hover performance using Taguchi method”, *J. Aircraft*, **55**(4), 1746-1753. https://doi.org/10.2514/1.C034539.

Beyer, H. and Sendhoff, B. (2007), “Robust optimization-A comprehensive survey”, *Comput. Meth. Appl. Mech. Eng.*, **196**(33-34), 3190-3218. https://doi.org/10.1016/j.cma.2007.03.003.

Bhattacharjya, S. and Chakraborty, S. (2011), “Robust optimization of structures subjected to stochastic earthquake with limited information on system parameter uncertainty”, *Eng. Optim.*, **43**(12), 1311-1330.

https://doi.org/10.1080/0305215X.2011.554545.

Bhattacharjya, S., Banerjee, S. and Datta, G. (2018), "Efficient robust design optimization of rail bridge hollow pier considering uncertain but bounded type parameters in metamodeling framework", *Asian J. Civil Eng.*, **19**(6), 679-692. https://doi.org/10.1007/s42107-018-0058-8.

Bhattacharjya, S., Chakraborti, S. and Das, S. (2015), "Robust optimization of reinforced concrete folded plate and shell roof structure incorporating parameter uncertainty", *Struct. Eng. Mech.*, **56**(5), 707-726. https://doi.org/10.12989/sem.2015.56.5.707.

Cheng, J., Liu, Z., Tang, M. and Tang, J. (2017), "Robust optimization of uncertain structures based on normalized violation degree of interval constraint", *Comput. Struct.*, **182**(1), 41-54. https://doi.org/10.1016/j.compstruc.2016.10.010.

Datta, G., Bhattacharjya, S. and Chakraborty, S. (2017), "Adaptive metamodel based efficient robust design optimization of offshore structure under stochastic wave loading", *J. Struct. Eng.*, **44**(3), 236-245. https://doi.org/10.1007/978-981-13-0362-3_37.

de Sensale, G.R. and Viacava, I.R. (2018), "A study on blended Portland cements containing residual rice husk ash and limestone filler", *Constr. Build. Mater.*, **166**(30), 873-888. https://doi.org/10.1016/j.conbuildmat.2018.01.113.

Deb, K., Pratap, A., Agarwal, S. and Meyariva, T. (2002), "A fast and elitist multi-objective genetic algorithm: NSGA-II", *IEEE Tran. Evol. Comput*., **6**(2), 182-197.

Doltsinis, I., Kang, Z. and Cheng, G. (2005), "Robust design of non-linear structures using optimization methods", *Comput. Meth. Appl. Mech. Eng.*, **194**, 1779-1795. https://doi.org/10.1016/j.cma.2004.02.027.

Ewa, D., Akeke, G.A. and Okoi, D. (2018), "Influence of rice husk ash source variability on road subgrade properties", *Nigerian J. Technol.*, **37**(3), 582-586.

Gazder, U., Al-Amoudi, O.S.B., Khan, S.M.S. and Maslehuddin, M. (2017), "Predicting compressive strength of bended cement concrete with ANNs", *Comput. Concrete*, **20**(6), 627-634. http://dx.doi.org/10.12989/cac.2017.20.6.627.

Indian Standard (IS) 10262 (2009), Concrete mix proportioning-guidelines, BIS, India

Indian Standard (IS) 1199 (1959), Methods of sampling and analysis of concrete, BIS, India.

Indian Standard (IS) 383 (1970), Specification for Coarse and Fine Aggregates from Natural Sources for Concrete, BIS, India.

Indian Standard (IS) 516 (1959), Methods of tests for strength of concrete, BIS, India.

Indian Standard (IS) 8112 (2013), Ordinary Portland Cement,43 Grade-Specification, BIS, India.

Indian Standard (IS) 9103-99 (2007), Specification For Concrete Admixtures, BIS, India.

Kim, C., Choi, K.K. and Wang, S. (2005), "Efficient response surface modeling by using moving least-square method and sensitivity", *AIAA J.*, **43**(11), 2404-2411.

Lee, K. and Park, G. (2001), "Robust optimization considering tolerances of design variable", *Comput. Struct.*, **79**(1), 77-86. https://doi.org/10.1016/S0045-7949(00)00117-6.

Li, Z., Feng, W., Yang, J. and Huang, Y. (2018), "An investigation on modeling and compensation of synthetic geometric errors on large machine tools based on moving least squares method", *Pr. Inst. Mech. Eng., Part-B, J. Eng. Manuf.*, **232**(3), 412-427. https://doi.org/10.1177/0954405416645985.

MATLAB, The MathWorks, Inc., Natick, Massachusetts, United States.

Myers, R.H. and Montgomery, D.C. (1995), *Response Surface Methodology: Process and Product in Optimization Using Designed Experiments*, John Wiley & Sons, Inc., New York, NY, USA.

Nunes, S., Milheiro-Oliveira, P., Coutinho, J. and Figueiras, J. (2013), "Robust SCC mixes through mix design", *J. Mater. Civil Eng.*, **25**(2), 183-193. https://doi.org/10.1061/(ASCE)MT.1943-5533.0000592.

Oti, J.E. and Kinuthia, J.M. (2015), "Development of stabilised brick and mortar using biomass waste", *Pr. Inst. Civil Eng.-Constr. Mater.*, **168**(5), 241-250. http://dx.doi.org/10.12989/cac.2018.21.5.559.

Ozturk, M., Cansiz, O.F., Sevim, U.K. and Bankir, M.B. (2018), "MLR & ANN approaches for prediction of compressive strength of alkali activated EAFS", *Comput. Concrete*, **21**(5), 559-567. https://doi.org/10.12989/cac.2018.21.5.559.

PWD WB (2017), Schedule of Rates, Public Works Department, Government of West Bengal, India.

Suhendra B. (2014), "Towards green concrete for better sustainable environment", *Procedia Eng.*, **95**, 305-320. https://doi.org/10.1016/j.proeng.2014.12.190.

Taflanidis, A.A. (2012), "Stochastic subset optimization incorporating moving least squares response surface methodologies for stochastic sampling", *Adv. Eng. Softw.*, **44**, 3-14. https://doi.org/10.1016/j.advengsoft.2011.07.009.

Venanzi, I., Materazzi, A.L. and Ierimonti, L. (2015), "Robust and reliable optimization of wind excited cable-stayed masts", *J. Wind Eng. Indus. Aerodyn.*, **147**, 368-379. https://doi.org/10.1016/j.jweia.2015.07.011.

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